MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE

SIMON KUZNETS KHARKIV NATIONAL UNIVERSITY OF ECONOMICS

Syllabus
of the academic discipline
"MATHEMATICAL ANALYSIS AND LINEAR ALGEBRA"
for students of training direction
6.030601 "Management"
specialization "Business Administration"
of all forms of study

Kharkiv
S. Kuznets KhNUE
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Затверджено на засіданні кафедри вищої математики й економіко-математичних методів.

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Compiled by Ie. Misiura

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The thematic plan of the academic discipline and its content according to the modules and themes are given. Plans of lectures and practical trainings, material for students' knowledge consolidation (control questions, tasks for independent work) as well as methods of students' knowledge assessment according to the credit transfer system of studies are presented.

Recommended for students of training direction 6.030601 "Management", specialization "Business Administration" of all forms of study.

Introduction

The fundamental base in the mathematical preparation of economists and managers is the academic discipline "Mathematical Analysis and Linear Algebra" which is a compulsory discipline of the natural scientific series and a component of the structural logical scheme which is provided for the educational professional program of bachelors of training direction 6.030601 "Management", specialization "Business Administration".

The basic problems of teaching the academic discipline is giving students knowledge of the basic parts of mathematical analysis and linear algebra; raising the level of the fundamental mathematical training of students with intensification of its applied direction, mastering the fundamentals of mathematical analysis and linear algebra and application of this knowledge to the economic investigations for solving economic problems, forming skills in the application of elements of mathematical analysis and linear algebra to investigations where higher mathematics is applied as an instrument of investigation and solving optimization economic problems for forming models of economic processes and developments, acquiring the necessary theoretical and practical knowledge for solving specific problems which are set in the process of forming economic and mathematical models, and obtaining the required mathematical knowledge for the study of other disciplines.

1. Description of the academic discipline

<table>
<tr><td rowspan="2">Name of indicators</td><td rowspan="2">Subject area;
training direction;
academic degree</td><td colspan="2">Academic discipline features</td></tr>
<tr><td>day-time form of studies</td><td>distant form of studies</td></tr>
<tr><td>Number of credits:
5 for the full-time form;
6 for the distant form</td><td rowspan="6">6.030601 "Management", specialization "Business Administration"</td><td colspan="2">Compulsory</td></tr>
<tr><td rowspan="2">Number of thematic modules: 2</td><td colspan="2">Academic year</td></tr>
<tr><td>1st</td><td>1st</td></tr>
<tr><td rowspan="3">Total number of hours:
150 for the day-time form;
180 for the distant form</td><td colspan="2">Term</td></tr>
<tr><td>1st</td><td>1st</td></tr>
<tr><td colspan="2">Lectures</td></tr>
<tr><td rowspan="14">The number of hours for the day-time form of studies per week:
in class: 4;
student's independent work: 5</td><td rowspan="14">Academic degree:
bachelor</td><td>32 hours</td><td>16 hours</td></tr>
<tr><td colspan="2">Practical studies</td></tr>
<tr><td>16 hours</td><td>16 hours</td></tr>
<tr><td colspan="2">Laboratory studies</td></tr>
<tr><td>16 hours</td><td>–</td></tr>
<tr><td colspan="2">Independent work</td></tr>
<tr><td>82 hours</td><td>144 hours</td></tr>
<tr><td colspan="2">Examination consultation</td></tr>
<tr><td>2 hours</td><td>2 hours</td></tr>
<tr><td colspan="2">Form of control:</td></tr>
<tr><td>Exam</td><td>Exam</td></tr>
<tr><td>2 hours</td><td>2 hours</td></tr>
</table>

Note. The ratio of the number of class hours and independent work is:
74 % for the full-time form of studies;
22 % for the distant form of studies.

2. The main purpose and tasks of the academic discipline

The main purpose of teaching is to form future specialists' basic mathematical knowledge for solving theoretical and practical problems in professional activity of a competent specialist in the service sphere, skills in analytical thinking and skills in using mathematical knowledge for formation of real processes and developments, and for solving economic problems.

The main tasks that should be carried out in the process of teaching the discipline are: giving students knowledge of the basic parts of mathematical analysis and linear algebra; definitions, theorems, rules; proving the main theorems; mastering the fundamentals of the methodology of mathematical investigation of the applied economic problems; independent broadening of knowledge, development of logical and algorithmical thinking; obtaining primary skills in independent learning of mathematical and applied library sources by students.

The subject of the academic discipline "Mathematical Analysis and Linear Algebra" is the fundamentals of mathematical analysis, linear and vector algebra.

In the process of learning the academic discipline "Mathematical Analysis and Linear Algebra" a student receives analytic and investigatory competences which are necessary for a modern economist in any sphere of his activity.

The syllabus of the academic discipline "Mathematical Analysis and Linear Algebra" is compiled according to the statements of the field standard of the higher education of Ministry of Education and Science of Ukraine based on the educational professional program of bachelor training, which is made by the Scientific Methodical Committee of Economics and Enterprise of the Ministry of Education and Science of Ukraine.

A student starts studying the academic discipline "Mathematical Analysis and Linear Algebra" in the first term of the first year of studies.

In the process of learning students obtain the required theoretical knowledge during lectures and acquire practical skills at the practical and laboratory studies and during independent work and fulfillment of individual tasks. Independent and individual work of students has a great value in the process of mastering the material and consolidating knowledge. All of these

types of studies were devised according to the statements of the Bolognese declaration.

As a result of studying the academic discipline a student ***must know***:

the elements of the limits theory: the limit of a sequence and the limit of a function; the limit of a function in a point, their equivalence; equivalent functions, their applications to finding the limit of a ratio of functions;

the first and second noteworthy limits, the table of basic limits, finding the limits of power-exponential functions;

the bases of limiting (marginal) analysis;

the differential calculus: the function of one variable and several variables;

ways to define the function and its illustration;

some special classes of functions; monotone, even and odd, convex and concave, bounded and unbounded functions;

the continuity of the function at the same point;

one-sided continuity of the function of one variable at the same point, necessary and sufficient conditions of continuity;

classification of points of discontinuity;

the differentiable function, its differential;

the derivative of the function of one variable, partial derivatives, the gradient of the function of several variables;

the derivative of the function of several variables in the direction, its relationship with the gradient; the elasticity of the function;

higher-order derivatives and differentials, higher-order derivatives of some elementary functions;

investigation of functions with the help of the differential calculus;

the notion of the differential of a function and its application to approximate calculation; the notion of the elasticity of a function;

the integral calculus: the notion of antiderivative, indefinite and definite integrals; methods of integration;

Newton – Leibnitz formula; the notion of improper integrals;

the elements of economic dynamics; the first-order ordinary differential equation, Cauchy problem;

the particular and general solutions; types of differential equations;

the higher-order differential equations and systems of differential equations;

solution of the second-order linear differential equations with constant coefficients;

numerical series, necessary and sufficient conditions of a convergence of numerical series with positive terms and alternating numerical series; absolute and conditional convergence;

power series, the convergence radius and the interval of power series; functional series, trigonometric Fourier series;

the bases of linear algebra: matrices and determinants, (facilities, possibilities) of their application to making a mathematical model of economic problems;

methods of solving the system with n linear algebraic equations with m unknowns;

the conditions of compatibility of the system of linear algebraic equations;

the notion of the basic solution;

the bases of vector algebra: the basis of space,

linear dependence and linear independence of vectors;

the notions of subspace, the linear vector space, the rank of finite systems of vectors, rules of its calculation;

***be able** to*:

learn mathematical literature by oneself;

calculate the mean values;

carry out the operations with vectors, matrices, calculation of the determinants;

solve the systems of linear equations;

investigate the forms and properties of straight lines and planes, second-order curves and quadratic surfaces;

classify the functions, numerical sequences;

find the limit of power-exponential functions;

investigate the function with the help of differential calculus;

carry out the integral calculus;

carry out calculation of numerical and power series;

solve first-order and higher-order differential equations, systems of differential equations;

form and use economic mathematical models;

broaden the knowledge, develop logical and algorithmic thinking by oneself.

A modern tendency in higher education is the reorientation of students of higher educational institutions from the process of education to the result, from knowledge to skills, forming definite competences.

In the process of learning the academic discipline "Mathematical Analysis and Linear Algebra" a student receives analytic and investigatory **competences** which are required for a modern economist in any sphere of his activity (Table 2.1).

Table 2.1

Competences which are formed as a result of mastering the academic discipline "Mathematical Analysis and Linear Algebra"

The code of the competence	The name of the competence	The components of the competence
1	2	3
AMI*1	Forming analytic thinking, the ability to explain the importance of complicated expressions with the help of mathematical symbols and operations. Development of the ability to solve problems with the help of calculation of limits and methods of differential calculus using mathematical symbolic variables, i.e. forming the initial skills in economic modelling	A student must 1) be able to define the type of function by its analytic recording; 2) calculate derivatives of elementary and composite functions and use the differential of a function for approximate calculus; 5) investigate the function with the help of differential calculus; 6) carry out the simplest calculations by optimization of production; 7) draw corresponding conclusions and independently analyze the obtained solution; 8) find partial and mixed derivatives of the function of several variables, 9) be able to investigate the local extremum of a function
AMI 2	Understanding a possibility to use the integral calculus for solving applied problems. Forming skills in independent formation of mathematical models for description of different processes. Forming the skill in independent work. Analysis and understanding of the	A student must 1) calculate definite and indefinite integrals; 2) be able to use definite integrals for independent calculation of areas and volumes of figures; 3) draw corresponding conclusions and independently analyze the obtained results

Table 2.1 (the end)

1	2	3
AMI 2	importance of the relationship between the definite and indefinite integral	
AMI 3	Forming inclinations to independent search of different ways of solving problems and understanding the necessity to use the knowledge of other themes (the function, the derivative, the integral)	A student must 1) be able to calculate the type of differential equation, the method of further solving it independently; 2) be able to use the knowledge for solving the simplest economic problems
AMI 4	Forming the ability to do analytic calculations	A student must 1) calculate the type of series; 2) be able to investigate the convergence of series independently; 3) find the convergence radius of power series
AMI 5	Forming the ability to prove independently the simplest statements with the help of elementary mathematical knowledge. Forming skills in the use of the instrument of the matrix calculus for modelling the simplest economical problems and situations. The ability to analyze the results of calculations	A student must 1) know the basic proofs and theorems of the theme; 2) give examples of using determinants, matrices and systems of linear equations in economics; 3) be able to use the instrument of matrix algebra for economic problems; 4) be able to model the simplest situations with the help of knowledge of the theme
AMI 6	Forming analytic thinking, the ability to explain the importance of complicated expressions with the help of mathematical symbols and operations	A student **must be able** to use vector algebra for calculation of the simplest problems of applied character (finding the area, the volume)

* Application of mathematical instruments (AMI)

The structure of professional competences and forming them according to the National Scale of Qualifications of Ukraine is given in Appendix A.

3. The syllabus of the academic discipline

Thematic module 1
The elements of mathematical analysis

Theme 1. The limit of a function and continuity

1.1. Sets, functions, their classification.

Basic notions. Numerical sets. Operations with sets. Numerical intervals, the neighborhood of a point. The notion of the function of one variable. Ways to define the function. The domain of a definition and a range of values of a function. A geometrical illustration of a function. Elementary functions and their graphs. Properties of a function: the boundedness and the unboundedness, an increasing and decreasing function, the oddness and the evenness, the periodicity. Classification of elementaty functions. The notion of an inverse function. Inverse trigonometric functions. The superposition of a function.

1.2. Numerical sequences and their limits.

A numerical sequence. The definition of the limit of a sequence. Infinitesimals. Infinitely large values. The relationship between infinitesimals and infinitely large values.

1.3. The limit of a function.

The definition of the limit of a function. One-sided limits. Properties of functions which have finite limits. Limiting processes in equalities and inequalities. Lemmas about infinitesimals. Arithmetical operations with functions which have finite limits. The limit of the function $\frac{\sin x}{x}$ at $x \to 0$. Indefinite expressions. The limit of a monotonic function. The number *e*. Natural logarithms.

1.4. The function continuity.

The definition of the function continuity at a point. Continuity of a function on a segment. Arithmetical operations with continuous functions. Classification of breaks. Properties of continuous functions. Continuity of elementary functions.

Theme 2. The differential calculus of the function of one variable

2.1. A derivative and a differential. Techniques of differentiation.

Application of a derivative to economic accounts. Limiting characteristics in microeconomics. Maximization of profit and marginal analysis. Optimi-

zation of taxation of enterprises. The definition of a derivative. The geometric, mechanical and economic meanings of a derivative. Derivatives of elementary functions. A derivative of an inverse function. A table of derivatives. The rules of calculation of derivatives. A derivative of a composite function. One-sided derivatives. Derivatives of higher orders. The definition of a differential. The differential of a sum, a product and a quotient. The invariance of the form of the first differential. Differentials of higher orders. Application of a differential to approximate calculations.

2.2. The main theorems of differential calculus and using them.

Fermat theorem. Rolle theorem. Lagrange theorem. Cauchy theorem. L'Hospital rule.

2.3. Application of derivatives to the investigation of functions.

The condition of a monotony of a function. The condition of increasing and decreasing a function on an interval. The maximum and the minimum of a function. Necessary and sufficient conditions of an extremum of a function. Convexity and concavity of a graph of a function, inflection points, asymptotes of a graph of a function. A general scheme of a plot of the graph of a function.

2.4. Application of a derivative to economics.

Marginal analysis. Elasticity of economic indicators. The economic meaning of Fermat theorem. Application of a derivative to economic calculations.

Theme 3. Analysis of the function of several variables

3.1. Basic notions.

The function of two variables, the domain of their definition. A graphical illustration of a function of two variables.

3.2. Partial derivatives. The differential.

A partial and a total increments of a function of two variables. Partial derivatives. A total differential. Derivatives of higher orders. The theorem about the equality of mixed derivatives. Differentials of higher orders.

3.3. The extremum of a function of several variables.

The necessary conditions of the function of two variables. The sufficient conditions of an extremum of the function of two variables. The conditions of absence of the extremum. The notion of a conditional extremum. The method of Lagrange multipliers. The least-squares method.

3.4. Application of the function of several variables to economics.

The function of several variables in problems of economics (the utility function, the expenditure function, the multifactor production function of Cobb and Douglas). Some problems of optimization (an optimal profit from production of goods of different types; the problem of price discrimination, an optimal distribution of resources; optimization of the choice of a consumer). The functional dependence between variables.

Theme 4. The indefinite integral

4.1. An antiderivative and an indefinite integral.

The notion of an antiderivative of a function and an indefinite integral. The geometrical and mechanical meanings of an integral. The table of basic integrals.

4.2. Basic methods of integration.

The simplest rules of integration. Direct integration. A change of a variable in an indefinite integral. Integration by parts.

4.3. Integration of some classes of functions.

Integration of rational fractions. Integration of irrational expressions and expressions which have trigonometric functions. Trigonometric substitutions.

Theme 5. The definite integral and its application

5.1. The notion and properties of a definite integral.

Integral sums. Conditions of the existence of a definite integral. Properties of a definite integral.

5.2. Calculation of a definite integral.

Newton – Leibnitz formula. A change of a variable in a definite integral. Integration by parts.

5.3. Improper integrals of the first and the second kinds.

The notion of an improper integral. Conditions of convergence of improper integrals. Euler – Poisson integral and its application.

5.4. Application of a definite integral.

The geometrical application of a definite integral: calculation of areas, volumes of the solid of a revolution, arc lengths of curves. An approximate calculus of a definite integral: formulas of rectangles, trapezoids, Simpson**.** Finding the volume of a productive production; consumer surplus, analysis of nonuniformity in the distribution of income from population with the help of the Lorenz curve.

Theme 6. Differential equations

6.1. The basic notions of the theory of differential equations. Solving the first-order differential equations.

The notion of a differential equation and its solutions. Application of differential equations to problems of economic dynamics. A model of increasing for a constant rate of an increment; a model of increasing under the conditions of competition; a dynamic model of Keynes; a neoclassic model of increasing; a marketing model with predicted prices. The order of a differential equation. Differential equations of the first order. A general solution and a general integral of a differential equation of the first order. Initial conditions. A particular solution and a particular integral of a differential equation of the first order with separable variables. Homogeneous equations of the first order. Linear differential equations of the first order. Differential equations of Bernoulli**.**

6.2. Differential equations of higher orders. Methods of solving the second-order differential equations.

The second-order linear differential equations with constant coefficients. Homogeneous and inhomogeneous differential equations. The notion of linearly independent solutions of a homogeneous differential equation of the second order. Initial conditions. The structure of a general solution of an inhomogeneous differential equation of the second order. Linear inhomogeneous differential equations of the second order with the right parts of a special form. The notion of the differential equation. The notion of a system of differential equations. The notion of an equilibrium of a solution.

6.3. Application of differential equations to economics.

Using differential equations for a construct of production functions. Models of economic dynamics. Solow model. A model of a natural increasing output. The dynamics of market prices. Application of differential equations to economics.

Theme 7. Series

7.1. Numerical series and their convergence.

Partial sums of series. The necessary condition of series convergence. Series with positive terms. The theorem of comparison of series. Sufficient conditions of series convergence with positive terms: D'Alembert criterion, Cauchy's criterion, Maclaurin – Cauchy integral criterion.

7.2. Alternating series and their convergence.

The notion of alternating series. Absolute and conditional convergence of series. Alternating series. Leibnitz theorem. A sign of a remainder of alternating series.

7.3. Power series.

Abel theorem. The convergence radius of power series. Differentiation and an integration of power series. Teylor and Maclaurin series. Decomposition of elementary functions in Teylor and Maclaurin series. Application of power series to an approximate calculus.

Thematic module 2
Linear algebra

Theme 8. The elements of the theory of matrices and determinants

8.1. Matrices.

The definition, types of matrices, basic matrices (square, triangular, diagonal, unit). Comparison of matrices. Basic operations with matrices: addition, multiplication of a matrix by a scalar, a vector, a matrix; properties of these operations. Transposition of a matrix. The notion of an inverse matrix, properties of a matrix inversion operation.

8.2. Determinants.

The definition of the determinant, rules of calculation of determinants of lower orders (schematic) and higher-orders (expansion by Laplace formulas). Properties of determinants. Calculation of some special determinants (triangular, diagonal, identity matrices, Vandermonde matrix). Calculation of an inverse matrix with the help of the determinants (algebraic cofactors).

8.3. The inverse matrix.

Calculation of an inverse matrix by two ways: with the help of a definition (as a transposed matrix of algebraic cofactors) and elementary row transformations with given and unit matrices.

A matrix rank and ways to define it.

Theme 9. The general theory of the system of linear algebraic equations

9.1. Systems of linear algebraic equations.

The definition of the system of linear algebraic equations, the augmented

and matrix forms of its entry. Definitions of a solution, consistent or inconsistent, determined or undetermined system.

9.2. Methods of solving systems of linear algebraic equations.

A solution of square systems of linear algebraic equations with the help of an inverse matrix, by Cramer formulas. Equivalent transformations, Gauss – Jordan method of sequential exclusion of unknowns for a solution of systems of linear algebraic equations, its realization with the help of tables. Finding an inverse matrix by Gauss – Jordan method. The notion of a matrix rank and its calculation. Kronecker – Capelli theorem, the particular and the general solutions of the system of linear algebraic equations.

9.3. Homogeneous systems of linear algebraic equations.

The notion of a homogeneous system of linear algebraic equations. The space of solutions of a homogeneous system, the relationship of its dimension and the matrix rank. A fundamental system of solutions of a homogeneous system of linear algebraic equations. Economic problems.

Theme 10. The elements of vector algebra

10.1. The basic notions of vector algebra.

The Cartesian coordinates of a vector and a point. Examples of economic problems, which are connected with using vector algebra and analytic geometry. Coordinates on a straight line. Coordinates on a plane. Coordinates in a space.

Linear operations with vectors in coordinates. Coordinates of a point of division of a segment. Coordinates of a vector which is given by two points. A sign of a colinearity of two vectors. A sign of a coplanarity of three vectors. Properties of a scalar product of two vectors. The expression of a scalar product through coordinates. A cross product of two vectors, its properties. The expression of a cross product through coordinates. A mixed product of three vectors, its properties. The expression of a mixed product through coordinates of vector factors.

10.2. The elements of the theory of linear spaces.

The definition of linear space. The definitions and main theorems of linear dependence and linear independence of linear space elements. The basis of linear space. The main theorems about the basis: uniqueness of

expansion, linear dependence of $n+1$ elements, a number of basic elements. The dimension of linear space. Coordinates of space elements in a given basis. The notion of subspace. The notion of linear vector space. The rank of finite systems of vectors, rules of its calculation.

10.3. Eigenvectors.

Eigenvalues and eigenvectors of a matrix. A characteristic equation. Methods of finding eigenvalues and eigenvectors for matrices of the second and the third orders. Economic examples.

10.4. Quadratic forms.

The notion of a quadratic form. Conditions of a determinacy of quadratic forms. The matrix of a quadratic form. Reducing quadratic forms to a canonical form. The curves of the second-order on a plane. A general equation of the second-order curve. Reducing the second-order curve to a canonical form.

4. The structure of the academic discipline

From the very beginning of studying the academic discipline each student has the possibility to learn both the discipline syllabus and forms of organization of education, as well as the structure, contents and volume of each of its educational modules, and all types of control and methods of the educational work assessment.

The educational process according to the syllabus of the academic discipline "Mathematical Analysis and Linear Algebra" is realized in such forms as: lectures, practical and laboratory studies; fulfillment of students' independent work; control activities.

A student's mastering of the academic discipline is carried out with the help of consecutive and thorough learning of the educational modules. An educational module is a relatively separate block of the given discipline, which logically unites its educational elements by content and interconnections. The assessment of knowledge and skills obtained by a student while learning the material of each module is effected at the final module control.

The thematic plan of the academic discipline consists of two thematic modules (Table 4.1).

Table 4.1

The structure of the test credit of the academic discipline

Names of thematic modules and themes	The number of hours											
	the day-time form of studies						the distant form of studies					
	total	which are allocated for					total	which are allocated for				
		lecture	practical	laboratory	final control	independent work: preparation for studies		lecture	practical	laboratory	final control	independent work: preparation for studies
1	2	3	4	5	6	7	8	9	10	11	12	13
Thematic module 1. The elements of mathematical analysis												
Theme 1. The limit of a function and the continuity	8	2	1	1	–	4	12	1	1	–	–	10
Theme 2. The differential calculus of the function of one variable	8	2	1	1	–	4	15	2	2	–	–	11
Theme 3. Analysis of the function of several variables	8	2	1	1	–	4	13	1	1	–	–	11
Theme 4. The indefinite integral	16	4	2	2	–	8	15	2	2	–	–	11
Theme 5. The definite integral and its application	10	2	1	1	–	6	13	1	1	–	–	11
Theme 6. Differential equations	18	4	2	2	–	10	15	2	2	–	–	11

Table 4.1 (the end)

1	2	3	4	5	6	7	8	9	10	11	12	13
Theme 7. Series	8	2	1	1	–	4	12	1	1	–	–	10
Total for module 1	**76**	**18**	**9**	**9**	–	**40**	**95**	**10**	**10**	**–**	**–**	**75**
Thematic module 2. Linear algebra												
Theme 8. The elements of the theory of matrices and determinants	17	4	2	2	–	9	23	2	2	–	–	19
Theme 9. The general theory of the system of linear algebraic equations	18	4	2	2	–	10	24	2	2	–	–	20
Theme 10. The elements of vector algebra	25	6	3	3	–	13	24	2	2	–	–	20
Total for module 2	**60**	**14**	**7**	**7**	**–**	**32**	**71**	**6**	**6**	**–**	**–**	**59**
Preparation for the exam	10	–	–	–	–	10	10	–	–	–	–	10
Consultations for the exam	2	–	–	–	2	–	2	–	–	–	2	–
Exam	2	–	–	–	2	–	2	–	–	–	2	–
Total number of hours	**150**	**32**	**16**	**16**	**4**	**82**	**180**	**16**	**16**	**–**	**4**	**144**

5. The plan of practical studies

5.1. The themes of practical studies

A practical study is a form of educational studies, at which the lecturer organizes a detailed consideration of separate theoretical statements of the academic discipline and forms the abilities and skills in their practical application through the students' individual accomplishment of the formulated tasks.

Conducting a practical study is based on the previously prepared material, i.e. tests designed to assess the mastery of the required theoretical statements, tasks of different complexity to be solved by students.

A practical study includes control of students' knowledge, abilities and skills, formulation of a general problem by the lecturer and discussing it with the students, solving control tasks, reviewing them, assessment.

The plan of the practical studies, their content and a bibliography for each theme are given in Table 5.1.

Table 5.1

The plan of practical studies

The name of the thematic module	The themes of the practical studies (by modules)	The number of hours	Recommended reading
1	2	3	4
Thematic module 1. The elements of mathematical analysis	*Theme 1. Calculation of the limits of functions and investigation of the continuity of functions.* 1. The limits of functions and their properties. 2. The types of indeterminations and methods for eliminating them. 3. The first remarkable limit. The table of equivalent infinitesimals. 4. The investigation of the continuity of a function	1	Main: [1 – 4]. Additional: [18 – 20; 22]. Methodological support: [23 – 25]
	Theme 2. The differential calculus of the function of one variable and applying it. 1. Techniques of differentiation: basic rules, a derivative of a composite function, logarithmic differentiation, derivatives of a parametric function and an implicit function. 2. The differential of a one-variable function and applying it. 3. Derivatives and differentials of higher orders. 4. The application of a derivative to the investigation of a function	1	Main: [1 – 4]. Additional: [18 – 20; 22]. Methodological support: [23 – 25]

Table 5.1 (continuation)

1	2	3	4
Thematic module 1. The elements of mathematical analysis	*Theme 3. The function of several variables* 1. Finding partial derivatives of the functions of several variables. 2. The differential of the function of several variables and its applying it. 3. The gradient and a derivative by direction. Level lines. 4. The investigation of an extremum of the function of two variables	1	Main: [1 – 4]. Additional: [18 – 20; 22]. Methodological support: [23 – 25]
	Theme 4. Finding indefinite integrals. 1. Direct integration. 2. The method of changing a variable (by substitution) of finding indefinite integrals. 3. The formula of integration by parts	2	Main: [1 – 4]. Additional: [18 – 20; 22]. Methodological support: [23 – 25]
	Theme 5. Finding definite integrals and applying them. 1. Calculation of definite integrals with Newton – Leibnitz formula. 2. The methods of changing a variable and integration by parts for definite integrals. 3. Calculation of improper integrals. 4. The application of definite integrals	1	Main: [1 – 4]. Additional: [18 – 20; 22]. Methodological support: [23 – 25]
	Theme 6. Solving differential equations. 1. Integration of differential equations with separated variables. 2. Linear differential equations of the first order. 3. Finding general and particular solutions of the second order linear differential equation with constant coefficients	2	Main: [1 – 4]. Additional: [15; 18 – 20; 22]. Methodological support: [23 – 25]
	Theme 7. Series. 1. The investigation of the convergence of constant-sign series. 2. The investigation of the convergence of alternate series. Absolute and a conditional convergences. 3. Power series and its convergence. 4. Decomposition of the basic elementary functions into Teylor and Macloren series	1	Main: [1 – 4]. Additional: [18 – 20; 22] Methodological support: [23 – 25]

Table 5.1 (the end)

1	2	3	4
Thematic module 2. Linear algebra	*Theme 8. The elements of the theory of matrices and determinants.* 1. Carrying out operations with matrices. 2. Calculation of determinants: lower order (schematic), higher orders (decomposition by Laplace's formulas). 3. Calculation of an inverse matrix as a transposed matrix of algebraic cofactors and with the help of the transformation of a matrix connected with the unit matrix	2	Main: [1 – 4; 6; 10]. Additional: [18; 20; 22]. Methodological support: [23 – 25]
	Theme 9. The general theory of the systems of linear algebraic equations. 1. Investigation of compatibility of the systems of linear algebraic equations and their definiteness. 2. Solving systems of linear algebraic equations with the help of the inverse matrix method and Cramer's method. 3. Solving systems of linear algebraic equations with the help of Gauss and Jordan – Gauss methods. 4. Finding a set of solutions of homogeneous and rectangular systems of linear algebraic equations	2	Main: [1 – 4; 6; 10]. Additional: [18; 20; 22]. Methodological support: [23 – 25]
	Theme 10. The elements of vector algebra. 1. The construction of a basis of n-dimension linear space, components of a vector in the given basis, transformation to other basis. 2. A scalar product of vectors. Checking the collinearity of vectors. Cross and mixed products of vectors, their properties and geometric meaning. 3. Checking the linear independence of vectors. 4. Finding eigenvalues and eigenvectors for matrices of the second and the thirs orders. 5. Investigation of the equation of the second order curves. 6. Economic examples	3	Main: [1 – 4; 6; 10]. Additional: [18; 20; 22]. Methodological support: [23 – 25]

5.2. Examples of typical tasks of a class written test according to the themes

Thematic module 2
Linear algebra

Written test No. 3

Theme 8. The elements of the theory of matrices and determinants.
Theme 9. The general theory of the system of linear algebraic equations.
Theme 10. The elements of vector algebra.

Task 1. Find the matrix $D = B \cdot A^T - 2A$, if $A = \begin{pmatrix} 2 & 8 & -3 \\ -1 & -7 & 4 \\ -3 & -6 & 2 \end{pmatrix}$,

$B = \begin{pmatrix} 2 & 3 & 3 \\ 1 & 0 & -2 \\ -1 & -4 & 1 \end{pmatrix}$.

Task 2. Find C^{-1} , if $C = A - B$. Check the condition $C^{-1} \cdot C = E$.

Task 3. Calculate the determinant of the matrix by obtaining zeros in any row (or any column)

$$A = \begin{pmatrix} -1 & -2 & 3 & 2 \\ 2 & 3 & 4 & -2 \\ -1 & 1 & 0 & 1 \\ 0 & -1 & 3 & 1 \end{pmatrix}.$$

Task 4. Solve the system using: a) Cramer method; b) Jordan – Gauss method:

$$\begin{cases} 3x_1 - x_2 + x_3 = -4 \\ x_1 + x_2 - 4x_3 = 0 \\ -3x_1 + x_2 - 5x_3 = 4 \end{cases}.$$

Task 5. Investigate the compatibility of the given system of equations by Kronecker – Capelli theorem and in the case of their compatibility solve it:

$$\begin{cases} x_1 + 7x_2 - 4x_3 - 5x_4 - 10x_5 = -7 \\ -x_1 - 4x_2 + 2x_3 + 2x_4 + 5x_5 = 2 \\ x_1 + 3x_2 - x_3 - 3x_5 = 1 \end{cases}.$$

6. The themes of laboratory studies

The educational plan provides conducting laboratory studies on the academic discipline "Mathematical Analysis and Linear Algebra" in the first term.

A laboratory study is a form of study when a student under the direction of a lecturer fulfills a practical task with the help of PC-programming (software MatLab). The plan of laboratory studies, their content and bibliography for each theme are given in Table 6.1.

Table 6.1

The plan of themes of laboratory studies

The theme name	The syllabus questions	Hours	Recommended reading
1	2	3	4
Thematic module 1. The elements of mathematical analysis			
Theme 1. The elements of the theory of limits	Learning the programming software MatLab. Elimination of different types of indeterminations. Investigation of continuity	1	Main: [4; 7] Methodological support: [23 – 25]
Theme 2. The differential calculus of the function of one variable	Finding derivatives of implicit, explicit and parametric functions of one variable. Fulfilment of the investigation of a function and plotting its graph	1	Main: [4; 7]. Methodological support: [23 – 25]
Theme 3. Functions of several variables	Finding partial derivatives and the gradient of the function of two variables. Investigation of the extremum of the function of two variables	1	Main: [4; 7]. Methodological support: [23 – 25]

Table 6.1 (the end)

1	2	3	4
Theme 4. The indefinite integral and its property	Finding indefinite integrals of rational, irrational and trigonometric functions	2	Main: [4; 7]. Methodological support: [23 – 25]
Theme 5. The definite integral and applying it to geometric and economic problems	Finding definite integrals with the help of Newton – Leibnitz formula. Calculation of the areas of plane figures and the volume of the revolution body with the help of the definite integral, solving economic problems which are reduced to the calculation of definite integrals	1	Main: [4; 7]. Methodological support: [23 – 25]
Theme 6. Differential equations and applying them to economics	Integration of differential equations, solving Caushy problem. Construction of mathematical models of economic problems with the help of solving differential equations	2	Main: [15]. Additional: [19]. Methodological support: [23 – 25]
Theme 7. Series	Investigation of the convergence of constant-sign, alternate and power series. Decomposition of the basic elementary functions into Taylor and Macloren series	1	Main: [4; 7]. Methodological support: [23 – 25]
Thematic module 2. Linear algebra			
Theme 8. The elements of the theory of matrices and determinants	Carrying out operations with matrices and calculation of determinants with the help of MatLab	1	Main: [6; 10]. Methodological support: [23 – 25]
Theme 9. The general theory of the systems of linear algebraic equations	Solving systems of linear algebraic equations by Cramer's formulas, the inverse matrix method, Jordan – Gausss method. Investigation of compatibility of the systems of linear algebraic equations	2	Main: [6; 10]. Methodological support: [23 – 25]
Theme 10. The elements of vector algebra	Investigation and plotting the graphs of the curves of the second order	3	Main: [6; 10]. Methodological support: [23 – 25]

Conducting a laboratory study on the defined theme is preceded by an analysis of the basic theoretical fundamentals forming practical skills. A laboratory study is fulfilled in the computer room with the use of the software MatLab and MS Excel. It favours the following: firstly, a student extends the knowledge of the basic formulas and relations fulfilling calculations by direct writing the corresponding formulas and, secondly, acquires skills in the use of built-in functions of the software MatLab. This kind of approach gives a possibility to pay more attention to economic explanation of mathematical transformations.

7. Independent work

7.1. Forms of independent work

Independent work is a scheduled educational and scientific work which is carried out on a lecture task under the methodical and scientific guidance of a lecturer, it is a specific form of the educational activity, its main objective is to form independence of a person.

Independent work is:

1) different forms of individual and group cognitive activity of students, which is fulfilled by them during practical studies and in the extracurricular time;

2) different types of educational tasks which are fulfilled under the guidance of a lecturer;

3) a system of work organization when management of the educational work of students is fulfilled in the absence of a lecturer and without his direct assistance;

4) work of students which is carried out according to a specific individual educational plan designed on the basis of taking into account individual characteristics and cognitive possibilities of students.

The types of independent work and forms of control are given in Table 7.1.

The educational time, which is intended for students' independent work of the day-time form of education, is defined according to the educational plan and makes 55 % (82 hours) out of the total educational time for learning the discipline. For students of the distant form of education this time equals 80 % (144 hours) out of the total educational time for learning the discipline.

During independent work students become active participants in the educational process, learn to master consciously theoretical and practical material, orientate easily in the information space, take responsibility for the quality of their professional training.

Table 7.1

Tasks for students' independent work and forms of control

The name of the theme	The content of students' independent work	The number of hours	Forms of control of IWS	Recommended reading
1	2	3	4	5
Thematic module 1. The elements of mathematical analysis				
Theme 1. **The limit of the function and continuity**	Learning the lecture material. Preparation for a practical study and laboratory work. Independent learning of the questions: 1) operations with sets; 2) classification of functions; 3) properties of the basic elementary functions and their graphs; 4) the second remarkable limit and its consequences in the form of the table of equivalences; 5) comparison of infinitesimals and using them for the calculation of limits; 6) classification of breakpoints; 7) application of limits to economic calculations. Carrying out homework and independent work	4	Homework	Main: [1 – 4; 7]. Additional: [18 – 20]
Theme 2. **The differential calculus of the function of one variable**	Learning the lecture material. Preparation for a practical study and laboratory work	4	Homework	Main: [1 – 4; 7]. Additional: [18 – 20]

Table 7.1 (continuation)

1	2	3	4	5
	Independent learning of the material by parts: 1) L'Hospital's rule for the elimination of indeterminations; 2) complete investigation of the functions and plotting their graphs; 3) finding the greatest and the least values of the function on the segment; 4) using a differential in approximate calculations; 5) application of a derivative to problems in economics. Carrying out homework and independent work			
Theme 3. **Analysis of the function of several variables**	Learning the lecture material. Preparation for a practical study and laboratory work. Independent learning of the questions: 1) a conditional extremum of the function of two variables, finding it by way of reducing to a function of one variable and Lagrange's factors; 2) using the functions of several variables for solving economic problems. Carrying out homework and independent work. Preparation for a written test. Preparation for the defence of laboratory works	4	Homework. An independent test on themes 1 – 3. A written test on themes 1 – 3	Main: [1 – 4; 7]. Additional: [18 – 20]
Theme 4. **The indefinite integral**	Learning the lecture material. Preparation for a practical study and laboratory work. Independent learning of questions:	8	Homework	Main: [1 – 4; 7]. Additional: [18 – 20]

Table 7.1 (continuation)

1	2	3	4	5
	1) integration of rational fractions using a general scheme; 2) integration of some irrational and trigonometric functions; 3) application of the indefinite integral to economic problems. Carrying out homework and an independent test			
Theme 5. **The definite integral and applying it**	Learning the lecture material. Preparation for a practical study and laboratory work. Independent learning of the material by parts: 1) approximate calculation of the definite integral; 2) geometrical applications of the definite integral: the area of a figure, the volume of a rotation body, the areas of the surface, the length of an arc; 3) application of the definite integral to economic calculations. Carrying out homework and an independent test. Preparation for the defence of laboratory works	6	Homework	Main: [1 – 4; 7]. Additional: [18 – 20]
Theme 6. **Differential equations**	Learning the lecture material. Preparation for a practical study and laboratory work. Independent learning of the questions: 1) homogeneous differential equations of the first order; 2) equations of higher orders with reducing their order;	10	Homework. A written test on themes 4 – 6	Main: [1 – 4; 7]. Additional: [15; 18 – 20]

Table 7.1 (continuation)

1	2	3	4	5
	3) linear nonhomogeneous differential equations of the second order with constant coefficients and the right part of a special form; 4) application of differential equations to the construction of economic models. Carrying out homework and an independent test. Preparation for a written test. Preparation for the colloquium			
Theme 7. **Series**	Learning the lecture material, preparation for a practical study and defence of laboratory works. Independent learning of the questions: 1) convergence signs of series with positive terms; 2) decomposition of basic functions in power series; 3) application of power series to approximate calculations. Carrying out homework and independent work	4	Homework. A competence-oriented task. Independent test on themes 4 – 7. A colloquium on themes 1 – 7	Main: [1 – 4; 7]. Additional: [18 – 20]
Total for thematic module 1		**40**		
Thematic module 2. Linear algebra				
Theme 8. **The elements of the theory of matrices and determinants**	Learning the lecture material. Preparation for a practical study and laboratory work. Independent learning of the material by parts: 1) properties of determinants; 2) properties of operations with matrices; 3) elementary transformations	9	Homework	Main: [1 – 4; 6]. Additional: [18; 22]

Table 7.1 (continuation)

1	2	3	4	5
	of matrices; the rank of a matrix, methods of finding it; 4) application of matricies to economic problems. Carrying out homework and an independent test			
Theme 9. **The general theory of the system of linear algebraic equations**	Learning the lecture material, preparation for practical studies and laboratory work. Independent learning of the material by parts: 1) Cramer's method and the inverse matrix method of solving systems of linear algebraic equations; 2) solving indefinite (rectangular) systems; 3) homogeneous systems; 4) application of systems of linear algebraic equations to economic problems. Carrying out homework and an independent test. Preparation for a written test	10	Homework. A written test on themes 8 – 9	Main: [1 – 4; 6]. Additional: [18; 22]
Theme 10**. The elements of vector algebra**	Learning the lecture material, preparation for a practical study and defence of laboratory works. Independent learning of the material by questions: 1) finding eigenvalues and eigenvectors of quadratic matricies; 2) application of quadratic forms; 3) linear ecomonic and mathematical models.	13	Homework. A competence-oriented task. An independent test on themes 8 – 10. A colloquim on themes 7 – 10.	Main: [1 – 4; 6]. Additional: [18; 22]

Table 7.1 (the end)

1	2	3	4	5
	Carrying out homework and an independent test. Preparation for the colloquim. Preparation for the presentation of an independent creative task		An independent creative task	
Total for thematic module 2		**32**		
Preparation for the exam		**10**	Exam	Main: [1 – 4; 6; 7; 10]. Additional: [18 – 20; 22]
Total for the academic discipline		**82**	–	**–**

The necessary element of successful mastering the material of the academic discipline is the students' independent work (SIW) with specifical mathematical and economic literature. The basic forms of tasks and control of independent work which are proposed to students for mastering the theoretical knowledge on the themes of the academic discipline are given in Table 7.1.

The educational module, the theme, within which tasks are carried out, and the periods of fulfillment, problems and checking the tasks for independent work are given in Table 7.2.

SIW includes: processing the lecture material (a lecture as a form of education provides theoretical knowledge besides being used for carrying out practical calculations); processing and learning the recommended literature, basic terms and concepts on the themes of the academic discipline; preparation for practical and laboratory studies; preparation for the defence of laboratory work; advanced study of particular themes or questions of lectures; carrying out practical homework, solving computational competence-oriented tasks on the given theme; choosing and consideration of literature sources on the given problem of the academic discipline; analytic consideration of scientific publications; self-control of students' knowledge by questions for self-diagnostics; carrying out independent work; carrying out independent creative

work; preparation for tests and other forms of current control; preparation for module control (a colloquium); systematization of the studied material for the purpose of preparation for term exams on each module of the academic discipline.

Table 7.2

The plan of carrying out independent work

The theme of the thematic module	Types of tasks	The allocated time (hours)	The ordinal number of the educational week which is given for		
			carrying out	defence	assessment
1	2	3	4	5	6
Thematic module 1. The elements of mathematical analysis					
Theme 1. The limit of the function and continuity	Answer the self-control questions on the theme. Carry out the practical homework. Solve the tasks of the independent test on the theme	4	1	3	3
Theme 2. The differential calculus of the function of one variable	Answer the self-control questions on the theme. Carry out the practical homework. Solve the tasks of the independent test on the theme	4	2	3	3
Theme 3. Analysis of the function of several variables	Answer the self-control questions on the theme. Carry out the practical homework. Solve the tasks of the independent test on the theme	4	3	5	5
Theme 4. The indefinite integral	Answer the self-control questions on the theme. Prepare for the written test on themes 1 – 3. Carry out the practical homework. Solve the tasks of the independent test on the theme	8	4 – 5	5	5
Theme 5. The definite integral and applying it	Answer the self-control questions on the theme. Carry out the practical homework. Solve the tasks of the independent test on the theme	6	6	7	7

Table 7.2 (the end)

2	3	4	5	6	7
Theme 6. Differential equations (continued)	Answer the self-control questions on the theme. Carry out the practical homework Solve the tasks of the independent test on the theme. Prepare for the written test on themes 4 – 6. Prepare for colloquium on themes 1 – 6	10	7 – 8	9	8; 9
Theme 7. Series	Answer the self-control questions on the theme. Carry out the practical homework. Solve the tasks of the independent test on the theme	4	9	11	11
Thematic module 2. Linear algebra					
Theme 8. The elements of the theory of matrices and determinants	Answer the self-control questions on the theme. Carry out the practical homework. Solve the tasks of the independent test on the theme	9	10 – 11	11	11; 15
Theme 9. The general theory of the system of linear algebraic equations	Answer the self-control questions on the theme. Carry out the practical homework. Solve the tasks of the independent test on the theme	10	12 – 13	13	13; 15
Theme 10. The elements of vector algebra	Answer the self-control questions on the theme. Prepare for the written test on themes 8 – 10. Carry out the practical homework. Solve the tasks of the independent test on the theme. Make a presentation of the independent creative work. Prepare for the colloquium on themes 8 – 10	13	14 – 16	15 – 16	16
Exam		10	17 – 20	19 – 20	18 – 20
Total time for the independent work		**82**	–	–	–

7.2. Examples of practical homework for independent work

Thematic module 1. The elements of mathematical analysis

Theme 1. The limit of the function and continuity

Calculate the limits of the following functions:

1.1. $\lim\limits_{x\to 1}\dfrac{3x^2-2x-1}{x^4-x}$. **1.2.** $\lim\limits_{x\to 4}\dfrac{x-\sqrt{3x+4}}{16-x^2}$. **1.3.** $\lim\limits_{x\to 64}\dfrac{\sqrt{x}-8}{4-\sqrt[3]{x}}$.

1.4. $\lim\limits_{x\to+\infty}\dfrac{\sqrt[3]{x^3+2x^2}+5}{3\sqrt{x^2-x}+x}$. **1.5.** $\lim\limits_{x\to\infty}\dfrac{3x^4+4x^2-2x}{5x^4-x^3+1}$.

1.6. Prove that at $x\to 0$ $\quad 1-\cos x \approx \dfrac{1}{2}x^2$.

1.7. $\lim\limits_{x\to\infty}\left(\dfrac{4x^4}{x^2+x+2}-4x^2\right)$. **1.8.** $\lim\limits_{x\to+\infty}\dfrac{3^x+2}{3^{x+1}-1}$. **1.9.** $\lim\limits_{x\to\pm\infty}\dfrac{2\cdot 5^x-3}{9\cdot 5^x+4}$.

Calculate the limits using the equivalencies:

1.10. $\lim\limits_{x\to 0}\dfrac{\sin 20x}{tg15x}$. **1.11.** $\lim\limits_{x\to 0}\dfrac{1-\cos 6x}{\sin^2 4x}$. **1.12.** $\lim\limits_{x\to 0}\dfrac{e^{\sin 2x}-1}{\ln(1+tg\,4x)}$.

1.13. Investigate the continuity of the function:

1) $f(x)=\begin{cases}3x+1, & x<0\\ 1\quad 4x, & x>0\\ e^2, & x=0\end{cases}$; 2) $f(x)=\dfrac{1}{2-x}$; $f(x)=3^{\frac{1}{x}}$.

Thematic module 2. Linear algebra

Theme 8. The elements of the theory of matrices and determinants

Task 8.1. For the pairs of matrices below say whether it is possible to add (subtract) them together and then, where it is possible, derive the matri-

ces $C = A + B$, $D = A - B$, $F = 3A - 4B$, $G = \left(-3\right)A + \frac{1}{2}B$:

1) $A = \begin{pmatrix} 0 & 3 \\ 4 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix}$; 2) $A = \begin{pmatrix} 1 & 4 & 2 & 3 \\ -3 & 2 & -5 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} -2 & 0 & 2 & -5 \\ 6 & 1 & 3 & 1 \end{pmatrix}$; 3) $A = \begin{pmatrix} 1 & 2 & 4 \\ 1 & -4 & -3 \\ -1 & 1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 2 & 0 \\ -2 & 6 & -2 \\ -1 & 5 & 3 \end{pmatrix}$.

Task 8.2. In each of the following cases, determine whether the products AB and BA are both defined; if so, also determine whether AB and BA have the same number of rows and the same number of columns; if so, also determine whether $AB = BA$:

a) $A = \begin{pmatrix} 0 & 3 \\ 4 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix}$; b) $A = \begin{pmatrix} 1 & -1 & 5 \\ 3 & 0 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 1 \\ 3 & 6 \\ 1 & 5 \end{pmatrix}$;

c) $A = \begin{pmatrix} 3 & 1 & -4 \\ -2 & 0 & 5 \\ 1 & -2 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & -1 \end{pmatrix}$.

Task 8.3. Calculate A^2 and A^3, where

1) $A = \begin{pmatrix} 2 & -5 \\ 3 & 1 \end{pmatrix}$; 2) $A = \begin{pmatrix} 3 & 4 & 2 \\ 1 & 3 & 2 \\ 0 & 2 & -7 \end{pmatrix}$.

Task 8.4. Carry out the operations $2\left(A + B\right)B^T$, $3B\left(B - 2A\right)$, $\left(3A - 2B\right)A^T$ on the given matrices and check the following properties: a) $\left(A + B\right)C = AC + BC$; b) $C\left(A + B\right) = CA + CB$; c) $A\left(BC\right) = \left(AB\right)C$.

$A = \begin{pmatrix} 1 & 2 & 4 \\ 1 & -4 & -3 \\ -1 & 1 & 1 \end{pmatrix}$; $B = \begin{pmatrix} 4 & -3 & 0 \\ 3 & 2 & -1 \\ -2 & -1 & 4 \end{pmatrix}$; $C = \begin{pmatrix} 1 & 3 & 4 \\ -2 & 0 & 3 \\ 5 & 6 & 4 \end{pmatrix}$.

Task 8.5. Consider six matrices: $G = \begin{pmatrix} 4 & 7 & 2 \end{pmatrix}$; $F = \begin{pmatrix} 1 & 7 & 2 & 9 \\ 9 & 2 & 7 & 1 \end{pmatrix}$;

$$A=\begin{pmatrix}2 & 5\\ 1 & 4\\ 1 & 2\end{pmatrix};\ B=\begin{pmatrix}1 & 0 & 4\\ 2 & 1 & 3\\ 1 & 1 & 5\\ 3 & 2 & 1\end{pmatrix};\ C=\begin{pmatrix}1 & 0 & 7\\ 2 & 1 & 2\\ 1 & 3 & 0\end{pmatrix};\ D=\begin{pmatrix}1\\ 2\\ 3\end{pmatrix}$$

and: a) calculate all possible products; b) transpose these matrices.

Task 8.6. Find the inverse matrix for the given matrix and check $A\cdot A^{-1}$ or $A^{-1}\cdot A$:

1) $A=\begin{pmatrix}2 & 8 & -3\\ -1 & -7 & 4\\ -3 & -6 & 2\end{pmatrix}$; 2) $A=\begin{pmatrix}3 & 2 & -3\\ 5 & 4 & 1\\ -6 & 3 & 1\end{pmatrix}$; 3) $A=\begin{pmatrix}2 & 2 & 1\\ 1 & -1 & 0\\ -1 & 1 & 1\end{pmatrix}$;

4) $A=\begin{pmatrix}-6 & 9 & 0\\ 1 & 2 & 3\\ 11 & 5 & 7\end{pmatrix}$; 5) $A=\begin{pmatrix}3 & -1 & 1\\ 1 & 0 & 2\\ 2 & 2 & 1\end{pmatrix}$; 6) $A=\begin{pmatrix}6 & 3 & 2\\ 7 & 1 & 2\\ 3 & 0 & 1\end{pmatrix}$.

Task 8.7. Calculate the determinant of the given matrices on the basis of Sarrus formula and check the result using the theorem concerning the decomposition of the determinant in row 1 (2 or 3) and column 1 (2 or 3):

1) $A=\begin{pmatrix}2 & 3 & 3\\ 1 & 0 & -2\\ -1 & -4 & 1\end{pmatrix}$; 2) $A=\begin{pmatrix}1 & 2 & 1\\ 0 & -2 & -3\\ -2 & -3 & 2\end{pmatrix}$; 3) $A=\begin{pmatrix}4 & 2 & 0\\ 2 & 1 & -2\\ -1 & -1 & 4\end{pmatrix}$.

Task 8.8. Calculate the following determinants:

1) $\begin{vmatrix}-1 & -2 & 3 & 2\\ 2 & 3 & 4 & -2\\ -1 & 1 & 0 & 1\\ 0 & -1 & 3 & 1\end{vmatrix}$; 2) $\begin{vmatrix}2 & 3 & 1 & -2\\ 1 & -2 & -2 & 1\\ -2 & 1 & 0 & -1\\ 3 & 0 & 3 & 1\end{vmatrix}$; 3) $\begin{vmatrix}2 & 1 & 2 & 1\\ 2 & 1 & 3 & -2\\ 0 & -1 & 2 & 2\\ -1 & 0 & 4 & -3\end{vmatrix}$;

4) $\begin{vmatrix}2 & 0 & 1 & 1\\ 1 & -1 & 0 & 2\\ 3 & 1 & -1 & -2\\ 4 & -2 & 3 & 1\end{vmatrix}$; 5) $\begin{vmatrix}-1 & 0 & 1 & 2\\ 2 & 1 & -1 & -4\\ 3 & 2 & 0 & 1\\ 4 & -2 & 1 & 2\end{vmatrix}$; 6) $\begin{vmatrix}2 & -1 & 1 & -3\\ 1 & -2 & -3 & 1\\ -2 & 2 & 0 & -1\\ 3 & 0 & 4 & 1\end{vmatrix}$.

7.3. Questions for self-control

Thematic module 1. The elements of mathematical analysis

Theme 1. The limit of the function and continuity

1. Numerical sets and operations with them.
2. The neighborhood of a point.
3. The definition of the function of one variable.
4. The domain of the definition and the range of values of a function.
5. Properties of a function.
6. The notion of the inverse function.
7. A superposition of functions.
8. A numerical sequence.
9. The definition of the limit of a sequence.
10. Infinitesimals and infinitely large values.
11. The definition of the limit of a function.
12. One-sided limits.
13. The first remarkable limit.
14. The second remarkable limit.
15. The definition of the function continuity at a point.
16. Classification of breaks.

Theme 2. Differential calculus of the function of one variable

1. The definition of a derivative.
2. A table of derivatives.
3. Rules of calculation of derivatives.
4. Derivatives of higher orders.
5. The definition of a differential.
6. Differentials of higher orders.
7. The main theorems of the differential calculus.
8. L'Hospital rule.
9. The condition of the monotony of a function.
10. The maximum and the minimum of a function.
11. Convexity and concavity of the graph of a function.
12. Inflection points.
13. Asymptotes of the graph of a function.
14. Marginal analysis.

15. Elasticity of economic indicators.
16. The economic meaning of Fermat theorem.

Theme 3. Analysis of the function of several variables

1. Functions of two variables.
2. The domain of the definition of functions.
3. Partial derivatives. Mixed derivatives.
4. The differential.
5. The necessary conditions of the function of two variables.
6. The sufficient conditions of the extremum of the function of two variables.
7. The notion of the conditional extremum.
8. The method of Lagrange factors.
9. The least-squares method.
10. The function of several variables in the problems of economics (the utility function, the expenditure function, the multifactor production function of Cobb and Douglas).

Theme 4. The indefinite integral

1. An antiderivative.
2. An indefinite integral.
3. A table of basic integrals.
4. Direct integration.
5. A change of the variable in an indefinite integral.
6. Integration by parts.
7. Integration of rational fractions.
8. Integration of irrational expressions and expressions which have trigonometric functions.

Theme 5. The definite integral and its application

1. The notion of the definite integral.
2. Integral sums.
3. Properties of the definite integral.
4. Newton – Leibnitz formula.
5. A change of the variable in a definite integral.
6. Integration by parts.
7. The notion of an improper integral.
8. The conditions of the convergence of improper integrals.

9. Euler – Poisson integral.
10. Calculation of areas, volumes of the solid of a rotation.
11. Calculation of the arc lengths of curves.
12. Formulas of rectangles, trapezoids, Simpson**.**
13. The volume of the productive production.
14. A consumer surplus.
15. Lorenz curve.

Theme 6. Differential equations

1. The notion of the differential equation.
2. The order of the differential equation.
3. Differential equations of the first order.
4. A general solution and a general integral of a differential equation.
5. Initial conditions.
6. A particular solution and a particular integral.
7. The differential equation of the first order with separable variables.
8. Homogeneous equations of the first order.
9. Linear differential equations of the first order.
10. Differential equations of Bernoulli**.**
11. The second-order linear differential equations with constant coefficients.
12. Homogeneous and inhomogeneous differential equations.
13. The notion of linearly independent solutions of a homogeneous differential equation of the second order.
14. The structure of a general solution of an inhomogeneous differential equation of the second order.
15. Linear inhomogeneous differential equations of the second order with the right parts of a special form.
16. The notion of the differential equation.
17. The notion of the system of differential equations.
18. The notion of the equilibrium of a solution.
19. Solow model.
20. The model of a natural increasing output.
21. The dynamics of market prices.

Theme 7. Series

1. Numerical series.

2. A partial sums of series.
3. The necessary condition of the series convergence.
4. Series with positive terms.
5. The theorem of a comparison of series.
6. D'Alembert criterion.
7. Cauchy's criterion.
8. Maclaurin – Cauchy integral criterion.
9. The notion of alternating series.
10. An absolute and conditional convergence of series.
11. Leibnitz theorem.
12. Power series.
13. The convergence radius of power series.
14. Differentiation and integration of power series.
15. Teylor and Maclaurin series.
16. Decomposition of elementary functions in Taylor and Maclaurin series.
17. Application of power series to an approximate calculus.

Thematic module 2. Linear algebra

Theme 8. The elements of the theory of matrices and determinants

1. The definition of a matrix.
2. Types of matrices (square, triangular, diagonal, unit).
3. Basic operations with matrices.
4. Properties of these operations.
5. Transposition of a matrix.
6. The notion of an inverse matrix.
7. The definition of the determinant.
8. The rule of the triangle (Sarrus formula).
9. The rules of the calculation of determinants of lower orders (schematic) and higher-orders (expansion by Laplace formulas).
10. The properties of determinants.
11. Calculation of some special determinants (triangular, diagonal, identity matrices, Vandermonde matrix).
12. Calculation of an inverse matrix with the help of the determinants (algebraic cofactors).
13. An inverse matrix.
14. The matrix rank and ways to define it.

Theme 9. The general theory of the system of linear algebraic equations

1. The definition of the system of linear algebraic equations.
2. An augmented matrix.
3. The definition of a solution.
4. The consistent or inconsistent system.
5. A determined or undetermined system.
6. The inverse matrix method of solving square systems of linear algebraic equations.
7. Cramer method of solving square systems of linear algebraic equations.
8. Gauss – Jordan method of sequential exclusion of unknowns for solving systems of linear algebraic equations.
9. The notion of the matrix rank.
10. Calculation of the matrix rank.
11. Kronecker – Capelli theorem.
12. The particular and general solutions of the system of linear algebraic equations.
13. A homogeneous system of linear algebraic equations.
14. The space of solutions of a homogeneous system.
15. A fundamental system of solutions of a homogeneous system of linear algebraic equations.

Theme 10. The elements of vector algebra

1. The definition of a vector.
2. The definition of a point.
3. Linear operations with vectors in coordinates.
4. Coordinates of the point of division of a segment.
5. Coordinates of the vector which is given by two points.
6. Properties of a scalar product of two vectors.
7. Expression of a scalar product through coordinates.
8. A cross product of two vectors, its properties.
9. Expression of a cross product through coordinates.
10. A mixed product of three vectors, its properties.
11. Expression of a mixed product through coordinates of vector factors.

12. The definition of linear space.
13. The basis of linear space.
14. The notion of subspace.
15. The notion of linear vector space.
16. The rank of the finite systems of vectors, rules of its calculation.
17. The definition of the eigenvalue of a matrix.
18. The definition of the eigenvector of a matrix.
19. A characteristic equation.
20. The notion of a quadratic form.
21. The conditions of determinacy of quadratic forms.
22. The matrix of a quadratic form.
23. Reducing quadratic forms to a canonical form.
24. A general equation of the second-order curve.
25. Reducing the second-order curve to a canonical form.
26. A parabola.
27. An ellipse.
28. A hyperbola.

7.4. The independent test

7.4.1. Basic requirements for carrying out the independent test

The purpose of carrying out an independent test is the formation of students' practical skills in the use of theoretical knowledge of the academic discipline "Mathematical Analysis and Linear Algebra" for solving economic problems and optimal decision making, obtaining skills in economic mathematical analysis and modelling for finding and explanation of the most effective solutions, as well as using the methods of quantitative and qualitative analysis of applied economic mathematical models.

The independent test should be carried out on the scheduled date. Besides, the description of each of the tasks for the independent test should be done (except the didactic analysis and the definition of corresponding elements of the independent work) according to the general technology of a fulfillment:

learning and citing the basic questions of the theoretical material out of the recommended sources;

the design of the report on carrying out the task for the independent test, answers to control questions;

handing in the fulfilled tasks of the independent test and the answers to the control questions to the lecturer.

the fulfillment of the tasks of the independent test on the academic discipline is assessed depending on:

the understanding, the degree of mastering the theory and methodology of the problems which are considered;

the degree of acquaintance with the recommended literature and the mastery of the factual material of the academic discipline;

the ability to connect theory and practice in the consideration of practical situations, solving problems, carrying out calculations, fulfillment of tasks given for independent work;

the completeness of taking into account the conditions for the fulfillment of the tasks;

the logic of the given material and correspondence of its structure to the provided thematic elements of the task; the availability and completeness of consideration of the key concepts (definitions, terms, varieties and so on) of the subject matter of the task; the availability and explanations of the student's final conclusions; illustration of the processed material with the help of student's own examples and graphical material.

7.5. Preparation of the independent creative work

Independent creative work of students is an integral part of the educational process. It forms the skills in the creation of major types of work (term papers, a diploma project). That is the reason why it is necessary for students to learn how to qualitatively prepare a creative work.

Within the framework of the given form of student's independent work it is proposed to prepare a presentation on the theme formulated for a student, in the electronic form (with the help of MS PowerPoint). An alternative may be a presentation at a next lecture or writing a scientific article.

Preparation of independent creative work provides for syste-matization, consolidation, broadening of the theoretical and practical knowledge of the academic discipline and using it in the process of solving a specific economic problem, development of skills in independent work and mastering the

methods of investigation and experiment, connected with the theme of the independent creative work.

Independent creative work provides for the availability of the following elements of scientific investigation: practical significance, a comprehensive systematic approach to solving the tasks of the investigation, the theoretical use of the progressive modern methodology and scientific developments, availability of the elements of creativity, the ability to use modern technologies.

A comprehensive systematic approach to disclosing the theme of the independent creative work implies consideration of the subject of the research from different points of view that is from the position of a theoretical basis and practical ground work, conditions of its realization, analysis, explanations of ways for improvement in close relationship and a common logic of exposition.

The use of modern technology consists in the fact that in the process of fulfillment of analysis and explanation of ways for improvement of particular aspects of the subject and the object of investigation, a student has to use information about high achievements in techniques and technologies of investigation, use varied mathematical methods and ways, approaches to the definition and explanation of indicators of analysis of a social economic system or its elements.

Students submit the independent creative work to the lecturer in the electronic form if it is a presentation or in the printed or electronic form if it is a scientific publication.

After the complex presentation or a scientific publication has been reviewed and corrected by the lecturer, students make their presentations in front of the audience, report on the results stated in the scientific publication, make reports at a student's scientific and practical conference and so on.

8. Individual consultative work

Individual consultative work is fulfilled according to the schedule of the individual consultative work in the following forms: individual studies, consultations, check of fulfillment of individual tasks, check and defence of the tasks presented for the current control and so on.

The forms of the individual consultative work are:

a) according to the mastery of the theoretical material:

consultations: individual (question-answer);
group (consideration of typical examples);

b) for complex assessment of the mastery of the program material: individual handing in of the fulfilled works.

9. Methods of study

To intensify the process of teaching the academic discipline "Mathematical Analysis and Linear Algebra" the following educational technologies are applied: problem lectures, mini-lectures, work in small groups, discussions, brainstorms, moderations, presentations, computer simulation (games), Delphi's method, the method of scenarios, banks of visual support (Tables 9.1 and 9.2).

The basic difference of active and interactive methods of education from traditional ones is not only defined by the methods and techniques of teaching, but also by high effectivity of the educational process, which reveals itself in the high motivation of students; consolidation of theoretical knowledge in practice; raised students' consciousness; forming the ability to make independent decisions; forming the ability to approve collective decisions; forming the ability for social integration; getting skills in resolving conflicts; development of the ability to reach compromises.

Problem lectures are directed at the development of students' logical thinking. The theme is confined to two or three key issues, students' attention is concentrated on the material which has not been represented in textbooks, the experience of foreign educational universities is used with handing out printed materials to students during the lecture and drawing basic conclusions as to the issues considered. In the course of lectures students are asked questions for independent reflection which a lecturer answers himself, without waiting for students' answers. This kind of system makes students concentrate and begin to actively think in search of a correct answer.

Mini-lectures provide for the delivery the educational material during a short-length segment of time and they are charactirized by a significant content, complexity of logical constructions, forms, proofs and generalizations. They are conducted, as a rule, as a part of a study-investigation. Mini-lectures differ from full-size lectures by a shoter duration. Usually, they last no more than 10 – 15 minutes and they are used to briefly give new information to all students. Mini lectures are often used as parts of a whole theme, which it is

desirable to teach as a full-size lecture in order to avoid the audience's getting tired. Then the information is given by turn as several particular fragments, between them other forms and methods of study are used.

Seminar-discussions provide for exchange of thoughts and ideas of students on the given theme and develop thinking, help to form ideas and beliefs, produce skills in formulating thoughts and expressing them, teach to assess other people's proposals, critically come to personal ideas.

Work in small groups gives an opportunity to structure practical studies in the form and content, gives a possibility for each student's partaking in the work on the theme under study, stimulates forming personal qualities and experience of social communication.

Brainstorming is a method of solving urgent tasks, its core lies in expressing as many ideas as possible in a short period of time, discussing and selecting them.

Presentations are speeches to students which are used for presenting certain achievements, group work results, reports of individual task fulfillment, instruction, demonstration of new goods and services.

Table 9.1

Distribution of forms and methods of intensification of the educational process according to the themes of the academic discipline

Theme	Practical application of educational technologies
1	2
Thematic module 1. The elements of mathematical analysis	
Theme 1. The limit of the function and continuity	A problem lecture on the theme: "Investigation of continuity of different types of functions"
Theme 2. The differential calculus of the function of one variable	A mini-lecture on the theme: "Application of the differential to approximate economic calculations"
Theme 3. Analysis of the function of several variables	A mini-lecture on the theme: "Investigation of the conditional extremum of the functions of two variables with the help of the Lagrange factors". Presentation of independent creative work
Theme 4. The indefinite integral	A mini-lecture on the theme: "Some classes of functions, integration of which is reduced to rational fractions"

Table 9.1 (the end)

Theme 5. The definite integral and applying it	A problem lecture on the theme: "Application of the definite integral to economic problems"
Theme 6. Differential equations	A problem lecture on the theme: "Mathematical modelling of the economic process with the help of ordinary and differential equations and systems"
Theme 7. Series	A mini-lecture on the theme: "Investigation of the convergence of series and calculation of a series sum". Work in small groups with further discussion of the results of laboratory works. Presentation of independent creative work
Thematic module 2. Linear algebra	
Theme 8. The elements of the theory of matrices and determinants	A mini-lecture on the theme: "Application of matrices to giving any information about the characteristics of the investigated economic process"
Theme 9. The general theory of the system of linear algebraic equations	A problem lecture on the theme: "Construction of an inverse matrix using Jordan − Gauss transformations"
Theme 10. The elements of vector algebra	A mini-lecture on the theme: "Application of vectors to the construction of economic problems". Work in small groups with discussion of the results of laboratory work

Moderation is a way to conduct a discussion, which leads quickly to concrete results, gives a possibility for all present students to take part in the process of search for a solution to a problem and take full responsibility for the result. The function of the moderator is to see to it that the rules of the discussion are observed, which gives a possibility to simplify the process of the search for a solution without interfering in its essence.

The Delphi method is used for the purpose of reaching a consensus in expert judgements. It gives a possibility for students to express their thoughts to a group of experts, which work individually in different places. To choose a management decision according to this method, the academic group is divided, for example, into five small groups. Four groups work, develop and make a management decision, and the fifth group is the expert team. This group carries out analysis of the variants of management decisions, which are proposed by the working groups, and assesses these variants. Within the expert group the distribution of its members according to specializations is fulfilled.

The method of scenarios is used for designing probabilistic models of behavior and development of concrete events in the long term.

Table 9.2

Using the methodologies of intensification of the educational process

The theme of the academic discilpline	Practical application of educational technologies	The methodologies of intensification of the educational process
1	2	3
Thematic module 1. The elements of mathematical analysis		
Theme 1. The limit of the function and continuity	*Practical study*. Task: Calculation of the limits of functions and investigation of the continuity of functions	A brainstorm on the question: "The choice of an effective method to eliminate indetermination in the calculation of limits of functions". Work in small groups, moderation
Theme 2. The differential calculus of the function of one variable	*Laboratory work.* Tasks: Finding derivatives of implicit, explicit and parametric functions of one variables. Fulfilment of investigation of the function and plotting its graph	Work in small groups with discussion of the results of laboratory work. Computer simulation and presentations of the tasks on the question: "Investigation of the functions and plotting their graphs"
Theme 3. Functions of several variables	*Laboratory work.* Task: Finding partial derivatives of the functions of several variables. The differential of the function of several variables and its application. The gradient and the derivative by direction. Level lines. Investigation of the extremum of the function of two variables. Application to economics	Computer simulation and presentations of the tasks on the question: "The extremums of the function of two variables and their geometric meaning". Work in small groups, the Delphi method
Theme 4. The indefinite integral	*Practical study*. *Laboratory work.* Task: Calculation of indefinite integrals	A brainstorm on the question: "The choice of an integration method to find the indefinite integral". Discussion, moderation
Theme 5. The definite integral and applying it	*Practical study*. *Laboratory work.* Taks: Calculation of definite and improper integrals and applying them to economic calculations	Work in small groups, brainstorms, the Delphi method

Table 9.2 (the end)

1	2	3
Theme 6. Differential equations	*Practical study*. Tasks: Solving differential equations and Cauchy problems. Construction of economic models using the apparatus of differential equations	A brainstorm on the question: "Defining the type of the differrential equation". Discussion about the choice of the method of integration of differential equations. The method of scenarios
Theme 7. Series	*Practical study*. Tasks: Investigation of the convergence of constant-sign series. Investigation of the convergence of alternate series. Absolute and conditional convergences. Power series and its convergence. Decomposition of the basic elementary functions into Taylor and Macloren series	Work in small groups, a discussion about the choice of the convergence sign of numerical series, brainstorms, moderation
Thematic module 2. Linear algebra		
Theme 8. The elements of the theory of matrices and determinants	*Practical study*. *Laboratory work.* Tasks: Solving economic problems of data analysis using matrices. Explanation of calculational results	Work in small groups, brainstorms, a computer simulation, a situational analysis
Theme 9. The general theory of the system of linear algebraic equations	*Practical study*. *Laboratory work.* Tasks: Construction of a mathematical model of production planning to follow technological relationships between separated products. Investigation of the model with changing its parameters	Work in small groups, brainstorms, computer simulation, presentations, discussion about the choice of the method of the systems of linear algebraic equations
Theme 10. The elements of vector algebra	*Practical study*. *Laboratory work.* Tasks: Carrying out arithmetic operations with vectors. Analysis of the geometric meaning of the linear dependence of the vector system. Classification of the second order curves as an example of investigation of a quadratic form	Work in small groups, brainstorms, computer simulation, discussion of the theoretical material, discussion about the method of investigation of the vectors dependence

A business game is a method of imitation of making administractive decisions in various situations by means of playing according to the rules which have been worked out or are worked out by the members themselves. This method is realized through students' independent solving the set problem provided a shortage of the necessary knowledge when students themselves are forced to master the new content or search new connections in the learnt material.

A computer simulation (game) is an education method, which is based on the use of a spesific computer program in order to get visual modelling of a process. Students can change the parameters and data, decisions and analyze the results of such decisions. The purpose of using this method is the development of systematic thinking of students, their ability to plan, form skills in identifying and analyzing problems, compare and estimate alternatives, make optimal decisions and work under the conditions of limited time.

Banks of a visual support help to intensify the education process of studying the themes of the academic discipline with the help of vizualization.

The interactive distant education is a set of educational technologies based on the principles of contact in the information educational space. They serve to organize the education of users distributed in the space and time.

10. Methods of control

The system of assessment of competences which were formulated for a student during the learning of the academic discipline (Table 2.1), takes into consideration the forms of studies which according to the syllabus of the academic discipline provide lectures, practical studies, laboratory work, fulfillment of students' independent work. The assessment of the formed competences of students is carried out on the accumulative 100-point system. According to the temporary provision "About the Order of Assessment of Students' Academic Performance on the Accumulative Point Rating System" of Simon Kuznets Kharkiv National University of Economics control ways include:

current control which is carried out within the term during lectures, practical studies and laboratory works and is assessed as a sum of accumulative points (the maximum equals 60 points; the minimum which makes it possible for a student to pass an exam, equals 35 points);

module control which is carried out in the form of a colloquium with taking into account the current control according to a corresponding thematic module, provides an integral assessment of student's results after learning the material of a logically completed part of the discipline (or a thematic module);

final/term control, which is carried out as a terminal exam, according to the schedule of the educational process.

Current control on the given academic discipline is carried out in the following forms:

active in-class work (lecture);

active in-class work (practical study);

active in-class work (laboratory study);

homework;

competence-oriented tasks (defence of laboratory work);

an independent test;

a written test;

independent creative work.

A colloquium is a form of reviewing and assessment of students' knowledge in the system of institutes of higher education. *The purpose* of carrying out of a colloquium is to clarify the theoretical and practical knowledge obtained by a student as a result of listening to lectures, attendance of practical and laboratory studies and independent learning of the material. Within the bounds of the assigned purpose the following *tasks* are fulfilled: evaluation of the quality and degree of student's understanding of the lecture material; the development and fixing of the skills in expressing thoughts; the development of student's ability for independent single-minded preparation; the development of skills in the generalization of different literary sources; giving a possibility for a student to compare different points of view on a given question. A colloquium is conducted as an intermediate mini-exam on the initiative of the lecturer and includes theoretical questions and practical tasks on the academic discipline. The list of questions, which are included into a colloquium on the themes of the thematic module, contains questions for self-diagnostics.

Final/term control is conducted in the form of a term exam. **Term exams** are a form of assessment of students' final mastery of the theoretical and practical material of a particular module of the academic discipline or the academic discipline on the whole, which are conducted as tests.

The order of conductiong the current assesssment of students' knowledge. Assessment of student's knowledge during practical studies and carrying out laboratory work is conducted on the accumulative system according to the following criteria:

understanding, the degree of the mastery of the theory and methodology of the problems which are considered;

the degree of the mastery of the factual material of the academic discipline; familiarizing with the recommended literary sources and modern literature on the questions which are considered;

the ability to connect theory and practice in the consideration of particular examples, solving problems, carrying out laboratory work, carrying out calculations in the process of doing homework and tasks which are considered in class;

the logic, structure, style of presenting the material in written papers and oral answers in class, the ability to ground one's own position, carry out generalization of the information and draw conclusions.

The maximum possible points which correspond to a particular task, are given on the condition of the correspondence of the solved task or the oral answer of a student to all the defined criteria. Lack of one or another component decreases the number of accumulative points. In the assessment of tasks set for independent work in laboratory and practical studies, the quality of fulfillment is also considered. Besides, handing in the performed task to the lecturer in accordance with the period defined by the schedule of the educational process plays an important role. If one of these conditions is not satisfied, the points are decreased.

A written test is carried out 3 times during a term and it includes practical tasks of different level of difficulty (complexity) according to the themes of the thematic module.

The criteria for assessment of the written test are as follows:

5 points, if the test has been carried out without mistakes and deficiencies, all the tasks contain the nesessary explanations, illustrations, an analysis of the results and conclusions;

4 points, if the test has been carried out, but there are no more than one mistake and no more than one deficiency or no more than three deficiencies;

3 points, if no less than 2/3 of the test have been carried out, there are no more than two mistakes and no more than two deficiencies;

2 points, if less than 2/3 of the test have been performed and the number of mistakes and deficiencies exceeds the norm for the mark of three points;

1 point, if the fulfillment of the tasks has not been begun, but there is a particular correct thinking;

0 point, if the task is unavailable.

Revision and marking of the competence-oriented tasks (defence of laboratory work on the themes which are combined into a corresponding thematic module) is carried out twice during a term in the form of work in small groups. Besides, the quality of fulfillment of the tasks for laboratory work and the ability to present the results of investigations, give reasonable answers to the questions of opponents, think critically, assess the results of the work of other participants must be assessed.

A colloquium is carried out twice during a term in the written form or in the form of an oral test for controlling students' knowledge of the theoretical material and the mastery of the categorical apparatus.

The criteria for assessment of a colloquium:

6 points, if deep knowledge of the syllabus material has been demonstrated, a sequential, complete and logical answer has been given, a correct decision has been made, the mastery of different methods and techniques in carrying out practical tasks has been demonstrated;

5 points, if knowledge of the syllabus material has been demonstrated, an answer without essential inaccuracies has been given, mastery of the necessary methods in carrying out practical tasks has been demonstrated;

4 points, if knowledge of the basic material has been demonstrated, an answer with inaccuracies has been given, mastery of the necessary methods in carrying out practical tasks has been demonstrated;

3 points, if knowledge of the basic material has been demonstrated, an answer with inaccuracies and quite incorrect formulations has been given, mistakes have been made in the use of the necessary methods in carrying out practical tasks;

2 points, if knowledge of the basic material has not been demonstrated, an answer with essential mistakes and incorrect formulations has been given, lack of skills in the use of the necessary methods in carrying out practical tasks has been demonstrated;

1 point, if an incorrect solution has been given, the fulfillment of the colloquium practical tasks has not been begun, but some particular correct thinking has been shown;

0 point, if the task is unavailable.

The criteria for assessment of independent work of students.

The general criteria for the assessment of independent work of students are profound and deep of knowledge, the level of thinking, skills in the systematization of knowledge on particular themes, skills in drawing conclusions, attainments and techniques of carrying out practical tasks, the ability to find necessary information, carry out its classification and processing, self-realization in practical and laboratory studies.

The criteria for assessment of independent creative work and independent tests are:

the ability to carry out a critical independent estimation of the defined problems;

skills in the explanation of alternative views and availability of a students' own point of view, position on the defined problem;

using the analytical approach; the quality and accuracy of expressing the thought;

the logic, structure and explanation of conclusions about a particular problem;

independence of carrying out the work;

grammatical correctness of the presentation of the material; using the methods of comparison, generalization of the concepts and facts;

the design of the work;

the quality of presentation.

The order of final control on the academic discipline. The final control of knowledge and competences of students on the academic discipline is carried out on the base of the term exam. The examination paper includes the syllabus of the discipline and provides for assessment of the knowledge level and degree of the mastery of corresponding competences of students (Table 2.1).

The purpose of the exam is to test student's understanding of the syllabus material on the whole, the logic and relations between its particular parts, the skills in the creative use of the gained knowledge, the ability to formulate their attitude to a particular problem of the academic discipline and so on. The competent approach to the assessment of the exam implies measuring the

level of the student's mastery of the competences provided by the qualifying requirements.

Each examination paper contains 5 practical tasks, including two first-level (diagnostic) tasks, two second-level (situational) tasks and one third-level (diagnostic and heuristic) task.

The structure of the examination paper is given in Table 10.1.

Table 10.1

The structure of the examination paper

Task level	The content of tasks according to the themes
First	Finding the limits of functions and investigation of the continuity of functions, finding asymptotes of the graphs of functions. Finding the derivatives of different types of functions. Application of the differential calculus of the function of one variable to the investigation of the monotonicity, concavity, existence of the extremums and the inflection point. Investigation of numerical series, basic rules of convergence of series, the comparison sign, D'Alembert's and Cauchy signs for constant-sign series, Leibnitz's sign for alternate series. Functional series: calculation of their radius and the convergence domain of power series. Solving the problems of vector algebra. Calculation of scalar, cross and mixed products, checking the complanarity and collinearity of vectors
Second	Solving the systems of linear algebraic equations by Cramer's method, the inverse matrix method and Jordan − Gauss method. Finding the basis of space and decomposition of the vector in this basis, carrying the transformation to a new basis. Calculation of eigenvalues and eigenvectors of a matrix. Investigation of the second order curves, fulfillment of the transformation of a curve equation to a canonical form and definition of its type. Finding the gradient and the plot of the level line of the function of two variables at the given point. Investigation of local and conditional extremums of the function of two variables
Third	Application of the knowledge according to the themes: theme 3 "Analysis of the function of several variables"; theme 5 "The definite integral and its application"; theme 6 "Differential equations"

The examination paper was formed according to the form No. H-5.05, "About the Statement of Forms of Documents for Personnel Training in Higher Educational Establishments of the I – IV Levels of Accreditation" which was approved by the Ministry of Education and Science. A sample examination paper is given below.

The assessment of the exam is carried out according to the temporary provision "About the Order of Assessment of Students' Academic Performance on the Accumulative Point Rating System" of Simon Kuznets Kharkiv National University of Economics.

In the case of irreproachable fulfillment of all the examination tasks with the demonstration of deep knowledge of the academic discipline, skills in the practical use of the formed competences which are based on the ability to analyze and solve a wide range of tasks, a high level of completing the written work the student obtains 40 points.

The assessment of a particular task is carried out according to the following criteria.

The task of the first level is assessed as follows:

7 points, in the case of the exact use of the scientific terminology and symbols in the necessary logical sequence; a creative approach to solving original problems which require a high level of knowledge;

6 points, in the case of the exact use of the mathematical terminology and symbols; irreproachable mastery of mathematical tools; correct use of mathematical methods, facts, formulas and relations for solving the task of the third level;

5 points, if a logically right sequence of steps of solution has been made. All the key moments of the solution have been grounded. 1 – 2 slight mistakes or slips are possible in the calculations which don't influence the correctness of the further solution;

4 points, if mastery of a small part of obligatory skills and attainments required by the syllabus of the academic discipline has been demonstrated in solving the tasks; the conclusions, reproduction of the syllabus material of the discipline has not always been shown;

3 points, if the task has been solved only partially with initial right considerations, but there are mistakes which considerably influenced the process of the right solution of the task;

2 points, if the task fulfillment has been begun, there are separate correct considerations, but a logical mistake has been made which resulted in an incorrect solution.

1 point, if the condition has been written;

0 point, if no task has been fulfilled.

The task of the second level is assessed as follows:

8 points, if solving the assigned tasks is characterized by a creative use of the theoretical instrument, logical correctness, precision, explanation of conclusions, rationality or using original approaches to solving the tasks;

7 points, if perfect mastery of the skill in the use of mathematical tools with application of information from other educational courses and disciplines has been demontrated; one slight mistake has been made; a high level of standards of carrying out the tasks has been shown;

6 points, if a logically right sequence of steps of solution has been chosen. All the key points of solution have been grounded. 1 – 2 slight mistakes or slips are possible in the calculations which don't influence the correctness of the further solution;

5 points, in the case of correct using the terminology of the discipline and the basic methods for solving standard problems; showing the ability to use theoretical knowledge for solving standard (multistep) problems, some mistakes or deficiencies on the calculating stage of presentation of the solution; the ability to conclude;

4 points, in the case of more than one mistake and one or two deficienies in the calculations, graphs, the choice of the method of solution, which have caused a wrong final result in some cases;

3 points, if the task fulfillment has been begun, there are separate correct considerations, but a logical mistake has been made which resulted in an incorrect solution;

2 points, if numerical gross mistakes have been made in the process of using the concepts of the discipline in the formulas which prove the absence of a minimum necessary part of the compulsory skills and the practical attainments provided for the discipline syllabus;

1 point, if no task fulfillment has been begun, but the condition has been written;

0 point, if no task fulfillment has been begun.

The task of the third level is assessed as follows:

10 points, if the ability for scientific investigative developments on the problems of the discipline has been shown; perfect skills in the use of mathematical tools and modern scientific theoretical approaches, a high level of standards of carrying out tasks have been demonstrated;

9 points, in the case of using scientific terminology and symbols in the necessary logical sequence; solving the assigned tasks characterized by precision, explanation; a creative approach; rationality of the choice of the method of solution; correct necessary calculations and transformations;

8 points, if systematic, deep and full knowledge of all the parts of the academic discipline and the basic questions which go beyond the discipline has been shown; a high level of standards of carrying out the tasks has been demonstrated;

7 points, in the case of sporadic slight deficiencies which don't influence the final result; correct use of mathematical methods, facts, formulas and relations for solving the task of different level of complexity;

6 points, if the ability to conclude and compare the theoretical and practical material has been demonstrated; correct (but not always rational) use of mathematical methods of solution, facts, formulas and relations has been shown;

5 points, if half of the tasks have been done, the interpretation of the obtained results is unavailable; the level of the standards of carrying out tasks is acceptable;

4 points, if the tasks have been carried out without any logical relationship of the mathematical concepts and practical solutions have not been given sufficient theoretical explanation;

3 points, if an acceptable volume of knowledge has been shown within the educational standard; the use of mathematical symbols and terminology has been insufficient and inexact, the knowledge of the basic formulas and concepts on the discipline has not been demonstrated;

2 points, in the case of solving the tasks with the theoretical material used only on the level of concepts; the inability to understand the connection of the theoretical material with the practical tasks;

1 point, if the condition has been written;

0 point, if no task fulfillment has been begun.

A sample examination paper

Form No. H-5.05

SIMON KUZNETS KHARKIV NATIONAL UNIVERSITY OF ECONOMICS

Educational degree: bachelor

Training direction: 6.030601(44) "Business and Administration"

Term 1

Academic discipline: "Mathematical Analysis and Linear Algebra"

Examination paper

Task 1 (diagnostic). Find the area of the figure bounded by the lines:

$$y = 3 + 2x - x^2; \quad y = x + 1.$$

Task 2 (diagnostic). Investigate the convergence of the series:

a) $\sum_{n=1}^{\infty} \frac{n!}{10^n}$;

b) $\sum_{n=1}^{\infty} \frac{-1^{\,n} \cdot n}{4n^3 + 10}$.

Task 3 (situational). Investigate and solve the system:

$$\begin{cases} x_1 + 4x_2 - 3x_3 = 5 \\ -2x_1 + x_2 - x_3 = -1. \\ 3x_1 - x_2 + 2x_3 = 2 \end{cases}$$

Task 4 (situational). Solve the differential equation and find a particular solution: $y' - y \operatorname{tg} x = \frac{1}{\cos x}$, $y\ 0\ = 0$.

Task 5 (heuristic). The laws of supply and demand have the form:

$$f(x) = 186 - x^2; \qquad g(x) = 20 + \frac{11}{6}x.$$

Find the point of market equilibrium, consumer benefit and supplier benefit under the condition of establishment of market equilibrium. Analyze the obtained values in the problem.

Approved at the meeting of the Department of Higher Mathematics and Economic Mathematical Methods.

Protocol No. 4 of November 30, 2015.

The chief of the department L. Malyarets

The lecturer Ie. Misiura

A student, who for a valid reason, attested documentally, hasn't have a possibility to take part in the forms of current control, that is, hasn't passed the thematic module, has the right to complete it during two weeks after coming back to studies according to the notice of the dean of the department subject to a given period.

A student can't be allowed to take the exam if the number of points obtained during the current and module control accoding to the thematic module during the term does not make 35. After the examination period the dean of the department gives a notice about sitting the failed exams. In a given period the student adds the required points.

The final mark on the academic discipline is calculated according to the points obtained during the exam and the points obtained during current control on the accumulative system.

The student's progress is assessed if the number of points obtained as a result of all forms of control equals or exceeds 60.

Accordingly the minimum possible number of points in the current and module control during the term equals 35 and the minimum possible number of points obtained in the exam equals 25.

The result of the terminal exam is assessed in points (the maximum is 40 points, the minimum possible number is 25 points) entered into a corresponding column of the *Examination Record List*.

The final mark on the academic discipline is calculated according to the points obtained during the exam and poins obtained during the current control on the accumulative system.

The total result in points during the term is: *"60 and more points mean passed", "59 and less points mean failed"* and it is entered into the *Examination Record List* on the academic discipline.

11. The distribution of points which students obtain

An example of a technological chart of accumulactive rating points and the system of assessment of the level of formed professional competences which a student of the day-time form of studies has to get, is given in Tables 11.1 and 11.2 according to the forms of study and methods of control which are used in teaching the academic discipline.

Table 11.1

The technological chart of the accumulactive rating system

Forms of a study		Weeks																Examination period 18 – 20	Σ
		2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17		
Total educational load of students, hours per week																			
Class hours	Lectures	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2		32
	Practical study	2		2		2		2		2		2		2		2			16
	Laboratory study		2		2		2		2		2		2		2		2		16
	Current consultations*	*c*	*c*	*c*	*c*	*c*	*c*	*c*	*c*	*c*	*c*	*c*	*c*	*c*	*c*	*c*	*c*	2	2
	Exam*																	2	2
Class hours		4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	68
IWS	Learning the theoretical material	2	2	2	2	2	3	3	2	2	2	2	3	2	3	2	2		36
	Fulfillment of practical tasks	2	2	2	2	2	3	3	2	2	3	2	2	3	2	3	1		36
	Preparation for the exam																	10	10
Independent work		4	4	4	4	4	6	6	4	4	5	4	5	5	5	5	3	10	82
Total number of hours		8	8	8	8	8	10	8	8	8	8	8	8	8	10	10	8	24	150
Assessment graph of hours per week																			
Methods of control	Class active work (lectures)	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2		3.2
	Class active work (practical study)	0.2		0.2		0.2		0.2		0.2		0.2		0.2		0.2			1.6
	Class active work (laboratory study)		0.2		0.2		0.2		0.2		0.2		0.2		0.2		0.2		1.6
	Competence-oriented tasks								5								5		10
	Homework			0.2		0.2		0.2		0.2		0.2		0.2		0.4			1.6
	Independent test					3						3				3			9
	Written test					5				5						5			15
	Independent creative work																6		6
	Colloquiums								6								6		12
	Exam																	40	40
Total sum of points per week		0.4	0.4	0.6	0.4	8.6	0.4	0.6	11.4	5.6	0.4	3.6	0.4	0.6	0.4	8.8	17.4	40	100
Accumulation of points		0.4	0.8	1.4	1.8	10.4	10.8	11.4	22.8	28.4	28.8	32.4	32.8	33.4	33.8	42.6	60.0	100	

*The lecturer carries out current and examination consultations according to the timetable; hours for examination consultations and the exam for a student are included into the independent work.

Table 11.2

The system of assessment of the professional competences formed

Profes-sional compe-tences		Educational week	Hours		Forms of study		Assessment of the level of the formed competences	
							Forms of control	Maxi-mal point
1		2	3		4		5	6
Thematic module 1. The elements of mathematical analysis								**28.4**
AMI 1	The ability to calculate and understand the meaning of the limit and use the derivative in economics	2	Class	2	**Lecture**	*Theme 1.* The limit of the function and continuity	Active class work	0.2
				2	**Practical study**	Calculation of the limits of functions and investigation of the continuity of functions	Active class work	0.2
			IWS	4	**Preparation for studies**	Search, choice and look-ing through literary sources on the themes of the academic discipline. Learning the lecture material and preparation for practical studies	There is no control of independent work	–
		3	Class	2	**Lecture**	*Theme 2*. Differential cal-culus of the function of one variable	Active class work	0.2
				2	**Laboratory study**	Learning the program-ming software MatLab. Using MatLab for calculation of the limits and investigation of the continuity of functions	Active class work	0.2
			IWS	4	**Preparation for studies**	Learning the lecture ma-terial and preparation for practical studies. Carrying out practical homework and the inde-pendent test. Search of the material for the independent creative task	Homework is marked at practical studies	–

Table 11.2 (continuation)

1		2	3		4		5	6
AMI 1	The ability to calculate and understand the derivative in economics	4	Class	2	**Lecture**	*Theme 3.* Analysis of the function of several variables	Active class work	0.2
				2	**Practical study**	Investigation of the continuity of a function, finding the types of breakpoints	Active class work	0.2
			IWS	4	**Preparation for studies**	Search, choice and looking through literary sources on the theme. Carrying out practical homework and the independent test	Homework	0.2
AMI 2	The ability to find indefinite integrals and explain their meaning in mathematical models of economic processes	5	Class	2	**Lecture**	*Theme 4.* The indefinite integral	Active class work	0.2
				2	**Laboratory study**	Using MatLab for investigation of the continuity of a function	Active class work	0.2
			IWS	4	**Preparation for studies**	Learning the lecture material and preparation for practical studies. Carrying out practical homework and the independent test. Preparation for a written test	–	–
		6	Class	2	**Lecture**	*Theme 4.* The indefinite integral (the end)	Active class work	0.2
				2	**Practical study**	Solving practical tasks on the lecture theme. Finding an antiderivative with the help of direct integration and integration by substitution	Active class work. Written test	0.2 + 5
			IWS	4	**Preparation for studies**	Learning the lecture material and preparation for practical studies. Carrying out practical homework and the independent test	Homework. Independent test	0.2 + 3

Table 11.2 (continuation)

1		2	3		4		5	6
AMI 2	The ability to find definite integrals and explain their meaning	7	Class	2	**Lecture**	*Theme 5.* The definite integral and its application	Active class work	0.2
				2	**Laboratory study**	Finding definite integrals in software MatLab	Active class work	0.2
			IWS	6	**Preparation for studies**	Search, choice and looking through literary sources on the theme. Carrying out practical homework and the independent test	–	–
AMI 3	The ability to define the types of differential equations and use the methods of solving them	8	Class	2	**Lecture**	Theme 6. Differential equations	Active class work	0.2
				2	**Practical study**	Solving differential equations of the first order	Active class work	0.2
			IWS	6	**Preparation for studies**	Learning the lecture material. Carrying out practical homework. Preparation for the colloquium	Homework	0.2
		9	Class	2	**Lecture**	*Theme 6.* Differential equations (the end)	Active class work. Colloquium	0.2 + 6
				2	**Laboratory study**	Solving differential equations of the second order	Active class work. Competentce-oriented task	0.2 + 5
			IWS	4	**Preparation for studies**	Learning the lecture material. Carrying out the practical homework and the independent test. Preparation for the written test	–	–

Table 11.2 (continuation)

1		2	3		4		5	6
AMI 4	The ability to use the methods of series investigation	10	Class	2	**Lecture**	*Theme 7.* Series	Active class work	0.2
				2	**Practical study**	Solving the tasks of investigation of the convergence of numerical series, finding the convergence domain and decomposition of the function into power series	Active class work. Written test	0.2 + 5
			IWS	4	**Preparation for studies**	Search, choice and looking through literary sources on the theme. Carrying out the practical homework and tasks of the independent test	Homework	0.2
Thematic module 2. Linear algebra								**31,6**
AMI 5	The ability to use matrices in the analysis of data in economics	11	Class	2	**Lecture**	*Theme 8*. The elements of the theory of matrices and determinants	Active class work	0.2
				2	**Laboratory study**	Operations with matrices and determinants in the software MatLab	Active class work	0.2
			IWS	5	**Preparation for studies**	Search, choice and looking through literary sources on the theme. Carrying out the practical homework and tasks of the independent test	–	–
		12	Class	2	**Lecture**	*Theme 8.* The elements of the theory of matrices and determinants	Active class work	0.2
				2	**Practical study**	Solving practical tasks with matrix operations and calculation of determinants	Active class work	0.2
			IWS	4	**Preparation for studies**	Search, choice and looking through literary sources on the theme. Carrying out the practical homework	Homework. Independent test	0.2 + 3

Table 11.2 (continuation)

1		2	3		4		5	6
AMI 5	The ability to use the systems of linear algebraic equations in the construction of economic and mathematical models	13	Class	2	**Lecture**	*Theme 9*. The general theory of the systems of linear algebraic equations	Active class work	0.2
				2	**Laboratory study**	Solving the systems of linear algebraic equations in MatLab	Active class work	0.2
			IWS	5	**Preparation for studies**	Learning the lecture material. Carrying out practical homework and the inde-pendent test	–	–
		14	Class	2	**Lecture**	*Theme 9.* The general theory of the system of linear algebraic equations (the end)	Active class work	0.2
				2	**Practical study**	Solving systems of linear algebraic equations	Active class work	0.2
			IWS	5	**Preparation for studies**	Learning the lecture material, preparation for practical studies. Carrying out practical homework and tasks of the independent test. Independent creative work	Homework	0.2
AMI 6	The ability to carry out basic operations with vectors	15	Class	2	**Lecture**	*Theme 10.* The elements of vector algebra	Active class work	0.2
				2	**Laboratory study**	Solving practical tasks of vector algebra	Active class work	0.2
			IWS	5	**Preparation for studies**	Search, choice and look-ing through literary sources on the theme. Carrying out the practical homework and tasks of the independent test. Preparation for the written test and independent creative work	–	–

Table 11.2 (the end)

1		2	3		4		5	6
AMI 6	The ability to carry out basic operations with vectors in the linear space, use vector algebra in economic investigations	16	Class	2	**Lecture**	Theme 10. The elements of vector algebra (continuation)	Active class work	0.2
				2	**Practical study**	Solving practical tasks of vector algebra	Active class work. Written test	0.2 + 5
			IWS	5	**Preparation for studies**	Search, choice and looking through literary sources on the theme. Carrying out practical homework. Preparation for the colloquium and the presentation of the independent creative work	Homework. Independent test	0.4 + 3
		17	Class	2	**Lecture**	*Theme 10.* The elements of vector algebra (the end)	Active class work. The independent creative work. Colloquium	0.2+ 6 + + 6
				2	**Laboratory study**	Calculation of eigenvalues and eigenvectors of matrices. Decomposition of the vector into the basis. Plotting the graphs of the second order curves	Active class work. Competence oriented task	0.2 + 5
			IWS	3	**Preparation for studies**	Preparation for the defence of laboratory work	–	–
Examination period			Class	2	**Consultation for the exam**	Solving practical tasks according to the themes which are included in the final control	**Total control**	40
				2	**Exam**	Carrying out the tasks of the examination paper		
			IWS	10	**Preparation for the exam**	Review of the material of thematic modules		
Total sum of hours			150	**Total maximal number of points for the discipline**				**100**
including								
class			**68**	45 %			*current control*	**60**
independent work			**82**	55 %			*total control*	**40**

The distribution of points according to the themes of thematic modules is given in Table 11.3.

Table 11.3

The distribution of points according to the themes

<table>
<tr><td colspan="10">Current testing and independent work</td><td>Final test (exam)</td><td>Sum</td></tr>
<tr><td colspan="7">Thematic module 1</td><td colspan="3">Thematic module 2</td><td rowspan="14">40</td><td rowspan="14">100</td></tr>
<tr><td>T1</td><td>T2</td><td>T3</td><td>T4</td><td>T5</td><td>T6</td><td>T7</td><td>T8</td><td>T9</td><td>T10</td></tr>
<tr><td>0.4</td><td>0.4</td><td>0.6</td><td>1</td><td>0.4</td><td>1</td><td>0.6</td><td>1</td><td>1</td><td>1.6</td></tr>
<tr><td colspan="7">Written test</td><td colspan="3">Written test</td></tr>
<tr><td colspan="5">5</td><td colspan="2">5</td><td colspan="3">5</td></tr>
<tr><td colspan="7">Independent test</td><td colspan="3">Independent test</td></tr>
<tr><td colspan="5">3</td><td colspan="2">3</td><td colspan="3">3</td></tr>
<tr><td colspan="7">Competence-oriented task</td><td colspan="3">Competence-oriented task</td></tr>
<tr><td colspan="7">5</td><td colspan="3">5</td></tr>
<tr><td colspan="7">Colloquium</td><td colspan="3">Colloquium</td></tr>
<tr><td colspan="7">6</td><td colspan="3">6</td></tr>
<tr><td colspan="10">Independent creative work</td></tr>
<tr><td colspan="10">6</td></tr>
</table>

Note. T1, T2, ..., T10 are themes of thematic modules.

The maximum number of points which a sudent can accumulate during a week according to the forms and methods of study and control is given in Table 11.4.

The final mark on the academic discipline is defined according to the temporary provision "About the Order of Assessment of Students' Academic Performance on the Accumulative Point Rating System" of Simon Kuznets Kharkiv National University of Economics (table 11.5). Marks according to this scale are entered in the Examination Record List, the individual educational plan of a student and other academic documents.

Table 11.4

The distribution of points by weeks

Themes of the thematic module			Lectures	Practical study	Laboratory study	Homework	Competence-oriented task	Independent test	Written test	Independent creative work	Colloquium	Total
Thematic module 1 Elements of mathematical analysis	Theme 1	week 2	0.2	0.2	–	–	–	–	–	–	–	0.4
	Theme 2	week 3	0.2	–	0.2	–	–	–	–	–	–	0.4
	Theme 3	week 4	0.2	0.2	–	0.2	–	–	–	–	–	0.6
	Theme 4	week 5	0.2	–	0.2	–	–	–	–	–	–	0.4
		week 6	0.2	0.2	–	0.2	–	–	–	–	–	0.6
	Theme 5	week 7	0.2	–	0.2	–	–	3	5	–	–	8.4
	Theme 6	week 8	0.2	0.2	–	0.2	–	–	–	–	–	0.6
		week 9	0.2	–	0.2	–	5	–	–	–	6	11.4
	Theme 7	week 10	0.2	0.2	–	0.2	–	–	5	–	–	5.6
Thematic module 2 Linear algebra	Theme 8	week 11	0.2	–	0.2	–	–	–	–	–	–	0.4
		week 12	0.2	0.2	–	0.2	–	3	–	–	–	3.6
	Theme 9	week 13	0.2	–	0.2	–	–	–	–	–	–	0.4
		week 14	0.2	0.2	–	0.2	–	–	–	–	–	0.6
	Theme 10	week 15	0.2	–	0.2	–	–	–	–	–	–	0.4
		week 16	0.2	0.2	–	0.4	–	3	5	–	–	8.8
		week 17	0.2	–	0.2	–	5	–	–	6	6	17.4
Total			3.2	1.6	1.6	1.6	10	9	15	6	12	60

Table 11.5

The scales of assessment: national and ECTS

Sum of points including all forms of study	Mark on the ECTS scale	Mark on the national scale	
		for an exam, a term paper, practice	for a test
90 – 100	A	excellent	passed
82 – 89	B	very good	
74 – 81	C	good	
64 – 73	D	satisfactory	
60 – 63	E		
35 – 59	FX	unsatisfactory	failed
1 – 34	F		

12. Recommended reading

12.1. Main

1. Васильченко Г. П. Вища математика для економістів : підручник / Г. П. Васильченко. – К. : Знання-Прес, 2002. – 454 с.

2. Малярець Л. М. Математика для економістів: практичний посібник. Ч. 1 / Л. М. Малярець, Л. Д. Широкорад. – Х. : Вид. ХНЕУ, 2008. – 304 с.

3. Малярець Л. М. Математика для економістів : практичний посібник. Ч. 2 / Л. М. Малярець, Л. Д. Широкорад. – Х. : Вид. ХНЕУ, 2008. – 476 с.

4. Травкін Ю. І. Математика для економістів : підручник / Ю. І. Травкін, Л. М. Малярець. – Х. : ВД "ІНЖЕК", 2005. – 816 с.

5. Guidelines for practical tasks in analytic geometry on the academic discipline "Higher and Applied Mathematics" for foreign and English-learning full-time students of the preparatory direction "Management" / compiled by Ie. Iu. Misiura. – Kh. : Publishing House of KhNUE, 2011. – 76 p. (English, Ukrainian)

6. Higher mathematics : handbook. Vol. 1 / under the editorship of L. V. Kurpa. – Kh. : NTU "KhPI", 2006. – 344 p.

7. Higher mathematics : handbook. Vol. 2 / under the editorship of L. V. Kurpa. – Kh. : NTU "KhPI", 2006. – 540 p.

8. Higher mathematics : handbook. Vol. 3 / under the editorship of L. V. Kurpa. – Kh. : NTU "KhPI", 2006. – 364 p.

9. Higher mathematics: handbook. Vol. 4 / under the editorship of L. V. Kurpa. – Kh. : NTU "KhPI", 2006. – 328 p.

10. Methodical recommendations for the conduct of the practical studies on the academic discipline "Higher mathematics" for foreign and English-learning students of the preparatory direction "Management" of the full-time education / complied by Ie. Iu. Misiura. – Kh. : Publishing House of KhNUE, 2010. – 44 p. (English, Ukrainian)

12.2. Additional

11. Англо-русский словарь математических терминов / под ред. П. С. Александрова. – М. : Мир, 1994. – 416 с.

12. Афанасьєва Л. М. Вища математика : конспект лекцій. Ч.1 / Л. М. Афанасьєва, Г. К. Снурнікова, О. К. Шевченко. – Х. : Вид. ХНЕУ, 2005. – 68 с.

13. Бузько Я. П. Вища математика : навч. посібник / Я. П. Бузько, В. Ф. Сенчуков, В. Г. Титарєв. – Х. : РВВ ХНЕУ, 1996. – 136 с.

14. Высшая математика для экономистов : учеб. пособ. для вузов / под ред. проф. Н. Ш. Кремера. – М. : Банки и биржи, ЮНИТИ, 1997. – 440 с.

15. Диференціальні рівняння. Ряди : текст лекцій з курсу "Математика для економістів" / укл. Я. П. Бузько, В. Г. Титарєв, І. Л. Лебедєва. – Х. : РВВ ХНЕУ, 1998. – 40 с.

16. Контрольні завдання з курсу "Математика для економістів" для студентів усіх форм навчання. Ч. 1. / укл. Е. Ю. Железнякова, А. В. Ігначкова, Л. Д. Широкорад. – Х. : Вид. ХНЕУ, 2005. – 52 с.

17. Коршунова Н. Н. Математика в экономике / Н. Н. Коршунова, В. С. Плясунова – М. : Изд. "Вита-Пресс", 1996. – 368 с.

18. Красс М. С. Математика для экономических специальностей : учебник / М. С. Красс. – М. : ИНФРА-М, 1999. – 464 с.

19. Панова Н. В. Елементи математичного аналізу з курсу "Математика для економістів" : тексти лекцій / Н. В. Панова, В. Г. Титарєв. – Х. : ХДЕУ, 2003. – 80 с.

20. Тевяшев А. Д. Высшая математика. Общий курс. Сборник задач и упражнений (Математика для экономистов) / А. Д. Тевяшев, А. Г. Литвин. – Х. : ХТУРЭ, 1997. – 192 с.

21. Borakovskiy A. B. Handbook for problem solving in higher mathematics / A. B. Borakovskiy, A. I. Ropavka. – Kh. : KNMA, 2008. – 195 p.

22. Handbook of mathematics / I. N. Bronshtein, K. A. Semendyaev, G. Musiol et. al. – Berlin : Springer, 2007. – 1097 p.

12.3. Methodological support

23. Математичний аналіз та лінійна алгебра: опорний конспект [Електронний ресурс]. – Режим доступу : http://www.ikt.hneu.edu.ua /course/view.php?id=929

24. Методичні рекомендації до виконання практичних завдань з навчальної дисципліни "Математичний аналіз та лінійна алгебра"

[Електронний ресурс]. – Режим доступу : http://www.ikt.hneu.edu.ua /course/ view. php?id=929.

25. Методичні рекомендації та завдання для виконання лабораторних робіт з навчальної дисципліни "Математичний аналіз та лінійна алгебра" [Електронний ресурс]. – Режим доступу : http://www.ikt. hneu.edu.ua/course/view.php?id=929.

Appendices

Appendix A
Table A.1

The structure of components of professional competences formed on mastering the academic discipline "Mathematical Analysis and Linear Algebra" according to Ukraine's national scale of qualifications

Competence formed within the theme	Minimal experience	Knowledge	Skills and abilities	Communication	Autonomy and responsibility
1	2	3	4	5	6
Theme 1. The limit of the function and continuity					
Forming analytic thinking, the ability to explain the importance of complicated expressions with the help of mathematical symbols and operations	The basic knowledge of the theme: forms of writing and ways to define the function, the range of values, the domain of definition, periodicity, evenness and so on	The definition of basic classes of functions, basic methods and ways of investigation of the function	1. The ability to define the type of the function by its analytic recording; 2. Plotting a graph of the function with the help of elementary mathematical calculations and transformations	Construction of mathematical models of economic processes using various production functions (the total input of production, profit, supply and demand and so on)	A student must: 1) investigate the function and independently carry out the analysis of the obtained results; 2) give examples of functional dependence

Appendix A (continuation)
Table A.1 (continuation)

1	2	3	4	5	6
Theme 2. Differential calculus of the function of one variable					
Development of the ability to solve problems with the help of the methods of differential calculus using mathematical symbolic variables, i.e. forming initial skills in economic modelling	1. The knowledge of the notions: the limit of the function, the limit of a numerical sequence. 2. Attainment of basic theorems of the limit of the function. 3. The ability to calculate derivatives of elementary functions	1. The notion of the derivative, the economic, geometric and mechanical meanings of the notion, basic theorems of the theme. 2. The notion of the differential of the function of one variable. 3. The notions of the function, the derivative of the function, the definition of the points of the extremum, monotonicity intervals, concavity and convexity	1. The ability to calculate derivatives of elementary and composite functions. 2. The ability to calculate derivatives, the differential of the function of one variable. 3. The ability to investigate the function in detail with the help of the acquired knowledge of the corresponding themes. 4. The ability to predict behavior of the function and plot its graph with the help of basic mathematical calculations	Understanding economic processes and an analyzing them with the help of differential calculus methods. Representation of the results of investigation of functions	A student must: 1) calculate derivatives of elementary and composite functions; 2) be able to use the differential of the function for approximate calculus; 3) investigate the function with the help of differential calculus; 4) carry out the simplest calculations by the optimization of production; 5) draw corresponding conclusions and independently analyze the obtained solution

1	2	3	4	5	6
Theme 3. Analysis of the function of several variables					
Forming the skills in the use of previous experience (the function of one variable) for further use in a more complex situation	Mastering the notion of the function of several variables in the simplest form: the function of two variables	Analytic recording of the function of several variables, definitions of the range of values and the domain of the definition, understanding partial derivatives	Obtaining the skills in the calculation of derivatives of the function of two variables and finding the domain of the definition	Using the function of several variables in economics for the description of the investigated processes and effects	A student must: 1) find partial and mixed derivatives; 2) be able to investigate the local extremum of the function; 3) be able to use the method of Lagrange multipliers and the least-squares method
Theme 4. The indefinite integral					
1. Understanding the possibility to use the integral calculus for solving applied problems. 2. Forming the skills in the independent formation of mathematical models for the description of different processes	Attainment of the table of integrals. The ability to calculate the simplest indefinite integrals which are directly reduced to the tabular form	The definition of the type of the integral relative to its integrand, attainment of more typical changes	Obtaining the skills in the calculation of the simplest integrals and reducing more composite integrals to the tabular form	Solving economic problems using the apparatus of integral calculus	A student must: 1) calculate indefinite integrals; 2) draw corresponding conclusions and independently analyze the obtained results

Appendix A (continuation)
Table A.1 (continuation)

1	2	3	4	5	6
Theme 5. The definite integral and its application					
Forming the skills in independent analysis and understanding the importance of the relationship between the elements of the examined material (the definite and indefinite integral)	The ability to calculate the simplest definite integrals which are directly reduced to the tabular form using Newton – Leibnitz formula	Using the definite integral for calculation of areas and volumes	Obtaining skills in the calculation of definite integrals, areas of figures and volumes of solids of revolution	Solving economic problems using the apparatus of integral calculus	A student must: 1) calculate definite integrals; 2) be able to use definite integrals for independent calculation of areas and volumes of figures; 3) draw corresponding conclusions and independently analyze the obtained results
Theme 6. Differential equations					
Forming an inclination to independent search of different ways of solving problems and understanding the necessity to use knowledge of other themes (the function, the derivative, the integral)	1. The ability to differentiate and integrate functions, find the derivative of a composite function. 2. The ability to solve the simplest differential equations of the first and the second orders	1. Basic ways to solve a differential equation of the first order. 2. Basic ways to solve a differential equation of the second order with constant coefficients	1. Skills in the calculation of the basic types of differential equations of the first order. 2. Skills in the calculation of the basic types of differential equations of the second order	Solving economic problems with differential equations as mathematical models, i.e. finding antiderivatives, the elasticity of the function and so on	A student must: 1) be able to calculate the type of the differential equation, the method of further independent solution; 2) be able to use the knowledge for solving the simplest economic problems

Appendix A (continuation)
Table A.1 (continuation)

1	2	3	4	5	6
Theme 7. Series					
Forming the ability to do analytic calculations	The ability to calculate limits using basic signs of convergence of series	Basic signs of convergence of series, the notion of a power series, alternating series	Skills in analytic calculations using the knowledge of the theme	Investigation of the convergence of numerical series, understanding the problem of the convergence of series and using series in approximate calculations	A student must: 1) calculate the type of series; 2) be able to independently investigate the convergence of series; 3) find the convergence radius of power series
Theme 8. The elements of the theory of matrices and determinants					
1. Forming the skills in the use of the instrument of matrix calculus for modelling the simplest economic problems and situations. 2. The ability to analyze the results of calculations from the mathematical and practical viewpoint	1. Carrying out the simplest mathematical calculations with the determinants of the second, third and n-th orders. 2. Carrying out the simplest mathematical calculations (addition, subtraction, multiplication)	1. Attainment of mathematical symbols, basic definitions and theorems of the theme. 2. Attainment of basic properties of determinants and basic notions of the theme	1. The ability to calculate the simplest determinants of matrices of the second and third orders with the help of basic methods and properties. 2. The ability to use the basic matrix operation	Using the available methods of work with numerical data, representing them in the matrix form and carrying out operations with them	A student must: 1) be able to use the instrument of matrix algebra for economic problems; 2) be able to model the simplest situations with the help of the knowledge of the theme

Appendix A (the end)
Table A.1 (the end)

1	2	3	4	5	6
Theme 9. The general theory of the system of linear algebraic equations					
Forming the ability to independently prove the simplest statements with the help of elementary mathematical knowledge	Using and attainment of the basic methods of solving the simplest systems of linear equations	Attainment of basic theorems and rules of solving systems of linear equations (Cramer method, the inverse matrix method, Jordan – Gauss method)	Solving matrix equations. Solving the systems with the help of matrices and determinants	Using the methods of solving the systems of linear equations with matrices of an arbitrary dimension	A student must know the basic proofs and theorems of the theme and give examples of using determinants, matrices and systems of linear equations in economics
Theme 10. The elements of vector algebra					
Forming analytic thinking, the ability to explain the importance of complicated expressions with the help of mathematical symbols and operations	The Cartesian system of coordinates on the plane and in the space. The notion of the vector, elementary operations with vectors	Basic operations and properties of vectors; scalar, cross and mixed products	The ability to use basic linear operations with vectors, the independence and dependence of vectors	Making a geometric presentation of economic problems	A student must be able to use vector algebra for the calculation of the simplest problems of applied character (finding the area, the volume)

Contents

НАВЧАЛЬНЕ ВИДАННЯ

Робоча програма навчальної дисципліни "МАТЕМАТИЧНИЙ АНАЛІЗ ТА ЛІНІЙНА АЛГЕБРА"

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