

MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE
SIMON KUZNETS KHARKIV NATIONAL UNIVERSITY OF ECONOMICS

HIGHER MATHEMATICS

Syllabus

**for Bachelor's (first) degree students
of speciality 056 "International Economic Relations"**

Kharkiv
S. Kuznets KhNUE
2017

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H 65

Затверджено на засіданні кафедри вищої математики й економіко-математичних методів.

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The thematic plan of the academic discipline and its content are given according to the modules and themes. Plans of lectures and practical trainings, material for students' knowledge consolidation (test questions, tasks for independent studies) as well as methods for students' knowledge assessment according to the credit transfer system of studies are presented.

For Bachelor's (first) degree students of speciality 056 "International Economic Relations".

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Introduction

The fundamental base in the mathematical training of economists and managers is the academic discipline "Higher Mathematics" which is a normative discipline of the natural science series and the component of the structural logical scheme which is provided for Bachelor's educational professional programme of speciality 056 "International Economic Relations" of all forms of study.

The basic problems of teaching the academic discipline are giving students knowledge of the basic parts of mathematical analysis, linear algebra, probability theory and mathematical statistics; raising the level of the fundamental mathematical training of students with intensification of its applied direction; mastering the fundamentals of mathematical analysis, linear algebra, probability theory and mathematical statistics and application of this knowledge to the economic investigations for solving economic problems; forming skills in the application of the elements of mathematical analysis, linear algebra, probability theory and mathematical statistics in investigations where higher mathematics is applied as an instrument of investigation, and solving optimization economic problems for modelling economic processes and developments; acquiring the necessary theoretical and practical knowledge for solving specific problems which are set in the process of forming economic and mathematical models, and obtaining the required mathematical knowledge for the study of other disciplines.

1. Description of the academic discipline

| Name of indicators | Subject area; speciality; academic degree | Academic discipline features | |
|--|---|------------------------------|-----------------------|
| | | full-time form of study | distant form of study |
| Number of credits: 5 for the full-time form; 6 for the distant form | specialty 056 "International Economic Relations" | Compulsory | |
| Number of thematic modules: 2 | | Academic year | |
| | | 1st | 1st |
| Total number of hours: 150 for the full-time form; 180 for the distant form | | Term | |
| | | 1st | 1st |
| | | Lectures | |
| | | 32 hours | 16 hours |
| The number of hours per week for the full-time form of study: in class: 4; student's independent work: 5 | | Practical studies | |
| | | 24 hours | 12 hours |
| | | Laboratory studies | |
| | 18 hours | – | |
| | Independent work | | |
| | 84 hours | 148 hours | |
| | Examination consultation | | |
| | – | 2 hours | |
| | Form of control: | | |
| Test | Exam | | |
| 2 hours | 2 hours | | |

Note. The ratio of the number of class hours to independent work is:
88 % for the full-time form of study;
19 % for the distant form of study.

2. The main purpose and tasks of the academic discipline

The main purpose of teaching is to form future specialists' basic mathematical knowledge for solving theoretical and practical problems in professional activity of a competent specialist in the sphere of international economic relations, skills in analytical thinking and skills in using mathematical knowledge for formation of real processes and developments, and for solving economic problems.

The main tasks that should be carried out in the process of teaching the discipline are:

giving students knowledge of the basic parts of mathematical analysis, linear algebra, probability theory and mathematical statistics;

learning definitions, theorems, rules;

proving the main theorems;

mastering the fundamentals of the methodology of mathematical investigation of the applied economic problems;

independent broadening of knowledge, development of logical and algorithmical thinking;

obtaining primary skills in independent learning of mathematical and applied library sources by students.

The subject of the academic discipline "Higher Mathematics" is the fundamentals of mathematical analysis, linear and vector algebra.

In the process of learning the academic discipline "Higher Mathematics" a student receives analytic and investigatory competences which are necessary for a modern economist in any sphere of his activity.

The syllabus of the academic discipline "Higher Mathematics" is compiled according to the statements of the field standard of the higher education of the Ministry of Education and Science of Ukraine based on the Bachelor's educational professional program of training, which is made by the Scientific Methodical Committee of Economics and Enterprise of the Ministry of Education and Science of Ukraine.

Students start studying the academic discipline "Higher Mathematics" in the first term of the first year of studies.

In the process of learning students obtain the required theoretical knowledge during lectures and acquire practical skills at the practical and laboratory studies and during independent work and fulfillment of individual

tasks. Independent and individual work of students has a great value in the process of mastering material and consolidation of knowledge. All of these types of studies were devised according to the statements of the Bolognese Declaration.

As a result of studying the academic discipline a student **must know**:

- the elements of the limits theory: the limit of a sequence and the limit of a function; the limit of a function in a point, their equivalence; equivalent functions, their applications to finding the limit of a ratio of functions;
- the first and second noteworthy limits, the table of basic limits, finding the limits of power-exponential functions;
- the bases of limiting (marginal) analysis;
- the differential calculus: a function of one variable and several variables; ways to define a function and its illustration;
- some special classes of functions; monotone, even and odd, convex and concave, bounded and unbounded functions;
- a continuity of a function at the same point;
- one-sided continuity of a function of one variable at the same point, necessary and sufficient conditions of continuity;
- classification of points of discontinuity;
- a differentiable function, its differential;
- the derivative of a function of one variable, partial derivatives, a gradient of a function of several variables;
- the derivative of a function of several variables in the direction, its relationship with a gradient; the elasticity of a function;
- higher-order derivatives and differentials, higher-order derivatives of some elementary functions;
- investigation of functions with the help of the differential calculus;
- the notion of a differential of a function and its application to approximate calculation; the notion of an elasticity of a function;
- the integral calculus: the notion of an antiderivative, indefinite and definite integrals; methods of integration;
- the Newton – Leibnitz formula; the notion of an improper integrals;
- the elements of economic dynamics; the first-order ordinary differential equation, the Cauchy problem;
- the particular and general solutions; types of differential equations;
- the higher-order differential equations and systems of differential equations;

solution of the second-order linear differential equations with constant coefficients;

numerical series, necessary and sufficient conditions of a convergence of numerical series with positive terms and alternating numerical series; absolute and conditional convergence;

power series, the convergence radius and the interval of power series; functional series, trigonometric Fourier series;

the bases of linear algebra: matrices and determinants, (facilities, possibilities), application to making a mathematical model of economic problems;

methods of solving the system with n linear algebraic equations with m unknowns;

the conditions of compatibility of the system of linear algebraic equations;

the notion of the basic solution;

the bases of vector algebra: the basis of space, linear dependence and linear independence of vectors;

the notions of subspace, the linear vector space, a rank of finite systems of vectors, rules of calculation;

the general notions of probability theory;

random persistent and impossible events; the notion of probability and methods of its definition;

dependent and independent events and basic formulas of addition and multiplication of probabilities for these events, formulas of total probability and Bayes;

a trial by Bernoulli's scheme;

discrete and continuous random variables;

basic laws of the distribution of discrete and continuous random variables and their basic numerical characteristics;

conditional laws of the distribution of probabilities of components of a discrete two-dimensional random variable;

the definition of basic numerical characteristics of two-dimensional random variable;

characteristics of the function of one random argument; limiting theorems of probability theory;

the general notions of mathematical statistics;

the sampling method;

statistical distribution and its basic numerical characteristics;
 statistical estimations of parameters of a population; requirements for statistical estimations;
 the notions of point and interval estimations and definition of their accuracy;
 the methods of parametric and nonparametric estimations of parameters; statistical hypotheses and statistical criteria for checking them;
 investigation of the form of correlation;
 construction of a model of pair regression using the least-squares method;
 the methods for checking parameter significance of a model of pair regression;
 estimation of the adequacy of a model in whole;
be able to:
 learn mathematical literature by oneself;
 calculate the mean values;
 carry out the operations with vectors, matrices, calculation of determinants;
 solve the systems of linear equations;
 investigate the forms and properties of straight lines and planes, second-order curves and quadratic surfaces;
 classify the functions, numerical sequences;
 find the limit of power-exponential functions;
 investigate the function with the help of differential calculus;
 carry out the integral calculus;
 carry out calculation of numerical and power series;
 solve first-order and higher-order differential equations, systems of differential equations;
 form and use economic mathematical models;
 broaden the knowledge, develop logical and algorithmic thinking by oneself.

A modern tendency in higher education is a reorientation of students of higher educational institutions from the process of education to a result, from knowledge to skills, forming definite competences.

In the process of learning the academic discipline "Higher Mathematics" a student receives analytic and investigatory **competences** which are required for a modern economist in any sphere of his activity (Table 2.1).

**Competences which are formed as a result of mastering
the academic discipline "Higher Mathematics"**

| The code of competence | The name of the competence | The components of the competence |
|------------------------|--|---|
| 1 | 2 | 3 |
| AMI*1 | Forming analytic thinking, the ability to explain the importance of complicated expressions with the help of mathematical symbols and operations. Development of the abilities to solve problems with the help of calculation of limits and methods of differential calculus using mathematical symbolic variables, i.e. forming the initial skills in economic modelling | A student must 1) be able to define the type of a function by its analytic recording; 2) calculate derivatives of elementary and composite functions and use the differential of a function for approximate calculus; 5) investigate a function with the help of differential calculus; 6) carry out the simplest calculations by an optimization of production; 7) draw corresponding conclusions and independently analyze the obtained solution; 8) find partial and mixed derivatives of the function of several variables, 9) be able to investigate the local extremum of a function |
| AMI 2 | Understanding a possibility of using the integral calculus for solving applied problems. Forming skills in independent formation of mathematical models for a description of different processes. Forming the skill in independent work. Analysis and understanding of the importance of a relationship between the definite and indefinite integral | A student must 1) calculate definite and indefinite integrals; 2) be able to use definite integrals for independent calculation of areas and volumes of figures; 3) draw corresponding conclusions and independently analyze the obtained results |
| AMI 3 | Forming inclinations to independent search of different ways of solving problems and understanding the necessity to use knowledge of other themes (a function, a derivative, an integral) | A student must 1) be able to calculate the type of a differential equation, the method of further solving it independently; 2) be able to use the knowledge for solving the simplest economic problems |

Table 2.1 (continuation)

| 1 | 2 | 3 |
|-------|---|--|
| AMI 4 | Forming the ability to do analytic calculations | A student must 1) calculate the type of series; 2) be able to investigate the convergence of series independently; 3) find the convergence radius of power series |
| AMI 5 | Forming the ability to prove independently the simplest statements with the help of elementary mathematical knowledge. Forming skills in the use of the instrument of the matrix calculus for modelling the simplest economical problems and situations. The ability to analyze the results of calculations | A student must 1) know the basic proofs and theorems of the theme; 2) give examples of using determinants, matrices and systems of linear equations in economics; 3) be able to use the instrument of matrix algebra for economic problems; 4) be able to model the simplest situations with the help of knowledge of the theme |
| AMI 6 | Forming analytic thinking, the ability to explain the importance of complicated expressions with the help of mathematical symbols and operations | A student must be able to use vector algebra for calculation of the simplest problems of applied character (finding the area, the volume) |
| AMI 7 | Using methods of probability theory for a prognosis of a probabilistic random events and making graphic interpretation of solutions of economic problems with the help of instruments of probability theory | A student must 1) use basic definitions and theorems for calculation of the probability of a random event; 2) define laws of distribution of discrete and continuous (one-dimensional) random variables, calculate their basic numerical characteristics, plot distribution functions; 3) find numerical characteristics of a function of a discrete and continuous random argument; 4) use the concept of the theory of random processes and the theory of queueing, the theory of modelling economic processes |
| AMI 8 | Identification of quantitative characteristics of economic processes with the help of a sampling method | A student must 1) understand the relationship between the instruments of the probability theory and mathematical statistics, form a representative sampling totality, plot a variational series and estimate basic numerical characteristics of a random variable using the results of investigation of a sample |

Table 2.1 (the end)

| 1 | 2 | 3 |
|-------|---|--|
| | | 2) check the statistical hypothesis of correspondence of properties of numerical characteristics and the distribution law of a random variable in a population and their estimations using the results of investigation of a sample; 3) understand the possibilities and restriction of using the instruments of mathematical statistics when solving real economic problems |
| AMI 9 | Using variance analysis for investigation of economic processes, using correlation and regression analysis when learning different economic phenomena, understanding the meaning of economic values which form the model of pair regression | A student must 1) understand the possibilities of using the single-factor analysis of variance when checking the existence of the difference between the investigated samples; 2) distinguish the types of dependences between economic factors and define the essence of correlation; 3) investigate the form of correlation and construct a model of pair regression using the least-squares method; 4) know the methods of checking the parameter significance of a model of pair regression and estimation of the adequacy of a model in whole |

*Note. Application of mathematical instruments (AMI)

The structure of professional competences to be formed according to the National Scale of Qualifications of Ukraine is given in Appendix A.

3. The syllabus of the academic discipline

Thematic module 1

The elements of mathematical analysis and linear algebra

Theme 1. Limits of functions and continuity

1.1. Sets, functions, their classification.

Basic notions. Numerical sets. Operations with sets. Numerical intervals, the neighborhood of a point. The notion of the function of one variable. Ways to define the function. The domain of the definition and the range of values of a function. A graph of a function. Basic elementary functions, their properties and graphs. Classification of elementary functions.

1.2. Numerical sequences and their limits.

The notion of a numerical sequence. Ways to define sequences. Arithmetic operations with sequences. The limit of a sequence, its geometrical meaning. Infinitesimals and infinitely large sequences, their properties. Basic theorems for limits of sequences.

1.3. Limits of functions.

The definition of the limit of a function at a point, its geometrical meaning. Infinite limits and limits if a limiting value approaches infinity. One-sided limits at a point. Basic theorems for limits of functions. The notions of indeterminate forms, their types. Methods of elimination of indeterminations. The first and the second remarkable limits, their consequences. Equivalent infinitesimals. Calculation of limits with the help of comparison of infinitesimals. Application of limits to solving economic problems.

1.4. The continuity of functions.

The definition of the function continuity at a point and on an interval. One-sided continuity. Continuity of elementary functions. Break points of functions and their classification. Basic properties of continuous functions.

Theme 2. The differential calculus of the function of one variable

2.1. A derivative and a differential. Techniques of differentiation.

The definition of a derivative, its economic and geometric meanings. The notion of differentiation of a function at a point. The relationship between the differentiation and continuity of a function. A table of derivatives of basic elementary functions.

Basic rules of differentiation. The theorem of a derivative of a composite function. Calculation of a derivative of a parametric function. Differentiation of implicit functions. The notion of a differential of a function, its geometric meaning and properties.

L'Hospital's rule for calculation of limits of functions.

2.2. Application of derivatives to the investigation of functions.

The condition of monotony of a function, finding local extremums of a function. The greatest and the least values of a function on a segment. Conditions of convexity and concavity of a graph of a function, finding inflection points. Vertical, horizontal and inclined asymptotes of a curve. A general scheme of investigation of a one-variable function and a plot of its graph.

2.3. Application of a derivative to economics.

Marginal analysis. Elasticity of economic indicators.

Theme 3. Analysis of the function of several variables

3.1. Basic notions.

The definition of a function of several variables. The domain of the definition of the function of two variables and its graph. Lines and surfaces of a level. The continuity and the limit of the function of two variables.

3.2. Partial derivatives. A gradient and a directional derivative.

Partial derivatives of a function, its geometrical and economic meaning. Partial derivatives and differentials of higher orders. The derivative of the function of several variables. The gradient of the function and its properties. The relationship between the gradient and level lines for the function of two variables.

3.3. The extremum of the function of two variables.

Basic notions. A local extremum of the function of two variables, the necessary and the sufficient conditions of an extremum. The greatest and the least values of a function in a closed domain.

A conditional extremum of the function of two variables. Reducing the problem of a conditional extremum of the function of two variables to the problem of the local extremum of the function of one variable. The method of Lagrange multipliers.

3.4. Application of the function of several variables to economics.

Application of the function of several variables to economic models: production functions, the function of Cobb and Douglas, the expenditure function, the demand function. Elasticity of the function of several variables.

Theme 4. The indefinite and definite integral

4.1. An antiderivative and an indefinite integral.

The notions of an antiderivative of a function and an indefinite integral. Properties of an indefinite integral. The table of basic integrals. The concept of integrals, which are not taken.

4.2. Basic methods of integration.

The method of direct integration. The method of a change of a variable (a substitution) in an indefinite integral. The formula of integration by parts, basic cases of using it.

4.3. The notion and properties of a definite integral.

The definition of a definite integral, its geometric meaning. The conditions of integrability of a function. The properties of a definite integral and application of the properties to calculation. The theorem of the mean value.

4.4. Calculation of a definite integral.

The Newton – Leibnitz formula. Change of a variable (substitution) in a definite integral. The formula of integration by parts for a definite integral.

4.5. Improper integrals of the first and the second kinds.

The notion of improper integrals with infinite limits of integration and improper integrals of unbounded functions. Conditions of convergence of improper integrals.

Theme 5. Differential equations

5.1. The basic notions of the theory of differential equations. Solving the first-order differential equations.

The general notions of the theory of ordinary differential equations. The first-order differential equations: basic definitions, the notions of general and particular solutions. The Cauchy problem, the theorem of existence and uniqueness of solution to it. Differential equations with separable variables, homogeneous equations of the first order, linear differential equations of the first order, Bernoulli's equation.

5.2. Differential equations of higher orders. Methods of solving the second-order differential equation.

The basic notions and definitions, general and particular solutions, the Cauchy problem. Differential equations of the second order. Equations, which reduce the order, methods of integration. Linear differential equations of the second order, the structure of a general solution. Homogeneous linear differential equations of the second order with constant coefficients.

Theme 6. Series

6.1. Numerical series and their convergence.

The definition of series, its sums. Numerical series and its convergence. The properties of convergent series. The necessary condition of convergence. The harmonic series, its divergence. The generalized harmonic series. Sufficient criteria of a convergence of series with positive terms: comparison criterion, D'Alembert criterion, Cauchy's radical criterion and Maclaurin – Cauchy integral criterion.

6.2. Alternating series and their convergence.

The notion of alternating series. Alternating numerical series. The sufficient sign of convergence. Absolute and conditional convergences. The Leibnitz criterion. Application of the Leibnitz theorem to finding an error of calculation of a sum of series.

6.3. *Functional series.*

The notion of functional series. The radius and the convergence domain of power series, formulas of calculation.

Theme 7. The elements of the theory of matrices and systems of linear algebraic equations

7.1. *Matrices.*

The definition of a matrix, its types. Operations with matrices: addition, multiplication of a matrix by a scalar, by a matrix. Transposition of a matrix. Equivalent transformations of matrices. Examples of using matrices.

7.2. *Determinants.*

The definition of the determinant, the rules of calculation of determinants of lower orders (schematic), higher orders (expansion by Laplace formulas). Properties of determinants. Calculation of some special determinants (triangular, diagonal, identity matrices).

7.3. *The inverse matrix.*

The notion of an inverse matrix, the properties of operation of a matrix inversion. Calculation of an inverse matrix by definition (as a transposed matrix of algebraic cofactors) and with the help of equivalent transformations of an adjoined unit matrix. The definition of a rank of a matrix and methods of definition.

7.4. *Systems of linear algebraic equations.*

The definition of the system of linear algebraic equations, the augmented and matrix forms of entry. Definitions of a solution to linear algebraic equations. The notion of consistent or inconsistent system of linear algebraic equations. Determined or undetermined systems of linear algebraic equations.

7.5. *Methods of solving systems of linear algebraic equations.*

Solving the system of linear algebraic equations with the help of an inverse matrix and Cramer's formulas. Kronecker – Capelli theorem. Investigation of compatibility of the system of linear algebraic equations.

General, particular and support solutions to the system of n algebraic equations with m unknowns. A fundamental system of solutions.

Solving the system of linear algebraic equations with the help of the method of sequential elimination of unknowns (the Gauss method). Application of the method to complete elimination of unknowns (the Gauss – Jordan method) for solving the system of linear algebraic equations, its realization with the help of tables.

7.6. Homogeneous systems of linear algebraic equations.

The notion of a homogeneous system of linear algebraic equations. The space of solutions to a homogeneous system, the relationship of its dimension and the matrix rank. A fundamental system of solutions to a homogeneous system of linear algebraic equations. Economic problems.

Theme 8. The elements of vector algebra

8.1. The basic notions of vector algebra.

Types of vectors, comparison of vectors. Linear operations with vectors in the geometric and coordinate forms, properties of these operations. A scalar product of vectors, its properties. An angle between vectors. Collinear vectors, the condition of collinearity. Vector (cross) and mixed products of vectors and their geometric meaning. Properties of vector and mixed products of vectors. The condition of coplanarity of vectors.

8.2. The elements of the theory of linear spaces.

The definition of the n th-dimensional vector and the n th-dimensional vector (linear) space. Linear independence of vectors. The definitions and main theorems of linear dependence and linear independence of linear space elements. The basis of linear space. Coordinates of a vector in a given basis. Transformation to other basis. Economic examples.

8.3. Eigenvectors.

Eigenvalues and eigenvectors of a matrix. A characteristic equation. Methods of finding eigenvalues and eigenvectors for matrices of the second and third orders. Economic examples.

8.4. Quadratic forms.

The notion of a quadratic form. The matrix of a quadratic form. Reducing quadratic forms to a canonical form. The curves of the second-order on a plane. A general equation of the second-order curve. Reducing the second-order curve to a canonical form.

Thematic module 2

The elements of probability theory and mathematical statistics

Theme 9. Empirical and logical bases of probability theory

9.1. The subject and problems of the discipline.

The role of the discipline as a theoretical base of mathematical modelling of economic processes and phenomena, which include possible risks.

9.2. A probabilistic model of an experiment.

Sure (certain), random and impossible events. Rules of operations with random events. The space of elementary events. The classical definition and calculation of probability. Basic formulas of combinatorics. The statistical definition of probability. The geometrical definition of probability.

9.3. Basic theorems of probability theory, their economic meaning.

Probabilistic space. Addition theorems of probabilities. Dependent and independent events. Conditional probability. Joint (compatible) and disjoint (incompatible) events. Multiplication theorems of probabilities. A complete group of events. Complementary events. The probability of at least one event. The probability that an event will occur at least once. The formula of total probability. Bayes' formula (the theorem of hypothesis).

Theme 10. The scheme of independent trials

10.1. Bernoulli's formulas.

The scheme of repeated independent trials.

10.2. The local theorem of Moivre – Laplace.

Gauss function, its properties, application to approximate calculations of the probability of occurrence of a random event a definite number times in the series of independent trials.

10.3. The integral theorem of Moivre – Laplace. Poisson's theorem.

Laplace's function, its properties and application to approximate calculations of the probability that values of a random variable lie in a definite interval. Poisson's theorem.

Theme 11. Random variables and their economic meaning

11.1. The definition of a random variable.

Discrete and continuous random variables. Distribution laws of probabilities for a random variable and ways of finding them.

11.2. Basic numerical characteristics of a random variable.

Mathematical expectation, variance and root-mean-square deviation. Properties of basic numerical characteristics. The function of distribution of probabilities, its properties. Distribution density and its probable explanation. A density function of the distribution of a random variable and its properties.

11.3. Additional numerical characteristics of distribution.

A mode, a median, an excess, initial and central theoretical moments of an arbitrary order.

11.4. Distribution laws of a discrete random variable.

Binomial distribution, geometrical distribution, hypergeometrical distribution. Specificities and properties of these distributions, their basic numerical characteristics and the economic meaning.

Theme 12. Basic distribution laws of a continuous random variable

12.1. Distribution laws of a continuous random variable.

Uniform distribution, normal distribution and exponential distribution. Properties of these distributions and their basic numerical characteristics. The influence of parameters of distribution on the density function of probabilities in the normal distribution law.

12.2. Student's distribution, Pearson's distribution and Fisher's distribution.

Specificities and properties of these distributions. The relationship of these distributions and the normal distribution law of a continuous random variable.

Theme 13. Preprocessing of statistical data

13.1. Basic problems of mathematical statistics.

The sampling method. Definitions of a population and its sample.

13.2. The empirical distribution law.

Ways of presentation of sampling totalities and representation of the results of observations. Discrete and interval variational series. A polygon and a histogram. Basic sampling characteristics and their asymptotic behavior.

Theme 14. Statistical estimation of the distribution parameters

14.1. Statistical estimations of distribution parameters of a population and their properties.

Unbiasedness, possibility and efficiency. Point estimations.

14.2. Interval estimations.

The confidence interval for mathematical expectation and the root mean square deviation of a normal population.

Theme 15. Checking the statistical hypothesis

15.1. Basic notions of checking the statistical hypothesis.

Main and alternative statistical hypothesis. A statistical criterion (test). Construction of critical domains for a statistical criterion (test). Errors of the first and second kinds. The concept of power of a criterion (test).

15.2. Checking the statistical hypothesis about defining the distribution law for a population using the results of investigation of a sample.

The Pearson fitting test. The fitting test relative to frequencies.

15.3. Checking the statistical hypothesis about the equality of two population means on the assumption of a normal distribution law.

Student's fitting test.

15.3. Comparison of variances.

The Fisher – Snedeker fitting test.

Theme 16. The elements of the theory of correlation and regression

16.1. Problems of correlation analysis.

The sampling coefficient of a correlation, its properties and the confidence interval. A coefficient of determination. The correlation ratio, its properties.

16.2. Problems of regression analysis.

The correlation dependence. The correlation table. Empirical lines of a regression. Estimation of parameters of a pair regression equation using the least-squares method. The accuracy of the estimation.

Checking the significance of parameters of a pair regression equation. The confidence interval for a line of a pair regression.

4. The structure of the academic discipline

From the very beginning of studying the academic discipline each student has an opportunity to learn both the discipline syllabus and forms of organization of education, as well as the structure, contents and volume of each of its educational modules, and all types of control and methods of the educational work assessment.

The educational process according to the syllabus of the academic discipline "Higher Mathematics" is realized in such forms as: lectures, practical and laboratory studies; fulfillment of students' independent work; control activities.

A student's mastering of the academic discipline is carried out with the help of consecutive and thorough learning of the educational modules. An educational module is a relatively separate block of the given discipline, which logically unites its educational elements by the content and interconnections. The assessments of knowledge and skills obtained by a student while learning the material of each module are effected in the final module control.

The thematic plan of the academic discipline consists of two thematic modules (Table 4.1).

Table 4.1

The structure of the test credit of the academic discipline

| Names of thematic modules and themes | The number of hours | | | | | | | | | | | |
|--|-----------------------------|-------------------------|-----------|------------|---------------|---|---------------------------|-------------------------|-----------|------------|---------------|---|
| | the full-time form of study | | | | | | the distant form of study | | | | | |
| | total | which are allocated for | | | | | total | which are allocated for | | | | |
| | | lecture | practical | laboratory | final control | independent work: preparation for studies | | lecture | practical | laboratory | final control | independent work: preparation for studies |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| Thematic module 1 | | | | | | | | | | | | |
| The elements of mathematical analysis and linear algebra | | | | | | | | | | | | |
| <i>Theme 1. Limits of functions and continuity</i> | 8 | 2 | 2 | – | – | 4 | 11 | 1 | 1 | – | – | 9 |
| <i>Theme 2. The differential calculus of the function of one variables</i> | 8 | 2 | 2 | – | – | 4 | 21 | 1 | 1 | – | – | 9 |
| <i>Theme 3. Analysis of the function of several variables</i> | 8 | 2 | 2 | – | – | 4 | | 1 | | – | – | 9 |
| <i>Theme 4. The indefinite and definite integral</i> | 8 | 2 | 2 | – | – | 4 | 11 | 1 | 1 | – | – | 9 |
| <i>Theme 5. Differential equations</i> | 8 | 2 | 2 | – | – | 4 | 20 | 1 | 1 | – | – | 9 |
| <i>Theme 6. Series</i> | 10 | 2 | 2 | – | – | 6 | | 1 | | – | – | 8 |
| <i>Theme 7. The elements of the theory of matrices and systems of linear algebraic equations</i> | 10 | 2 | 2 | – | – | 6 | 11 | 1 | 1 | – | – | 9 |
| <i>Theme 8. The elements of vector algebra</i> | 10 | 2 | 2 | 2 | – | 4 | 11 | 1 | 1 | – | – | 9 |
| Total for module 1 | 70 | 16 | 16 | 2 | – | 36 | 85 | 8 | 6 | – | – | 71 |

Table 4.1 (the end)

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
|---|------------|-----------|-----------|-----------|----------|-----------|------------|-----------|-----------|----------|----------|------------|
| Thematic module 2 | | | | | | | | | | | | |
| The elements of probability theory and mathematical statistics | | | | | | | | | | | | |
| <i>Theme 9.</i> Empirical and logical bases of probability theory | 10 | 2 | 2 | 2 | – | 4 | 19 | 1 | 1 | – | – | 8 |
| <i>Theme 10.</i> The scheme of independent trials | 8 | 2 | – | 2 | – | 4 | | 1 | | – | – | 8 |
| <i>Theme 11.</i> Random variables and their economic meaning | 10 | 2 | 2 | 2 | – | 4 | 11 | 1 | 1 | – | – | 9 |
| <i>Theme 12.</i> Basic distribution laws of a continuous random variable | 8 | 2 | – | 2 | – | 4 | 11 | 1 | 1 | – | – | 9 |
| <i>Theme 13.</i> Preprocessing of statistical data | 12 | 2 | 2 | 2 | – | 6 | | 1 | | – | – | 8 |
| <i>Theme 14.</i> Statistical estimation of the distribution parameters | 10 | 2 | – | 2 | – | 6 | 20 | 1 | 1 | – | – | 9 |
| <i>Theme 15.</i> Checking the statistical hypothesis | 12 | 2 | 2 | 2 | – | 6 | 10 | 1 | 1 | – | – | 8 |
| <i>Theme 16.</i> The elements of the theory of correlation and regression | 10 | 2 | | 2 | – | 6 | 10 | 1 | 1 | – | – | 8 |
| Total for module 2 | 80 | 16 | 8 | 16 | – | 40 | 81 | 8 | 6 | – | – | 67 |
| <i>Preparation for the exam</i> | – | – | – | – | – | – | 10 | – | – | – | – | 10 |
| <i>Consultations for the exam</i> | – | – | – | – | – | – | 2 | – | – | – | 2 | – |
| <i>Exam</i> | – | – | – | – | – | – | 2 | – | – | – | 2 | – |
| Total number of hours | 150 | 32 | 24 | 18 | – | 76 | 180 | 16 | 12 | – | 4 | 148 |

5. The plan of the practical studies

5.1. The themes of the practical studies

A **practical study** is a form of educational studies at which the lecturer organizes a detailed consideration of separate theoretical statements of the academic discipline and forms the abilities and skills in their practical application through students' individual accomplishment of the formulated tasks.

Conducting a practical study is based on the previously prepared material, i.e. tests designed to assess the mastery of the required theoretical statements, tasks of different complexity to be solved by students.

A practical study includes control of students' knowledge, abilities and skills, formulation of a general problem by the lecturer and discussing it with the students, solving control tasks, reviewing them, assessment.

The plan of the practical studies, their content and a bibliography for each theme are given in Table 5.1.

Table 5.1

The plan of the practical studies

| The name of the thematic module | The themes of the practical studies (according to the modules) | The number of hours | Recommended reading |
|---|--|---------------------|---|
| 1 | 2 | 3 | 4 |
| Thematic module 1 The elements of mathematical analysis and linear algebra | <i>Theme 1. Calculation of limits of functions and investigation of the continuity of functions.</i> 1. Limits of functions and their properties. 2. Types of indeterminations and methods for eliminating them. 3. The first remarkable limit. The table of equivalent infinitesimals. 4. Investigation of the continuity of a function | 2 | Main: [1; 4 – 8]. Additional: [14; 16; 17; 20; 21; 25; 27; 29] |
| | <i>Theme 2. The differential calculus and application of the function of one variable.</i> 1. Techniques of differentiation. 2. The differential and application of a function of one variable. 3. Application of a derivative to the investigation of a function | 2 | Main: [1; 4 – 8]. Additional: [14; 16; 17; 20; 21; 25; 27; 29] |
| | <i>Theme 3. The function of several variables.</i> 1. Calculation of partial derivatives of the function of several variables. 2. The differential of the function of several variables. 3. A gradient and a directional derivative. Level lines. 4. Investigation of the extremum of the function of several variables | 2 | |

Table 5.1 (continuation)

| 1 | 2 | 3 | 4 |
|---|---|---|---|
| | <p><i>Theme 4. Calculation of indefinite and definite integrals.</i></p> <ol style="list-style-type: none"> 1. Direct integration. The method of change of a variable (substitution) for calculation of indefinite integrals. 2. The formula of integration by parts. 3. Calculation of definite integrals with the help of the Newton – Leibnitz formula. 4. Methods of the change of a variable and integration by parts for definite integrals | 2 | <p>Main: [1; 4 – 8]. Additional: [14; 16; 17; 20; 21; 25; 27; 29]</p> |
| | <p><i>Theme 5. Solving differential equations.</i></p> <ol style="list-style-type: none"> 1. Differential equations of the 1st order. 2. Finding a general and a particular solution to a linear differential equation of the second order with constant coefficients | 2 | |
| | <p><i>Theme 6. Series.</i></p> <ol style="list-style-type: none"> 1. Investigation of convergence of series with positive terms. 2. Investigation of convergence of alternating series. Absolute and conditional convergences. 3. Power series and its convergence | 2 | |
| | <p><i>Theme 7. The elements of the theory of matrices and systems of linear algebraic equations.</i></p> <ol style="list-style-type: none"> 1. Fulfilment of operations with matrices. 2. Calculation of determinants. 3. Investigation of compatibility and determinacy of systems of linear algebraic equations. 4. Solving systems of linear algebraic equations with the help of an inverse matrix, Cramer's formulas and the Jordan – Gauss method | 2 | <p>Main: [1; 4 – 8]. Additional: [14; 16; 17; 20; 21; 25; 27; 29]</p> |
| | <p><i>Theme 8. The elements of vector algebra.</i></p> <ol style="list-style-type: none"> 1. A scalar product of vectors. Verifying collinearity of vectors. A vector (cross) and a mixed product of vectors, their properties and geometrical meaning. 2. Verifying the linear independence of vectors. 3. Finding eigenvalues and eigenvectors of matrices of the second order. 4. Investigation of the equation of the second order curves | 2 | |

Table 5.1 (the end)

| 1 | 2 | 3 | 4 |
|---|--|---|--|
| Thematic module 2 The elements of probability theory and mathematical statistics | <p><i>Theme 9. Empirical and logical bases of probability theory.</i></p> <p>1. Solving tasks using the classical definition of the probability of a random event and elements of combinatorics.</p> <p>2. Calculation of conditional probability, using the theorems of multiplication and addition of probabilities, the total probability formula and Bayes' formula</p> | 2 | <p>Main: [2; 9 – 11; 13].</p> <p>Additional: [15; 18; 19; 22 – 26]</p> |
| | <p><i>Theme 11. Random variables and their economic meaning.</i></p> <p>1. Plotting a distribution function.</p> <p>2. According to the definition, calculation of basic and additional characteristics of a random variable</p> | 2 | |
| | <p><i>Theme 13. Preprocessing of statistical data.</i></p> <p>1. Construction of variational series, a polygon and a histogram.</p> <p>2. Calculation of basic numerical characteristics of an empirical distribution</p> | 2 | |
| | <p><i>Theme 15. Checking the statistical hypothesis.</i></p> <p>1. Checking the statistical hypothesis about defining the distribution law for a population using the results of investigation of a sample. Pearson's fitting test.</p> <p>2. Checking the statistical hypothesis about the equality of two population means at an assumption of a normal distribution law and variances</p> | 2 | |

5.2. Examples of typical tasks of a class written test according to the themes

Thematic module 1

The elements of mathematical analysis and linear algebra

Written test No. 1

Theme 1. Limits of functions and continuity.

Theme 2. The differential calculus of the function of one variable.

Theme 3. Analysis of the function of several variables.

Theme 4. The indefinite and definite integral.

Theme 5. Differential equations.

Theme 7. The elements of the theory of matrices and systems of linear algebraic equations.

Theme 8. The elements of vector algebra.

Level 1.

1. Calculate the limits:

$$\text{a) } \lim_{x \rightarrow \infty} \frac{5x^3 + 3x - 1}{2x^3 + 9x^2 - x}; \quad \text{b) } \lim_{x \rightarrow 0} \frac{x \arcsin 2x}{\text{tg}^2 3x}.$$

2. Find the derivatives:

$$\text{a) } y = x^2 \cdot 2^x - \frac{x+2}{\sin x} + \frac{3\pi}{4}; \quad \text{b) } y = \sin \sqrt{5-x}.$$

3. Calculate the second order partial derivatives of the function:

$$z(x, y) = x \ln y + \sin x - 2\sqrt{3}.$$

4. Investigate the extremum of the function of two variables:

$$z(x, y) = 3x^2 + 2y^2 + 6x - 8y - 1.$$

5. Find the indefinite integrals:

$$\text{a) } \int \left(\frac{2}{\sqrt{5-x^2}} - \sqrt[3]{x} \right) dx; \quad \text{b) } \int 2^{1-x} dx; \quad \text{c) } \int (4 - \cos x)^7 \sin x dx.$$

$$\text{6. Find the definite integrals: a) } \int_0^{\pi} (\sin x + 1) dx; \quad \text{b) } \int_0^1 x e^{2x} dx.$$

7. Find the general solution to the differential equation:

$$y' = x \sqrt{1-y^2}.$$

8. Solve the Cauchy problem:

$$y'' - 8y' - 9y = 0, \quad y(0) = 1, \quad y'(0) = -2.$$

9. Calculate the product of the matrices:

$$A = \begin{pmatrix} 2 & -1 & 4 & 0 \\ 3 & 5 & -1 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & -3 \\ 3 & -4 \\ 2 & -5 \end{pmatrix}.$$

10. Solve the matrix equation:

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \cdot X = \begin{pmatrix} 3 & 5 \\ 5 & 9 \end{pmatrix}.$$

11. Find the matrix rank. Define the basic minor.

$$\begin{pmatrix} -2 & 0 & 8 & 1 & -5 \\ 3 & -1 & 7 & 2 & 4 \\ -8 & 2 & -6 & -3 & -13 \\ 11 & -3 & 13 & 5 & 17 \end{pmatrix}.$$

Level 2.

1. Calculate the limits of the functions:

$$\text{a) } \lim_{x \rightarrow -2} \frac{2x^2 + 3x - 2}{x^3 + 8}; \quad \text{b) } \lim_{x \rightarrow \infty} \frac{4x^3 + x^2 + 3}{x^4 + 9x^3 - x^2}; \quad \text{c) } \lim_{x \rightarrow 0} \frac{\sin x - \sin 3x}{e^{2x} - 1}.$$

2. Find the derivatives of the functions:

$$\text{a) } y = \frac{x^3}{\arccos x} - 7^x \cdot \operatorname{tg} x + \frac{2}{\sqrt{x}} - \sqrt{3}; \quad \text{b) } y = \ln \sqrt{\frac{1 - \sin x}{1 + \sin x}}.$$

3. The function $y = x^4 - 2x^2 - 1$ is given. Find: a) the local extremums and intervals of monotonicity; b) the greatest and least values on the interval $[0; 3]$; c) the intervals of convexity and concavity of the graph of the function and the inflection points; d) the equation of a tangent line and a normal at the point with the abscissa $x_0 = 0.5$.

4. Investigate of the extremum of the function of two variables:

$$z(x, y) = x^3 + y^2 - 3x + 4y + 5.$$

5. Find the indefinite integrals:

$$\text{a) } \int x \sin \frac{x}{2} dx; \quad \text{b) } \int \frac{x dx}{x^2 + 2x + 10}.$$

6. Plot the figure, bounded by the given curves, and calculate its area:

$$y = 3x - x^2, \quad 5x - y - 8 = 0.$$

7. Find the general solution to the differential equation:

$$xy' = y + 2\sqrt{xy}.$$

8. Solve the Cauchy problem:

$$y'' + 6y' - 7y = 5x + 2, \quad y(0) = -1, \quad y'(0) = 3.$$

9. Solve the system of linear algebraic equations using Cramer's formulas and the inverse matrix method:

$$\begin{cases} x_1 - 2x_2 + x_3 = -2; \\ 5x_1 + 4x_2 - x_3 = 0; \\ 3x_1 + x_2 + x_3 = 2. \end{cases}$$

10. Investigate the system of linear algebraic equations using the Kronecker – Capelli theorem:

$$\begin{cases} 2x_1 - x_2 + 4x_3 - x_4 = 4; \\ x_1 + 3x_2 + x_3 + 2x_4 = 6; \\ 3x_1 + x_2 - 4x_3 - 3x_4 = 7. \end{cases}$$

If it is compatible, define the number of its general solutions, find one of them and its corresponding basic solution. Is it a support solution?

11. The general equation of the curve on a plane is given as:

$$x^2 + 9y^2 - 10x + 18y - 2 = 0.$$

Write a matrix of its quadratic form and reduce it to the canonical form using Lagrange's method.

Define the type of the curve using its canonical equation and plot the graph of this curve.

Level 3.

1. Calculate the limits of the functions and check the result using L'Hospital's rule:

$$\text{a) } \lim_{x \rightarrow 1} \frac{x^4 - 3x + 2}{5x^2 - x - 4}; \quad \text{b) } \lim_{x \rightarrow 3} \frac{\lg(x-2)}{\sin 6 - \sin 2x}; \quad \text{c) } \lim_{x \rightarrow \infty} \left(\frac{3x-2}{3x+1} \right)^{7-x^2}.$$

2. Investigate the continuity of functions, define the type of break points and plot their graphs:

$$\text{a) } y = \begin{cases} x^2, & x < 0 \\ \arcsin x, & 0 \leq x < 1; \\ 1-x, & x \geq 1 \end{cases}; \quad \text{b) } y = 2^{\frac{1}{x-3}}.$$

3. Carry out investigation of the function $y = \left(\frac{x+2}{x-1} \right)^2$ and plot its graph. Find and draw the tangent and the normal at the inflection point of the graph of the function.

4. The function is given as $z = \ln(2x^3 + 3y^4)$. Find the gradient and the directional derivative if the direction $\vec{a} = (-3; 4)$ and the point $M_0 = (1; -1)$ are given. Calculate elasticities $E_z(x)$, $E_z(y)$ according to each argument at $x = y = 1$. Draw conclusions.

5. Calculate the integrals:

$$\text{a) } \int \frac{x^2 dx}{x^6 - 9}; \quad \text{b) } \int x^2 \sin 2x dx; \quad \text{c) } \int_1^e \frac{\ln^2 x}{x} dx; \quad \text{d) } \int_{-1}^{+\infty} \frac{dx}{x^2 + 2x + 10}.$$

6. Find the general solution to the differential equations:

$$\text{a) } xy' - y - \sqrt{x^2 + y^2} = 0; \quad \text{b) } y'' = \frac{xe^{2x} - 2\sqrt{x} + 1}{x}.$$

7. Solve the Cauchy problem:

$$\text{a) } y' + 2xy = x^3 e^{-x^2}, \quad y(0) = 1; \\ \text{b) } y'' + 6y' - 7y = 5x + 2, \quad y(0) = -1, \quad y'(0) = 3.$$

8. Let the function of the productivity have the form: $f(t) = \frac{2}{3t+4} + 5$

(parts per hour), where t is a time segment from the beginning of the day.

a) Determine the number of parts produced by an employee over the third working hour.

b) Find the mean value of the number of parts produced by an employee during a working day (8 hours).

c) Make analysis of the obtained values in the problem.

9. Find the fundamental set of solutions to the homogeneous system of linear equations

$$\begin{cases} 2x_1 + 2x_2 - 2x_3 + x_4 - 3x_5 = 0; \\ 3x_1 - x_2 + 2x_3 - x_4 + 2x_5 = 0; \\ x_1 - 3x_2 + 4x_3 - 2x_4 + 5x_5 = 0. \end{cases}$$

10. Solve the system of equations:

$$\begin{cases} -4x_1 - 2x_2 - 9x_3 - 4x_4 - 17x_5 = -9; \\ 12x_1 + 6x_2 + 33x_3 + 24x_4 + 68x_5 = 21; \\ 16x_1 + 8x_2 + 42x_3 + 24x_4 + 82x_5 = 34; \\ 4x_1 + 2x_2 + 9x_3 + 8x_4 + 20x_5 = 5. \end{cases}$$

11. Find the eigenvalues and eigenvectors of the matrix:

$$A = \begin{pmatrix} 5 & 2 & -3 \\ 4 & 5 & -4 \\ 6 & 4 & -4 \end{pmatrix}.$$

6. The themes of the laboratory studies

The educational plan provides conducting laboratory studies on the academic discipline "Higher Mathematics" in the first term.

A laboratory study is a form of study when a student under the direction of a lecturer fulfills a practical task with the help of PC-programming (MS Excel). The plan of laboratory studies, their content and bibliography for each theme are given in Table 6.1.

Table 6.1

The plan of themes of laboratory studies

| The theme name | The syllabus questions | Hours | Recommended reading |
|---|---|-------|---|
| 1 | 2 | 3 | 4 |
| Thematic module 1 | | | |
| The elements of mathematical analysis and linear algebra | | | |
| <i>Theme 1.</i> The elements of linear algebra in <i>MS Excel</i> | Learning the built-in functions of <i>MS Excel</i> . Using built-in functions for solving a system of linear algebraic equations of $n \times n$ by Cramer's formulas and with the help of the inverse matrix method and the Jordan – Gauss method | 2 | Main: [12]. Methodical support: [32] |
| Thematic module 2 | | | |
| The elements of probability theory and mathematical statistics | | | |
| <i>Theme 2.</i> Empirical and logical bases of probability theory | Using built-in functions for <i>MS Excel</i> for calculation of probability of random events with the help of theorems of multiplication and addition of probabilities. Using the total probability formula (a priori probability) and Bayes' formula (a posteriori probability) | 2 | Main: [3]. Methodical support: [32] |
| <i>Theme 3.</i> The scheme of independent trials | Calculation of probabilities for definite values p and n on the base of a model of repeated trials according to Bernoulli's scheme. Limit theorems of Moivre – Laplace ($n \rightarrow \infty$) and Poisson ($p \rightarrow \infty$ at $n \rightarrow \infty$) | 2 | |
| <i>Theme 4.</i> Random variables and their economic meaning | Calculation of basic and additional numerical characteristics of a discrete random variable by the definition and with the help of built-in functions of <i>MS Excel</i> . Construction of a distribution law of a sum and a difference of random variables, calculation of their basic numerical characteristics | 2 | |

Table 6.1 (the end)

| 1 | 2 | 3 | 4 |
|--|---|---|---|
| <i>Theme 5.</i> Basic distribution laws of a continuous random variable | Calculation of numerical characteristics of a continuous random variable, the function and density of distribution of a continuous random variable and numerical characteristics of uniform, exponential and normal distribution laws | 2 | |
| <i>Theme 6.</i> Preprocessing of statistical data | Construction of a continuous statistic series, presenting it with the help of a histogram and a polygon | 2 | |
| <i>Theme 7.</i> Statistical estimation of parameters of a distribution | Calculation of point and interval estimations of basic numerical characteristics of a distribution with the help of built-in functions of <i>MS Excel</i> | 2 | |
| <i>Theme 8.</i> Checking the statistical hypothesis | Using uniform, exponential and normal distribution laws checking statistical hypothesis relative to the correspondence of the distribution law in a population to a definite type | 2 | |
| <i>Theme 9.</i> The elements of the theory of correlation and regression | Construction of an empirical equation of a regression and a confidence interval of a regression line with the help of built-in functions of <i>MS Excel</i> | 2 | |

7. Independent work

7.1. Forms of independent work

Independent work is a scheduled educational and scientific work which is carried out at a lecture under the methodical and scientific guidance of the lecturer. It is a specific form of the educational activity, its main objective is forming independence of a person.

Independent work is:

1) different forms of individual and group cognitive activity of students, which are fulfilled by them during practical studies and in the extracurricular time;

2) different types of educational tasks which are fulfilled under the guidance of the lecturer;

3) a system of work organization when management of the educational work of students is fulfilled in the absence of the lecturer and without his direct assistance;

4) work of students which is carried out according to a specific individual educational plan designed on the basis of taking into account individual characteristics and cognitive possibilities of students.

The types of independent work and forms of control are given in Table 7.1.

Table 7.1

Tasks for students' independent work and forms of control

| The name of the theme | The content of students' independent work | The number of hours | Forms of control of IWS | Recommended reading |
|--|--|---------------------|-------------------------|---|
| 1 | 2 | 3 | 4 | 5 |
| Thematic module 1 | | | | |
| The elements of mathematical analysis and linear algebra | | | | |
| <i>Theme 1.</i> Limits of functions and continuity | Learning the lecture material. Preparation for a practical study. Carrying out homework | 4 | Homework | Main: [1; 4 – 8]. Additional: [17; 21; 29] |
| <i>Theme 2.</i> The differential calculus of the function of one variable. <i>Theme 3.</i> Analysis of the function of several variables. <i>Theme 4.</i> The indefinite and the definite integral. <i>Theme 5.</i> Differential equations. <i>Theme 6.</i> Series | Learning the lecture material, Preparation for a practical study. Carrying out homework and independent work | 22 | Homework | Main: [1; 4 – 8]. Additional: [14; 16; 17; 20; 21; 25; 27; 29]. Methodical support: [30; 31] |

Table 7.1 (continuation)

| 1 | 2 | 3 | 4 | 5 |
|---|---|-----------|---|---|
| <i>Theme 7.</i> The elements of the theory of matrices and systems of linear algebraic equations | Learning the lecture material. Preparation for a practical study. Carrying out homework and an independent work. Preparation for the defence of a competence oriented task. Preparation for the test. Preparation for a colloquium | 6 | Homework. An independent test on themes 1 – 7. A written test on themes 1 – 7 | Main: [1; 4 – 8]. Additional: [14; 16; 17; 20; 21; 25; 27; 29]. Methodical support: [30; 31] |
| <i>Theme 8.</i> The elements of vector algebra | Learning the lecture material. Preparation for a practical study and a laboratory work. Carrying out homework. Preparation for the defence of a competence oriented task. Preparation for a colloquium | 4 | Homework. Competence oriented task. Colloquium on themes 1 – 8 | |
| Total for thematic module 1 | | 36 | – | – |
| Thematic module 2 | | | | |
| The elements of probability theory and mathematical statistics | | | | |
| <i>Theme 9.</i> Empirical and logical bases of probability theory. <i>Theme 10.</i> The scheme of independent trials. <i>Theme 11.</i> Random variables and their economic meaning. <i>Theme 12.</i> Basic distribution laws of a continuous random variable | Learning the lecture material. Preparation for a practical study and laboratory work. Carrying out homework and independent work | 16 | Homework | Main: [2; 9 – 11; 13]. Additional: [15; 18; 19; 22 – 26]. Methodical support [30; 31] |

Table 7.1 (the end)

| 1 | 2 | 3 | 4 | 5 |
|---|--|-----------|---|---|
| <p><i>Theme 13.</i> Pre-processing of statistical data.</p> <p><i>Theme 14.</i> Statistical estimation of the distribution parameters</p> | <p>Learning the lecture material. Preparation for a practical study and laboratory work.</p> <p>Carrying out homework and independent work.</p> <p>Preparation for the defence of a competence oriented task</p> | 12 | Homework. Competence oriented task | <p>Main: [2; 9 – 11; 13].</p> <p>Additional: [15; 18; 19; 22 – 26].</p> <p>Methodical support: [30; 31]</p> |
| Theme 15. Checking the statistical hypothesis | <p>Learning the lecture material. Preparation for practical studies and laboratory work. Carrying out homework and independent work. Preparation for a written test</p> | 6 | Homework. An independent test and a written test on themes 9 – 15 | |
| <p><i>Theme 16.</i> The elements of the theory of correlation and regression</p> | <p>Learning the lecture material. Preparation for a laboratory study. Preparation for a colloquium. Preparation for presentation of the independent creative work</p> | 6 | A colloquium on themes 9 – 16. Independent creative work | |
| Total for thematic module 2 | | 40 | – | – |
| Total sum for modules | | 76 | – | – |

The educational time which is intended for full-time students' independent work is defined according to the educational plan and makes 55 % (84 hours) out of the total educational time for learning the discipline.

For students of the distant form of education this time equals 80 % (148 hours) out of the total educational time for learning the discipline.

During independent work a student becomes an active participant in the educational process, learns to master conscientiously theoretical and practical knowledge, orientate easily in the information space, has to take responsibility for the quality of his own professional training.

7.2. Examples of practical tasks for independent work

Thematic module 1

The elements of mathematical analysis and linear algebra

Theme 1. Limits of functions and continuity

1. Calculate the limits of the sequences:

$$\text{a) } \lim_{n \rightarrow \infty} \frac{(n^2 + 2)^2 - (n^2 - 1)^2}{(n + 2)(n + 5)}; \quad \text{b) } \lim_{n \rightarrow \infty} \frac{2^n + 5^n}{2^n - 5^n}.$$

2. Calculate the limits of the functions:

$$\begin{array}{ll} \text{a) } \lim_{x \rightarrow 1} \frac{4x^2 - x - 3}{x^2 - 1}; & \text{b) } \lim_{x \rightarrow 2} \frac{x^4 - 16}{2x^3 - 6x - 4}; \\ \text{c) } \lim_{x \rightarrow \infty} \frac{3x^4 + 4x^2 - 2x}{5x^4 - x^3 + 1}; & \text{d) } \lim_{x \rightarrow 0} \frac{3 \operatorname{tg}^2 x}{\cos 2x - 1}. \end{array}$$

3. Investigate the continuity of the function:

$$\begin{array}{ll} \text{a) } y = \frac{x^3 - 3x^2 + 3x - 1}{x^2 - 2x + 1}; & \text{b) } y = \begin{cases} -3x, & x < 0 \\ e^x - 1, & 0 \leq x < 1; \\ x - 1, & x \geq 1 \end{cases} \\ \text{c) } y = \frac{x + 6}{x^2 - 4}. & \end{array}$$

Theme 2. The differential calculus of the function of one variable

1. Calculate the derivatives of the given functions:

$$\begin{array}{ll} \text{a) } y = x \sin x - \frac{\lg x}{x + 1} + 2e; & \text{b) } y = \ln \cos^2 x; \\ \text{c) } y = \arcsin \sqrt{\frac{x}{x + 1}} + \operatorname{arctg} \sqrt{x}; & \text{d) } y = (x^2 + 3x - 4)^{\operatorname{tg} x}; \\ \text{e) } y^2 - \ln x = x^2 \ln y; & \text{f) } x = \log_2 t; \quad y = te^t. \end{array}$$

2. Calculate the limits with the help of L'Hospital's rule:

a) $\lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{\sin x - x \cos x}$; b) $\lim_{x \rightarrow \infty} \frac{\log_4 x}{x^3 - 1}$.

3. Write the equation of a tangent and a normal of the graph of the function $y = \frac{x}{x^2 + 1}$ at the point $x_0 = 0$.

4. Find the monotonicity intervals and local extremums of the function $y = x + 2\sqrt{x}$.

5. Find the intervals of a concavity (convexity) and inflection points of the graph of the function $y = x^7 + 7x + 1$.

Theme 3. Analysis of the function of several variables

1. Find the partial derivatives of the first order for the given functions:

a) $z(x, y) = y^7 \sin x + e^x - 5y + \sqrt{2}$; b) $z(x, y) = y^{\ln x}$;

c) $z(x, y) = \text{ctg}(2x + 3y)$; d) $u(x, y, z) = \text{arctg}\left(\frac{y}{x}\right) + xz^2$.

2. Calculate the gradient and the directional derivative for $\vec{a} = (-1; 2; 2)$ at the point $M = \left(4; \frac{1}{2}; 1\right)$ for the function $u(x, y, z) = 3x^3y + \sqrt{xz}$.

3. Investigate the extremum of the function

$$z(x, y) = 9x^2 + 2y^2 - 12y + 5.$$

Theme 4. The indefinite and definite integral

1. Calculate the indefinite integrals:

a) $\int \left(\frac{1}{\sqrt{x^3}} + 2 \sin x \right) dx$; b) $\int \left(\frac{2}{x^6} - \frac{1}{4 + x^2} \right) dx$;

c) $\int \sqrt{3x + 7} dx$; d) $\int (5x + 1)^5 dx$;

e) $\int \sin x(4 + \cos x)^7 dx$; f) $\int \frac{e^x dx}{e^{2x} + 1}$;

g) $\int \frac{dx}{\sqrt{(1-x^2)\arccos x}}$; h) $\int x^2 e^{-3x} dx$;

i) $\int (2x-3)\ln x dx$; j) $\int x^2 \operatorname{arctg} x dx$.

2. Calculate the definite integrals:

a) $\int_1^8 (1 - 7\sqrt[3]{x}) dx$; b) $\int_{-1}^0 \left(2x - \frac{1}{9-x^2} \right) dx$;

c) $\int_0^4 \frac{dx}{\sqrt{x+1}}$; d) $\int_0^{\pi/2} \frac{\sin x dx}{\cos^2 x - 9}$;

e) $\int_1^3 \log_3 x dx$; f) $\int_{-1}^0 (1-x)\sin \pi x dx$.

3. Calculate the improper integrals or prove their divergence:

a) $\int_{-\infty}^{-1} \frac{dx}{(x-2)^2}$; b) $\int_{-\infty}^{+\infty} \frac{dx}{x^2 + 4x + 5}$; c) $\int_0^2 \frac{dx}{x-1}$.

Theme 5. Differential equations

1. Find a general solution to the differential equation of the first order:

a) $y' = e^{2x-y}$; b) $xy' = y + x \sin^2 \frac{y}{x}$; c) $y' + y \operatorname{ctg} x = \sin x$.

2. Solve the Cauchy problem:

a) $y' = 2^x \sin^2 y$, $y(1) = \frac{\pi}{4}$; b) $y' - \frac{y}{x} = 3 - \sqrt{x}$, $y(0) = -1$.

3. Find a general solution to the differential equation of the second order:

a) $y'' = e^{-x} + 6x$; b) $y'' - 12y' + 36y = 0$; c) $y'' - 4y' + 8y = 0$.

Theme 6. Series

1. Prove the convergence or divergence of the series:

a) $\frac{1}{2!} + \frac{1}{3!} + \dots + \frac{1}{(2n+1)!} + \dots;$ b) $1 + \frac{2}{3} + \dots + \frac{n}{2n-1} + \dots;$

c) $\sum_{n=1}^{\infty} \frac{n}{2^n};$ d) $\sum_{n=1}^{\infty} \left(\frac{n}{2n+1}\right)^n.$

2. Find the convergence intervals of the power series:

a) $x - \frac{x^2}{2} + \dots + (-1)^{n+1} \frac{x^n}{n} + \dots;$ b) $1 + 3x + \dots + (n-1)3^{n-1}x^{n-1} + \dots;$

c) $\frac{x}{1 \cdot 2} + \frac{x^2}{2 \cdot 3} + \dots + \frac{x^n}{n \cdot (n+1)} + \dots$

Theme 7. The elements of the theory of matrices and systems of linear algebraic equations

1. Calculate the value of the expression $3A^3 - 2A + 5E$, where

$$A = \begin{pmatrix} 1 & -2 & 3 \\ 2 & -4 & 1 \\ 3 & -5 & 2 \end{pmatrix}.$$

2. Calculate the determinant: $\begin{vmatrix} 1 & 2 & 3 & 4 \\ -1 & 0 & -1 & 2 \\ 1 & 3 & 1 & -1 \\ 2 & 1 & 6 & 0 \end{vmatrix}.$

3. Find the value λ if the matrix A does not have the inverse matrix:

$$A = \begin{pmatrix} \lambda & 4 & 1 \\ 2 & 5 & -1 \\ 0 & \lambda & 1 \end{pmatrix}.$$

4. Using elementary row operations find the matrix which is inverse for the given matrix

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 6 & 10 \\ 1 & 4 & 10 & 20 \end{pmatrix}.$$

5. Find the rank and the basic minor of the matrix: $\begin{pmatrix} 1 & 3 & 5 & -1 \\ 2 & -1 & -3 & 4 \\ 5 & 1 & -1 & 7 \\ 7 & 7 & 9 & 1 \end{pmatrix}.$

6. Solve the system with the help of Cramer's method and the inverse matrix method:

$$\begin{cases} x_1 + 2x_2 + x_3 = 8 \\ -2x_1 + 3x_2 - 3x_3 = -5. \\ 3x_1 - 4x_2 + 5x_3 = 10 \end{cases}$$

7. Solve the matrix equation:

$$X \cdot \begin{pmatrix} 5 & 3 & 1 \\ 1 & -3 & -2 \\ -5 & 2 & 1 \end{pmatrix} = \begin{pmatrix} -8 & 3 & 0 \\ -5 & 9 & 0 \\ -2 & 15 & 0 \end{pmatrix}.$$

8. Investigate the compatibility, find a general solution to the system of equations and one particular solution.

$$\begin{cases} x_1 + x_2 + x_3 + x_4 + x_5 = 7 \\ 3x_1 + 2x_2 + x_3 + x_4 - 3x_5 = -2 \\ x_2 + 2x_3 + 2x_4 + 6x_5 = 23 \\ 5x_1 + 4x_2 + 3x_3 + 3x_4 - x_5 = 12 \end{cases}.$$

Theme 8. The elements of vector algebra

1. The angle between \vec{a} and \vec{b} equals 120° , their modules $|\vec{a}| = 3$ and $|\vec{b}| = 4$. Calculate $\vec{a} \cdot \vec{b}$, $(\vec{a} - \vec{b})^2$.

2. Find the value λ if vectors $\vec{a} + \lambda\vec{b}$ and \vec{c} are collinear.

$$\vec{a} = (2; 3), \quad \vec{b} = (3; 5), \quad \vec{c} = (-1; 3).$$

3. Check the coplanarity of vectors \vec{a} , \vec{b} and \vec{c} :

$$\vec{a} = (2; 3; 1), \quad \vec{b} = (-1; 0; -1), \quad \vec{c} = (2; 2; 2).$$

4. Check that vectors $\vec{a}_1 = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$; $\vec{a}_2 = \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}$; $\vec{a}_3 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ form the basis, and

find coordinates of the vector $\vec{b} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ into this basis.

5. Check linear dependence or independence of the vectors:

$$\vec{a}_1 = (4; -5; 2; 6), \quad \vec{a}_2 = (2; -2; 1; 3), \quad \vec{a}_3 = (6; -3; 3; 9), \quad \vec{a}_4 = (4; -1; 5; 6).$$

6. Find eigenvectors and eigenvalues of the matrix:

$$A = \begin{pmatrix} 1 & 2 & -2 \\ 1 & 0 & 3 \\ 1 & 3 & 0 \end{pmatrix}.$$

7. Check the determinacy of the quadratic form:

$$x_1^2 + x_2^2 + 3x_3^2 + 4x_1x_2 + 2x_1x_3 + 6x_2x_3.$$

8. Indicate the type of the second order curve and reduce this equation to a canonical form:

$$4x^2 + 3y^2 - 8x + 12y - 32 = 0.$$

9. Using the orthogonal transformation reduce this equation to a canonical form:

$$7x^2 + 16xy - 23y^2 - 14x - 16y - 218 = 0.$$

Thematic module 2

The elements of probability theory and mathematical statistics

Theme 9. Empirical and logical bases of probability theory

1. One digit is randomly chosen from 1 to 9. What is the probability that the chosen number is: 1) even; 2) odd; 3) prime; 4) greater than 7; 5) less than 4; 6) composite; 7) even or prime; 8) odd or prime; 9) composite or even; 10) composite or odd; 11) composite or prime; 12) composite and prime.

2. Three students are going to take an exam. The probability that the first student will pass it equals 0.6; for the second and third ones it is 0.7 and 0.75 respectively. What is the probability that

- a) all of the three students will pass the exam;
- b) the first student will only do it;
- c) the second and the third ones will only do it;
- d) one student will do it;
- e) no student will do it;
- f) at least one student will do it?

3. Three machines produce the same type of product at a factory. The first one gives 200 articles, the second one makes 300 articles and the third one produces 500 articles. It is known that the first machine produces 1 % of defective articles, the second one does 2 %, the third one does 4 %.

a) What is the probability that the article selected randomly from the total products will be defective?

b) It is known that the selected article is defective. What is the probability that this article was made by the second machine?

Theme 10. The scheme of independent trials

Theme 11. Random variables and their economic meaning

Theme 12. Basic distribution laws of a continuous random variable

1. The probability of hitting the target with 1 shot equals 0.6. Find the probability of the following events:

- a) with 12 shots the target will be hit 7 times;
- b) with 15 shots the target will be hit from 5 to 8 times;
- c) with 200 shots the target will be hit
 - 1) 120 times;
 - 2) from 90 to 110 times;
 - 3) no less than 111 and no more than 130 times;
 - 4) no more than 110 times;

5) no less than 115 times;

d) find the most probable number of hitting the target with 1 shot out of 12 shots and the probability of the most probable number;

e) find the most probable number of hitting the target with 1 shot out of 200 shots and the probability of the most probable number.

2. The probability of the birth of a boy is equal to 0.51. Make up the distribution law of a number of newborn boys out of 10 newborns. Find the probability that among 10 newborns there will be from 3 to 7 boys. Calculate $M(X)$, $D(X)$ and $\sigma(X)$. Find the distribution function of the random variable X .

3. There is a random variable X :

a)

| | | | | |
|-------|-----|-----|-----|-----|
| x_i | 5 | 10 | 15 | 20 |
| p_i | 0.2 | 0.3 | 0.4 | 0.1 |

b)

| | | | | | |
|-------|-----|------|------|-----|---|
| x_i | 0 | 1 | 2 | 3 | 4 |
| p_i | 0.1 | 0.25 | 0.35 | 0.2 | ? |

c)

| | | | | | |
|-------|------|------|------|------|------|
| x_i | -1 | 1 | 3 | 5 | 7 |
| p_i | 0.10 | 0.19 | 0.31 | 0.25 | 0.15 |

d)

| | | | | | |
|-------|-----|---|-----|-----|-----|
| x_i | 1 | 2 | 3 | 4 | 5 |
| p_i | 0.2 | ? | 0.1 | 0.3 | 0.1 |

Draw a distribution polygon. Calculate $M(X)$, $D(X)$ and $\sigma(X)$. Find the distribution function of the random variable X .

4. The parameters a, b of the uniform law of distribution are given: $a = 2$ and $b = 6$.

Find: a) functions $f(x)$ and $F(x)$; b) the mathematical expectation $M(X)$, the variance $D(X)$ and the root-mean-square deviation $\sigma(X)$; c) $P(0 < X < 3)$.

5. The probability density function of the exponential law of distribution

$$f(x) = \begin{cases} 0, & x < 0 \\ 0.05 \cdot e^{-0.05x}, & x \geq 0 \end{cases} \text{ is given.}$$

Find: a) the function $F(x)$; b) the mathematical expectation $M(X)$, the variance $D(X)$ and the root-mean-square deviation $\sigma(X)$; c) $P(2 < X < 10)$.

6. The probability density function of the normal law of distribution

$$f(x) = \frac{1}{2\sqrt{2\pi}} \cdot e^{-\frac{(x-3)^2}{8}} \text{ is given.}$$

Find $F(x)$, calculate $M(X)$, $D(X)$ and $P(1 < X < 7)$.

7. A man's height is distributed by the normal law. The mathematical expectation of a normal random variable X equals 170 centimeters, the root-mean-square deviation equals 5 centimeters.

Find the probability that the men' height will be

- a) less than 160 centimeters;
- b) greater than 180 centimeters;
- c) from 160 to 175 centimeters.

Theme 13. Preprocessing of statistical data

Theme 14. Statistical estimation of the distribution parameters

Theme 15. Checking the statistical hypothesis

1. A random variable X has a sample of 40 elements: 10, 13, 10, 9, 9, 12, 12, 6, 7, 9, 8, 9, 11, 9, 14, 13, 9, 8, 8, 7, 10, 10, 11, 11, 11, 12, 8, 7, 9, 10, 14, 13, 8, 8, 9, 10, 11, 11, 12, 12.

1) Construct a discrete statistic series for this sample, plot a polygon of relative frequencies and $F(x)$;

2) calculate $\bar{x}_s, S_x, R, v, A_s, E_s$;

3) find M_o and M_e .

2. Plot a polygon, a cumulative function, a histogram of relative frequencies for this series.

| | | | | | |
|-----------------|---------|---------|---------|---------|---------|
| $x_i - x_{i+1}$ | 10 – 15 | 15 – 20 | 20 – 25 | 25 – 30 | 30 – 35 |
| m_i | 5 | 15 | 20 | 35 | 25 |

Find the empirical distribution function and plot its graph. Find $\bar{x}_s, S_x, R, v, A_s, E_s, M_o$ and M_e .

3. The interval statistical series of x_i of a number of defective articles is given:

| | | | | | |
|-----------------|--------|---------|---------|---------|---------|
| $x_i - x_{i+1}$ | 5 – 10 | 10 – 15 | 15 – 20 | 20 – 25 | 25 – 30 |
| m_i | 2 | 6 | 10 | 8 | 4 |

Construct the confidence intervals for \bar{x}_{pop} and S_{pop} with the confidence probability $\gamma = 0.9, 0.93, 0.95, 0.99$ and 0.999 .

4. Using χ^2 test and Romanovskiy's test for goodness of fit check the assumption about the normal distribution of the population if there are empirical (m_i) and theoretical (\tilde{m}_i) frequencies.

| | | | | | | | | |
|---------------|---|----|----|----|-----|----|----|----|
| m_i | 6 | 13 | 38 | 74 | 106 | 85 | 30 | 14 |
| \tilde{m}_i | 3 | 14 | 42 | 82 | 99 | 76 | 37 | 13 |

Theme 16. The elements of the theory of correlation and regression

1. The results of the values of a two-dimensional random variable are given as the correlation table.

| | | | | | |
|------------------|---|---|----|---|---|
| $Y \backslash X$ | 1 | 2 | 3 | 4 | 5 |
| 4 | 2 | 3 | | | |
| 6 | | 4 | 10 | 1 | |
| 8 | | 3 | 12 | 4 | |
| 10 | | | 3 | 6 | 2 |

1. Calculate numerical characteristics.
2. Construct the theoretical regression line for this dependence.
3. Calculate the correlation coefficient.
4. Make analysis of the obtained values.

2. The dependence between the variables x and y was obtained with the help of experiment and presented by the table:

a)

| | | | | | | | |
|---|------|------|------|------|------|------|------|
| X | 1.5 | 3.0 | 4.5 | 6.0 | 7.5 | 9.0 | 10.5 |
| Y | 14.0 | 24.0 | 33.0 | 37.0 | 46.0 | 51.0 | 63.0 |

b)

| | | | | | | | |
|---|-----|-----|-----|-----|-----|-----|-----|
| X | 2.0 | 3.5 | 4.0 | 5.5 | 7.0 | 8.5 | 9.0 |
| Y | 10 | 21 | 25 | 33 | 41 | 51 | 63 |

Form a regression line for this dependence. Make analysis of the obtained value.

7.3. Questions for self-assessment

Thematic module 1

The elements of mathematical analysis and linear algebra

Theme 1. Limits of functions and continuity

1. Numerical sets and operations with them.
2. The neighborhood of a point.
3. The definition of the function of one variable.
4. The domain of the definition and the range of values of a function.
5. Properties of a function.
6. The notion of the inverse function.
7. A superposition of functions.
8. A numerical sequence.
9. The definition of the limit of a sequence.
10. Infinitesimals and infinitely large values.
11. The definition of the limit of a function.
12. One-sided limits.
13. The first remarkable limit.
14. The second remarkable limit.
15. The definition of the function continuity at a point.
16. Classification of breaks.

Theme 2. The differential calculus of the function of one variable

1. The definition of a derivative.
2. A table of derivatives.
3. Rules of calculation of derivatives.
4. Derivatives of higher orders.
5. The definition of a differential.
6. Differentials of higher orders.
7. The main theorems of differential calculus.
8. L'Hospital rule.
9. The condition of the monotony of a function.
10. The maximum and the minimum of a function.
11. Convexity and concavity of the graph of a function.
12. Inflection points.
13. Asymptotes of the graph of a function.

14. Marginal analysis.
15. Elasticity of economic indicators.
16. The economic meaning of Fermat's theorem.

Theme 3. Analysis of the function of several variables

1. Functions of two variables.
2. The domain of their definition.
3. Partial derivatives. Mixed derivatives.
4. The differential.
5. The necessary conditions of the function of two variables.
6. The sufficient conditions of the extremum of the function of two variables.
7. The notion of the conditional extremum.
8. The method of Lagrange factors.
9. The least-squares method.
10. The function of several variables in the problems of economics (the utility function, the expenditure function, the multifactor production function of Cobb and Douglas).

Theme 4. The indefinite and the definite integral

1. An antiderivative.
2. An indefinite integral.
3. A table of basic integrals.
4. Direct integration.
5. A change of the variable in an indefinite integral.
6. Integration by parts.
7. Integration of rational fractions.
8. Integration of irrational expressions and expressions which have trigonometric functions.
9. The notion of the definite integral.
10. Integral sums.
11. Properties of the definite integral.
12. The Newton – Leibnitz formula.
13. A change of the variable in a definite integral.
14. Integration by parts.
15. The notion of an improper integral.
16. The conditions of the convergence of improper integrals.
17. The Euler – Poisson integral.

18. Calculation of areas, volumes of the solid of revolution.
19. Calculation of the arc lengths of curves.
20. Formulas of rectangles, trapezoids, Simpson.
21. The volume of efficient production.
22. A consumer surplus.
23. The Lorenz curve.

Theme 5. Differential equations

1. The notion of the differential equation.
2. The order of the differential equation.
3. Differential equations of the first order.
4. A general solution and a general integral of a differential equation.
5. Initial conditions.
6. A particular solution and a particular integral.
7. The differential equation of the first order with separable variables.
8. Homogeneous equations of the first order.
9. Linear differential equations of the first order.
10. Differential equations of Bernoulli.
11. The second-order linear differential equations with constant coefficients.
12. Homogeneous and inhomogeneous differential equations.
13. The notion of linearly independent solutions of a homogeneous differential equation of the second order.
14. The structure of a general solution of an inhomogeneous differential equation of the second order.
15. Linear inhomogeneous differential equations of the second order with the right parts of a special form.
16. The notion of the differential equation.
17. The notion of the system of differential equations.
18. The notion of the equilibrium of a solution.
19. The Solow model.
20. The model of a natural increasing output.
21. The dynamics of market prices.

Theme 6. Series

1. Numerical series.
2. Partial sums of series.
3. The necessary condition of the series convergence.

4. Series with positive terms.
5. The theorem of comparison of series.
6. D'Alembert criterion.
7. Cauchy's criterion.
8. The Maclaurin – Cauchy integral criterion.
9. The notion of alternating series.
10. Absolute and conditional convergence of series.
11. The Leibnitz theorem.
12. Power series.
13. The convergence radius of power series.
14. Differentiation and integration of power series.
15. The Taylor and Maclaurin series.
16. Decomposition of elementary functions in the Taylor and Maclaurin series.
17. Application of power series to an approximate calculus.

**Theme 7. The elements of the theory of matrices and systems
of linear algebraic equations**

1. The definition of a matrix.
2. Types of matrices (square, triangular, diagonal, unit).
3. Basic operations with matrices.
4. Properties of these operations.
5. Transposition of a matrix.
6. The notion of an inverse matrix.
7. The definition of the determinant.
8. The rule of the triangle (Sarrus' rule).
9. The rules of the calculation of determinants of lower orders (schematic) and higher orders (expansion by Laplace formulas).
10. The properties of determinants.
11. Calculation of some special determinants (triangular, diagonal, identity matrices, the Vandermonde matrix).
12. Calculation of an inverse matrix with the help of determinants (algebraic cofactors).
13. An inverse matrix.
14. The matrix rank and ways to define it.
15. The definition of the system of linear algebraic equations.
16. An augmented matrix.

17. The definition of a solution.
18. A consistent or inconsistent system.
19. A determined or undetermined system.
20. The inverse matrix method of solving square systems of linear algebraic equations.
21. The Cramer method of solving square systems of linear algebraic equations.
22. The Gauss – Jordan method of sequential exclusion of unknowns for solving systems of linear algebraic equations.
23. The notion of the matrix rank.
24. Calculation of the matrix rank.
25. The Kronecker – Capelli theorem.
26. Particular and general solutions of a system of linear algebraic equations.
27. A homogeneous system of linear algebraic equations.
28. The space of solutions of a homogeneous system.
29. A fundamental system of solutions of a homogeneous system of linear algebraic equations.

Theme 8. The elements of vector algebra

1. The definition of a vector.
2. The definition of a point.
3. Linear operations with vectors in coordinates.
4. Coordinates of the point of division of a segment.
5. Coordinates of the vector which is given by two points.
6. A scalar product.
7. Properties of a scalar product of two vectors.
8. Expression of a scalar product through coordinates.
9. A cross product of two vectors.
10. Properties of a cross product.
11. Expression of a cross product through coordinates.
12. A mixed product of three vectors, its properties.
13. Expression of a mixed product through coordinates of vectors-factors.
14. The definition of linear space.
15. The basis of linear space.
16. The notion of subspace.
17. The notion of linear vector space.

18. The rank of the finite systems of vectors, the rules of calculation.
19. The definition of the eigenvalue of a matrix.
20. The definition of the eigenvector of a matrix.
21. A characteristic equation.
22. The notion of a quadratic form.
23. The conditions of determinacy of quadratic forms.
24. The matrix of a quadratic form.
25. A canonical form.
26. Reducing quadratic forms to a canonical form.
27. A general equation of the second-order curve.
28. Reducing the second-order curve to a canonical form.
29. A parabola.
30. An ellipse.
31. A hyperbola.

Thematic module 2

The elements of probability theory and mathematical statistics

Theme 9. Empirical and logical bases of probability theory

1. The subject of this discipline.
2. A probabilistic model of an experiment.
3. Sure (certain) events.
4. Random events.
5. Impossible events.
6. Rules of operations with random events.
7. The space of elementary events.
8. The classical definition of probability and its calculation.
9. Basic formulas of combinatorics.
10. The statistical definition of probability.
11. Axiomatics of Kolmogorov.
12. The geometrical definition of probability.
13. The Venn – Euler diagram.

Theme 10. The scheme of independent trials

1. Probabilistic space.
2. Addition theorems of probabilities.
3. Dependent and independent events.

4. Conditional probability.
5. Joint (compatible) and disjoint (incompatible) events.
6. Multiplication theorems of probabilities.
7. A complete group of events.
8. Complementary events.
9. The probability of at least one event.
10. The probability that an event will occur at least once.
11. The formula of total probability.
12. Bayes' formula (the theorem of hypothesis).

Theme 11. Random variables and their economic meaning

1. A scheme of repeated independent trials.
2. Bernoulli's formulas.
3. The local theorem of Moivre – Laplace.
4. Gauss's function, its properties.
5. Application of the probability of occurrence of a random event a definite number times in a series of independent trials to approximate calculations.
6. The integral theorem of Moivre – Laplace.
7. Laplace's function, its properties.
8. Application of the probability that values of a random variable lie in a definite interval to approximate calculations.
9. Poisson's theorem.

Theme 12. Basic distribution laws of a continuous random variable

1. The definition of a random variable.
2. Discrete and continuous random variables.
3. Distribution laws of probabilities for a random variable and ways of finding them (tabular, graphic and analytical).
4. The distribution function of probabilities, its properties.
5. Basic numerical characteristics of a random variable.
6. Mathematical expectation.
7. A variance.
8. A root-mean-square deviation.
9. Properties of basic numerical characteristics.
10. Additional numerical characteristics of a distribution: a mode, a median, an excess.
11. Initial and central theoretical moments of an arbitrary order.

12. Calculation of numerical characteristics of a distribution of a random variable using its theoretical moments.
13. The definition of a continuous random variable.
14. The distribution density and its probable explanation.
15. The density function of the distribution of a random variable and its properties.
16. Distribution laws of a discrete random variable, which are often used in social and economic phenomena.
17. A binomial distribution.
18. A geometrical distribution.
19. A hypergeometrical distribution.
20. Specificities and properties of these distributions, their basic numerical characteristics and the economic meaning.
21. A flow of events.
22. The simplest flow of events and its properties.
23. Distribution laws of a continuous random variable, which are often used in social and economic phenomena.
24. A uniform distribution.
25. A normal distribution.
26. An exponential distribution.
27. Properties of these distributions and their basic numerical characteristics.
28. The influence of parameters of a distribution on the density function of probabilities in a normal distribution law.
29. Student's distribution.
30. Pearson's distribution.
31. Fisher's distribution.
32. Specificities and properties of these distributions.
33. The relationship of these distributions and the normal distribution law of a continuous random variable.

Theme 13. Preprocessing of statistical data

1. Basic problems of mathematical statistics.
2. The sampling method.
3. Definitions of a population and its sample.
4. The empirical distribution law.

5. Ways of presentation of sampling totalities and representation of the results of observations.
6. Discrete and interval variational series.
7. A polygon and a histogram.
8. Basic sampling characteristics and their asymptotic behavior.

Theme 14. Statistical estimation of the distribution parameters

1. Statistical estimations of the distribution parameters of a population and their properties: unbiasedness, possibility and efficiency.
2. Asymptotic efficiency of maximally plausible estimations.
3. The method of moments.
4. Point and interval estimations.
5. The confidence interval for mathematical expectation of a normal population.

Theme 15. Checking the statistical hypothesis

1. Main and alternative statistical hypothesis.
2. A statistical test.
3. Construction of critical domains for a statistical test.
4. Errors of the first kind.
5. Errors of the second kind.
6. The concept of power of a test.
7. Checking the statistical hypothesis about defining the distribution law for a population using the results of investigation of a sample.
8. Pearson and Kolmogorov fitting tests.
9. The fitting test relative to a frequency.
10. Checking the statistical hypothesis about the equality of two population means in the assumption of a normal distribution law.
11. Student's fitting test.
12. Comparison of variances.
13. The Fisher – Snedeker fitting test.
14. Checking the hypothesis about the equality of a sampling mean and mathematical expectation.

Theme 16. The elements of the theory of correlation and regression

1. Problems of correlation analysis.
2. The sampling coefficient of a correlation, its properties.
3. The confidence interval.

4. The coefficient of determination.
5. The correlation ratio, its properties.
6. Problems of regression analysis.
7. The correlation dependence.
8. The correlation table.
9. Empirical lines of a regression.
10. Estimation of parameters of a pair regression equation using the least-squares method.
11. Point estimations.
12. Checking the significance of parameters of a pair regression equation.
13. The confidence interval for a line of a pair regression.
14. The simplest cases of a nonlinear regression.
15. The concept of a multiple regression.

7.4. The independent test

7.4.1. Basic requirements for carrying out the independent test

The purpose of carrying out an independent test is the formation of students' practical skills in the use of theoretical knowledge of the academic discipline "Higher Mathematics" for solving economic problems and optimal decision making, obtaining skills in economic mathematical analysis and modelling for finding and explanation of the most effective solutions, as well as using the methods of quantitative and qualitative analysis of applied economic mathematical models.

The independent test should be carried out on the scheduled date. Besides, a description of each of the tasks for the independent test should be done (except the didactic analysis and the definition of corresponding elements of the independent work) according to the general technology of fulfillment:

learning and citing the basic questions of the theoretical material out of the recommended sources;

the design of the report on carrying out the task for the independent test, answers to control questions;

handing in the fulfilled tasks of the independent test and the answers to the control questions to the lecturer.

the fulfillment of the tasks of the independent test on the academic discipline is assessed based on:

the understanding, the degree of the mastery of the theory and methodology of the problems which are considered;

the degree of acquaintance with the recommended literature and the mastery of the factual material of the academic discipline;

the ability to connect theory and practice in the consideration of practical situations, solving problems, carrying out calculations, fulfillment of tasks, given for independent work;

the completeness of taking into account the conditions for the fulfillment of the task;

the logic of the given material and correspondence of its structure to the provided thematic elements of the task; the availability and completeness of consideration of the key concepts (definitions, terms, varieties and so on) of the subject area of the task; the availability and explanations of the student's final conclusions; illustration of the processed material with the help of student's own examples and the graphical material.

7.4.2. Examples of typical tasks of independent tests

Thematic module 1

The elements of mathematical analysis and linear algebra

The independent work on themes 1 – 7

1. Calculate the limits of the numerical sequences:

$$\text{a) } \lim_{n \rightarrow \infty} (\sqrt{n+1} - \sqrt{n}); \quad \text{b) } \lim_{n \rightarrow \infty} \frac{(n+4)!}{(n+2)! + (n+3)!};$$

$$\text{c) } \lim_{n \rightarrow \infty} \frac{\sqrt[3]{2n^3 + 1} + \sqrt[3]{n^6 + 4}}{2n(3+n)}.$$

2. Calculate the limits of the given functions:

$$\text{a) } \lim_{x \rightarrow 4} \frac{2x^2 - 7x - 4}{x^2 - 2x - 8};$$

$$\text{b) } \lim_{x \rightarrow -2} \frac{x^4 + 8x}{x^3 + 5x^2 + 8x + 4};$$

$$\text{c) } \lim_{x \rightarrow 1} \frac{3 - \sqrt{7x^2 + x + 1}}{\sqrt[4]{x} - 1};$$

$$\text{d) } \lim_{x \rightarrow \infty} \frac{2x + 3x\sqrt{x} + 1}{\sqrt[3]{x^5} - 5x};$$

$$\text{e) } \lim_{x \rightarrow 0} \frac{\arcsin 4x}{3^{-x} - 3^x};$$

$$\text{f) } \lim_{x \rightarrow \infty} \left(\frac{2x+1}{2x+5} \right)^{2-9x};$$

$$\text{g) } \lim_{x \rightarrow -1} \frac{\ln(3+2x)\cos(x+1)}{4\operatorname{arctg}(x+1)}.$$

3. Investigate the continuity of the function $y = f(x)$, define the type of break points. Plot a graph of the function.

$$\text{a) } y = \frac{x^3 + x^2 - 2x}{x + 2}; \quad \text{b) } y = \begin{cases} x^2, & x \leq 1 \\ \log_2 x, & 1 < x \leq 2; \\ 2x - 3, & x > 2 \end{cases}; \quad \text{c) } y = e^{\frac{1}{3x-6}}.$$

4. Calculate the derivatives of the given functions:

$$\text{a) } y = \sqrt{x} \cdot \log_7 x - \frac{\arccos x}{2x + 3} + \sqrt[3]{2};$$

$$\text{b) } y = \ln \ln^3 \ln^2 x;$$

$$\text{c) } y = x \cdot (2^x + 1) \cdot \sqrt{x^2 + 1} - \ln(x + \sqrt{x^2 + 1});$$

$$\text{d) } \operatorname{arctg}(y^2 - 3x) = \frac{y}{x};$$

$$\text{e) } x = \sqrt{2t + 1}; \quad y = t^4 - 4^{-t}.$$

5. Calculate the limits of the functions using L'Hospital's rule:

$$\text{a) } \lim_{x \rightarrow +0} \frac{\ln \sin x - 1}{\ln \sin 5x};$$

$$\text{b) } \lim_{x \rightarrow \infty} \sqrt{x} \cdot e^{-2x}.$$

6. Carry out a full investigation of the function $y = \frac{x^4}{x^3 + 1}$ and plot its graph. Find the greatest and the least values of the function on the segment $[-1; 2]$.

7. Calculate the partial derivatives of the functions:

a) $z(x, y) = (2x - y)e^x + \frac{\ln y}{x + y} - \cos 2$; b) $z(x, y) = \sin^2\left(\frac{x}{y} - 1\right)$;

c) $z(x, y) = \frac{y \operatorname{tg} x}{1 + \sqrt{x}} - x 2^y \cos x$; d) $z(x, y) = \arcsin^x(2x + 3y)$.

8. For the given function $u(x, y, z) = y \cos z - x e^y$ find a gradient and a directional derivative for the direction $\vec{a} = (4; 0; 3)$ at the point

$$M_0 = \left(0; 1; \frac{\pi}{2}\right).$$

9. Investigate the extremum of the function of two variables:

$$z(x, y) = x^3 + y^2 - 3x + 4y + 5.$$

10. Calculate the indefinite integrals and check their correctness with the differentiation:

a) $\int e^x \sqrt{\frac{e^x + 1}{2}} dx$; b) $\int \frac{\ln x dx}{x \sqrt{1 + \ln x}}$.

11. Calculate the definite integrals:

a) $\int_0^{\pi} \frac{\cos x dx}{1 + \sin^2 x}$; b) $\int_0^{\frac{\sqrt{3}}{2}} \sqrt{1 - x^2} dx$.

12. Calculate or prove the divergence of the improper integrals:

$$\text{a) } \int_1^{\infty} \frac{\ln x}{x^2} dx; \quad \text{b) } \int_1^2 \frac{xdx}{\sqrt{x-1}}.$$

13. Indicate the type of the differential equations, find a general solution or a general integral:

$$\text{a) } (x-y)y' = x+y; \quad \text{b) } y'' - 2\frac{(y')^2}{y} + y' = 0.$$

14. Investigate the convergence of the numerical series:

$$\text{a) } \sum_{n=1}^{\infty} \frac{3n^2+2}{n^2+5}; \quad \text{b) } \sum_{n=1}^{\infty} \frac{\sqrt[3]{n+3}}{n+1}; \quad \text{c) } \sum_{n=1}^{\infty} \frac{n^4}{(n+2)!}; \quad \text{d) } \sum_{n=1}^{\infty} \left(\frac{n+1}{n+2}\right)^{n^2}.$$

15. Find the value of the polynomial $f(x) = 3x^2 + 2x - 1$ of the matrix

$$A = \begin{pmatrix} 1 & 1 & 2 \\ -1 & 3 & -7 \\ 7 & 2 & 2 \end{pmatrix}.$$

16. Solve the system of the matrix equations

$$\begin{cases} X - Y = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}; \\ 2X + 3Y = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \end{cases}$$

Calculate $7 \cdot X \cdot Y^T$.

17. Calculate the determinant:

$$\begin{vmatrix} 2 & -1 & 3 & 6 \\ 4 & 1 & 2 & -1 \\ -3 & 0 & 4 & 1 \\ 1 & 1 & 0 & 3 \end{vmatrix}.$$

Calculate the determinant decomposing it into the elements of the second order.

Calculate the determinant getting zeros in any row or column.

Verify that the determinant changes its sign with the change of places of any two rows or columns with the help of direct calculation.

18. The system of linear equations is given as:

$$\begin{cases} x_1 + 5x_2 - x_3 + x_4 + x_5 = -3; \\ 3x_1 + x_2 + 3x_3 + 3x_4 - 3x_5 = -3; \\ -x_1 + x_3 - x_4 + 3x_5 = 2; \\ -x_1 + 2x_2 - 2x_3 - x_4 + 2x_5 = 0. \end{cases}$$

Prove the compatibility of this system. Find its general and particular solutions.

19. The vectors are given as

$$\bar{a} = (1; 2; -1), \quad \bar{b} = (2; 1; 3), \quad \bar{c} = (1; 1; 0), \quad \bar{d} = (0; -1; 1).$$

Check whether the vectors $\bar{a} - 2\bar{b}$, $3\bar{c} - 2\bar{d}$, $-2\bar{d} + \bar{a}$ are coplanar.

20. Find eigenvalues and eigenvectors of the matrix

$$A = \begin{pmatrix} 6 & 5 & -5 \\ -11 & -4 & 11 \\ -1 & 5 & 2 \end{pmatrix}.$$

Check the result using the definition.

Thematic module 2

The elements of probability theory and mathematical statistics

The independent work on themes 9 – 15

1. A symmetrical dice is rolled four times. What is the probability of getting the faces with the odd, even, odd, even points respectively?
2. A student is choosing 3 questions out of 30 at an exam. There are 10 questions from algebra, 15 from analysis and 5 from geometry. What is the probability that he will choose at least two questions from the same area?
3. A box of fuses which are all of the same shape and size comprises 23 2A fuses, 47 5A fuses and 69 13A fuses. Determine the probability of selecting 2 or more 5A fuses.
4. A phone company has found that 75 % of customers want text messaging, 80 % photo capabilities and 65 % both. What is the probability that a customer will want at least one of these?
5. On five cards there are written letters "s", "m", "e", "t", "a". After shuffling they take five cards one by one and put them near in turn. What is the probability that the word results in: 1) "steam"; 2) "team"; 3) "tea"?
6. There are N white and M black balls (N good and M defective articles on the shelf). If k balls are chosen randomly, what is the probability of getting this way exactly n white balls?
7. From the set of 52 playing cards 5 cards were chosen randomly. What is the probability of getting exactly two hearts among them?
8. Suppose the probability that a married man votes is 0.45, the probability that a married woman votes is 0.4, and the probability a woman votes given that her husband votes is 0.6. What is the probability that: a) both vote; b) a man votes given that his wife votes?
9. Three girls, Alice, Betty and Charlotte, wash the family dishes. Since Alice is the oldest, she does the job 40 % of the time. Betty and Charlotte share the other 60 % equally. The probability that at least one dish will be broken when Alice is washing them is 0.02; for Betty and Charlotte the probabilities are 0.03 and 0.02. The parents do not know who is washing the dishes, but one night they hear one break. What is the probability that Alice was washing up? Betty? Charlotte?
10. Three machines produce the same type of product at a factory. The first one produces 150 articles, the second one makes 600 articles and the third machine manufactures 250 articles. It is known that the share of defective

articles is 2 % with the first machine, 4 % with the second one, 8 % with the third one. (a) What is the probability that an article selected randomly from the total products will be defective? (b) It is known that a selected article is defective. What is the probability that this article was made by the third machine?

11. The probability that the part is produced with a defect equals 0.2. Find the probability that among 400 randomly selected details there will be: a) from 70 to 100 defective ones; b) 90 defective ones; c) no less than 60 defective ones; d) no more than 100 defective ones. Find the most probable number of defective (standard) articles.

12. The probability of hitting the target with 1 shot equals 0.6. Find the probability of the following events: a) with 600 shots hitting frequency will deviate from the probability 0.6 by the absolute value of no more than 0.03; b) find the boundaries of hitting the target with 600 shots in order that the probability $P = 0.993$; c) find such a number of shots that the probability $P = 0.993$ gives the deviation of frequency of hits from the probability 0.6 by the absolute value no more 0.03; d) the accuracy ε , which probability of deviation of relative frequency from the probability $p = 0.7$ is $P = 0.996$.

13. A factory sent 5000 products of high quality to the warehouse. The probability of damaging the products on the way is equal to 0.0008. Find the probability that: a) 5 damaged products will be received at the warehouse; b) from 3 to 6 damaged products will be received at the warehouse; c) at least one damaged product will be received at the warehouse.

14. The probability that the student will pass a test at the first attempt is equal to 0.9. Find the probability that among 7 students of the same knowledge level: a) 5 students will pass the test; b) from 4 to 6 will pass the test. Find the most probable number of students who will pass the test at the first attempt.

15. The probability of train arriving at the station on time is equal to 0.8. Make up the distribution law to a number of train arrivals at the station on time out of 4 expected trains. Find the probability that no less than 2 and no more than 3 trains will arrive on time. Find the probability that at least one train will arrive on time. Calculate $M(X)$, $D(X)$ and $\sigma(X)$. Find the distribution function of the random variable X .

16. Measurements of scientific systems are always subject to variation, some more than others. There are many structures for measuring error and statisticians spend a great deal of time modelling these errors. Suppose the

error X of a certain physical quantity is calculated by the density function

$$f(x) = \begin{cases} 0, & x \leq -1 \\ c(3 - x^2), & -1 < x \leq 1. \text{ Determine } c \text{ that renders } f(x) \text{ a valid den-} \\ 0, & x > 1 \end{cases}$$

sity function. Find the probability that a random error in the measurement is less than 1/2. For this particular measurement, it is undesirable that the magnitude of the error should exceed 0.8. What is the probability that this occurs?

17. The waiting time, in hours, between successive speeders spotted by a radar unit is an exponential continuous random variable with cumulative

$$\text{distribution function } F(x) = \begin{cases} 0, & x < 0 \\ 1 - e^{-8x}, & x \geq 0 \end{cases}.$$

Find: a) the density function $f(x)$;

b) $M(X)$, $D(X)$ and $\sigma(X)$;

c) the probability of waiting less than 12 minutes between successive speeders.

18. The voltage (volts) in an electricity network for each hour is given with the help of series: 222, 219, 224, 220, 218, 217, 221, 220, 215, 218, 223, 225, 220, 226, 221, 216, 211, 219, 220, 221, 222, 218, 221, 219.

1. Construct a discrete statistic series for this sample, plot a polygon of relative frequencies and $F(x)$.

2. Calculate \bar{x}_s , S_x , R , v , A_s , E_s .

3. Find M_o and M_e .

19. For calculation of an average productivity x_i the area of 2000 hectares is divided into 20 equal lots m_i . The real productivity is given:

| | | | | | |
|-------------------------|----|----|----|----|----|
| x_i , centner/hectare | 25 | 30 | 35 | 40 | 45 |
| m_i | 2 | 3 | 8 | 4 | 3 |

1. Construct a discrete statistic series for this sample, plot a polygon of relative frequencies and $F(x)$.

2. Calculate $\bar{x}_s, S_x, R, \nu, A_s, E_s$.

3. Find M_o and M_e .

20. The loading of a telephone line in the period from 9:00 till 16:00 (one day) is characterized with the help of the table:

| Hours | 9 – 10 | 10 – 11 | 11 – 12 | 12 – 13 | 13 – 14 | 14 – 15 | 15 – 16 |
|-----------------|--------|---------|---------|---------|---------|---------|---------|
| Number of calls | 16 | 21 | 18 | 10 | 12 | 15 | 13 |

Plot a histogram and a cumulative function. Calculate \bar{x}_s, S_x, R, ν and A_s, E_s, M_o, M_e .

21. The interval statistical series of x_i of the number of defective articles is given:

| | | | | | |
|-----------------|--------|---------|---------|---------|---------|
| $x_i - x_{i+1}$ | 5 – 10 | 10 – 15 | 15 – 20 | 20 – 25 | 25 – 30 |
| m_i | 2 | 6 | 10 | 8 | 4 |

Construct the confidence intervals for \bar{x}_{pop} and S_{pop} with the confidence probability $\gamma = 0.9, 0.93, 0.95, 0.99$ and 0.999 .

22. Define the number of lamps for the sample, if the root mean square deviation of lifetime of a lamp equals 60 hours, the deviation from the population mean is less than 8 hours with the probability 0.9545.

23. To determine the ash content of coal a number of tests should be conducted so that the deviation from the sample mean were less than 3 %. The root mean square deviation equals 10 %. The confidence probability is 0.9973. How many tests do you need to conduct?

24. Using χ^2 test and Romanovskiy's test for goodness of fit check the assumption about the normal distribution of the population if there are empirical (m_i) and theoretical (\tilde{m}_i) frequencies.

| | | | | | | | | |
|-------------|---|---|----|----|----|---|---|---|
| m_i | 1 | 4 | 10 | 14 | 12 | 6 | 2 | 1 |
| \hat{m}_i | 2 | 3 | 10 | 13 | 12 | 7 | 2 | 1 |

25. The results of measuring the values of the two-dimensional random variable (X, Y) are given as the correlation table.

| $Y \backslash X$ | 10 | 15 | 20 | 25 | 30 | 35 | 40 |
|------------------|----|----|----|----|----|----|----|
| 20 | 2 | 3 | | | | | |
| 40 | | 8 | 12 | 8 | | | |
| 60 | | 4 | 15 | 10 | 8 | 2 | |
| 80 | | | 3 | 4 | 4 | 4 | |
| 100 | | | | | 5 | 4 | 1 |
| 120 | | | | | | 2 | 1 |

Construct conjugate empirical lines of regression. Using their form and arrangement make an assumption about the significance of correlation between the factors X and Y and its form.

7.5. Performing the independent creative work

Independent creative work of students is an integral part of the educational process. It forms the skills in the performance of major works (term papers, a diploma project). That is the reason why it is necessary for students to learn how to qualitatively prepare a creative work.

Within the framework of the given form of student's independent work it is proposed to prepare a presentation on the theme formulated for a student, in the electronic form (with the help of MS PowerPoint). An alternative may be a presentation at the next lecture or writing a scientific article.

Preparation of an independent creative work provides for systematization, consolidation, broadening of the theoretical and practical knowledge of the academic discipline and using it in the process of solving a specific economic problem, development of skills in independent work and mastering the methods of investigation and experiment, connected with the theme of the independent creative work.

An independent creative work provides for the availability of the following elements of scientific investigation: practical significance, a comprehensive systematic approach to solving the tasks of the investigation, the theoretical use of the progressive modern methodology and scientific developments, availability of the elements of creativity, the ability to use modern technologies.

A comprehensive systematic approach to developing the theme of the independent creative work implies consideration of the subject of the research from different points of view that is with the positions of a theoretical basis and practical groundworks, conditions of its realization, analysis, explanations of ways for improvement both in a close relationship and a common logic of exposition.

The use of modern technology consists in the fact that in the process of fulfillment of analysis and explanation of ways for improvement of particular aspects of the subject and the object of investigation a student has to use information about high achievements in techniques and technologies of investigation, use varied mathematical methods and ways, approaches to the definition and explanation of indicators of analysis of a social economic system or its elements.

Students submit the independent creative work to the lecturer in the electronic form if it is a presentation or in the printed or electronic form if it is a scientific publication.

After the complex presentation or a scientific publication has been reviewed and corrected by the lecturer, students make their presentations in front of the audience, report on the results stated in the scientific publication, make reports on a student's scientific and practical conference and so on.

8. Individual consultative work

Individual consultative work is fulfilled according to the schedule of the individual consultative work in the following forms: individual studies, consultations, checking the fulfillment of individual tasks, checking and defending the tasks presented for the current control and so on.

The forms of the individual consultative work are:

a) as to the mastery of the theoretical material:

consultations: individual (question – answer);

group (consideration of typical examples);

b) for complex assessment of the mastery of the syllabus material: individual handing in of the fulfilled work.

9. Methods of study

To intensify the process of teaching the academic discipline "Higher Mathematical" the following educational technologies are applied: problem lectures, minilectures, work in small groups, discussions, brainstorming, moderations, presentations, computer simulation (games), the Delphi method, the method of scenarios, banks of visual support (Table 9.1).

The basic difference of active and interactive methods of education from traditional ones is not only defined by the methods and techniques of teaching, but also by high effectivity of the educational process which reveals itself in: the high motivation of students; consolidation of theoretical knowledge in practice; raised students' conscientiousness; forming the ability to make independent decisions; forming the ability to approve collective decisions; forming the ability for social integration; getting the skills in resolving conflicts; development of the ability to reach compromise.

Problem lectures aim to develop students' logical thinking. The theme is confined to two or three key issues, students' attention is concentrated on the material which has not been represented in textbooks, the experience of foreign educational universities is used with handing out printed material to students during the lecture and drawing basic conclusions as to the issues considered. In the course of lectures students are asked questions for independent reflection, which the lecturer answers himself without waiting for students' answers. This kind of system makes students concentrate and begin to actively think in the search of a correct answer.

Table 9.1

Distribution of forms and methods of intensification of the educational process according to the themes of the academic discipline

| Theme | Practical application of educational technologies |
|--|---|
| 1 | 2 |
| Thematic module 1. The elements of mathematical analysis and linear algebra | |
| <i>Theme 1.</i> Limits of functions and continuity | A problem lecture on the theme: "Investigation of the continuity of different types of functions" |
| <i>Theme 2.</i> The differential calculus of the function of one variable | A minilecture on the theme: "Application of the differential to approximate calculations of the economic meaning" |

Table 9.1 (continuation)

| 1 | 2 |
|--|---|
| <i>Theme 3.</i> Analysis of the function of several variables | A minilecture on the theme: "Investigation of the conditional extremum of the functions of two variables with the help of the Lagrange factors". Presentation of the independent creative work |
| <i>Theme 4.</i> The indefinite and definite integral | A minilecture on the theme: "Some classes of functions, with integration reduced to rational fractions". A problem lecture on the theme: "Application of the definite integral to economic problems" |
| <i>Theme 5.</i> Differential equations | A problem lecture on the theme: "Mathematical modelling of the economic process with the help of ordinary and difference differential equations and systems" |
| <i>Theme 6.</i> Series | A minilecture on the theme: "Investigation of the convergence of series and calculation of a series sum". Work in small groups with discussion of the results of laboratory work. Presentation of the independent creative work |
| <i>Theme 7.</i> The elements of the theory of matrices and systems of linear algebraic equations | A minilecture on the theme: "Application of matrices to giving any information about the characteristics of the investigated economic process". A problem lecture on the theme: "Construction of an inverse matrix using Jordan – Gauss transformations" |
| <i>Theme 8.</i> The elements of vector algebra | A minilecture on the theme: "Application of vectors to the construction of economic problems". Work in small groups with discussion of the results of laboratory work |
| Thematic module 2 | |
| The elements of probability theory and mathematical statistics | |
| <i>Theme 9.</i> Empirical and logical bases of probability theory | A minilecture on the theme: "Formation of probability theory as a science". A mini-lecture on the theme: "Relationship between the theory of sets and Kolmogorov's axiomatics" |
| <i>Theme 10.</i> The scheme of independent trials | A minilecture on the theme: "Bernuolli and the classical definition of probability" |
| <i>Theme 11.</i> Random variables and their economic meaning | A minilecture on the theme: "Management decision-making under the conditions of risk". A problem lecture on the theme: "Current trends in the calculation of numerical characteristics of a random variable". Computer simulation: "Investigation of the influence of parameters of the random variable distribution on its distribution polygon" |
| <i>Theme 12.</i> Basic distribution laws of a continuous random variable | A minilecture on the theme: "The normal distribution law and the theory of errors" |

Table 9.1 (the end)

| 1 | 2 |
|---|--|
| <i>Theme 13.</i> Preprocessing of statistical data | A problem lecture on the theme: "Some differences between the theoretical definition of distribution characteristics and realization of calculation with the help of standard software" |
| <i>Theme 14.</i> Statistical estimation of the distribution parameters | A minilecture on the theme: "Comparative viewing of applied programs for statistical data processing" |
| <i>Theme 15.</i> Checking the statistical hypothesis | A minilecture on the theme: "Parametric and nonparametric statistical criteria" |
| <i>Theme 16.</i> The elements of the theory of correlation and regression | Application of visual support to the construction of empirical distribution of a two-dimensional random variable. A minilecture on the theme: "Application of regression models to the investigation of economic processes". Work in small groups for discussion of the results of laboratory work |

Minilectures provide for the delivery of the educational material during a short-length period of time and they are characterized by a significant content, complexity of logical constructions, forms, proofs and generalizations. They are conducted, as a rule, as a part of a study-investigation. Minilectures differ from full-size lectures in a shorter duration. Usually, they last no more than 10 – 15 minutes and they are used in order to briefly give new information to all students. Minilectures are often used as parts of a whole theme, which is desirable to teach as a full-size lecture in order to avoid the audience's getting tired. Then the information is given by turn as several particular fragments, with other forms and methods of study used between them.

Moderation is a way to conduct a discussion, which quickly leads to concrete results, gives a possibility for all present students to take part in the process of search for a solution to a problem and take full responsibility for the result. The function of the moderator is to see to it that the rules of the discussion are observed, which gives a possibility to simplify the process of the search for a solution without interfering in its essence.

The Delphi method is used for the purpose of reaching a consensus in expert judgements. It gives a possibility for students to express their thoughts to a group of experts, who work individually in different places. To choose a management decision according to this method, the academic group is divided, for example, into five small groups. Four groups work, develop and make a management decision, and the fifth group is the expert team. This

group carries out analysis of the variants of management decisions, which are proposed by the working groups, and assesses these variants. Within the expert group the distribution of its members according to specialities is fulfilled.

The method of scenarios is used for designing probabilistic models of behavior and development of concrete events in the long term.

Seminar-discussions provide for exchange of thoughts and ideas of students on the given theme and develop thinking, help to form ideas and beliefs, produce skills in formulating thoughts and expressing them, teach to assess other people's proposals, critically come to personal ideas.

Work in small groups gives an opportunity to structure practical studies in the form and content, gives a possibility for each student's partaking in the work on the theme under study, stimulates forming personal qualities and experience of social communication.

Brainstorming is a method of solving urgent tasks, its core lies in expressing as many ideas as possible in a short period of time, discussing and selecting them.

Presentations are speeches to students which are used for presenting certain achievements, group work results, reports of individual task fulfillment, instruction, demonstration of new goods and services.

10. Methods of control

The system of assessment of competences which were formulated by students during the learning of the academic discipline (Table 2.1), takes into consideration the forms of studies which, according to the syllabus of the academic discipline, provide lectures, practical studies, laboratory work, fulfillment of students' independent work. The assessment of the formed competences of students is carried out on the accumulative 100-point system. According to the temporary regulations "About the Order of Assessment of Students' Academic Performance on the Accumulative Point Rating System" of Simon Kuznets Kharkiv National University of Economics, control measures include:

current control which is carried out within a term during lectures, practical studies and laboratory work and assessed as a sum of cumulative points (the maximum equals 100 points; the minimum which makes it possible for a student to pass a test, equals 60 points);

module control which is carried out in the form of a colloquium with taking into account current control according to a corresponding thematic module, provides an integral assessment of student's results after learning the material of a logically completed part of the discipline (or a thematic module);

final/term control, which is carried out as a terminal exam, according to the schedule of the educational process.

Current control on the given academic discipline is carried out in the following forms:

active in-class work (lecture);

active in-class work (practical study);

active in-class work (laboratory study);

homework;

competence-oriented tasks (defence of laboratory work);

an independent test;

a written test;

independent creative work.

A colloquium is a form of reviewing and assessment of students' knowledge in the system of institutes of higher education. *The purpose* of carrying out a colloquium is to clarify the theoretical and practical knowledge obtained by a student as a result of listening to lectures, attendance of practical and laboratory studies and independent learning of the material. Within the bounds of the assigned purpose the following *tasks* are fulfilled: clarification of the quality and degree of student's understanding of the lecture material; the development and consolidation of students' skills in expressing thoughts; expansion of student's independent single-minded preparation; the development of skills in the generalization of different literary sources; giving a possibility for a student to compare different points of view on a given question. A colloquium is conducted as an intermediate miniexam on the initiative of a lecturer and includes theoretical questions and practical tasks on the academic discipline. The list of questions which are included into a colloquium on the themes of the thematic module, contains questions for self-assessment.

Final/term control is conducted in the form of a term exam. **Term exams** are a form of assessment of students' final mastery of the theoretical and practical material of a particular module of the academic discipline or the academic discipline on the whole, which is conducted as a test.

The order of conducting the current assessment of students' knowledge. Assessment of student's knowledge during practical studies and carrying out laboratory work is conducted on the accumulative system according to the following criteria:

understanding, the degree of the mastery of the theory and methodology of problems which are considered;

the degree of the mastery of the factual material of the academic discipline; familiarizing with the recommended literary sources and modern literature on the questions which are considered;

the ability to connect theory and practice in the consideration of particular examples, solving problems, carrying out laboratory work, carrying out calculations in the process of doing homework and tasks which are considered in class;

the logic, structure, style of presenting the material in written papers and in oral answers in class, the ability to ground their position, carry out generalization of the information and draw conclusions.

The maximal possible points which correspond to a particular task, are given on the condition that the solved problem or oral answer of a student correspond to all the defined criteria. Lack of one or another component decreases the number of accumulative points. The quality of fulfillment is also considered in the assessment of tasks set for independent work at laboratory and practical studies. Besides, handing in the performed task to the lecturer in accordance with the period defined by the schedule of the educational process plays an important role. If one of these conditions is not satisfied, the points are decreased.

A written test is carried out twice during a term and it includes practical tasks of different level of complexity according to the themes of the thematic module.

The criteria for assessment of the written test are as follows:

7 points – if the test has been carried out without mistakes and deficiencies, all the tasks contain the necessary explanations, illustrations, analysis of the results and conclusions;

6 points – if the test has been carried out, but there are no more than one mistake and no more than one deficiency or no more than three deficiencies;

5 points – if no less than 2/3 of the test has been carried out, there are no more than two mistakes and no more than two deficiencies;

4 points – if no less than 2/3 of the test has been carried out, there are more than two mistakes and more than two deficiencies;

3 points – if less than 2/3 of the test has been performed and the number of mistakes and deficiencies does not exceed the norm for the mark of three points;

2 points – if the fulfillment of the tasks has not been begun, but there is a particular correct thinking;

1 point – if the fulfillment of the tasks has not been begun, but there is no particular correct thinking;

0 point – if the task is unavailable.

Revision marking of the competence-oriented tasks (defence of laboratory work on the themes which are combined into a corresponding thematic module) is carried out twice during a term in the form of work in small groups. Besides, the quality of fulfillment of the tasks for laboratory work and the ability to present the results of investigations, give reasonable answers to the questions of opponents, think critically, assess the results of the work of other participants must be assessed.

A colloquium is carried out twice during a term in the written form or in the form of an oral test for controlling students' knowledge of the theoretical material and the mastery of the categorical apparatus.

The criteria for assessment of a colloquium:

8 points – if deep knowledge of the syllabus material has been demonstrated, a sequential, complete and logical answer has been given, a correct decision has been made, the mastery of different methods and techniques in carrying out practical tasks has been demonstrated;

6 points – if knowledge of the syllabus material has been demonstrated, an answer without essential inaccuracies has been given, mastery of the necessary methods in carrying out practical tasks has been demonstrated;

4 points – if knowledge of the basic material has been demonstrated, an answer with inaccuracies has been given, mastery of the necessary methods in carrying out practical tasks has been demonstrated;

3 points – if knowledge of the basic material has been demonstrated, an answer with inaccuracies and quite incorrect formulations has been given, mistakes have been made in the use of the necessary methods in carrying out practical tasks;

2 points – if knowledge of the basic material has not been demonstrated, an answer with essential mistakes and incorrect formulations has been given, lack of skills in the use of the necessary methods in carrying out practical tasks has been demonstrated;

1 point – if an incorrect solution has been given, the fulfillment of the colloquium practical tasks has not been begun, but there has been shown some particular correct thinking;

0 point – if the task is unavailable.

The criteria for assessment of independent work of students.

The general criteria for the assessment of independent work of students are profound and deep knowledge, the level of thinking, skills in systematization of knowledge on particular themes, skills in drawing conclusions, attainments and techniques of carrying out practical tasks, the ability to find, classify and process necessary information, self-realization in practical and laboratory studies.

The criteria for assessment of independent creative work and independent tests are:

the ability to carry out a critical and independent estimation of the defined problems;

skills in the explanation of alternative views and availability of a students' own point of view, position on the defined problem;

using the analytical approach; the quality and accuracy of expressing the thought;

the logic, structure and explanation of conclusions about a particular problem;

independence of carrying out the work;

grammatical correctness of the presentation of the material; using the methods of comparison, generalization of the concepts and facts;

the design of the work;

the quality of presentation.

The order of the final control on the academic discipline. The final control of students' knowledge and competences on the academic discipline is carried out according to the temporary regulations "About the Order of Assessment of Students' Academic Performance on the Accumulative Point Rating System" of Simon Kuznets Kharkiv National University of Economics.

A student, who for a valid reason, attested documentally, hasn't had a possibility to take part in the forms of current control, that is, hasn't passed the thematic module, has the right to complete it during two weeks after coming back to studies according to the notice of the dean of the department subject to a given period.

The final mark on the academic discipline is calculated according to the points obtained during the current control on the accumulative system.

The student's progress is assessed if the number of points obtained as a general result of all forms of control equals or exceeds 60.

11. The distribution of points which students obtain

An example of the technological chart of accumulative rating points and the system of assessment of professional competences which a full-time student has to form, is given in Tables 11.1 and 11.2 according to the forms of study and methods of control used in teaching the academic discipline. The distribution of points according to the themes of thematic modules is given in Table 11.3.

Table 11.1

The system of assessment of the professional competences formed

| Professional competences | Educational week | Hours | Forms of study | | Assessment of the level of the formed competences | | | |
|---|--|-------|--------------------------------|---|---|---|---|-----|
| | | | | | Forms of control | Maximal point | | |
| 1 | 2 | 3 | 4 | | 5 | 6 | | |
| Thematic module 1 | | | | | | | | |
| The elements of mathematical analysis and linear algebra | | | | | | 43.5 | | |
| AMI 1 | The ability to calculate and understand the meaning of the limit and use the derivative in economics | 2 | Cla | 2 | Lecture | <i>Theme 1.</i> Limits of functions and continuity | Active class work | 0.5 |
| | | | IWS | 4 | Preparation for studies | Search, choice and looking through literary sources on the theme. Learning the lecture material and preparation for practical studies | There is no control of independent work | – |
| | 3 | Class | 2 | Lecture | <i>Theme 2.</i> The differential calculus of the function of one variable | Active class work | 0.5 | |
| | | | 2 | Practical study | The differential calculus of the function of one variable and applying it | Active class work | 0.5 | |
| | IWS | 4 | Preparation for studies | Learning the lecture material. Preparation for practical studies. Carrying out homework and independent work. Search, choice and looking through literary sources for independent creative work | There is no control of independent work | – | | |

Table 11.1 (continuation)

| 1 | 2 | 3 | 4 | | 5 | 6 | | |
|-------|--|---|-------|---|--------------------------------|--|-------------------|-----|
| AMI 1 | The ability to use the function of several variables | 4 | Class | 2 | Lecture | <i>Theme 3.</i> Analysis of the function of several variables | Active class work | 0.5 |
| | | | | 2 | Practical study | Investigation of functions of two variables | Active class work | 0.5 |
| | | | IWS | 4 | Preparation for studies | Search, choice and looking through literary sources on the theme. Carrying out homework and independent work | Homework | 1 |
| AMI 2 | The ability to calculate integrals | 5 | Class | 2 | Lecture | <i>Theme 4.</i> The indefinite and definite integrals | Active class work | 0.5 |
| | | | | 2 | Practical study | Integration. Calculation of definite integrals | Active class work | 0.5 |
| | | | IWS | 4 | Preparation for studies | Learning the lecture material. Preparation for practical studies. Carrying out homework and independent work | Homework | 1 |
| AMI 3 | The ability to solve differential equations | 6 | Class | 2 | Lecture | <i>Theme 5.</i> Differential equations | Active class work | 0.5 |
| | | | | 2 | Practical study | Solving differential equations of the first and second order | Active class work | 0.5 |
| | | | IWS | 6 | Preparation for studies | Learning the lecture material. Carrying out practical homework and independent work | Homework | 1 |
| AMI 4 | The ability to use the methods of series investigation | 7 | Class | 2 | Lecture | <i>Theme 6.</i> Series | Active class work | 0.5 |
| | | | | 2 | Practical study | Solving practical tasks: investigation of convergence of numerical series, finding the convergence domain of a power series | Active class work | 0.5 |
| | | | IWS | 6 | Preparation for studies | Search, choice and looking through literary sources on the theme. Carrying out practical homework and tasks of independent test | Homework | 1 |

Table 11.1 (continuation)

| 1 | 2 | 3 | 4 | | 5 | 6 |
|--|---|-------|---|--|--|---------|
| AMI 5 The ability to use matrices for analysis of data in economics | 8 | Class | 2 | Lecture <i>Theme 7.</i> The elements of the theory of matrices and systems of linear algebraic equations | Active class work | 0.5 |
| | | | 2 | Practical study Solving practical tasks: operations with matrices, calculation of determinants, solving systems of linear algebraic equations | Active class work. Written test | 0.5 + 7 |
| | | IWS | 4 | Preparation for studies Search, choice and looking through literary sources on the theme. Carrying out practical homework. Preparation for a written test and a colloquium | Homework. Independent test | 1 + 7 |
| AMI 6 The ability to carry out basic operations with vectors | 9 | Class | 2 | Lecture <i>Theme 8.</i> The elements of vector algebra | Active class work. Colloquium | 0.5 + 8 |
| | | | 2 | Laboratory study Solving practical tasks of linear algebra in <i>MS Excel</i> | Active class work | 0.5 |
| | | | 2 | Practical study Solving practical tasks of vector algebra | Active class work. Competence oriented task | 0.5 + 7 |
| | | IWS | 4 | Preparation for studies Search, choice and looking through literary sources on the theme. Carrying out practical homework. Preparation for a colloquium | Homework. Independent test | 1 + 7 |

Table 11.1 (continuation)

| 1 | 2 | 3 | 4 | | 5 | 6 | | |
|---|--|----|--------------------------------|---|-------------------------|---|-------------------|-----|
| Thematic module 2 | | | | | | 56.5 | | |
| The elements of probability theory and mathematical statistics | | | | | | | | |
| AMI 7 | The ability to use basic definitions of probability theory, theorems of multiplication and addition of probabilities | 10 | Class | 2 | Lecture | <i>Theme 9.</i> Empirical and logical bases of probability theory | Active class work | 0.5 |
| | | | | 2 | Laboratory study | Calculation of the probability of random events using theorems of multiplication and addition of probabilities. Using the total probability formula and Bayes' formula | Active class work | 1 |
| | | | | 2 | Practical study | A classical definition of the probability of a random event. Using theorems of multiplication and addition of probabilities, the total probability formula and Bayes' formula | Active class work | 0.5 |
| | | 4 | Preparation for studies | Search, choice and looking through literary sources on the theme. Carrying out practical homework and independent work. | Homework | 1 | | |
| | The ability to use basic formulas of Bernoulli's scheme | 11 | Class | 2 | Lecture | <i>Theme 10.</i> The scheme of independent trials | Active class work | 0.5 |
| | | | | 2 | Practical study | Using the total probability formula and Bayes' formula | Active class work | 0.5 |
| | | | | 2 | Laboratory study | Calculation of the probabilities using a model of repeating trials according to Bernoulli's scheme. The theorems of Moivre – Laplace and Poisson | Active class work | 1 |
| | | 4 | Preparation for studies | Search, choice and looking through literary sources on the theme. Carrying out homework | – | – | | |

Table 11.1 (continuation)

| 1 | 2 | 3 | 4 | | 5 | 6 | | |
|-------|--|-----|-------|--------------------------------|--|--|-------------------|-----|
| AMI 7 | The ability to defied distribution laws of a discrete random variable, calculate their basic numerical characteristics | 12 | Class | 2 | Lecture | <i>Theme 11.</i> Random variables and their economic meaning | Active class work | 0.5 |
| | | | | 2 | Practical study | Construction of a distribution function. Calculation of basic and additional characteristics of a random variable | Active class work | 0.5 |
| | | | | 2 | Laboratory study | Calculation of a discrete random variable. Construction of the distribution law of the sum or difference of random variables | Active class work | 1 |
| | | IWS | 4 | Preparation for studies | Learning the lecture material. Carrying out practical homework and the independent creative task | Homework | 1 | |
| | The ability to define distribution laws of a discrete random variable, calculate their basic numerical characteristics | 13 | Class | 2 | Lecture | <i>Theme 12.</i> Basic distribution laws of a continuous random variable | Active class work | 0.5 |
| | | | | 2 | Laboratory study | Calculation of numerical characteristics of a continuous random variable and numerical characteristics of basic distribution law | Active class work | 1 |
| | | | IWS | 4 | Preparation for studies | Learning the lecture material. Preparation for practical studies. Carrying out homework and tasks of the independent test | – | – |

Table 11.1 (continuation)

| 1 | 2 | 3 | 4 | | 5 | 6 | | | |
|-------|---|-----|-------|--------------------------------|--|--|---|---|-------|
| AMI 8 | The ability to form a sample and calculate characteristics of an empirical distribution | 14 | Class | 2 | Lecture | <i>Theme 13.</i> Preprocessing of statistical data | Active class work | 0.5 | |
| | | | | 2 | Practical study | Construction of a statistical series. Calculation of numerical characteristics of empirical distribution | Active class work | 0.5 | |
| | | | | 2 | Laboratory study | Construction of a continuous statistical series, presenting it with the help of a histogram and a polygon | Active class work | 1 | |
| | The ability to find interval estimations | 15 | IWS | 6 | Preparation for studies | Learning the lecture material. Preparation for practical studies. Carrying out homework and tasks of independent work. Preparation for the defence of the competence-oriented task | Homework | 1 | |
| | | | | Class | 2 | Lecture | <i>Theme 14.</i> Statistical estimation of the distribution parameters | Active class work | 0.5 |
| | | | | | 2 | Laboratory study | Finding point and interval estimations of basic numerical characteristics of a distribution | Active class work. Competence oriented task | 1 + 7 |
| | | IWS | 6 | Preparation for studies | Learning the lecture material. Preparation for practical studies. Carrying out homework and tasks of the independent work. Preparation for a written test and a colloquium | – | – | | |

Table 11.1 (the end)

| 1 | 2 | 3 | 4 | | 5 | 6 | |
|---------------------------|---|------------|--|--------------------------------|---|---|---------|
| AMI 8 | The ability to carry out checking the statistical hypothesis | Class | 2 | Lecture | <i>Theme 15.</i> Checking the statistical hypothesis | Active class work. Colloquium | 0.5 + 8 |
| | | | 2 | Practical study | Checking the statistical hypothesis | Active class work. Written test | 0.5 + 7 |
| | | | 2 | Laboratory study | Checking the statistical hypothesis relative to the correspondence of the distribution law in a population to a define type | Active class work | 1 |
| | | IWS | 6 | Preparation for studies | Learning the lecture material. Preparation for practical studies. Carrying out homework | Homework. Independence work | 1 + 7 |
| AMI 9 | The ability to use methods of checking the significance of parameters of a model of a pair regression | Class | 2 | Lecture | <i>Theme 16.</i> The elements of the theory of correlation and regression | Active class work. The independent creative task | 0.5 + 8 |
| | | | 2 | Laboratory study | Construction of an empirical equation of regression and the confidence interval of a regression line with the help of built-in functions of <i>MS Excel</i> | Active class work | 1 |
| | | IWS | 6 | Preparation for studies | Forming a report of carrying out independent creative work | – | – |
| Total sum of hours | | 160 | Total maximal number of points for the discipline | | | 100 | |
| including | | | | | | | |
| <i>Class</i> | | 74 | 45 % | | | | |
| <i>Independent work</i> | | 86 | 55 % | <i>total control</i> | | 100 | |

Table 11.2

The distribution of points according to the themes

| Current testing and independent work | | | | | | | | | | | | | | | Sum |
|--------------------------------------|----|----|----|----|----|----|----|------------------------------|-----|-----|-----|-----|-----|-----|-----|
| Thematic module 1 | | | | | | | | Thematic module 2 | | | | | | | |
| T1 | T2 | T3 | T4 | T5 | T6 | T7 | T8 | T9 | T10 | T11 | T12 | T13 | T14 | T15 | T16 |
| 1 | 2 | 2 | 2 | 2 | 2 | 2 | 3 | 3 | 1.5 | 3 | 1.5 | 3 | 1.5 | 3 | 1.5 |
| Written test – 7 | | | | | | | | Written test – 7 | | | | | | | 100 |
| Independent test – 7 | | | | | | | | Independent test – 7 | | | | | | | |
| Competence oriented task – 7 | | | | | | | | Competence oriented task – 7 | | | | | | | |
| Colloquium – 8 | | | | | | | | Colloquium – 8 | | | | | | | |
| Independent creative work – 8 | | | | | | | | | | | | | | | |

Note. T1, T2, ..., T16 are themes of thematic modules.

Table 11.3

The distribution of points across weeks

| Themes of the thematic module | | | Lectures | Practical study | Laboratory study | Homework | Competence oriented task | Independent test | Written test | Independent creative work | Colloquium | Total |
|---|---------|--------|----------|-----------------|------------------|----------|--------------------------|------------------|--------------|---------------------------|------------|-------|
| 1 | | | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| Thematic module 1 The elements of mathematical analysis and linear algebra | Theme 1 | Week 2 | 0.5 | - | - | - | - | - | - | - | - | 0.5 |
| | Theme 2 | Week 3 | 0.5 | 0.5 | - | - | - | - | - | - | - | 1 |
| | Theme 3 | Week 4 | 0.5 | 0.5 | - | 1 | - | - | - | - | - | 2 |
| | Theme 4 | Week 5 | 0.5 | 0.5 | - | 1 | - | - | - | - | - | 2 |
| | Theme 5 | Week 6 | 0.5 | 0.5 | - | 1 | - | - | - | - | - | 2 |
| | Theme 6 | Week 7 | 0.5 | 0.5 | - | 1 | - | - | - | - | - | 2 |
| | Theme 7 | Week 8 | 0.5 | 0.5 | - | 1 | - | 7 | 7 | - | - | 16 |
| | Theme 8 | Week 9 | 0.5 | 0.5 | - | 1 | 7 | - | - | - | 8 | 18 |

Table 11.3 (the end)

| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|----------|---------|----------|----------|----------|-----------|-----------|-----------|-----------|----------|-----------|------------|
| Thematic module 2 The elements of probability theory and mathematical statistics | Theme 9 | Week 10 | 0.5 | 0.5 | 1 | 1 | – | – | – | – | – | 3 |
| | Theme 10 | Week 11 | 0.5 | 0.5 | 1 | 1 | – | – | – | – | – | 3 |
| | Theme 11 | Week 12 | 0.5 | 0.5 | 1 | 1 | – | – | – | – | – | 3 |
| | Theme 12 | Week 13 | 0.5 | – | 1 | – | – | – | – | – | – | 1.5 |
| | Theme 13 | Week 14 | 0.5 | 0.5 | 1 | 1 | – | – | – | – | – | 3 |
| | Theme 14 | Week 15 | 0.5 | – | 1 | – | 7 | – | – | – | – | 8.5 |
| | Theme 15 | Week 16 | 0.5 | 0.5 | 1 | 1 | – | 7 | 7 | – | 8 | 25 |
| | Theme 16 | Week 17 | 0.5 | – | 1 | – | – | – | – | 8 | – | 9.5 |
| Total | | | 8 | 6 | 9 | 11 | 14 | 14 | 14 | 8 | 16 | 100 |

The maximum number of points which a student can accumulate during a week according to the forms and methods of study and control is given in Table 11.3.

The final mark on the academic discipline is defined according to the temporary regulations "About the Order of Assessment of Students' Academic Performance on the Accumulative Point Rating System" of Simon Kuznets Kharkiv National University of Economics (Table 11.4).

Marks according to this scale are entered in the mark sheets, the individual educational plan of a student and other academic documents.

Table 11.4

The scales of assessment: national and ECTS

| Sum of points including all forms of study | Mark on the ECTS scale | Mark on the national scale | |
|--|------------------------|-------------------------------------|------------|
| | | for an exam, a term paper, practice | for a test |
| 90 – 100 | A | excellent | passed |
| 82 – 89 | B | very good | |
| 74 – 81 | C | good | |
| 64 – 73 | D | satisfactory | |
| 60 – 63 | E | | |
| 35 – 59 | FX | unsatisfactory | failed |
| 1 – 34 | F | | |

12. Recommended reading

12.1. Main

1. Вища математика : базовий підручник для вузів / під ред. В. С. Пономаренка. – Харків : Фоліо, 2014. – 669 с.
2. Збірник вправ з розділу "Теорія ймовірностей та математична статистика" навчальної дисципліни "Математика для економістів" для студентів галузі знань "Економіка і підприємництво" усіх форм навчання / уклад. Е. Ю. Железнякова, А. В. Ігначкова, З. Г. Попова та ін. – Харків : Вид. ХНЕУ, 2009. – 116 с.
3. Лабораторний практикум із розділу "Теорія ймовірностей та математична статистика" навчальної дисципліни "Математика для економістів" : навч.-практ. посіб. / І. Л. Лебедева, О. О. Єгоршин, Е. Ю. Железнякова та ін. – Харків : Вид. ХНЕУ, 2009. – 116 с.
4. Малярець Л. М. Вища математика для економістів у прикладах, вправах і задачах : навч. посіб. / Л. М. Малярець, А. В. Ігначкова. – Харків : ВД "ІНЖЕК", 2006. – 544 с.
5. Малярець Л. М. Математика для економістів : практик. посіб. до розв'язання задач / Л. М. Малярець, Л. Д. Широкоград. – Харків : Вид. ХНЕУ, 2008. – 476 с.
6. Малярець Л. М. Математика для економістів : навч. посіб. / Л. М. Малярець ; під ред. Л. М. Малярець. – Харків : Вид. ХНЕУ, 2011. – 568 с.
7. Малярець Л. М. Математика для економістів : навч. посіб. У 2-х ч. Ч. 1 / Л. М. Малярець, Л. М. Афанасьєва, А. В. Ігначкова. – Харків : Вид. ХНЕУ, 2011. – 393 с.
8. Малярець Л. М. Математика для економістів : навч. посіб. У 2-х ч. Ч. 2 / Л. М. Малярець, Л. М. Афанасьєва, А. В. Ігначкова. – Харків : Вид. ХНЕУ, 2011. – 368 с.
9. Малярець Л. М. Математика для економістів. Теорія ймовірностей та математична статистика : навч. посіб. У 3-х ч. Ч. 3 / Л. М. Малярець, І. Л. Лебедева, Л. Д. Широкоград – Харків : Вид. ХНЕУ, 2011. – 568 с.
10. Малярець Л. М. Практикум з теорії ймовірностей та математичної статистики в Excel : навч.-практ. посіб. / Л. М. Малярець, І. Л. Лебедева, Е. Ю. Железнякова. – Харків : Вид. ХНЕУ, 2007. – 160 с.
11. Малярець Л. М. Математика для економістів : практик. посіб. до розв'язання задач економічних досліджень в MatLab / Л. М. Малярець, Є. В. Резнік, О. Г. Тижненко. – Харків : Вид. ХНЕУ, 2008. – 212 с.

12. Малярець Л. М. Теория вероятностей и математическая статистика в примерах и задачах. Учебное пособие для студентов-иностранцев отрасли знаний 0305 "Экономика и предпринимательство" / Л. М. Малярець, Е. Ю. Железнякова, А. В. Игначкова. – Харків : ХНЕУ. – 2012. – 124 с.

13. Теорія ймовірностей та математична статистика : навч. посіб. / Л. М. Малярець, І. Л. Лебедева, Е. Ю. Железнякова та ін. – Харків : Вид. ХНЕУ, 2010. – 404 с.

12.2. Additional

14. Барковський В. В. Математика для економістів : навч. посіб. / В. В. Барковський, Н. В. Барковська. – Київ : НАУ, 1999. – 448 с.

15. Барковський В. В. Теорія ймовірностей та математична статистика / В. В. Барковський, Н. В. Барковська, О. К. Лопатін. – 5-е вид. – Київ : Центр учбової літератури, 2010. – 424 с.

16. Вища математика для економістів : підручник / під ред. О. І. Ляшенка, О. І. Черняка. – Київ : Видавничо-поліграфічний центр "Київський університет", 2008. – 497 с.

17. Высшая математика для экономистов / под ред. Н. Ш. Кремера. – Москва : ЮНИТИ, 2002. – 440 с.

18. Гмурман В. Е. Руководство по решению задач по теории вероятностей и математической статистике / В. Е. Гмурман. – Москва : Высшая школа, 2001. – 576 с.

19. Гмурман В. Е. Теория вероятностей и математическая статистика : учеб. пособ. для вузов / В. Е. Гмурман. – 6-е изд. – Москва : Высшая школа, 1998. – 480 с.

20. Данко П. Е. Высшая математика в упражнениях и задачах. У 2-х ч. Ч. 1 / П. Е. Данко, А. Г. Попов, Т. Я. Кожевникова. – Москва : Высшая школа, 2003. – 304 с.

21. Данко П. Е. Высшая математика в упражнениях и задачах. У 2-х ч. Ч. 2 / П. Е. Данко, А. Г. Попов, Т. Я. Кожевникова. – Москва : Высшая школа, 2003. – 416 с.

22. Ермаков В. И. Общий курс высшей математики для экономистов / В. И. Ермаков, Б. М. Рудык, Р. К. Гринцевичюс. – Москва : ИНФРА-М, 2007. – 657 с.

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Appendices

Appendix A

Table A.1

The structure of components of professional competences formed in mastering the academic discipline "Higher Mathematics" according to Ukraine's National Scale of Qualifications

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| Competence formed within the theme | Minimal experience | Knowledge | Skills and abilities | Communication | Autonomy and responsibility |
|--|--|---|---|---|--|
| 1 | 2 | 3 | 4 | 5 | 6 |
| Theme 1. Limits of functions and continuity | | | | | |
| Forming analytic thinking, the ability to explain the importance of complicated expressions with the help of mathematical symbols and operations | Availability of basic knowledge of the theme: forms of writing and ways to define the function, the range of values, the domain of the definition, periodicity, evenness and so on | The definition of basic classes of functions, basic methods and ways of investigation of the function | 1. The ability to define the type of function by its analytic recording. 2. Plotting a graph of the function with the help of elementary mathematical calculations and transformations | Construction of mathematical models of economic processes using various production functions (the total input of production, profit, supply and demand and so on) | A student must 1) investigate the function and independently carry out the analysis of the obtained results; 2) give examples of functional dependence |

Table A.1 (continuation)

| 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|--|--|---|--|
| Theme 2. The differential calculus of the function of one variable | | | | | |
| Development of the ability to solve problems with the help of the methods of differential calculus using mathematical symbolic variables, i.e. forming initial skills in economic modelling | <p>1. Knowledge of the notions: the limit of the function, the limit of a numerical sequence.</p> <p>2. Attainment of basic theorems of the limit of the function.</p> <p>3. The ability to calculate derivatives of elementary functions</p> | <p>1. The notion of the derivative, the economic, geometric and mechanical meanings of the notion, basic theorems of the theme.</p> <p>2. The notion of the differential of the function of one variable.</p> <p>3. The notions of the function, the derivative of the function, the definition of the points of the extremum, monotonicity intervals, concavity and convexity</p> | <p>1. The ability to calculate derivatives of elementary and composite functions.</p> <p>2. The ability to calculate derivatives, the differential of the function of one variable.</p> <p>3. The ability to investigate the function in detail with the help of the acquired knowledge of the corresponding themes.</p> <p>4. The ability to predict the behavior of the function and plot its graph with the help of basic mathematical calculations</p> | Understanding economic processes and analysis of them with the help of differential calculus methods. Representation of the results of investigation of functions | <p>A student must:</p> <p>1) calculate derivatives of elementary and composite functions;</p> <p>2) be able to use the differential of the function for approximate calculus;</p> <p>3) investigate the function with the help of differential calculus;</p> <p>4) carry out the simplest calculations by the optimization of production;</p> <p>5) draw corresponding conclusions and independently analyze the obtained solution</p> |

Table A.1 (continuation)

| 1 | 2 | 3 | 4 | 5 | 6 |
|---|--|---|--|--|--|
| Theme 3. Analysis of the function of several variables | | | | | |
| Forming the skills in the use of previous experience (the function of one variable) for further use in a more complex situation | Mastering the notion of the function of several variables in the simplest form: the function of two variables | Analytic recording of the function of several variables, definitions of the range of values and the domain of the definition, understanding partial derivatives | Obtaining the skills in the calculation of derivatives of the function of two variables and finding the domain of the definition | Using the function of several variables in economics for description of the investigated processes and effects | A student must be able: 1) to find partial and mixed derivatives; 2) to investigate the local extremum of the function; 3) to use the method of Lagrange multipliers and the least-squares method |
| Theme 4. The indefinite and definite integral | | | | | |
| 1. Understanding the possibility of using the integral calculus for solving applied problems. 2. Forming the skills in independent formation of mathematical models for description of different processes | Attainment of the table of integrals. The ability to calculate the simplest indefinite integrals which are directly reduced to the tabular form | The definition of the type of integral relative to its integrand, attainment of more typical changes | Obtaining the skills in the calculation of the simplest integrals and reducing more composite integrals to the tabular form | Solving economic problems using the apparatus of integral calculus | A student must be able: 1) to calculate indefinite integrals; 2) to draw corresponding conclusions and independently analyze the obtained results |

Table A.1 (continuation)

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| 1 | 2 | 3 | 4 | 5 | 6 |
|--|---|---|--|--|--|
| Forming the skills in independent analysis and understanding the importance of the relationship between the examined material (the definite and indefinite integral) | The ability to calculate the simplest definite integrals which are directly reduced to the tabular form, using the Newton – Leibnitz formula | Using the definite integral for calculation of areas and volumes | Obtaining skills in the calculation of definite integrals, areas of figures and volumes of solids of revolution | Solving economic problems using the apparatus of integral calculus | A student must be able: 1) to calculate definite integrals; 2) to use definite integrals for independent calculation of areas and volumes of figures; 3) to draw corresponding conclusions and independently analyze the obtained results |
| Theme 5. Differential equations | | | | | |
| Forming an inclination to independent search of different ways of solving problems and understanding the necessity to use knowledge of other themes (the function, the derivative, the integral) | 1. The ability to differentiate and integrate functions, find the derivative of a composite function. 2. The ability to solve the simplest differential equations of the first and second orders | 1. Basic ways to solve a differential equation of the first order. 2. Basic ways to solve a differential equation of the second order with constant coefficients | 1. Skills in the calculation of the basic types of differential equations of the first order. 2. Skills in the calculation of the basic types of differential equations of the second order | Solving economic problems with differential equations as mathematical models, i.e. finding antiderivatives, the elasticity of the function and so on | A student must be able: 1) to calculate the type of the differential equation, the method of further independent solution; 2) to use the knowledge for solving the simplest economic problems |

Table A.1 (continuation)

| 1 | 2 | 3 | 4 | 5 | 6 |
|---|--|---|---|---|---|
| Theme 6. Series | | | | | |
| Forming the ability to do analytic calculations | The ability to calculate limits using basic signs of convergence of series | Basic signs of convergence of series, the notion of a power series, alternating series | Skills relative to analytic calculations the using knowledge of the theme | Investigation of the convergence of numerical series, understanding the problem of the convergence of series and using series in approximate calculations | A student must be able: 1) to calculate the type of series; 2) to independently investigate the convergence of series; 3) to find the convergence radius of power series |
| Theme 7. The elements of the theory of matrices and systems of linear algebraic equations | | | | | |
| 1. Forming the skills in the use of the instrument of matrix calculus for modelling the simplest economic problems and situations. 2. The ability to analyze the results of calculations from the mathematical and practical viewpoint | 1. Carrying out the simplest mathematical calculations with the determinants of the second, third and n th orders. 2. Carrying out the simplest mathematical calculations (addition, subtraction, multiplication) | 1. Attainment of the knowledge of mathematical symbols, basic definitions and theorems of the theme. 2. Attainment of the knowledge of basic properties of determinants and basic notions of the theme | 1. The ability to calculate the simplest determinants of matrices of the second and third orders with the help of basic methods and properties. 2. The ability to use the basic matrix operation | Using the available methods of work with numerical data, their representation in the matrix form and carrying out operations with them | A student must be able: 1) to use the instrument of matrix algebra for economic problems; 2) to model the simplest situations with the help of the knowledge of the theme |

Table A.1 (continuation)

| 1 | 2 | 3 | 4 | 5 | 6 |
|--|--|--|---|--|---|
| Forming the ability to independently prove the simplest statements with the help of elementary mathematical knowledge | Using and attainment of the knowledge of basic methods of solving the simplest systems of linear equations | Attainment of the knowledge of basic theorems and rules of solving systems of linear equations (the Cramer method, the inverse matrix method, the Jordan – Gauss method) | Solving matrix equations. Solving the systems with the help of matrices and determinants | Using the methods of solving the systems of linear equations with matrices of an arbitrary dimension | A student must know the basic proofs and theorems of the theme and give examples of using determinants, matrices and systems of linear equations in economics |
| Theme 8. The elements of vector algebra | | | | | |
| Forming analytical thinking, the ability to explain the importance of complicated expressions with the help of mathematical symbols and operations | The Cartesian system of coordinates on the plane and in the space. The notion of the vector, elementary operations with vectors | Basic operations and properties of vectors; scalar, cross and mixed products | The ability to use basic linear operations with vectors, the independence and dependence of vectors | Making a geometric presentation of economic problems | A student must be able to use vector algebra for the calculation of the simplest problems of applied character (finding the area, the volume) |

Table A.1 (continuation)

| 1 | 2 | 3 | 4 | 5 | 6 |
|---|--|---|--|---|--|
| Theme 9. Empirical and logical bases of probability theory | | | | | |
| Theme 10. The scheme of independent trials | | | | | |
| <p>1. Skills in the independent calculation of basic notions of probability theory, combinatorial analysis and mathematical statistics.</p> <p>2. Independent carrying out and analysis of the obtained problem.</p> <p>3. Definition of the type or class.</p> <p>4. Forming possible directions of solving a problem on the basis of modern scientific attainments.</p> <p>5. Using basic methods of probability theory for solving a</p> | <p>1. Availability of the basic knowledge of the course of a secondary educational institution or institution of the first and the second level of accreditation.</p> <p>2. Carrying out simplest mathematical calculations; solving simplest combinatoric problems.</p> <p>3. Understanding the notions: a probability, a probability of dependent or independent events; a definition of a complete group of events; the notion of a conditional probability</p> | <p>1. Revision of the material of the academic discipline.</p> <p>2. Knowledge of mathematical symbols and basic definitions and theorems on the theme.</p> <p>3. The formula of the total probability, the formula of Bayes, the notion of Bayes' approach.</p> <p>4. The theorem of Bernoulli, theorems of Moivre – Laplace, Poisson, limits and conditions of using them</p> | <p>1. Completing the course of mathematics by basic notions.</p> <p>2. The ability to define a problem by type, to solve a task independently with the help of basic theorems of probability theory.</p> <p>3. The ability to solve problems with the help of the formula of the total probability and the formula of Bayes; to distinguish between dependent and independent events.</p> <p>4. The ability to define the type of problem, to reasonably use the corresponding theorem</p> | <p>Understanding the role and place of probability theory and mathematical statistics in modern scientific researches and their significance in further solving professional problems</p> | <p>A student must</p> <p>1) know the basic theorems and statements of the theme;</p> <p>2) give examples of using basic notions of probability theory and use them for solving different problems;</p> <p>3) develop and improve the ability to reasonably construct an optimal analytic solution to a problem</p> |

Table A.1 (continuation)

| 1 | 2 | 3 | 4 | 5 | 6 |
|---|--|---|---|--|---|
| composite problem and reducing it to simple problems | 4. Understanding the notions of repeated trials, an independent event, revision of formulas of combinatorial theory | | to independently explain and analyze the obtained solution | | |
| Theme 11. Random variables and their economic meaning | | | | | |
| Theme 12. Basic distribution laws of a continuous random variable | | | | | |
| Using the laws of distribution (discrete and continuous) random variables, reasoning suitability and necessity of using them; prediction of processes which will occur in real economic models on the basis of knowledge of the theme | 1. Understanding the notion of a discrete random variable, series of distribution, a mean (a mathematical expectation), a variance, a root-mean-square deviation. 2. Understanding the notion of a continuous random variable, a function and the density of distribution | 1. The basic notions: series of distribution, the numerical characteristics, the relationship between them, graphic interpretation. 2. The basic notions: the function of distribution, the density of distribution, the relationship between them, graphic interpretation | 1. The ability to calculate the numerical characteristics of a discrete random variable and analyze the obtained result. 2. The ability to calculate the numerical characteristics of a continuous random variable and analyze the obtained result | Solving economic problems with the use of the apparatus of probability theory and random variables | A student must be able: 1) to find the numerical characteristics of a random variable by the given law of distribution; 2) to calculate the probability of lying in the given interval, estimate the obtained result, give it a corresponding economic or mathematical explanation; |

Table A.1 (continuation)

| 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|--|---|---|
| | | | | | 3) to plot a polygon, a histogram, graphs of an integral function and the density of a probability |
| Theme 13. Preprocessing of statistical data Theme 14. Statistical estimation of the distribution parameters Theme 15. Checking the statistical hypothesis | | | | | |
| 94 Attainment of theoretical and probabilistic fundamentals of mathematical statistics, understanding the role of basic mathematical assumptions when making a statement and solving statistical problems | Availability of basic knowledge of probability theory | Basic fundamentals of the sampling method, distribution, statistical estimations and their properties | Attainment of basic fundamentals of the sampling method and basic methods of modern statistical researches | Understanding the role and place of mathematical statistics in modern scientific researches and its significance in further solving professional problems | A student must 1) know basic notions, definitions, types of basic mathematical and statistical calculations, basic models and methods of statistical researches; 2) be able to systematize the obtained data in the form of a table or presentation of data |

Table A.1 (continuation)

| 1 | 2 | 3 | 4 | 5 | 6 |
|--|--|---|---|---|--|
| Theme 16. The elements of the theory of correlation and regression | | | | | |
| 1. Attainment of skills in using correlation analysis for defining the influence of the factor-argument on the functional factor. | Knowledge of the problems of correlation analysis, definition of the sample coefficient of correlation and the method of estimation. Understanding the meaning of economic values, which are included in the pair regression model | Knowledge of the methods of defining point estimations of basic numerical characteristics of a two-dimensional random variable using sampling data. | The ability to use built-in functions and the superstructure of <i>MS Excel</i> for estimation of numerical characteristics of a two-dimensional random variable and for construction of an equation of a pair linear regression, carry out a prognosis using this equation and define its accuracy | Presentation and discussion of the results of statistical investigations into the strength of the correlation. The ability to present and reasonably give results of the application of regression analysis to the construction of a pair regression model | The ability to use the acquired experience for investigation of the real economic processes. The ability to choose the function which is used as an approximation for investigation of economic processes |
| 2. Attainment of skills in using correlation analysis and regression analysis for construction of economic models and making model-based predictions | | Knowledge of the properties of construction of a pair regression model, using the methods of checking the significance of parameters of the regression model and estimating the adequacy of the model | | | |

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НАВЧАЛЬНЕ ВИДАННЯ

ВИЩА МАТЕМАТИКА

**Робоча програма
для студентів спеціальності
056 "Міжнародні економічні відносини"
першого (бакалаврського) рівня
(англ. мовою)**

Самостійне електронне текстове мережеве видання

Укладачі: **Місюра Євгенія Юріївна**
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Подано тематичний план навчальної дисципліни та її зміст за модулями й темами. Вміщено плани лекцій і практичних занять, матеріал для закріплення знань (контрольні запитання, завдання для самостійної роботи), а також методику оцінювання знань студентів відповідно до вимог кредитно-трансферної системи процесу навчання.

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