МІНІСТЕРСТВО ОСВІТИ І НАУКИ УКРАЇНИ

ХАРКІВСЬКИЙ НАЦІОНАЛЬНИЙ ЕКОНОМІЧНИЙ УНІВЕРСИТЕТ ІМЕНІ СЕМЕНА КУЗНЕЦЯ

"ЗАТВЕРДЖУЮ"
Заступник керівника
(проректор з науково-педагогічної роботи)
_______________ М.В. Афанасьєв

МІНІСТЕРСТВО ОСВІТИ І НАУКИ УКРАЇНИ ХАРКІВСЬКИЙ НАЦІОНАЛЬНИЙ ЕКОНОМІЧНИЙ УНІВЕРСИТЕТ ІМЕНІ СЕМЕНА КУЗНЕЦЯ №02071211

ВИЩА ТА ПРИКЛАДНА МАТЕМАТИКА

робоча програма навчальної дисципліни

Галузь знань	**24 Сфера обслуговування**
Спеціальність	**242 Туризм**
Освітній рівень	**перший (бакалаврський)**
Освітня програма	**Туризм**

Вид дисципліни	**базова**
Мова викладання, навчання та оцінювання	**іноземна (англійська)**

Завідувач кафедри вищої математики та економіко-математичних методів _______ Малярець Л.М.

Харків
ХНЕУ ім. С. Кузнеця
2018

ЗАТВЕРДЖЕНО
на засіданні кафедри вищої математики та економіко-математичних методів
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Лист оновлення та перезатвердження робочої програми навчальної дисципліни

Навчальний рік	Дата засідання кафедри – розробника РПНД	Номер протоколу	Підпис завідувача кафедри

1. Introduction

The fundamental base in the mathematical preparation is the discipline “Higher and applied mathematics” which is the basic discipline of the natural scientific series and the component of the structurally logical scheme which is provided for educationally professional program of the preparation of bachelors of the speciality 242 "Tourism".

The basic problems of teaching the discipline is giving students knowledge of the basic parts of higher and applied mathematics; rise of the level of the fundamental mathematical preparation of students with intensification of its applied direction, mastering the fundamentals of probability theory and application of this knowledge in the economic investigations for solving problems of mathematical statistics, forming skills of application of elements of higher mathematics, probability theory and mathematical statistics in investigations where higher mathematics is applied as instrument of investigation and solving optimization problems of mathematical programming and operations research for forming models of economic processes and developments, the obtainment of necessary totality of theoretical and practical knowledge for solving specific problems, which are set in a process of forming economic and mathematical models, and the obtainment of required mathematical preparation for study of other disciplines.

Owing to study of the discipline a student is obliged to receive the basic knowledge of linear algebra, analytical geometry, mathematical analysis, probability theory and mathematical statistics, mathematical programming and operations research and skills of using them for forming economic mathematical models. This makes it possible for him to apply the acquired knowledge and skills for solving many practical problems of economics and business.

The annotation of the academic discipline:

The main purpose of teaching is forming future specialists’ basic mathematical knowledge for solving theoretical and practical problems in professional activity of competent specialist in a service sphere, skills of analytical thinking and skills of using mathematical knowledge for formation of real processes and developments, and for solving economic problems.

The main tasks that should be solved in the process of teaching the discipline are: giving students knowledge of the basic parts of higher mathematics, probability theory and mathematical statistics, mathematical programming and operations research; definitions, theorems, rules; proving of the main theorems; mastering the fundamentals of a methodology of a mathematical investigation of the applied economic problems; independent broadening of knowledge, development of logical and algorithmical thinking; the obtainment of primary skills for independent-learning of mathematical and applied Bibliography by students.

The subject of the discipline "Higher and applied mathematics" is the fundamentals of linear and vector algebra, analytical geometry, mathematical analysis, basic laws of mass random phenomenon, principles of recording, a description and analysis of results of statistical observations, solving optimization problems.

A student starts studying educational discipline "Higher and applied mathematics" in the first term of the first year of studies.

In the process of learning the students obtain the required theoretical knowledge during lectures and acquire practical skills at the practical and laboratory studies and during independent work and fulfillment of individual tasks. Independent and individual work of students have a great value in the process of mastering material and fixing of knowledge. All of these types of studies were devised according to the statements of Bolognese declaration.

As the result of the educational discipline study a student must **know**:

the bases of linear algebra: matrices and determinants, (facilities, possibilities) of their application at making a mathematical model of economic problems;

methods of solving the system with n linear algebraic equations with m unknowns; the conditions of compatibility of the system of linear algebraic equations;

the notion of basic solution;

the bases of vector algebra: a basis of a space, linear dependence and linear independence of vectors; notions of subspace, linear vector space, a rank of finite systems of vectors, rules of its calculation;

the bases of analytic geometry: equation of straight line at R^2 and R^3, their properties; the different types of plane equations in three-dimensional space; second-order curves, general equation of second-order curve, its reduction to a canonical form; its reduction to a canonical form;

elements of limits theory: a limit of a sequence and a limit of a function; the limit of a function in a point, their equivalence; equivalent functions, their applications at finding limit of a ratio of functions;

the first and second noteworthy limits, table of basic limits, finding limits of power-exponential functions; bases of limiting (marginal) analysis;

differential calculus: a function of one variable and several variables; ways to define a function and its illustration; some special classes of functions; monotone, even and odd, convex and concave, bounded and unbounded functions;

a continuity of a function at the same point; one-sided continuity of a function of one variable at the same point, necessary and sufficient conditions of continuity; classification of points of discontinuity; a differentiable function, its differential;

derivative of a function of one variable, partial derivatives, a gradient of a function of several variables; a derivative of a function of several variables in the direction, its relationship with a gradient; an elasticity of a function;

higher-order derivatives and differentials, higher-order derivatives of some elementary functions; investigation of functions with the help of differential calculus; a notion of a differential of a function and its application for approximate calculation; a notion of an elasticity of a function;

integral calculus: notion of an antiderivative, indefinite and definite integrals; methods of integration; Newton–Leibnitz formula; a notion of an improper integrals;

elements of economic dynamics; the first-order ordinary differential equation, Cauchy problem; the particular and general solutions; types of the differential equations;

the higher-order differential equations and systems of differential equations; solution of the second-order linear differential equations with constant
coefficients;

numerical series, necessary and sufficient conditions of a convergence of numerical series with positive terms and alternating numerical series; absolute and conditional convergence;

power series, convergence radius and interval of power series; functional series, trigonometric Fourier series;

general notions of probability theory; random persistent and impossible events; a notion of a probability and methods of its definition; dependent and independent events and basic formulas of an addition and a multiplication of probabilities for these events, formulas of a total probability and Bayes;

a trial by Bernoulli's scheme; discrete and continuous random variables; basic laws of a distribution of discrete and continuous random variables and their basic numerical characteristics; conditional laws of a distribution of probabilities of components of discrete two-dimensional random variable;

a definition of basic numerical characteristics of two-dimensional random variable; characteristics of a function of one random argument; limiting theorems of probability theory;

general notions of mathematical statistics; a sampling method; a statistical distribution and its basic numerical characteristics; statistical estimations of parameters of a population; requirements for statistical estimations;

notions of point and interval estimations and a definition of their accuracy; methods of parametric and nonparametric estimations of parameters; statistical hypotheses and statistical criteria for their checking;

a statement and a formalization of optimization problems according to a general technology of modeling in economics; a solution of problems of linear programming with the help of a graphical and a simplex-methods;

a model of a dual problem; a solution of a primal problem by a solution of a dual problem and economic interpretation of dual estimations;

a formulation of a transportation problem and the main point of a method of potentials;

solving of problems of integer programming with using a method of Gomory;

modern mathematical methods and inventory management model; the main point of a question of optimal inventory management;

a classification of costs which are connected with making and saving inventory;

a statement of optimization problem of current inventory at different conditions of a supplier; methods of definition of optimal insurance inventory on a base of an investigation of fluctuations; using methods of statistical modeling;

a formulation of queuing problems; a characteristic of elements of a queuing system;

a formulation of an optimization problem of a sequence of processing products; using methods of integer programming and combinatorial for solving a sequencing problem;

a classification of a network planning and management; using network methods of planning and management; a characteristic of basic types of network models; using optimization methods of a network graph by a criterion of time;

a formulation and a classification of replacement problems of an equipment of a continuous usage; an optimization of a term of an equipment replacement; a dynamic model of an equipment replacement;

a formulation and a characteristic of problems of stochastic programming; a formulation and a characteristic of problems of game theory;

a formulation and a characteristic of optimization multicriterion problems; methods of a multicriterion optimization of managerial decisions.

be able to:

– learn mathematical literature by oneself;
– calculate of the mean values;
– carry out the operations with vectors, matrices, calculation of the determinants;
– solve the systems of linear equations;
– investigate forms and properties of the straight lines and planes, second-order curves and quadratic surfaces;
– classify the functions, numerical sequences;
– find the limit of power-exponential functions;
– investigate the function with the help of differential calculus;
– carry out integral calculus;
– carry out calculation of numerical and power series;
– solve first-order and higher-order differential equations, systems of differential equations;
– form and use economic mathematical models;
– broaden of knowledge, develop of logical and algorithmic thinking by oneself.

A modern tendency in higher education is a reorientation of students of higher educational institutions from a process of education to a result, from knowledge to skills, forming definite competences.

The purpose of the academic discipline:

Academic year	**1st**	
Term	**1st and 2nd**	
Number of credits	**4 and 5**	
The form of studies	**lectures**	**28/32**
	practical studies	**16/16**
	laboratory studies	**16/16**
Independent work		**60/86**
Form of final control	**test / exam**	

Structural and logical scheme of studying the academic discipline:

Previous academic disciplines	**Next academic disciplines**
Algebra, Geometry (Mathematics)	Statistics

2. Competences and result of mastering the academic discipline:

Competences	**Results of mastering the academic discipline**
1) Forming ability to prove independently the simplest statements with the help of elementary mathematical knowledge 2) forming skills to use the instrument of matrix calculus for modeling the simplest economical problems and situations; 3) ability to analyze the results of calculations from the mathematical and practical viewpoint	A student **must** 1) **know** the basic proofs and theorems of the theme; 2) **give** examples of using determinants, matrices and systems of linear equations in economics; 3) **be able to** use instrument of matrix algebra for economical prob lems; 4) **be able to** model the simplest situations with the help of knowledge of the theme
Forming analytic thinking, ability to explain an importance of complicated expressions with the help of mathematical symbols and operations	A student **must** 1) **be able to** use vector algebra for a calculation of the simplest problems of applied character (finding an area, a volume); 2) **carry out** analytic analysis of relationship of straight lines (planes, a straight line and a plane) in a space or on a plane and understand direct relationship of analytic geometry and matrix algebra; 3) **define** lines of the second order according to their canonical equations; 4) **be able to** use material according to defined themes for economical researches (analysis of curves of demand and supply, definition of equilibrium point, arachnoidal model making of market)
Forming analytic thinking, ability to explain an importance of complicated expressions with the help of mathematical symbols and operations	A student **must** 1**) investigate** a function and independently carry out an analysis of the obtained results; 2) **give** examples of functional dependence

Competences	Results of mastering the academic discipline
Development of abilities to solve problems with the help of methods of differential calculus using mathematical symbolic variables, i.e. forming initial skills of economic modeling	A student **must** 1**) calculate** derivatives of elementary and composite functions; 2) **be able to use** a differential of a function for approximate calculus; 3) **investigate** a function with the help of differential calculus; 4) **carry out** the simplest calculations by an optimization of production; 5) **make** corresponding conclusions and independently analyze the obtained solution
Forming skills to use previous experience (a function of one variable) for further using in more complex situation	A student **must** 1**) find** partial and mixed derivatives, 2) **be able to investigate** a local extremum of a function; 3) **be able to use** method of Lagrange factors and the least-squares method
1) understanding a possibility to use integral calculus for solving applied problems; 2) forming skills to independently form mathematical models for a description of different processes; 3) forming a skill of independent analysis and understanding an importance of a relationship between the examined material (a definite and an indefinite integral)	A student **must** 1**) calculate** definite and indefinite integrals; 2) **be able to use** definite integrals for a calculation of areas and volumes of figures independently; 3) **make** the corresponding conclusions and independently analyze the obtained results
Forming inclinations to independent search of different ways of solving problems and understanding of necessity to use knowledge of other themes (a function, a derivative, an integral)	A student **must** 1**) be able to** calculate a type of a differential equation, a method of further solving independently; 2**) be able to** use the knowledge for solving the simplest economic problems
Forming ability to analytic calculations	A student **must** 1**) calculate** a type of series; 2) **be able to** investigate a convergence of series independently; 3) **find** a convergence radius of power series
1) Skills to calculate basic notions of probability theory, combinatorial analysis and mathematical statistics independently; 2) independent carrying out and analysis of the obtained problem; 3) a definition of a type or a class; 4) forming possible directions of solving a problem on a base of modern scientific attainments; 5) using basic methods of probability theory for solving a composite problem and	Understanding a role and a place of probability theory and mathematical statistics in modern scientific researches and their significance in further solving professional problems. A student **must** 1) **know** the basic theorems and statements of the theme; 2) **give** examples of using basic notions of probability theory and use them for solving different problems; 3) **develop** and improve abilities to

Competences	Results of mastering the academic discipline
reducing it to simple problems	reasonably construct analytic solving a prob lem in a direction of a search of an optimal way of its solving
Using laws of a distribution (discrete and continuous) random variables, reasoning suitability and necessity of their using; prediction of processes which will occur in real economic models on a base of knowledge of the theme	A student **must** 1) **be able to** find the numerical characteristics of random variable by the given law of a distribution, 2) **be able to** calculate a probability to lie in the given interval, estimate the obtained result, give it the corresponding economic or mathematical explanation; 3) **plot** a polygon, a histogram, graphs of an integral function and a density of a probability
1) Reasonable using laws of a distribution of continuous random variable, reasoning suitability of their using in real situation; 2) ability to unite knowledge of different themes, understand their relationship and further common using (elements of vector analysis and theory of random variable)	A student **must** 1) **be able to** calculate the numerical characteristics of random variable by the given law, understand and estimate the obtained result; 2) **be able to** calculate a probability to lie in the given interval, estimate the obtained result, give it the corresponding economic or mathematical explanation; 3) **plot** a polygon, a histogram, graphs of an integral function and a density of a probability
Prediction to analyze processes which will occur in real economic models	A student **must** 1) **use** the acquired knowledge for fundamental analysis of the obtained problem; 2) **carry out** detailed analysis and corresponding mathematical calculations; 3) **predict** and analyze the obtained results using the obtained knowledge by a course of probability theory
Attainment of theoretical and probabilistic fundamentals of mathematical statistics, understanding a role of basic mathematical assumptions at a statement of a question and solving statistical problems	A student **must** 1) **know** basic notions, definitions, types of basic mathematical and statistical calculations, basic models and methods of statistical researches; 2) **be able to** systematize the obtained data in a form of a table or a form of a presentation of data
Independent organization of a statistical observation, scientific explanation of a method of data collection and analysis; a comparison of possible ways of solving, prediction and reasonable carrying out mathematical calculations in the chosen direction	A student **must** 1) **distinguish** independent and dependent factors, carry out an analysis of intermediate results with a purpose of a refinement of a type and a relationship between variables; 2) **give** the predicted recommendations on a base of calculations and model consequences of further development of a process

Competences	Results of mastering the academic discipline
1) Forming skills to use the instrument of mathematical programming for solving economic problems; 2) ability to form and solve linear programming problems; 3) ability to use duality theorems of finding a solution of a dual problem; 4) ability to analyze economic problems which use principles of solving a transportation problem; 5) forming skills to solve economic problems using integer programming	A student **must** 1) **be able to** solve problems of linear programming; 2) **form** a dual problem for the given primal problem; 3) **find** a solution of a dual problem knowing a solution of a primal problem with the help of duality problems; 4) **give** economic interpretations of solutions of a primal and a dual problem; 5) **be able to** solve and find an optimal solution problems of transportation problems; 6) **be able to** solve problems of integer programming
1) forming skills to use the instrument of operations research for solving economic problems; 2) ability to form, solve and analyze optimization inventory problems, queuing problems, sequencing and coordination problems, replacement problems, problems with conditions of uncertainty and conflict, multicriterion problems	A student **must** 1) **use** the obtained knowledge for fundamental analysis of the obtained problem; 2) **know** basic notions, definitions, types of basic mathematical calculations, basic models and methods for economic researches; 3) **form**, **solve** and **analyze** optimization inventory problems, queuing problems, sequencing and coordination problems, replacement problems, problems with conditions of uncertainty and conflict, multicriterion problems; 4) **give** examples of using basic notions and models of operations research for solving

3. The syllabus of the academic discipline

Thematic module 1. The elements of mathematical analysis

Theme 1. The limit of a function and continuity

1.1. Sets, functions, their classification.

Basic notions. Numerical sets. Operations with sets. Numerical intervals, the neighborhood of a point. The notion of a function of one variable. Ways to define a function. The domain of a definition and a range of values of a function. A geometrical illustration of a function. Elementary functions and their graphs. Properties of a function: the boundedness and the unboundedness, an increasing and decreasing function, the oddness and the evenness, the periodicity. Classification of elementary functions. The notion of an inverse function. Inverse trigonometric functions. The superposition of a function.

1.2. Numerical sequences and their limits.

A numerical sequence. The definition of the limit of a sequence. Infinitesimals. Infinitely large values. The relationship between infinitesimals and infinitely large values.

1.3. A limit of a function.

The definition of the limit of a function. One-sided limits. Properties of functions which have finite limits. Limiting processes in equalities and inequalities. Lemmas about infinitesimals. Arithmetical operations with functions which have finite limits. The limit of a function $\frac{\sin x}{x}$ at $x \to 0$. Indefinite expressions. The limit of a monotonic function. The number *e*. Natural logarithms.

1.4. A function continuity.

The definition of a function continuity at a point. Continuity of a function on a segment. Arithmetical operations with continuous functions. Classification of breaks. Properties of continuous functions. Continuity of elementary functions.

Theme 2. Differential calculus of the function of one variable

2.1. A derivative and a differential. Techniques of differentiation.

Application of a derivative to economic accounts. Limiting characteristics in microeconomics. Maximization of profit and marginal analysis. Optimization of a taxation of enterprises. The definition of a derivative. The geometric, mechanical and economic meanings of a derivative. Derivatives of elementary functions. A derivative of an inverse function. A table of derivatives. The rules of calculation of derivatives. A derivative of a composite function. One-sided derivatives. Derivatives of higher orders. The definition of a differential. The differential of a sum, a product and a quotient. The invariance of the form of the first differential. Differentials of the higher orders. Application of a differential to approximate calculations.

2.2. The main theorems of differential calculus and using them.

Fermat theorem. Rolle theorem. Lagrange theorem. Cauchy theorem. L'Hospital rule.

2.3. Application of derivatives to the investigation of functions.

The condition of a monotony of a function. The condition of increasing and decreasing of a function on an interval. The maximum and the minimum of a function. Necessary and sufficient conditions of an extremum of a function. Convexity and concavity of a graph of a function, inflection points, asymptotes of a graph of a function. A general scheme of a plot of the graph of a function.

2.4. Application of a derivative to economics.

Marginal analysis. Elasticity of economic indicators. The economic meaning of Fermat theorem. Application of a derivative to economic calculations.

Theme 3. Analysis of the function of several variables

3.1. Basic notions.

The function of two variables, the domain of their definition. A graphical illustration of a function of two variables.

3.2. Partial derivatives. The differential.

A partial and a total increments of a function of two variables. Partial derivatives. A total differential. Derivatives of the higher orders. The theorem about the equality of mixed derivatives. Differentials of the higher orders.

3.3. The extremum of a function of several variables.

The necessary conditions of a function of two variables. The sufficient conditions of an extremum of a function of two variables. The conditions of an absence of the extremum. The notion of a conditional extremum. The method of Lagrange multipliers. The least-squares method.

3.4. Application of a function of several variables to economics.

The function of several variables in problems of economics (the utility function, the expenditure function, the multifactor production function of Cobb and Douglas). Some problems of optimization (an optimal profit from production of goods of different types; the problem of price discrimination, an optimal distribution of resources; optimization of the choice of a consumer). The functional dependence between variables.

Theme 4. The indefinite integral

4.1. An antiderivative and an indefinite integral.

The notion of an antiderivative of a function and an indefinite integral. The geometrical and mechanical meanings of an integral. The table of basic integrals.

4.2. Basic methods of integration.

The simplest rules of an integration. Direct integration. A change of a variable in an indefinite integral. Integration by parts.

4.3. Integration of some classes of functions.

Integration of rational fractions. Integration of irrational expressions and expressions which have trigonometric functions. Trigonometric substitutions.

Theme 5. The definite integral and its application

5.1. The notion and properties of a definite integral.

Integral sums. Conditions of the existence of a definite integral. Properties of a definite integral.

5.2. Calculation of a definite integral.

Newton–Leibnitz formula. A change of a variable in a definite integral. Integration by parts.

5.3. Improper integrals of the first and the second kinds.

The notion of an improper integral. Conditions of convergence of improper integrals. Euler-Poisson integral and its application.

5.4. Application of a definite integral.

The geometrical application of a definite integral: calculation of areas, volumes of the solid of a revolution, arc lengths of curves. An approximate calculus of a definite integral: formulas of rectangles, trapezoids, Simpson**.** Finding the volume of a productive production; a consumer surplus, an analysis of a nonuniformity in the distribution of income from population with the help of Lorenz curve.

Theme 6. Differential equations

6.1. The basic notions of the theory of differential equations. Solving the first-order differential equations.

The notion of a differential equation and its solutions. Application of differential equations to problems of economic dynamics. A model of increasing for a constant rate of an increment; a model of increasing under the conditions of a competition; a dynamic model of Keynes; a neoclassic model of increasing; a marketing model with predicted prices. The order of a differential equation. Differential equations of the first order. A general solution and a general integral of a differential equation of the first order. Initial conditions. A particular solution and a particular integral of a differential equation of the first order with separable variables. Homogeneous equations of the first order. Linear differential equations of the first order. Differential equations of Bernoulli**.**

6.2. Differential equations of higher orders. Methods of solving the second-order differential equations.

The second-order linear differential equations with constant coefficients. Homogeneous and inhomogeneous differential equations. The notion of linearly independent solutions of a homogeneous differential equation of the second order. Initial conditions. The structure of a general solution of an inhomogeneous differential equation of the second order. Linear inhomogeneous differential equations of the second order with the right parts of a special form. The notion of the differential equation. The notion of a system of differential equations. The notion of an equilibrium of a solution.

6.3. Application of differential equations to economics.

Using differential equations for a construct of production functions. Models of economic dynamics. Solow model. A model of a natural increasing output. The dynamics of market prices. Application of differential equations to economics.

Theme 7. Series

7.1. Numerical series and their convergence.

Partial sums of series. The necessary condition of series convergence. Series with positive terms. The theorem of a comparison of series. Sufficient conditions of series convergence with positive terms: D'Alembert criterion, Cauchy's criterion, Maclaurin – Cauchy integral criterion.

7.2. Alternating series and their convergence

The notion of alternating series. Absolute and a conditional convergence of series. Alternating series. Leibnitz theorem. A sign of a remainder of alternating series.

7.3. Power series.

Abel theorem. The convergence radius of power series. A differentiation and an integration of power series. Taylor and Maclaurin series. Decomposition of elementary functions in Taylor and Maclaurin series. Application of power series to an approximate calculus.

Thematic module 2. Linear algebra and analytical geometry

Theme 8. The elements of the theory of matrices and determinants

8.1. Matrices.

The definition, types of matrices, basic matrices (square, triangular, diagonal, unit). Comparison of matrices. Basic operations with matrices: addition, multiplication of a matrix by a scalar, a vector, a matrix; properties of these operations. Transposition of a matrix. The notion of an inverse matrix, properties of a matrix inversion operation.

8.2. Determinants.

The definition of the determinant, rules of calculation of determinants of lower orders (schematic) and higher-orders (expansion by Laplace formulas). Properties of determinants. Calculation of some special determinants (triangular, diagonal, identity matrices, Vandermonde matrix). Calculation of an inverse matrix with the help of the determinants (algebraic cofactors).

8.3. The inverse matrix

Calculation of an inverse matrix by two ways: with the help of a definition (as a transposed matrix of algebraic cofactors) and elementary row transformations with a given and unit matrices. A matrix rank and ways to define it.

Theme 9. The general theory of the system of linear algebraic equations

9.1. Systems of linear algebraic equations.

The definition of the system of linear algebraic equations, the augmented and matrix forms of its entry. Definitions of a solution, consistent or inconsistent, determined or undetermined system.

9.2. Methods of solving systems of linear algebraic equations.

A solution of square systems of linear algebraic equations with the help of an inverse matrix, by Cramer formulas. Equivalent transformations, Gauss - Jordan method of sequential exclusion of unknowns for a solution of systems of linear algebraic equations, its realization with the help of tables. Finding an inverse matrix by Gauss - Jordan method. The notion of a matrix rank and its calculation. Kronecker - Capelli theorem, the particular and the general solutions of the system of linear algebraic equations.

9.3. Homogeneous systems of linear algebraic equations

The notion of a homogeneous system of linear algebraic equations. The space of solutions of a homogeneous system, a relationship of its dimension and a matrix rank. A fundamental system of solutions of a homogeneous system of linear algebraic equations.

Economic problems.

Theme 10. The elements of vector algebra

10.1. The basic notions of vector algebra.

The Cartesian coordinates of a vector and a point. Examples of economic problems, which are connected with using vector algebra and analytic geometry. Coordinates on a straight line. Coordinates on a plane. Coordinates in a space.

Linear operations with vectors in coordinates. Coordinates of a point of division of a segment. Coordinates of a vector which is given by two points. A sign of a colinearity of two vectors. A sign of a complanarity of three vectors. Properties of a scalar product of two vectors. The expression of a scalar product through coordinates. A cross product of two vectors, its properties. An expression of a cross product through coordinates. A mixed product of three vectors, its properties. The expression of a mixed product through coordinates of vectors-factors.

10.2. The elements of the theory of linear spaces.

The definition of linear space. The definitions and main theorems of linear dependence and linear independence of linear space elements. A basis of linear space. The main theorems about the basis: uniqueness of expansion, linear dependence of $(n+1)$ elements, the number of basic elements. The dimension of linear space. Coordinates of space elements in a given basis. The notion of subspace. The notion of linear vector space. The rank of finite systems of vectors, rules of its calculation.

10.3. Eigenvectors.

Eigenvalues and eigenvectors of a matrix. A characteristic equation. Methods of finding eigenvalues and eigenvectors for matrices of the second and the third orders. Economic examples.

10.4. Quadratic forms.

The notion of a quadratic form. Conditions of a determinacy of quadratic forms. The matrix of a quadratic form. Reducing quadratic forms to a canonical form. The curves of the second-order on a plane. A general equation of the second-order curve. Reducing the second-order curve to a canonical form.

Thematic module 3. Probability theory and Mathematical statistics

Theme 11. Empirical and logical bases of probability theory

11.1. The subject and problems of this course.

The role of this course as a theoretical base of a mathematical modelling of economic processes and phenomenons, which take into consideration of possible risks.

11.2. Basic definitions, rules and types of events

A probabilistic model of an experiment. Sure (certain), random and impossible events. Rules of operations with random events. A space of elementary events.

11.3. Basic concepts and formulas

A classical definition of a probability and its calculation. Basic formulas of combinatorics. A statistical definition of a probability. Axiomatics of Kolmogorov.

A geometrical definition of a probability. Venn-Euler diagram.

Theme 12. Basic theorems of probability theory, their economic meaning

12.1. Basic definitions and multiplication theorems of probabilities

Probabilistic space. Dependent and independent events. A conditional probability. Multiplication theorems of probabilities.

12.2. Basic concepts and addition theorems of probabilities

A complete group of events. Complementary events.

Joint (compatible) and disjoint (incompatible) events. Addition theorems of probabilities.

12.3. The probability of at least one event

The probability of at least one event. The probability that an event will occur at least once. Calculation of a necessary number of trials, which occur with a definite reliability in order to guarantee an occurrence of a random event at least once.

12.4. Formula of a total probability and Bayes formula

Formula of a total probability. Bayes formula (the theorem of hypothesis).

Theme 13. Scheme of independent trials

13.1. Repeated independent trials and Bernoulli's scheme

A scheme of repeated independent trials. Bernoulli's formulas.

13.2. Local theorem of Moivre – Laplace

Local theorem of Moivre – Laplace. Gauss's function, its properties, an application to approximate calculations of a probability of an occurrence of a random event of a definite number times in series of independent trials.

13.3. Integral theorem of Moivre – Laplace

Integral theorem of Moivre – Laplace. Laplace's function, its properties and an application to approximate calculations of a probability that values of a random variable lies in a definite interval.

A relationship between Gauss and Laplace functions.

13.4. Poisson's theorem

Low-probability events. Poisson's theorem.

Theme 14. Random variables and their economic meaning

14.1. Definitions and types of random variables

A definition of a random variable. Discrete and continuous random variables.

14.2. Distribution laws and distribution function of probabilities

Distribution laws of probabilities for a random variable and ways of finding (tabular, graphic and analytical).

A distribution function of probabilities, its properties.

14.3. Basic numerical characteristics

Basic numerical characteristics of a random variable: a mathematical expectation, a variance and a root-mean-square deviation. Properties of basic numerical characteristics.

14.4. Additional numerical characteristics

Additional numerical characteristics of a distribution: a mode, a median, an excess.

Initial and central theoretical moments, their application to a calculation of numerical characteristics of a distribution of a random variable.

Theme 15. Distribution laws and numerical characteristics of discrete random variables

15.1. Basic types and properties of distribution laws of a discrete random variable

Distribution laws of a discrete random variable, which are often used in social and economic investigations: a binomial distribution, a geometrical distribution, a hypergeometrical distribution. Specificities and properties of these distributions, their basic numerical characteristics and the economic meaning.

15.2. A flow of events

A flow of events. The simplest flow of events and its properties.

Theme 16. Basic distribution laws of a continuous random variable

16.1. A density distribution function and its properties

A density of a distribution and its probable meaning. A density distribution function of a random variable and its properties.

16.2. Basic types and properties of distribution laws of a continuous random variable

Distribution laws of a continuous random variable, which are often used in social and economic investigations: a uniform distribution, a normal distribution and an exponential distribution. Properties of these distributions and their basic numerical characteristics.

An influence of parameters of a distribution on a density function of probabilities at a normal distribution law.

16.3. Student's, Pearson's and **Fisher's** *distributions*

Student's distribution, Pearson's distribution and ***Fisher's*** distribution. Specificities and properties of these distributions. A relationship of these distributions and a normal distribution law of a continuous random variable.

Theme 17. Preprocessing of statistical data

17.1. Basic definitions and problems of mathematical statistics

Basic problems of mathematical statistics. A sampling method. Definitions of a population and its sample.

17.2. Discrete and interval variational series

An empirical distribution law. Ways of a presentation of sampling totalities and a representation of results of observations. Discrete and interval variational series, rules of its construction. A defining limits of an interval by Sturges's formula.

17.3. A graphical presentation of a statistical distribution

A graphical presentation of a statistical distribution. A polygon and a histogram.

17.4. Basic numerical characteristics

Basic sampling numerical characteristics and their asymptotic behavior.

Theme 18. Statistical estimation of parameters of a distribution

18.1. Basic statistical estimations and their properties

Statistical estimations of parameters of a distribution of a population and their properties: an unbiasedness, possibility and an efficiency.

An asymptotic efficiency of maximally plausible estimations.

18.2. Types of estimations

The method of moments. Point and interval estimations. An confidence interval for a mathematical expectation of a normal population.

Theme 19. Checking of statistical hypothesis

19.1. Types of statistical hypothesis and kinds of errors

Main and alternative statistical hypothesis. A statistical test. A construction of critical domains for a statistical test. Errors of the first and the second kinds. A concept of power of a test.

19.2. Different types of tests for checking of a statistical hypothesis

A checking of a statistical hypothesis about a defining of a distribution law for a population using results of an investigation of a sample. Pearson and Kolmogorov fitting tests. The fitting test relative to a frequency.

A checking of a statistical hypothesis about an equality of two population means at an assumption of a normal distribution law. Student's fitting test.

A comparison of variances. ***Fisher-Snedeker*** fitting test.

A checking of a hypothesis about an equality of a sampling mean and a mathematical expectation.

Thematic module 4. Correlation, regression and Operation research

Theme 20. Elements of correlation theory

20.1. Basic definitions and problems of correlation analysis

Problems of correlation analysis.

20.2. Basic coefficients and their properties

A sampling coefficient of a correlation, its properties and a confidence interval. A coefficient of determination. A correlation ratio, its properties.

Theme 21. Elements of regression theory

21.1. Basic concepts of regression analysis and LSM

Problems of regression analysis. A correlation dependence. A correlation table. Empirical lines of a regression. An estimation of parameters of a pair regression equation using the least-squares method (LSM). Point estimations.

21.2. A significance and a confidence interval of a pair regression line

A checking of a significance of parameters of a pair regression equation. A confidence interval for a line of a pair regression.

Theme 22. Problems of linear programming and methods for solving them

22.1. Statements and basic definitions of linear programming problems

Economic and mathematical statements of linear programming problems (LPP). The system of hypothesis. Basic definitions. A standard form of a linear optimization model. A set of feasible solutions and an optimal solution to LPP.

22.2. The graphical method of solving LPP

The graphical method of solving LPP. The geometrical meaning of LPP. The graphical method of solving LPP, its possibilities and the field of application. Examples of problems, which can be solved by the graphical method.

22.3. The simplex method of solving LPP

The simplex method of solving LPP. The canonical (basic) form of LPP. The construction (plotting) of support solutions. The optimization criterion. Searching an optimal solution using the algorithm of the simplex method. The geometrical meaning of the simplex method. The theoretical aspects of the simplex method. A problem with mixed constraints.

22.4. The method of artificial basis

The method of artificial basis. The features of solving LPP, which are given in the general form of LPP for solving economic problems.

Theme 23. The transportation problem

23.1. The basic statement of the transportation problem and methods of finding the support basic solution

Solving the transportation problem using the criterion of costs. The statement of the transportation problem using the criterion of the transportation cost. Finding the support basic solution. Transformation of the other basic solution. The problem of solution degeneracy of the transportation problem and ways to eliminate the degeneracy.

23.2. The method of potentials

Finding an optimal solution using the method of potentials. The optimality criterion of the solution. The method of potentials. The economic meaning of potentials.

23.3. Investigation of stability of an optimal solution

Investigation of stability of an optimal solution as a problem of parametric programming. Transportation problems with additional conditions.

23.4. Solving a transportation problem using the criterion of time

Solving a transportation problem using the criterion of time. Problems with economic content which are reduced to transportation problems.

Theme 24. Game theory. Analysis and risk management in eco-nomics on the basis of the concept of game theory

24.1. The basic definitions and models of game theory

The basic notions of game theory, a mathematical model of the matrix game as a particular case of probabilistic models of economic systems.

24.2. The basic notions and the basic theorem of two players' matrix games

Matrix games of two players. The payoff matrix. Minimax and maximin criteria. The game price. The game in pure strategies. A saddle point. The game in mixed strategies. The basic theorem of game theory (Neumann theorem). Reducing a matrix game of two players to a linear programming problem.

24.3. A graphical method of solving a matrix game

The geometrical meaning of the matrix game of two players. Finding active strategies of players. The content of basic ways of quantitative pricing risk. The system of quantitative estimations of the measure of an economic risk. A game price as the risk appraisal.

4. The order of assessment of studying results

The system of assessment of competences which were formulated for a student during the learning of the academic discipline, takes into consideration the forms of studies which according to the syllabus of the academic discipline provide lectures, practical studies, laboratory works, fulfillment of students' independent work. The assessment of the formed competences of students is carried out on the accumulative 100-point system. According to the temporary provision "About the Order of Assessment of Students Academic performance on the Accumulative Point Rating System" of Simon Kuznets Kharkiv National University of Economics control ways include:

current control which is carried out within a term during lectures, practical studies and laboratory works and it is assessed as a sum of accumulative points (the maximum equals 100 points; the minimum which makes it possible for a student to pass a test, equals 60 points (term 1); the maximum equals 60 points; the minimum which makes it possible for a student to pass an exam, equals 35 points (term 2));

module control which is carried out in the form of a colloquium with taking into account the current control according to a corresponding thematic module, provides an integral assessment of student's results after learning the material of a logically completed part of the discipline (or a thematic module);

final/term control, which is carried out as a terminal exam, according to the schedule of the educational process.

Current control on the given academic discipline is carried out in the following forms: active in-class work (lecture); active in-class work (practical study); active in-class work (laboratory study); homework; competence oriented tasks (defence of laboratory works); an independent test; a written test; independent creative work.

Final/term control (term 1)*:* the final mark on the academic discipline is calculated according to the points obtained during the current control on the accumulative system.

Final/term control (term 2) is conducted in the form of a term exam. **Term exams** are a form of assessment of students' final mastery of the theoretical and practical material of a particular module of the academic discipline or the academic discipline on the whole, which is conducted as a test.

Assessment of student's knowledge during practical studies and carrying out laboratory works is conducted on the accumulative system according to the following criteria: understanding, the degree of the mastery of the theory and methodology of problems which are considered; the degree of the mastery of the factual material of the academic discipline; familiarizing with the recommended literary sources and modern literature on the questions which are considered; the ability to connect theory and practice in the consideration of particular examples, solving problems, carrying out laboratory works, carrying out calculations in the process of doing homework and tasks which are considered in class; the logic, structure, style of presenting the material in written works and in oral answers in class, the ability to ground one's position, carry out generalization of the information and draw conclusions.

The general criteria for the assessment of ***independent work*** of students are profound and deep of knowledge, the level of thinking, skills in systematization knowledge on particular themes, skills in drawing conclusions, attainments and techniques of carrying out practical tasks, the ability to find necessary information, carry out its classification and processing, self-realization in practical and laboratory studies.

The criteria for assessment of independent creative work and independent tests are: the ability to carry out a critical and an independent estimation of the defined problem questions; skills in the explanation of alternative views and availability of a students' own point of view, position on the defined problem question; using the analytical approach; the quality and accuracy of expressing the thought; the logic, structure and explanation of conclusions about a particular problem; independence of carrying out of the work; grammatical correctness of the presentation of the material; using the methods of comparison, generalization of the concepts and facts; the design of the work; the quality of presentation.

The final control of knowledge and competences of students on the academic discipline is carried out on the base of the term exam. The examination paper includes the syllabus of the discipline and provides for assessment of the knowledge level and a degree of the mastery of corresponding competences of students.

The purpose of ***the exam*** is to test student's understanding of the syllabus material on the whole, the logic and relations between its particular parts, the skills in the creative use of the stored knowledge, the ability to formulate one's attitude to a particular problem of the academic discipline and so on. The competent approach to the assessment of the exam implies measuring the level of the student's mastery of the competences provided by the qualifying requirements.

Each examination paper contains 5 practical tasks, including two first-level (diagnostic) tasks, two second level (situational) tasks and one third level (diagnostic and heuristic) task.

The assessment of the exam is carried out according to the temporary provision "About the Order of Assessment of Students' Academic Performance on the Accumulative Point Rating System" of Simon Kuznets Kharkiv National University of Economics.

In the case of irreproachable fulfillment of all the examination tasks with the demonstration of deep knowledge of the academic discipline, skills in the practical use of the formed competences which are based on the ability to analyze and solve a wide range of tasks, a high level of completing the written work the student obtains 40 points.

A student can't be allowed to take the exam, if the number of points, obtained during the current and module control according to the thematic module during the term, does not make 35 points. After the examination period the dean of the department gives a notice about sitting the failed exams. In a given period the student adds the required points.

The final mark on the academic discipline is calculated according to points, obtained during an exam, and points, obtained during a current control by an accumulative system.

It should be assessed student's progress, if a sum of points, obtained as the total result of an assessment by all forms of a control, equals or exceeds 60. Accordingly the minimal possible quantity of points by a current and a module control during a term equals 35 and the minimal possible quantity of points, obtained on an exam, equals 25.

The result of a terminal exam is assessed in points (the maximum is 40 points, the minimum of a quantity, which is passed, equals 25 points) and it is entered into the corresponding column of an *examination «Mark sheet»*.

The final mark of the academic discipline is calculated according to the points obtained during the exam and points obtained during the current control on the accumulative system.

The total result in points during the term is "*60 and more points mean passed*", "*59 and less points mean failed*" and it is entered into the "*Mark sheet*" on the academic discipline.

The distribution of points by weeks (term 1)

Themes of the thematic module			Lectures	Practical study	Laboratory study	Homework	Competence oriented task	Independent test	Written test	Independent creative work	Colloquium	Total
Thematic module 1 Elements of mathematical analysis	Theme 1	week 2	1	1	–	–	–	–	–	–	–	2
	Theme 2	week 3	1	–	1	0,5	–	–	–	–	–	2,5
	Theme 3	week 4	1	1	–	–	–	–	–	–	–	2
	Theme 4	week 5	1	–	1	0,5	–	–	–	–	–	2,5
		week 6	1	1	–	–	–	5	10	–	–	17
	Theme 5	week 7	1	–	1	0,5	–	–	–	–	–	2,5
	Theme 6	week 8	1	1	–	–	5	–	–	–	10	17
		week 9	1	–	1	0,5	–	–	–	–	–	2,5
	Theme 7	week 10	1	1	–	–	–	–	–	–	–	2
Thematic module 2 Linear algebra	Theme 8	week 11	1	–	1	0,5	–	–	–	–	–	2,5
	Theme 9	week 12	1	1	–	–	–	–	–	–	–	2
		week 13	1	–	1	0,5	5	–	–	–	–	7,5
	Theme 10	week 14	1	1	–	–	–	–	10	–	–	12
		week 15	1	–	1	0,5	–	–	–	–	10	12,5
		week 16	–	1	–	–	–	5	–	6	–	12
		week 17	–	–	1	0,5	–	–	–	–	–	1,5
Total			14	8	8	4	10	10	20	6	20	100

The distribution of points by weeks (term 2)

Themes of the thematic module			Lectures	Practical study	Laboratory study	Homework	Competence oriented task	Independent test	Written test	Independent creative work	Colloquium	Total
Thematic module 3 Probability theory and mathematical statistics	Theme 11	week 1	0.3	0.3	–	–	–	–	–	–	–	0.6
	Theme 12	week 2	0.3	–	0.3	–	–	–	–	–	–	0.6
	Theme 13	week 3	0.3	0.3	–	0.2	–	–	–	–	–	0.8
	Theme 14	week 4	0.3	–	0.3	–	–	–	–	–	–	0.6
	Theme 15	week 5	0.3	0.3	–	0.2	–	4	–	–	–	4.8
	Theme 16	week 6	0.3	–	0.3	–	–	–	7	–	–	7.6
	Theme 17	week 7	0.3	0.3	–	0.2	–	–	–	–	–	0.8
	Theme 18	week 8	0.3	–	0.3	–	5	–	–	–	6	11.6
	Theme 19	week 9	0.3	0.3	–	0.2	–	–	–	–	–	0.8
Thematic module 2 Correlation, regression and Operation research	Theme 20	week 10	0.3	–	0.3	–	–	–	–	–	–	0.6
	Theme 21	week 11	0.3	0.3	–	0.2	–	–	–	–	–	0.8
	Theme 22	week 12	0.3	–	0.3	–	–	4	–	–	–	4.6
		week 13	0.3	0.3	–	0.2	–	–	7	–	–	7.8
	Theme 23	week 14	0.3	–	0.3	–	–	–	–	–	–	0.6
		week 15	0.3	0.3	–	0.2	–	–	–	–	6	6.8
	Theme 24	week 16	0.3	–	0.3	–	5	–	–	5	–	10.6
Exam			–	–	–	–	–	–	–	–	–	40
Total			4.8	2.4	2.4	1.4	10	8	14	5	12	100

The scales of assessment: national and ECTS

Sum of points including all forms of study	Mark on the ECTS scale	Mark on the national scale	
		for an exam, a term paper, practice	for a test
90 – 100	A	excellent	passed
82 – 89	B	very good	
74 – 81	C	good	
64 – 73	D	satisfactory	
60 – 63	E		
35 – 59	FX	unsatisfactory	failed
1 – 34	F		

5. Recommended reading

5.1. Main

1. Вища математика : базовий підручник для вузів / під ред. В. С. Пономаренка. – Харків : Фоліо, 2014. – 669 с.
2. Guidelines for practical tasks in analytic geometry of the academic discipline "Higher and Applied Mathematics" for foreign and English-learning full-time students of the preparatory direction "Management" / compiled by Ie. Iu. Misiura. – Kh. : Publishing House of KhNUE, 2011. – 76 p. (English, Ukrainian)
3. Higher mathematics : handbook Vol. 1 / under edition of Kurpa L. V. – Kh. : NTU "KhPI", 2006. – 344 p.
4. Higher mathematics : handbook Vol. 2 / under edition of Kurpa L. V. – Kh. : NTU "KhPI", 2006. – 540 p.
5. Higher mathematics : handbook Vol. 3 / under edition of Kurpa L. V. – Kh. : NTU "KhPI", 2006. – 364 p.
6. Higher mathematics: handbook Vol. 4 / under edition of Kurpa L. V. – Kh. : NTU "KhPI", 2006. – 328 p.
7. Methodical recommendations for the conduct of the practical studies in the academic discipline "Higher mathematics" for foreign and English-learning students of the preparatory direction "Management" of the full-time education / complied by Ie. Iu. Misiura. – Kh. : Publishing House of KhNUE, 2010. – 44 p. (English, Ukrainian)

5.2. Additional

8. Англо-русский словарь математических терминов / под ред. П. С. Александрова. – М. : Мир, 1994. – 416 с.
9. Малярець Л. М. Математика для економістів : навч. посіб. У 2-х ч. Ч. 1 / Л. М. Малярець, Л. М. Афанасьєва, А. В. Ігначкова. – Харків : Вид. ХНЕУ, 2011. – 393 с.
10. Малярець Л. М. Математика для економістів : навч. посіб. У 2-х ч. Ч. 2 / Л. М. Малярець, Л. М. Афанасьєва, А. В. Ігначкова. – Харків : Вид. ХНЕУ, 2011. – 368 с.
11. Borakovskiy A. B. Handbook for problem solving in higher mathematics / A. B. Borakovskiy, A. I. Ropavka. – Kh. : KNMA, 2008. – 195 p.
12. Handbook of mathematics / I. N. Bronshtein, K. A. Semendyaev, G. Musiol et. al. – Berlin : Springer, 2007. – 1097 p.
13. Gibbons R. Game theory for applied economics / Robert Gibbons. – New Jersey : Princeton University Press, 2002. – 142 p.
14. Weber R. Mathematics for operations research / Richard Weber. – Chichester : Wiley, 2000. – 99 p.

5.3. Methodical support

15. Сайт персональних навчальних систем: https://pns.hneu.edu.ua/enrol/index.php?id=864