MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE

SIMON KUZNETS KHARKIV NATIONAL UNIVERSITY OF ECONOMICS

PROBABILITY THEORY AND MATHEMATICAL STATISTICS

Syllabus

for Bachelor's (first) degree students of all specialities

Kharkiv
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P93

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The thematic plan of the academic discipline and its content are given according to the modules and themes. Plans of lectures and practical trainings, material for students' knowledge consolidation (test questions, tasks for independent work), as well as methods of students' knowledge assessment according to the credit transfer system of studies are presented.

For Bachelor's (first) degree students of all specialities.

UDC 519.2(07.034)

Introduction

The fundamental base of the mathematical training of economists and managers is the academic discipline "Probability Theory and Mathematical Statistics" which is the normative discipline of the natural science series and a component of the structurally logical scheme which is provided for the educational professional program of the bachelor training of all specialities.

The basic problems of teaching the discipline are: giving students knowledge of the basic parts of probability theory and mathematical statistics, raising the level of the fundamental mathematical training of students with intensification of its applied direction, mastering the fundamentals of probability theory and mathematical statistics and application of this knowledge to economic investigations for solving economic problems, forming skills in the application of elements of probability theory and mathematical statistics to investigations which are applied as an instrument of investigation and solving optimization economic problems for forming models of economic processes and developments, the obtainment of the necessary totality of theoretical and practical knowledge for solving specific problems which are set in the process of forming economic and mathematical models, and the obtainment of the required mathematical training for studying other disciplines.

1. Description of the academic discipline

<table>
<tr><td rowspan="2">Name of indicators</td><td rowspan="2">Speciality,
academic degree</td><td colspan="2">Academic discipline features</td></tr>
<tr><td>day-time form of studies</td><td>distant form of studies</td></tr>
<tr><td>Number of credits:
5 for the full-time form;
6 for the distant form</td><td>All specialities</td><td colspan="2">Compulsory</td></tr>
<tr><td rowspan="3">Number of thematic modules: 2</td><td rowspan="6">All specialities</td><td colspan="2">Academic year</td></tr>
<tr><td>1st</td><td>2nd</td></tr>
<tr><td colspan="2">Term</td></tr>
<tr><td rowspan="3">Total number of hours:
150 for the day-time form;
180 for the distant form</td><td>2nd</td><td>3rd</td></tr>
<tr><td colspan="2">Lectures</td></tr>
<tr><td>32 hours</td><td>8 hours</td></tr>
<tr><td rowspan="10">The number of hours for the day-time form of studies per week:
in class: 4;
student's independent work: 5</td><td rowspan="10">Academic degree:
Bachelor's (first)</td><td colspan="2">Practical studies</td></tr>
<tr><td>16 hours</td><td>12 hours</td></tr>
<tr><td colspan="2">Laboratory studies</td></tr>
<tr><td>16 hours</td><td>–</td></tr>
<tr><td colspan="2">Independent work</td></tr>
<tr><td>86 hours</td><td>160 hours</td></tr>
<tr><td colspan="2">Form of control</td></tr>
<tr><td colspan="2">Exam</td></tr>
</table>

Note. The ratio of the number of class hours and independent work is:
74 % for the full-time form of studies;
11 % for the distant form of studies.

2. The main purpose and tasks of the academic discipline

The main purpose of teaching is forming future specialists' basic mathematical knowledge for solving theoretical and practical problems in professional activity of a competent specialist in the service sphere, skills in analytical thinking and skills in using mathematical knowledge for formation of real processes and developments, and for solving economic problems.

The main tasks that should be solved in the process of teaching the discipline are: giving students knowledge of the basic parts of probability theory and mathematical statistics; definitions, theorems, rules; proving the main theorems; mastering the fundamentals of the methodology of mathematical investigation of the applied economic problems; independent broadening of knowledge, development of logical and algorithmical thinking; the obtainment of primary skills in independent learning of mathematical and applied publications.

The subject of the discipline "Probability Theory and Mathematical Statistics" is the fundamentals of probability theory and mathematical statistics.

In the process of learning the discipline "Probability Theory and Mathematical Statistics" a student receives analytic and investigatory competences which are necessary for a modern economist in any sphere of his activity.

The syllabus of the academic discipline "Probability Theory and Mathematical Statistics" is compiled according to the statements of the field standard of higher education of the Ministry of Education and Science of Ukraine based on the educational professional program of bachelor training which has been made by the Scientific Methodical Committee of Economics and Enterprise of the Ministry of Education and Science of Ukraine.

A student starts studying the academic discipline "Probability Theory and Mathematical Statistics" in the first term of the first year of studies.

In the process of learning the students obtain the required theoretical knowledge during lectures and acquire practical skills at the practical and laboratory studies and during independent work and fulfillment of individual tasks. Independent and individual work of students has a great value in the process of mastering the material and consolidation of knowledge. All of these types of studies were devised according to the statements of the Bolognese Declaration.

As a result of studying the academic discipline a student **must know**:

the general notions of probability theory;

random, persistent and impossible events;

the notion of probability and the methods of calculation of probability;

compatible and incompatible events;

dependent and independent events;

conditional and unconditional probabilities;

addition theorems;

multiplication theorems;

the total probability formula and Bayes' formula;

a trial by Bernoulli's scheme;

discrete and continuous random variables;

the basic laws of the distribution of discrete random variables and their basic numerical characteristics;

the basic laws of the distribution of continuous random variables and their basic numerical characteristics;

theoretical initial and central moments;

conditional laws of the distribution of probabilities of components of a discrete two-dimensional random variable;

calculation of basic numerical characteristics of a two-dimensional random variable;

characteristics of the function of one random argument;

the random process;

the simplest flow of events;

Markov chains;

limiting theorems of probability theory;

the general notions of mathematical statistics;

the sampling method;

statistical distribution and its basic numerical characteristics;

statistical estimations of parameters of a population;

requirements for statistical estimations;

the notions of point and interval estimations and calculation of their accuracy;

the methods of parametric and nonparametric estimations of parameters;

the types of statistical hypotheses;

statistical hypotheses and statistical criteria for checking them;

***be able to*:**

calculate the probability of an elementary random event using classical and geometrical definitions of probability;

use the Venn − Euler diagram for representation of random events on a sample space;

compute the probability of a complete random event with the help of addition and multiplication theorems;

calculate the total probability and the posteriori probability of a definite hypothesis using Bayes' formula;

compute the probability that a random event will occur a definite number of times in a series of repeated trials (using Bernoulli's scheme);

calculate the basic characteristics of a discrete random variable which is distributed by a binomial law and use the built-in function of MS Excel for calculations;

use the local and integral theorems of Moivre – Laplace for approximate calculation of probabilities of random events if a random event is distributed by a binomial law and the number of independent trials increases unboundedly;

use Poisson formula for approximate calculation of probabilities of random events if a random event is distributed by a binomial law, the number of independent trials increases unboundedly and the occurrence of an event in one trial is low-probable;

define the distribution function of a discrete random variable and plot its graph;

define the distribution function and the density function of a continuous random variable and plot these graphs;

calculate the basic numerical characteristics of the distribution of a continuous random variable: the mathematical expectation; the variance; the root-mean-square deviation;

calculate additional numerical characteristics of the distribution of random variables: the mode, the median, the theoretical initial and central moments, the asymmetry, the excess; also use the built-in functions of MS Excel for calculations;

construct conditional laws of the distribution of probabilities of a composite discrete two-dimensional random variable;

compute the basic numerical characteristics of a discrete two-dimensional random variable and use the built-in functions of MS Excel for calculations;

construct laws of the distribution of a random variable which is the function of one random argument;

compute the matrix of transition probabilities and the presentation of Markov chain as a graph;

plot a histogram and a polygon of frequencies and relative frequencies in the investigation of a sample;

plot graphs of functions with the help of the built-in functions of MS Excel;

define the statistical distribution function and plot its graph;

calculate the basic numerical characteristics of a statistical distribution: the sample mean; the sample variance; the corrected variance; the root-mean-square deviation;

compute additional numerical characteristics of a statistical distribution: the mode, the median, the theoretical initial and central moments, the variation coefficient;

define the point statistical estimations of numerical characteristics of a population and the confidence intervals for statistical estimations of the mathematical expectation, the variance and the root-mean-square deviation using the built-in functions of MS Excel;

calculate the sample size which is necessary for defining the basic numerical characteristics of the distribution of a population with a definite reliability;

compute the estimations of parameters of the distribution of a random variable supposing normal, random and exponential laws in a population using the data of a sample;

use statistical criteria for checking the statistical hypothesis about the significance of the difference between the values of means of two samples;

use statistical criteria for checking statistical hypothesis about the significance of the difference between the values of variances of two samples;

use statistical criteria for checking the statistical hypothesis about the significance of the difference between empirical frequencies and frequencies which are calculated supposing a normal distribution of a random variable in a population;

classify empirical data and construct empirical regression lines in a pair correlation;

conduct an estimation of equation parameters supposing a correlation dependence between two random variables and use the built-in functions of MS Excel for this purpose;

check the statistical hypothesis about the significance of a correlation using the Fisher fitting test;

construct a confidence of a regression line;

learn mathematical literature by oneself;

form and use economic mathematical models for an estimation of the state and the forecast of the development of economic processes;

apply the skills at using the built-in functions of MS Excel to statistical investigations;

broaden the knowledge, develop logical and algorithmic thinking by oneself.

A modern tendency in higher education is a reorientation of students of higher educational institutions from the process of education to the result, from knowledge to skills, forming definite competences.

In the process of learning the academic discipline "Probability Theory and Mathematical Statistics" students receive analytic and investigatory **competences** which are required for a modern economist in any sphere of their activity (Table 2.1).

Table 2.1

Competences which are formed as a result of mastering the academic discipline "Probability Theory and Mathematical Statistics"

Code of compe-tence	Name of competence	Components of competence
1	2	3
PT&MS*1	Using the methods of probability theory for a prognosis of probabilistic random events and making a graphic interpretation of solutions to economic problems with the help of instruments of probability theory	Using basic definitions and theorems to calculate the probability of a random event. Defining laws of the distribution of discrete and continuous (one-dimensional) random variables, calculating their basic numerical characteristics, plotting distribution functions. Finding the numerical characteristics of the function of a discrete and continuous random argument

Table 2.1 (the end)

1	2	3
		Using the concept of the theory of random processes and the queueing theory for modeling of economic processes
PT&MS 2	Identification of quantitative characteristics of economic processes with the help of the sampling method	Understanding the relationship between the instruments of probability theory and mathematical ones. Forming a representative sampling totality. Plotting a variational series and estimating basic numerical characteristics of a random variable using the results of investigation of a sample. Checking the statistical hypothesis of the correspondence of properties of numerical characteristics and the distribution law of a random variable in a population and their estimations using the results of the investigation of a sample. Understanding the possibilities and restrictions on the use of instruments of mathematical statistics when solving real economic problems
PT&MS 3	Using the variance analysis for investigation of economic processes, using correlation and regression analysis when studying different economic phenomena, understanding the meaning of economic values which form a model of a pair regression	Understanding possibilities of the use of the single-factor analysis of variance when checking the existence of the difference between the investigated samples. Distinguishing the types of dependences between the economic factors and defining the essence of a correlation. Investigating the form of correlation and constructing a model of a pair regression using the least-squares method (LSM). Knowledge of the methods of checking the significance of parameters of a model of pair regression and estimation of the adequacy of the model on the whole

* Application of probability theory and instruments of mathematical statistics (PT&MS)

The structure of professional competences and formation of these competences according to the National Framework of Qualifications of Ukraine is given in Appendix A.

3. The syllabus of the academic discipline

Thematic module 1. Probability theory

Theme 1. The empirical and logical bases of probability theory

1.1. The subject and problems of this course.

The role of this course as a theoretical basis for mathematical modelling of economic processes and phenomena, which take into consideration possible risks.

1.2. Basic definitions, rules and types of events.

A probabilistic model of an experiment. Sure (certain), random and impossible events.

Rules of operations with random events. The space of elementary events.

1.3. Basic concepts and formulas.

A classical definition and calculation of a probability. Basic formulas of combinatorics. A statistical definition of a probability.

Axiomatics of Kolmogorov. A geometrical definition of a probability.

The Venn − Euler diagram.

Theme 2. The basic theorems of probability theory, their economic meaning

2.1. Basic definitions and multiplication theorems of probabilities.

Probabilistic space. Dependent and independent events. A conditional probability. Multiplication theorems of probabilities.

2.2. Basic concepts and addition theorems of probabilities.

A complete group of events. Complementary events. Joint (compatible) and disjoint (incompatible) events. Addition theorems of probabilities.

2.3. The probability of at least one event.

The probability of at least one event. The probability that an event will occur at least once.

Calculation of the necessary number of trials, which occur with a definite reliability in order to guarantee the occurrence of a random event at least once.

2.4. The formula of a total probability and Bayes formula.

The total probability formula. Bayes' formula (the theorem of hypothesis).

Theme 3. The scheme of independent trials

3.1. Repeated independent trials and Bernoulli's scheme.

The scheme of repeated independent trials. Bernoulli's formulas.

3.2. The local theorem of Moivre – Laplace.

The local theorem of Moivre – Laplace. Gauss's function, its properties, application to approximate calculations of the probability of occurrence of a random event a definite number of times in a series of independent trials.

3.3. The integral theorem of Moivre – Laplace.

The integral theorem of Moivre – Laplace. Laplace's function, its properties and application to approximate calculations of the probability that the values of a random variable lie in a definite interval.

The relationship between the Gauss and Laplace functions.

3.4. Poisson's theorem.

Low-probability events. Poisson's theorem.

Theme 4. Random variables and their economic meaning

4.1. Definitions and types of random variables.

The definition of a random variable. Discrete and continuous random variables.

4.2. Distribution laws and the distribution function of probabilities.

Distribution laws of probabilities for a random variable and ways of finding (tabular, graphic and analytical).

The distribution function of probabilities, its properties.

4.3. The basic numerical characteristics.

The basic numerical characteristics of a random variable: the mathematical expectation, the variance and the root-mean-square deviation. Properties of the basic numerical characteristics.

4.4. Additional numerical characteristics.

Additional numerical characteristics of a distribution: the mode, the median, the excess.

The application of initial and central theoretical moments to the calculation of numerical characteristics of the distribution of a random variable.

Theme 5. Distribution laws and numerical characteristics of discrete random variables

5.1. The basic types and properties of distribution laws of a discrete random variable.

Distribution laws of a discrete random variable which are often used in social and economic investigations: a binomial distribution, a geometrical distribution, a hypergeometrical distribution. Specificities and properties of these distributions, their basic numerical characteristics and the economic meaning.

5.2. A flow of events.

A flow of events. The simplest flow of events and its properties.

Theme 6. The basic distribution laws of a continuous random variable

6.1. The density distribution function and its properties.

The density of a distribution and its probable meaning. The density distribution function of a random variable and its properties.

6.2. The basic types and properties of distribution laws of a continuous random variable.

Distribution laws of a continuous random variable which are often used in social and economic investigations: a uniform distribution, a normal distribution and an exponential distribution. Properties of these distributions and their basic numerical characteristics. The influence of parameters of a distribution on the density function of probabilities in the normal distribution law.

6.3. Student's, Pearson's and Fisher's distributions.

Student's distribution, Pearson's distribution and Fisher's distribution. Specificities and properties of these distributions. The relationship of these distributions and the normal distribution law of a continuous random variable.

Theme 7. Multidimensional random variables

7.1. The system of random variables.

The concept of the system of random variables. The distribution function and the density of distribution of the system of two random variables.

7.2. Conditional distribution laws of the system of random variables.

Conditional distribution laws of the components of the system of discrete random variables, their basic numerical characteristics.

7.3. *The basic numerical characteristics of the system of two random variables.*

The basic numerical characteristics of the system of two random variables.

The correlation moment, its properties.

The coefficient of a correlation, its properties.

Theme 8. The functions of the random argument

8.1. The basic definitions and the distribution law of the function of a random argument.

The general concepts of the function of a random argument.

The construction of the distribution law of the function of a discrete random argument using the distribution law of its argument.

8.2. Basic numerical characteristics of the function of a random argument.

Basic numerical characteristics of the function of a random argument.

The function of a continuous random argument and its basic numerical characteristics. The function of two random variables.

Theme 9. The elements of the theory of random processes and the theory of queuing problems

9.1. Basic definitions of the queuing theory.

The subject of the queuing theory. Random functions. Random processes with a finite set of states.

9.2. Numerical characteristics and the correlation function.

Numerical characteristics and the correlation function of a random process. Properties of the correlation function of a random process. The correlation function of the dependence between two random processes.

9.3. A flow of events and the elements of the queuing theory.

A flow of events. The simplest flow of events and its properties. Markov random process and Markov chains. The elements of the queuing theory.

Thematic module 2. Mathematical statistics

Theme 10. Limit theorems of probability theory

10.1. The statistical definition of the probability and statistical stability.

Bernoulli's theorem. The statistical definition of a probability. The statistical stability of sampling characteristics.

10.2. Basic theorems and their consequences.

Laws of large numbers. Convergence in probability and convergence in distribution.

Chebishev's inequality. The theorem of Chebishev and its consequences. The theorem of Lyapunov (the central limiting theorem) and its consequences. The theorem of Glivenko.

Theme 11. Preprocessing of statistical data

11.1. Basic definitions and problems of mathematical statistics.

Basic problems of mathematical statistics. A sampling method. Definitions of the population and its sample.

11.2. Discrete and interval variational series.

The empirical distribution law. The ways of the presentation of sampling totalities and representation of the results of observations.

Discrete and interval variational series, rules of construction. The defining limits of an interval by Sturges's formula.

11.3. The graphical presentation of a statistical distribution.

The graphical presentation of a statistical distribution. A polygon and a histogram.

11.4. Basic numerical characteristics.

Basic sampling numerical characteristics and their asymptotic behavior.

Theme 12. Statistical estimation of the parameters of a distribution

12.1. Basic statistical estimations and their properties.

Statistical estimations of parameters of the distribution of a population and their properties: the unbiasedness, the possibility and the efficiency.

The asymptotic efficiency of maximally plausible estimations.

12.2. Types of estimations.

The method of moments. Point and interval estimations. The confidence interval for mathematical expectation of a normal population.

Theme 13. Checking the statistical hypothesis

13.1. Types of statistical hypothesis and kinds of errors.

The main and alternative statistical hypothesis. A statistical test. Construction of critical domains for a statistical test. Errors of the first and second kinds. The concept of power of a test.

13.2. Different types of tests for checking a statistical hypothesis.

Checking the statistical hypothesis about the definition of the distribution law for a population using the results of the investigation of a sample. Pearson and Kolmogorov fitting tests. The fitting test relative to a frequency.

Checking the statistical hypothesis about the equality of two population means on the assumption of the normal distribution law.

Student's fitting test. Comparison of variances. The Fisher – Snedeker fitting test.

Checking the hypothesis about the equality of the sampling mean and the mathematical expectation.

Theme 14. The elements of the correlation theory

14.1. Basic definitions and problems of correlation analysis.

The problems of correlation analysis.

14.2. Basic coefficients and their properties.

The sampling coefficient of a correlation, its properties and the confidence interval.

The coefficient of determination. The correlation ratio, its properties.

Theme 15. The elements of the variance analysis

15.1. Basic definitions and problems of the variance analysis.

Problems of the variance analysis. The role of the variance analysis in economic investigations.

15.2. The single-factor variance analysis.

The single-factor analysis as a procedure for checking the hypotheses about the lack of the factor influence on the feature which is investigated. The general, external and intrinsical group variances.

15.3. The multidimensional variance analysis.

The concept of the multidimensional variance analysis.

Theme 16. The elements of the regression theory

16.1. The basic concepts of regression analysis and LSM.

The problems of regression analysis. The correlation dependence. The correlation table. The empirical lines of a regression. The estimation of parameters of a pair regression equation using the least-squares method (LSM). Point estimations.

16.2. The significance and the confidence interval of a pair regression line.

Checking the significance of parameters of a pair regression equation. The confidence interval for a line of a pair regression.

4. The structure of the academic discipline

From the very beginning of studying the academic discipline each student has the possibility to learn both the discipline syllabus and the forms

of organization of education, as well as the structure, contents and volume of each of its educational modules, and all types of control and the methods of assessment of the educational work. The educational process according to the syllabus of the academic discipline "Probability Theory and Mathematical Statistics" is realized in such forms as: lectures, practical and laboratory studies; fulfillment of students' independent work; control activities.

A student's mastering of the academic discipline is carried out through consecutive and thorough learning of the educational modules. An educational module is a relatively separate block of the given discipline, which logically unites its educational elements by content and interconnections. The assessment of knowledge and skills obtained by a student while learning the material of each module is effected at the final module control.

The thematic plan of the academic discipline consists of two thematic modules (Table 4.1).

Table 4.1

The structure of the test credit of the academic discipline

Names of thematic modules and themes	The number of hours											
	the day-time form of studies						the distant form of studies					
	total	which are allocated for					total	which are allocated for				
		lectures	practical studies	laboratory studies	final control	independent work: preparation for studies		lectures	practical studies	laboratory studies	final control	independent work: preparation for studies
1	2	3	4	5	6	7	8	9	10	11	12	13
Thematic module 1 Probability theory												
Theme 1. The empirical and logical bases of probability theory	8	2	1	1	–	4	7	1	1	–	–	5

Table 4.1 (continuation)

1	2	3	4	5	6	7	8	9	10	11	12	13
Theme 2. The basic theorems of probability theory, their economic meaning	8	2	1	1	–	4	23	1	1	–	–	10
Theme 3. The scheme of independent trials	5.5	1	0.5	1	–	3			1	–	–	10
Theme 4. Random variables and their economic meaning	3.5	1	0.5	1	–	1	8	1	1	–	–	6
Theme 5. Distribution laws and numerical characteristics of discrete random variables	9	2	1	2	–	4	12	1	1	–	–	10
Theme 6. The basic distribution laws of a continuous random variable	8	2	2	–	–	4	12	1	1	–	–	10
Theme 7. Multidimen-sional random variables	9	2	–	2	–	5	10	–	–	–	–	10
Theme 8. The functions of the random argument	8	2	1	–	–	5						

Table 4.1 (continuation)

1	2	3	4	5	6	7	8	9	10	11	12	13
Theme 9. The elements of the theory of random processes and the theory of queuing problems	8	2	1	–	–	5	10	–	–	–	–	10
Total hours for module 1	**67**	**16**	**8**	**8**	–	**35**	**82**	**5**	**6**	**–**	**–**	**71**
Thematic module 2 Mathematical statistics												
Theme 10. Limit theorems of probability theory	4	1	1	–	–	2	11	–	1	–	–	10
Theme 11. Preprocessing of statistical data	6	1	1	1	–	3	18	1	1	–	–	15
Theme 12. Statistical estimation of the parameters of a distribution	8	2	1	1	–	4			1			
Theme 13. Checking the statistical hypothesis	15	4	1	2	–	8	17	1	1	–	–	15
Theme 14. The elements of the corre-lation theory	9	2	1	1	–	5	21	1	1	–	–	20
Theme 15. The elements of the variance analysis	14	2	1	1	–	10	22		1	–	–	20
Theme 16. The elements of	18	4	2	2	–	10				–	–	

Table 4.1 (the end)

1	2	3	4	5	6	7	8	9	10	11	12	13
the regression theory												
Total hours for module 2	**74**	**16**	**8**	**8**	**–**	**42**	**89**	**3**	**6**	**–**	**–**	**80**
Preparation for the exam	5	–	–	–	–	5	5	–	–	–	–	5
Preexam consultations	2	–	–	–	2	–	–	–	–	–	–	**–**
Exam	2	–	–	–	2	–	2	–	–	–	2	**–**
Total hours	**150**	**32**	**16**	**16**	**86**		**180**	**8**	**12**	**–**	**160**	

5. The plan of the practical studies

5.1. The themes of the practical studies

A practical study is a form of educational studies at which the lecturer organizes a detailed consideration of separate theoretical issues of the academic discipline and forms the abilities and skills in the practical application of the theory through students' individual accomplishment of the formulated tasks.

Conducting a practical study is based on the previously prepared material, i. e. tests aiming to assess the degree of the mastery of the required theoretical statements, and tasks of different complexity to be solved by students.

A practical study includes conducting previous control of students' knowledge, abilities and skills, formulation of a general problem by the lecturer and discussion of the problem with the students, solving test tasks, reviewing and assessment of the tasks.

The plan of the practical studies, their content and a bibliography for each theme are given in Table 5.1.

Table 5.1

The plan of practical studies

The name of the thematic module	The themes of the practical studies (within the modules)	The number of hours	Recom-mended reading
1	2	3	4
Thematic module 1. Probability theory	*Theme 1. The empirical and logical bases of probability theory.* 1. Solving of problems using a classical definition of the probability of random events and the elements of combinatorics. *Theme 2. The basic theorems of probability theory, their economic meaning*. 1. Calculation of a conditional probability, using the theorems of multiplication and addition of probabilities. 2. The total probability formula and Bayes' formula	2	Main: [1 – 6]. Additional: [7 – 17]. Methodological support [18 – 19]
	Theme 3. The scheme of independent trials. 1. Solving of problems using Bernoulli's formulas. 2. Calculation of the most probable number of the occurrences of a random event. 3. Application of the asymptotic theorems of Moivre – Laplace and Poisson. *Theme 5. Distribution laws and numerical characteristics of discrete random variables*. 1. Calculation of the basic numerical characteristics of a random variable using the definitions. 2. Calculation of additional characteristics of a random variable using the definitions	2	Main: [1 – 6]. Additional: [7 – 17]. Methodological support [18 – 19]
	Theme 6. The basic distribution laws of a continuous random variable. 1. Solving of problems with random variables which are distributed by the normal law. 2. Solving of problems with random variables which are distributed by the uniform law. 3. Solving of problems with random variables which are distributed by the exponential law	2	Main: [1 – 6]. Additional: [7 – 17]. Methodological support [18 – 19]

Table 5.1 (continuation)

1	2	3	4
Thematic module 1. Probability theory	*Theme 8. The functions of the random argument.* 1. Calculation of the numerical characteristics of the function of a random argument. *Theme 9. The elements of the theory of random processes and the theory of queuing problems*. 1. Construction of the space of states with the matrix of probabilities of transformations	2	Main: [1 – 6]. Additional: [7 – 17]. Methodological support [18 – 19]
Thematic module 2. Mathematical statistics	*Theme 10. Limit theorems of probability theory.* 1. Solving of problems using Bernoulli's theorem. 2. Solving of problems with the help of the theorems of Chebishev and Lyapunov. *Theme 11. Preprocessing of statistical data.* 1. Construction of a variational series, a polygon and a histogram. 2. Calculation of basic numerical characteristics of an empirical distribution	2	Main: [1 – 6]. Additional: [7 – 17]. Methodological support [18 – 19]
	Theme 12. Statistical estimation of the parameters of a distribution. 1. Calculation of basic numerical characteristics of an empirical distribution. *Theme 13. Checking the statistical hypothesis.* 1. Checking the statistical hypothesis about the definition of the distribution law in a population using the results of investigation of a sample according to Pearson's fitting test. 2. Using the Student's test to check the statistical hypothesis about the equality of two population means with the assumption of the normal distribution law and variances	2	Main: [1 – 6]. Additional: [7 – 17]. Methodological support [18 – 19]
	Theme 14. The elements of the correlation theory. 1. Calculation of the sample correlation coefficient. 2. Construction of the confidence interval of the sample correlation coefficient. *Theme 15. The elements of the variance analysis.* 1. Using the single-factor analysis for checking the hypothesis of the lack of the factor of influence on the feature which is investigated	2	Main: [1 – 6]. Additional: [7 – 17]. Methodological support [18 – 19]

Table 5.1 (the end)

1	2	3	4
Thematic module 2. Mathematical statistics	*Theme 16. The elements of the regression theory.* 1. Estimation of parameters of a pair regression with the help of the least squares method (LSM). 2. Defining the accuracy of estimation. 3. Checking the significance of parameters of the equation of a pair regression. 4. Construction of the confidence interval for a pair regression line. 5. Investigation of the simplest cases of nonlinear regression	2	Main: [1 – 6]. Additional: [7 – 17]. Methodological support [18 – 19]

5.2. Examples of typical tasks of a class written test according to the themes

Thematic module 1
Probability theory

Theme 1. The empirical and logical bases of probability theory

Level 1. ***Task 1.*** Paul has 3 green shirt, 5 red shirts, 1 pink shirt, and 9 striped shirts. He randomly draws one out of his drawer.

1. What is the probability that the shirt is green? 2. What is the probability that the shirt is red? 3. What is the probability that the shirt is striped? 4. What is the probability that the shirt is pink? 5. What is the probability that the shirt is not striped? 6. What is the probability that the shirt is red and striped? 7. What is the probability that the shirt is green or pink?

Theme 2. The basic theorems of probability theory, their economic meaning

Level 1. **Task 2.** A jar contains black and white marbles. Two marbles are chosen without replacement. The probability of selecting a black marble and then a white marble is 0.34, and the probability of selecting a black marble on the first draw is 0.47.

What is the probability of selecting a white marble on the second draw, given that the first marble drawn was black?

Level 1. **Task 3 (1 – without replacement, 2 – with replacement)** The table below lists the results of a student's survey pertaining to favorite types of music.

Music	Rock	Jazz	Country	Classical	Rap
Number	51	24	33	5	15

Using the given table find the probability that

a) 2 students like jazz;

b) exactly 1 student likes rock and 1 student does rap;

c) at least one of two students likes classical music;

d) the first student likes country.

Theme 3. The scheme of independent trials

Level 2. **Task 4.** The probability of train's arrival at the station on time is equal to 0.8. What is the probability that out of 4 expected trains 2 trains will arrive on time?

Theme 5. Distribution laws and numerical characteristics of discrete random variables

Level 2. **Task 5.** The probability of passing an exam excellently for each of three students equals 0.4. Make up a distribution law of a number of excellent marks which are got by the students at the exam. Find a mathematical expectation, a variance and a root-mean square deviation of a discrete random variable.

Theme 6. The basic distribution laws of a continuous random variable

Level 3. **Task 6.** The density function of a continuous variable X is given:

$$f(x)=\begin{cases}0, & x\le 0\\ c\left(4x-2x^2\right), & 0<x\le 2\\ 0, & x>2\end{cases}.$$

a) What is the value of c?

b) Find $P(X>1)$.

c) Find the mathematical expectation $M(X)$, the variance $D(X)$ and the root-mean-square deviation $\sigma(X)$;

d) $P(2<X<10)$.

Thematic module 2
Mathematical statistics

Theme 11. Preprocessing of statistical data

Level 1. **Task 1.** Fifty students who study computer science, the following number of points is given:

12,	14,	19,	15,	14,	18,	13,	16,	17,	12,
20,	17,	15,	13,	17,	16,	20,	14,	14,	13,
17,	16,	15,	19,	16,	15,	18,	17,	15,	14,
16,	15,	15,	18,	15,	15,	19,	14,	16,	18,
18,	15,	15,	17,	15,	16,	16,	14,	14,	17.

1) Construct a discrete statistic series for this sample, plot a polygon of relative frequencies and $F(x)$;

2) calculate $\overline{x_s}, S_x, R, v, A_s, E_s$;

3) find M_o and M_e.

Theme 12. Statistical estimation of the parameters of a distribution

Level 2. **Task 2.** The interval statistical series of x_i of a number of defective articles is given:

$x_i - x_{i+1}$	5 − 10	10 − 15	15 − 20	20 − 25	25 − 30
m_i	2	6	10	8	4

Construct the confidence intervals for $\overline{x}_{pop}$ and S_{pop} with the confidence probability $\gamma =$0.9, 0.93, 0.95, 0.99 and 0.999.

Theme 13. Checking the statistical hypothesis

Level 2. **Task 3.** Using χ^2 test for goodness of fit, check the assumption about the normal distribution of the population if there are empirical (m_i) and theoretical ($\widetilde{m}_i$) frequencies.

m_i	6	13	38	74	106	85	30	14
$\widetilde{m}_i$	3	14	42	82	99	76	37	13

Theme 14. The elements of the correlation theory

Theme 16. The elements of the regression theory

Level 3. The results of observations of variables X and Y are given as:

X	0.25	2.25	4.25	6.25	8.25	10.25	12.25
Y	6.0	9.5	16.0	18.0	23.5	27.0	33.0

It is necessary to:

1) calculate the numerical characteristics (the mean, the variance, the root-mean square deviation for variables X and Y, the coefficient of the covariation);

2) compute the empirical correlation coefficient (the strength) r_{xy}, explain it;

3) calculate the determination coefficient R^2, explain it;

4) compute the elasticity coefficient: $\overline{\mathrm{E}}$, explain it;

5) construct the theoretical regression line Y upon X as $\hat{y}_x = b_0 + b_1 x$ using the method of least squares;

6) explain the economic meaning of the regression coefficients;

7) estimate the significance of the regression equation using the F(Fisher)-criterion;

8) estimate the statistical significance of the coefficients using the t-test (Student test);

9) analyze the constructed model.

6. The themes of laboratory studies

The educational plan provides conducting laboratory studies on the academic discipline "Probability Theory and Mathematical Statistics" in the first term.

A laboratory study is a form of study when the student under the supervision of the lecturer fulfills a practical task with the help of PC-programming (software Excel). The plan of laboratory studies, their content and bibliography for each theme are given in Table 6.1.

Conducting a laboratory study on the defined theme is preceded by the analysis of the basic theoretical fundamentals forming practical skills. A laboratory study is fulfilled in the computer room with the use of MS Excel. It favours the following: firstly, the student extends the knowledge of the basic formulas and relations fulfilling calculations by direct writing of the corresponding formulas and, secondly, acquires skills in the use of built-in functions of MS Excel. This kind of approach gives a possibility to pay more attention to economic explanation of mathematical transformations.

Table 6.1

The plan of themes of laboratory studies

The theme name	The syllabus questions	Hours	Recommended reading
1	2	3	4
Thematic module 1 **Probability theory**			
Theme 1. The empirical and logical bases of probability theory. *Theme 2. The basic theorems of probability theory,* their economic meaning	Learning the software MS Excel. Learning the built-in functions of MS Excel. Using built-in functions of MS Excel for calculation of the probability of random events with the help of the theorems of multiplication and addition of probabilities. Using the formula of the total probability (a priori probability) and Bayes' formula (a posteriori probability)	2	Main: [1 – 3]. Additional: [4; 6; 8; 9]. Methodological support: [10; 11]

Table 6.1 (continuation)

1	2	3	4
Theme 3. The scheme of independent trials. *Theme 4.* Random variables and their economic meaning	Calculation of basic and additional numerical characteristics of a discrete random variable according to the definition and with the help of the built-in functions of MS Excel. Construction of the distribution of a sum, the difference and the product of random variables, calculation of their basic numerical characteristics according to the definition and with the help of properties of mathematical expectation and variance	2	Main: [1 – 3]. Additional: [4; 6; 8; 9]. Methodological support [10; 11]
Theme 5. Distribution laws and numerical characteristics of discrete random variables	Construction of the binomial law of a distribution for the definite values p and n. Calculation of the basic numerical characteristics of a distribution. Investigation of the influence of factors p and n on the form of the polygon of a distribution	2	Main: [1 – 3]. Additional: [4; 6; 8; 9]. Methodological support: [10; 11]
Theme 6. The basic distribution laws of a continuous random variable. *Theme 7.* Multidimensional random variables	Calculation of the basic numerical characteristics of a discrete random variable. Calculation of the basic numerical characteristics of the continuous random variable, a function and the density of a distribution of a continuous random variable. Construction of the theoretical law of the distribution of a multidimentional random variable (based on the example of a two-dimentional random variable), calculation of its basic numerical	2	Main: [1 – 3]. Additional: [4; 6; 8; 9]. Methodological support: [10; 11]

Table 6.1 (continuation)

1	2	3	4
	characteristics with the help of the definition and the built-in functions of MS Excel		
Thematic module 2 **Mathematical statistics**			
Theme 11. Preprocessing of statistical data. *Theme 12.* Statistical estimation of the parameters of a distribution	Construction and representation of a statistical interval series with the help of a histogram and a polygon. According to a set sample, carrying out the estimation of the basic numerical characteristics of the distribution of a random variable. Defining the point and interval (the confidence intervals) estimations of the basic numerical characteristics of a distribution with the help of the built-in functions of MS Excel	2	Main: [1 – 3]. Additional: [4; 5; 7 – 9]. Methodological support: [10; 11]
Theme 13. Checking the statistical hypothesis	According to the criterion, checking the hypothesis of the distribution law of a population based on the examples of uniform, exponential and normal laws of distribution. Plotting a histogram of relative frequencies and a cumulative curve with the help of the built-in functions of MS Excel	2	Main: [1 – 3]. Additional: [4; 5; 7 – 9]. Methodological support: [10; 11]
Theme 14. The elements of the correlation theory. *Theme 15.* The elements of the variance analysis	Calculation of basic numerical characteristics of the distribution of a two-dimensional random variable according to the definition and with the help of the built-in functions. Construction of an empirical equation of a regression equation with the help of the built-in functions and superstructures of MS Excel	2	Main: [1 – 3]. Additional: [4; 5; 7 – 9]. Methodological support: [10; 11]

Table 6.1 (the end)

1	2	3	4
Theme 16. The elements of the regression theory	Construction of the confidence interval of a regression line with the help of the built-in functions of MS Excel. Using the variance analysis in order to check the significance of the correlation between two random variables	2	Main: [1 – 3]. Additional: [4; 5; 7 – 9]. Methodological support: [10; 11]

7. Independent work

7.1. Forms of independent work

Independent work is the scheduled educational and scientific work which is carried out under the methodical and scientific supervision of the lecturer, it is a specific form of the educational activity designed to form independence of a person.

Independent work is:

1) different forms of individual and group cognitive activity of students, which is fulfilled by them during practical studies and in extracurricular time;

2) different types of educational tasks which are fulfilled under the supervision of the lecturer;

3) the system of organization of students' work in the absence and without direct assistance of the lecturer;

4) work of students which is carried out according to a specific individual educational plan designed on the basis of taking into account individual characteristics and cognitive possibilities of students.

The types of independent work and forms of control are given in Table 7.1.

The educational time which is intended for students' independent work of the day-time form of study is defined according to the educational plan and makes 57 % (86 hours) out of the total educational time for learning of the discipline.

For students of the distant form of study this time equals 89 % (160 hours) out of the total educational time for learning of the discipline.

During independent work a student becomes an active participant in the educational process, learns to consciously master the theoretical and practical knowledge, orientate easily in the methodological space, has to take responsibility for the quality of their own professional training.

Table 7.1

Tasks for students' independent work and forms of control

The name of the theme	The content of students' independent work	The number of hours	Forms of control of IWS	Recom-mended reading
1	2	3	4	5
Thematic module 1 Probability theory				
Theme 1. **The empirical and logical bases of probability theory**	Learning the lecture material. Preparation for the practical study and laboratory work. Preparation for the defence of the laboratory work. Independent learning of the questions of the theme: 1) statistical definition of a probability; 2) axiomatics of Kolmogorov. 3) the geometrical definition of a probability. Venn – Euler diagram. Carrying out the homework and independent work	4	Homework	Main: [1 – 6]. Additional: [7 – 17]
Theme 2. **The basic theo-rems of prob-ability theory, their economic meaning**	Learning the lecture material. Preparation for the practical study and laboratory work. Preparation for the defence of the laboratory work. Independent learning of the questions of the theme: 1) the probability of at least one event	4	Homework	Main: [1 – 6]. Additional: [7 – 17]

Table 7.1 (continuation)

1	2	3	4	5
	2) the probability that an event will occur at least once; 3) calculation of the necessary number of trials which occur with a definite reliability in order to guarantee the occurrence of the random event at least once. Carrying out the homework and independent work. Preparation for an independent test			
Theme 3. **The scheme of independent trials**	Learning the lecture material. Preparation for the practical study and laboratory work. Preparation for the defence of the laboratory work. Independent learning of the questions of the theme: 1) the relationship between the Gauss and Laplace functions; 2) application to approximate calculations of the probability that the values of a random variable lie in a definite interval; 3) low-probability events. Carrying out the homework and independent work	3	Homework	Main: [1 – 6]. Additional: [7 – 17]
Theme 4. **Random variables and their economic meaning**	Learning the lecture material. Preparation for the practical study and laboratory work. Preparation for the defence of the laboratory work. Independent learning of the questions of the theme:	1	Homework. Independent test	Main: [1 – 6]. Additional: [7 – 17]

Table 7.1 (continuation)

1	2	3	4	5
	1) the distribution laws of probabilities for a random variable and ways of finding (tabular, graphic and analytical); 2) the distribution function of probabilities, its properties; 3) the properties of basic numerical characteristics; 4) additional numerical characteristics of a distribution: the mode, the median, the excess; 5) initial and central theoretical moments, their application of these moments to the calculation of the numerical characteristics of a distribution of a random variable. Carrying out the homework and independent work			
Theme 5. **The distribution law and numerical characteristics of discrete random variables**	Learning the lecture material. Preparation for the practical study and laboratory work. Preparation for the defence of the laboratory work. Independent learning of the questions of the theme: 1) the binomial distribution; 2) the geometrical distribution; 3) the hypergeometrical distribution; 4) the specificities and properties of these distributions, their basic numerical characteristics and the economic meaning; 5) the flow of events	4	Homework	Main: [1 – 6]. Additional: [7 – 17]

Table 7.1 (continuation)

1	2	3	4	5
	6) the simplest flow of events and its properties. Carrying out the homework and independent work. Preparation for a written test			
Theme 6. **The basic distribution laws of a continuous random variable**	Learning the lecture material. Preparation for the practical study and laboratory work. Preparation for the defence of the laboratory work. Independent learning of the questions of the theme: 1) the influence of the parameters of a distribution on the density function of probabilities in the normal distribution law; 2) Student's distribution; 3) Pearson's distribution; 4) Fisher's distribution; 5) the specificities and properties of these distributions; 6) the relationship of these distributions and the normal distribution law of a continuous random variable. Carrying out the homework and independent work	4	Homework. A written test on the themes	Main: [1 – 6]. Additional: [7 – 17]
Theme 7. **Multi-dimensional random vari-ables**	Learning the lecture material. Preparation for the practical study and laboratory work. Preparation for the defence of the laboratory work. Independent learning of the questions of the theme: 1) the distribution function of	5	Homework. A compe-tence-oriented task	Main: [1 – 6]. Additional: [7 – 17]

Table 7.1 (continuation)

1	2	3	4	5
	the distribution of the system of two random variables; 2) the density of the distribution of a system of two random variables; 3) the conditional distribution laws of components of a system of discrete random variables; 4) their basic numerical characteristics; 5) the basic numerical characteristics of a system of two random variables; 6) the correlation moment, its properties; 7) the coefficient of a correlation, its properties. Carrying out the homework and independent work. Preparation for the defence of the competence-oriented task			
***Theme 8*. The functions of the random argument**	Learning the lecture material. Preparation for the practical study and laboratory work. Preparation for the defence of the laboratory work. Independent learning of the questions of the theme: 1) construction of the distribution law of the function of a discrete random argument using the distribution law of its argument; 2) basic numerical characteristics of the function of a random argument	5	Homework. A compe-tence-oriented task	Main: [1 – 6]. Additional: [7 – 17]

Table 7.1 (continuation)

1	2	3	4	5
	3) the function of the continuous random argument and its basic numerical characteristics; 4) the function of two random variables. Carrying out the homework and independent work. Preparation for a colloquium			
Theme 9. **The elements of the theory of random processes and the theory of queuing problems**	Learning the lecture material. Preparation for the practical study and laboratory work. Preparation for the defence of the laboratory work. Independent learning of the questions of the theme: 1) random functions; 2) random processes with a finite set of states; 3) the numerical characteristics and the correlation function of the random process; 4) the properties of the correlation function of a random process; 5) the correlation function of the dependence between two random processes; 6) the simplest flow of events and its properties; 7) the Markov random process and the Markov chains; 8) the elements of the queuing theory. Carrying out the homework Preparation for a colloquium	5	Homework. Colloquium	Main: [1 – 6]. Additional: [7 – 17]
Total for thematic module 1		**35**	–	–

Table 7.1 (continuation)

1	2	3	4	5
Thematic module 2 Mathematical statistics				
Theme 10. **Limit theorems of probability theory**	Learning the lecture material. Preparation for the practical study and laboratory work. Preparation for the defence of the laboratory work. Independent learning of the questions of the theme: 1) the statistical definition of a probability; 2) statistical stability of sampling characteristics; 3) the laws of large numbers; 4) convergence in probability; 5) convergence in distribution; 6) Chebishev's inequality; 7) the theorem of Chebishov and its consequences; 8) the theorem of Lyapunov (the central limiting theorem) and its consequences; 9) the theorem of Glivenko. Carrying out the homework and independent work	2	Homework	Main: [1 – 6]. Additional: [7 – 17]
Theme 11. **Pre-processing of statistical data**	Learning the lecture material. Preparation for the practical study and laboratory work. Preparation for the defence of the laboratory work. Independent learning of the questions of the theme: 1) the sampling method; 2) ways of the presentation of sampling totalities and	3	Homework	Main: [1 – 6]. Additional: [7 – 17]

Table 7.1 (continuation)

1	2	3	4	5
	the representation of the results of observations; 3) discrete and interval variational series, rules of its construction; 4) the defining limits of the interval by the Sturges's formula; 5) graphical presentation of a statistical distribution; 6) the basic sampling numerical characteristics and their asymptotic behavior. Carrying out the homework and independent work			
Theme 12. **Statistical estimation of the parameters of a distribution**	Learning the lecture material. Preparation for the practical study and laboratory work. Preparation for the defence of the laboratory work. Independent learning of the questions of the theme: 1) statistical estimations of the parameters of the distribution of a population; 2) unbiasedness; 3) possibility; 4) efficiency; 5) asymptotic efficiency of maximally plausible estimations; 6) the method of moments; 7) point and interval estimations; 8) the confidence interval for mathematical expectation of a normal population. Carrying out the homework and independent work	4	Homework	Main: [1 – 6]. Additional: [7 – 17]

Table 7.1 (continuation)

1	2	3	4	5
Theme 13. **Checking the statistical hypothesis**	Learning the lecture material. Preparation for the practical study and laboratory work. Preparation for the defence of the laboratory work. Independent learning of the questions of the theme: 1) main and alternative statistical hypothesis; 2) a statistical test; 3) construction of critical domains for a statistical test; 4) errors of the first and second kinds; 5) the concept of power of a test; 6) checking the statistical hypothesis about the defining of the distribution law for a population using the results of investigation of a sample; 7) Pearson fitting test; 8) Kolmogorov fitting test; 9) the fitting test relative to a frequency; 10) checking the statistical hypothesis about the equality of two population means based on the assumption of the normal distribution law; 11) Student's fitting test; 12) comparison of variances; 13) Fisher – Snedeker fitting test; 14) checking the hypothesis about the equality of a sampling mean and mathematical expectation	8	Homework. An independent test on the themes	Main: [1 – 6]. Additional: [7 – 17]

Table 7.1 (continuation)

1	2	3	4	5
	Carrying out the homework and independent work. Preparation for an independent test and a written test			
Theme 14. **The elements of correlation theory**	Learning the lecture material. Preparation for the practical study and laboratory work. Preparation for the defence of the laboratory work. Independent learning of the questions of the theme: 1) the problems of the correlation analysis; 2) the sampling coefficient of a correlation, its properties and the confidence interval; 3) the coefficient of determination; 4) the correlation ratio, its properties. Carrying out the homework and independent work. Preparation for a written test	5	Homework. A written test on the themes	Main: [1 – 6]. Additional: [7 – 17]
Theme 15. **The elements of the variance analysis**	Learning the lecture material. Preparation for the practical study and laboratory work. Preparation for the defence of the laboratory work. Independent learning of the questions of the theme: 1) problems of the variance analysis; 2) the role of the variance analysis in economic investigations	10	Homework. A competence-oriented task on the themes	Main: [1 – 6]. Additional: [7 – 17]

Table 7.1 (continuation)

1	2	3	4	5
	3) the single-factor analysis as a procedure for checking the hypotheses about the lack of the factor influence on the feature which is investigated; 4) the general variance; 5) the external group variance; 6) intrinsical group variances; 7) the concept of multi-dimensional variance analysis. Carrying out the homework and independent work. Preparation for the defence of the competence-oriented task on the themes and the colloquium			
Theme 16. **The elements of the regression theory**	Learning the lecture material. Preparation for the practical study and laboratory work. Preparation for the presentation of the independent creative task. Independent learning of the questions of the theme: 1) problems of regression analysis; 2) the correlation dependence; 3) the correlation table; 4) empirical lines of a regression; 5) estimation of the parameters of a pair regression equation using the least-squares method (LSM); 6) point estimations; 7) checking the significance of parameters of a pair regression equation	10	Homework. A competence-oriented task on the themes	Main: [1 – 6]. Additional: [7 – 17]

Table 7.1 (the end)

1	2	3	4	5
	8) the confidence interval for a line of a pair regression. Carrying out the homework and independent work			
Total for thematic module 2		**42**	–	–
Preparation for the exam		**9**	Exam	Main: [1 – 6]. Additional: [7 – 17]
Total on the discipline		**86**	–	–

The necessary element of successful mastering of the material of the academic discipline is the students' independent work (SIW) with specifical mathematical and economic literature. The basic forms of tasks and control of independent work which are proposed to students for mastering the theoretical knowledge on the themes of the academic discipline are given in Table 7.1.

SIW includes: studying the lecture material (a lecture as a form of studies provides theoretical knowledge besides being used for carrying out practical calculations); studying and learning the recommended literature, basic terms and concepts on the themes of the academic discipline; preparation for practical and laboratory studies; preparation for the defence of laboratory work; advanced study of particular themes or questions of lectures; carrying out practical homework, solving computational competence-oriented tasks on the given theme; choosing and consideration of literature sources on the given problem of the academic discipline; analytic consideration of scientific publications; self-assessment of students' knowledge by questions for self- assessment; carrying out independent work; carrying out independent creative work; preparation for tests and other forms of current control; preparation for module control (a colloquium); systematization of the studied material for the purpose of preparation for term exams on each module of the academic discipline.

7.2. Examples of practical homework for independent work

Thematic module 1
Probability theory

Theme 2. The basic theorems of probability theory, their economic meaning

Task 2.1. A symmetrical dice is rolled four times. What is the probability of getting the faces with the odd, even, odd, even points respectively?

Task 2.2. A student is choosing 3 questions out of 30 at an exam. There are 10 questions from algebra, 15 from analysis and 5 from geometry. What is the probability that he will choose at least two questions from the same area?

Task 2.3. A box of fuses, which are all of the same shape and size, comprises 23 2A fuses, 47 5A fuses and 69 13A fuses. Determine the probability of selecting at least 2 or more 5A fuses.

Task 2.4. A phone company found out that 75 % of the customers want text messaging, 80 % photo capabilities and 65 % both. What is the probability that the customer will want at least one of these?

Task 2.5. On five cards there are written letters "s", "m", "e", "t", "a". After shuffling they take five cards one by one and put them near in turn. What is the probability that the word results in (1) "steam"? (2) "team"? (3) "tea"?

Task 2.6. There are N white and M black balls (N good and M defective articles on the shelf). If k balls are chosen randomly, what is the probability of getting this way exactly n white balls?

Task 2.7. From the set of 52 playing cards, 5 cards were chosen randomly. What is the probability of getting exactly two hearts among them?

Task 2.8. Suppose the probability that a married man votes is 0.45, the probability that a married woman votes is 0.4, and the probability a woman votes given that her husband votes is 0.6. What is the probability that: (a) both vote; (b) a man votes given that his wife votes?

Task 2.9. Three girls, Alice, Betty and Charlotte, wash the family dishes. Since Alice is the oldest, she does the job 40 % of the time. Betty and Charlotte share the other 60 % equally. The probability that at least one dish will be broken when Alice is washing them is 0.02; for Betty and Charlotte the

probabilities are 0.03 and 0.02. The parents do not know who is washing the dishes, but one night they hear one break. What is the probability that Alice was washing up? Betty? Charlotte?

Task 2.10. Three machines produce the same type of product at a factory. The first one produces 150 articles, the second one makes 600 articles and the third machine manufactures 250 articles. It is known that the share of defective articles is 2 % with the first machine, 4 % with the second one, 8 % with the third one.

(a) What is the probability that an article selected randomly from the total products will be defective?

(b) It is known that a selected article is defective. What is the probability that this article was made by the third machine?

Theme 3. The scheme of independent trials

Task 3.1. The probability that the part is produced with a defect is equal to 0.2. Find the probability that among 400 randomly selected articles there will be:

a) from 70 to 100 defective ones;

b) 90 defective ones;

c) no less than 60 defective ones;

d) no more than 100 defective ones.

Find the most probable number of defective (standard) articles.

Task 3.2. The probability of the hitting the target with 1 shot equals 0.6. Find the probability of the following events:

a) with 600 shots the hitting frequency will deviate from the probability 0.6 by the absolute value of no more than 0.03;

b) find the boundaries of hitting the target with 600 shots in order that the probability $P = 0.993$;

c) find such a number of shots that the probability $P = 0.993$ gives the deviation of frequency of hits from the probability 0.6 by the absolute value no more than 0.03;

d) the accuracy ε, whose probability of deviation of relative frequency from the probability $p = 0.7$ is $P = 0.996$.

Task 3.3. The factory sent 5000 products of high quality to the warehouse. The probability of damaging the products on the way is equal to 0.0008. Find the probability that

a) 5 damaged products will be received at the warehouse;

b) from 3 to 6 damaged products will be received at the warehouse;

c) at least one damaged product will be received at the warehouse.

Task 3.4. The probability that the student will pass a test at the first attempt is equal to 0.9. Find the probability that out of 7 students of the same knowledge level

a) 5 students will pass the test;

b) from 4 to 6 will pass the test.

Find the most probable number of students who will pass the test at the very first attempt.

Theme 5. Distribution laws and numerical characteristics of discrete random variables

Task 5.1. Statisticians use sampling plans to either accept or reject batches or lots of material. Suppose one of these sampling plans involves sampling independently 10 items from a lot of 100 items in which 12 are defective. Make up a distribution law of the number of items found defective in the sample of 10 (in this case, the random variable takes on the values 0, 1, 2, ..., 9, 10).

Task 5.2. The factory sent 5 000 products of high quality to the warehouse. The probability of damaging the products on the way is equal to 0.0004. Make up a distribution law of a number of damaged products received at the warehouse.

Find the probability that

a) from 2 to 3 damaged products will be received at the warehouse;

b) at least one damaged product will be received at the warehouse.

Calculate the numerical characteristics $M(X)$, $D(X)$ and $\sigma(X)$. Find the distribution function of the random variable X.

Task 5.3. There are 500 details. The probability of producing defective details is equal to 0.01. Make up a distribution law of the number of defective details.

Find the probability that there will be: a) 3 defective details; b) from 2 to 4 defective ones.

Calculate $M(X)$, $D(X)$ and $\sigma(X)$.

Task 5.4. A coin is tossed 5 times. Make up a distribution law of the number of occurrences of heads. Find the probability that the coin will land with a head at least 2 times. Calculate $M(X)$, $D(X)$ and $\sigma(X)$. Find the distribution function of the random variable X.

Task 5.5. The probability of the train arrival at the station on time is equal to 0.8. Make up a distribution law of the number of train arrival at the station on time out of 4 expected trains. Find the probability that no less than 2 and no more than 3 trains will arrive on time. Find the probability that at least one train will arrive on time. Calculate $M(X)$, $D(X)$ and $\sigma(X)$. Find the distribution function of the random variable X.

Task 5.6. The probability of the birth of a boy is equal to 0.51. Make up a distribution law of the number of newborn boys out of 10 newborns. Find the probability that among 10 newborns there will be from 3 to 7 boys. Calculate $M(X)$, $D(X)$ and $\sigma(X)$. Find the distribution function of the random variable X.

Task 5.7. The probability that the student will pass a test on the very first try is equal to 0.9. Make up a distribution law of the number of students who will pass the test on the very first try among 7 students of the same knowledge level. Find the probability that from 4 to 6 students will pass the test.

Task 5.8. There is a random variable X:

a)

x_i	5	10	15	20
p_i	0.2	0.3	0.4	0.1

b)

x_i	0	1	2	3	4
p_i	0.1	0.25	0.35	0.2	?

c)

x_i	-1	1	3	5	7
p_i	0.10	0.19	0.31	0.25	0.15

d)

x_i	1	2	3	4	5
p_i	0.2	?	0.1	0.3	0.1

Draw a distribution polygon. Calculate $M(X)$, $D(X)$ and $\sigma(X)$. Find the distribution function of the random variable X.

a)

x_i	2	4	6	8	10
p_i	0.05	0.18	0.32	0.28	0.17

b)

x_i	-1	2	5	8	11
p_i	0.12	0.17	0.18	0.38	0.15

Theme 6. The basic distribution laws of a continuous random variable

Task 6.1. Measurements of scientific systems are always subject to variation, some more than others. There are many structures for measuring error and statisticians spend a great deal of time modelling these errors.

Suppose the error X of a certain physical quantity is calculated by the density function $f(x)=\begin{cases}0, & x\le -1\\ c\left(3-x^2\right), & -1<x\le 1.\\ 0, & x>1\end{cases}$

a) Determine c that renders $f(x)$ a valid density function.

b) Find the probability that a random error in the measurement is less than 1/2. For this particular measurement, it is undesirable that the magnitude of the error should exceed 0.8. What is the probability that this occurs?

Task 6.2. The waiting time, in hours, between successive speeders spotted by a radar unit is an exponential continuous random variable with the cumulative distribution function $F(x)=\begin{cases}0, & x<0\\ 1-e^{-8x}, & x\ge 0\end{cases}$.

Find: a) the density function $f(x)$;

b) $M(X)$, $D(X)$ and $\sigma(X)$;

c) the probability of waiting less than 12 minutes between successive speeders.

Theme 11. Preprocessing of statistical data

Task 11.1. A voltage (volts) in an electricity network for each hour is given with the help of series: 222, 219, 224, 220, 218, 217, 221, 220, 215, 218, 223, 225, 220, 226, 221, 216, 211, 219, 220, 221, 222, 218, 221, 219.

1) Construct a discrete statistic series for this sample, plot a polygon of relative frequencies and $F(x)$;

2) calculate $\overline{x_s}, S_x, R, v, A_s, E_s$;

3) find M_o and M_e.

Task 11.2. For calculation of an average productivity x_i the area of 2000 hectares is divided into 20 equal lots m_i. Real productivity is given:

x_i, centner/hectare	25	30	35	40	45
m_i	2	3	8	4	3

1) Construct a discrete statistic series for this sample, plot a polygon of relative frequencies and $F(x)$;

2) calculate $\overline{x_s}, S_x, R, v, A_s, E_s$;

3) find M_o and M_e.

Task 11.3. The loading of a telephone line between two points from 9:00 till 16:00 hours (one day) is characterized as shown in the table:

Hours	9 – 10	10 – 11	11 – 12	12 – 13	13 – 14	14 – 15	15 – 16
Number of calls	16	21	18	10	12	15	13

a) Plot a histogram and a cumulative function.

b) Calculate $\overline{x_s}, S_x, R, v$ and A_s, E_s, M_o, M_e.

Theme 12. Statistical estimation of the parameters of a distribution

Task 12.1. The interval statistical series of x_i of the number of defective articles is given:

$x_i - x_{i+1}$	5 – 10	10 – 15	15 – 20	20 – 25	25 – 30
m_i	2	6	10	8	4

Construct the confidence intervals for $\overline{x}_{pop}$ and S_{pop} with the confidence probability $\gamma =$0.9, 0.93, 0.95, 0.99 and 0.999.

Task 12.2. Define the number of lamps for the sample, if the root-mean square deviation of the lifetime of a lamp equals 60 hours, the deviation from the population mean is less than 8 hours with the probability 0.9545.

Task 12.3. To determine the ash content of coal, a number of tests should be conducted so that the deviation from the sample mean were less than 3 %. The root-mean square deviation equals 10 %. The confidence probability is 0.9973. How many tests do you need to conduct?

Theme 13. Checking the statistical hypothesis

Task 13.1. Using χ^2 test and Romanovskiy's test for goodness of fit, check the assumption about the normal distribution of the population if there are empirical (m_i) and theoretical ($\tilde{m}_i$) frequencies.

m_i	1	4	10	14	12	6	2	1
$\hat{m}_i$	2	3	10	13	12	7	2	1

Theme 14. The elements of the correlation theory

Task 14.1. The results of measuring the values of the two-dimensional random variable (X , Y) are given as the correlation table:

Y \ X	10	15	20	25	30	35	40
20	2	3					
40		8	12	8			
60		4	15	10	8	2	
80			3	4	4	4	
100					5	4	1
120						2	1

Construct conjugate empirical lines of regression. Using their form and arrangement make an assumption about the significance of correlation between factors X and Y and its form.

Theme 16. The elements of the regression theory

Task 16.1. The results of observations of variables X and Y are given as:

a)

X	1.5	3.0	4.5	6.0	7.5	9.0	10.5
Y	14	24	33	37	46	51	63

b)

X	2.0	3.5	4.0	5.5	7.0	8.5	9.0
Y	10	21	25	33	41	51	63

1) Calculate the coefficients of a pair liner regression.
2) Construct the theoretical regression line Y upon X .
3) Calculate the correlation coefficient, the determination coefficient.
4) Estimate the statistical significance of the coefficients and a model.

7.3. Questions for self-assessment

Thematic module 1. Probability theory

Theme 1. The empirical and logical bases of probability theory

1. The subject of this discipline.
2. A probabilistic model of an experiment.
3. Sure (certain) events.
4. Random events.
5. Impossible events.
6. Rules of operations with random events.
7. The space of elementary events.
8. The classical definition and calculation of a probability.
9. Basic formulas of combinatorics.
10. The statistical definition of a probability.
11. Axiomatics of Kolmogorov.
12. The geometrical definition of a probability.
13. The Venn – Euler diagram.

Theme 2. The basic theorems of probability theory, their economic meaning

1. Probability space.
2. Addition theorems of probabilities.
3. Dependent and independent events.
4. A conditional probability.
5. Joint (compatible) and disjoint (incompatible) events.
6. Multiplication theorems of probabilities.
7. A complete group of events.
8. Complementary events.
9. The probability of at least one event.
10. The probability that an event will occur at least once.
11. The total probability formula.
12. Bayes formula (the theorem of hypothesis).

Theme 3. The scheme of independent trials

1. A scheme of repeated independent trials.
2. Bernoulli's formulas.

3. The local theorem of Moivre – Laplace.

4. Gauss's function, its properties.

5. Application of the formulas to approximate calculations of a probability of the occurrence of a random event a definite number of times in series of independent trials.

6. The integral theorem of Moivre – Laplace.

7. Laplace's function, its properties.

8. Application of the formulas to approximate calculations of a probability that values of a random variable lie in a definite interval.

9. Poisson's theorem.

Theme 4. Random variables and their economic meaning

1. The definition of a random variable.

2. Discrete and continuous random variables.

3. Distribution laws of probabilities for a random variable and ways of finding (tabular, graphic and analytical).

4. The distribution function of probabilities, its properties.

5. The basic numerical characteristics of a random variable.

6. A mathematical expectation.

7. A variance.

8. A root-mean-square deviation.

9. The properties of basic numerical characteristics.

10. Additional numerical characteristics of a distribution: the mode, the median, the excess.

11. The initial and central theoretical moments of an arbitrary order.

12. Calculation of numerical characteristics of the distribution of a random variable using its theoretical moments.

13. The definition of a continuous random variable.

14. The distribution density and its probable explanation.

15. The density function of the distribution of a random variable and its properties.

Theme 5. Distribution laws and numerical characteristics of discrete random variables

1. Distribution laws of a discrete random variable which are often used to describe social and economic phenomena.

2. A binomial distribution.

3. A geometrical distribution.

4. A hypergeometrical distribution.

5. Specificities and properties of these distributions, their basic numerical characteristics and the economic meaning.

6. A flow of events.

7. The simplest flow of events and its properties.

Theme 6. The basic distribution laws of a continuous random variable

1. Distribution laws of a continuous random variable which are often used to describe social and economic phenomena.

2. A uniform distribution.

3. A normal distribution.

4. An exponential distribution.

5. Properties of these distributions and their basic numerical characteristics.

6. The influence of the parameters of a distribution on the density function of probabilities in the normal distribution law.

7. Student's distribution.

8. Pearson's distribution.

9. Fisher's distribution.

10. Specificities and properties of these distributions.

11. The relationship of these distributions and the normal distribution law of a continuous random variable.

Theme 7. Multidimensional random variables

1. The concept of the system of random variables.

2. The distribution function and the density of the distribution of the system of two random variables.

3. Conditional distribution laws of the components of the system of discrete random variables, their basic numerical characteristics.

4. The basic numerical characteristics of the system of two random variables.

5. The correlation moment, its properties.

6. The coefficient of a correlation, its properties.

Theme 8. The functions of the random argument

1. The general concepts of the function of a random argument.
2. Construction of the distribution law of the function of a discrete random argument using the distribution law of its argument.
3. Basic numerical characteristics of the function of a random argument.
4. The function of a continuous random argument.
5. The basic numerical characteristics of the function of a continuous random argument.
6. The function of two random variables.

Theme 9. The elements of the theory of random processes and the theory of queuing problems

1. The subject of the queuing theory.
2. Random functions.
3. Random processes with a finite set of states.
4. Numerical characteristics and the correlation function of a random process.
5. Properties of the correlation function of a random process.
6. The correlation function of the dependence between two random processes.
7. Markov random process and Markov chains.
8. The elements of the queuing theory.

Thematic module 2
Mathematical statistics

Theme 10. Limit theorems of probability theory

1. Bernoulli's theorem.
2. The statistical definition of a probability.
3. The statistical stability of sampling characteristics.
4. Laws of large numbers.
5. Convergence in probability.
6. Convergence in distribution.
7. Chebishev's inequality.
8. The theorem of Chebishev and its consequences.
9. The theorem of Lyapunov (the central limiting theorem) and its consequences.

10. The theorem of Glivenko.

Theme 11. Preprocessing of statistical data

1. The basic problems of mathematical statistics.
2. The sampling method.
3. Definitions of the population and its sample.
4. An empirical distribution law.
5. Ways of presentation of sampling totalities and representation of the results of observations.
6. Discrete and interval variational series.
7. A polygon and a histogram.
8. The basic sampling characteristics and their asymptotic behavior.

Theme 12. Statistical estimation of the parameters of a distribution

1. Statistical estimations of the distribution parameters of a population and their properties: unbiasedness, possibility and efficiency.
2. The asymptotic efficiency of maximally plausible estimations.
3. The method of moments.
4. Point and interval estimations.
5. The confidence interval for mathematical expectation of a normal population.

Theme 13. Checking the statistical hypothesis

1. The main and alternative statistical hypothesis.
2. A statistical test.
3. Construction of critical domains for a statistical test.
4. Errors of the first kind.
5. Errors of the second kind.
6. The concept of power of a test.
7. Checking the statistical hypothesis about the defining of a distribution law for a population using the results of investigation of a sample.
8. Pearson and Kolmogorov fitting tests.
9. The fitting test relative to a frequency.
10. Checking the statistical hypothesis about the equality of two population means on the assumption of the normal distribution law.
11. Student's fitting test.

12. The comparison of variances.

13. Fisher − Snedeker fitting test.

14. Checking the hypothesis about the equality of the sampling mean and mathematical expectation.

Theme 14. The elements of the correlation theory

1. The problems of the correlation analysis.
2. The sampling coefficient of a correlation.
3. The properties of the sampling coefficient of a correlation.
4. The confidence interval.
5. The coefficient of determination.
6. The correlation ratio.
7. The properties of the correlation ratio.

Theme 15. The elements of the variance analysis

1. The problems of the variance analysis.
2. The role of the variance analysis in economic investigations.
3. The single-factor analysis as a procedure for checking the hypotheses about the lack of the factor influence on the feature which is investigated.
4. A general variance.
5. An external group variance.
6. An intrinsical group variance.
7. The concept of the multidimensional variance analysis.

Theme 16. The elements of the regression theory

1. The problems of the regression analysis.
2. The correlation dependence.
3. The correlation table.
4. The empirical lines of a regression.
5. The theoretical lines of a regression.
6. The least-squares method.
7. Estimation of the parameters of a pair regression equation using the least-squares method.
8. Point estimations.
9. Checking the significance of the parameters of a pair regression equation.

10. The confidence interval for a line of a pair regression.
11. The simplest cases of a nonlinear regression.
12. The concept of a multiple regression.

7.4. The independent test

7.4.1. The basic requirements for carrying out the independent test

The purpose of carrying out an independent test is the formation of students' practical skills in the use of theoretical knowledge of the academic discipline "Probability Theory and Mathematical Statistics" for solving economic problems and optimal decision making, obtaining skills in economic mathematical analysis and modelling for finding and explanation of the most effective solutions, as well as using the methods of quantitative and qualitative analysis of applied economic mathematical models.

The independent test should be carried out on the scheduled date. Besides, the description of each of the tasks for the independent test should be done (except the didactic analysis and the definition of corresponding elements of the independent work) according to the general technology of fulfillment:

learning and citing the basic questions of the theoretical material out of the recommended sources;

the design of the report on carrying out the task for the independent test, answers to test questions;

handing in the fulfilled tasks of the independent test and the answers to the test questions to the lecturer.

The fulfillment of the tasks of the independent test on the academic discipline is assessed depending on:

the understanding, the degree of the mastery of the theory and methodology of the problems which are considered;

the degree of acquaintance with the recommended literature and the mastery of the factual material of the academic discipline;

the ability to connect theory and practice in the consideration of practical situations, solving problems, carrying out calculations, fulfillment of tasks given for independent work;

the completeness of taking into account the conditions for the fulfillment of the tasks;

the logic of the given material and correspondence of its structure to the provided thematic elements of the task; the availability and completeness of consideration of the key concepts (definitions, terms, varieties and so on) of the subject matter of the task; the availability and explanations of the student's final conclusions; illustration of the processed material with the help of student's own examples and graphical material.

7.5. Preparation of the independent creative work

Independent creative work of students is an integral part of the educational process. It forms the skills in the creation of major types of work (term papers, a diploma project). That is the reason why it is necessary for students to learn how to qualitatively prepare a creative work.

Within the framework of the given form of student's independent work it is proposed to prepare a presentation on the theme formulated for a student, in the electronic form (with the help of MS PowerPoint). An alternative may be a presentation at a next lecture or writing a scientific article.

Preparation of independent creative work provides for systematization, consolidation, broadening of the theoretical and practical knowledge of the academic discipline and using it in the process of solving a specific economic problem, development of skills in independent work and mastering the methods of investigation and experiment, connected with the theme of the independent creative work.

Independent creative work provides for the availability of the following elements of scientific investigation: practical significance, a comprehensive systematic approach to solving the tasks of the investigation, the theoretical use of the progressive modern methodology and scientific developments, availability of the elements of creativity, the ability to use modern technologies.

A comprehensive systematic approach to the development of the theme of the independent creative work implies consideration of the subject of the research from different points of view that is from the position of a theoretical basis and practical ground work, conditions of its realization, analysis, explanations of ways for improvement in close relationship and a common logic of exposition.

The use of modern technology consists in the fact that in the process of fulfillment of analysis and explanation of ways for improvement of particular aspects of the subject and the object of investigation, a student has to use information about high achievements in techniques and technologies of investigation, use varied mathematical methods and ways, approaches to the definition and explanation of indicators of analysis of a social economic system or its elements.

Students submit the independent creative work to the lecturer in the electronic form if it is a presentation or in the printed or electronic form if it is a scientific publication.

After the complex presentation or a scientific publication has been reviewed and corrected by the lecturer, students make their presentations in front of the audience, report on the results stated in the scientific publication, make reports at a student's scientific and practical conference and so on.

8. Individual consultative work

Individual consultative work is fulfilled according to the schedule of the individual consultative work in the following forms: individual studies, consultations, check of fulfillment of individual tasks, check and defence of the tasks presented for the current control and so on.

The forms of the individual consultative work are:

a) according to the mastery of the theoretical material:
consultations: individual (question – answer);
group (consideration of typical examples);

b) for complex assessment of the mastery of the program material: individual handing in of the fulfilled work.

9. Methods of study

To intensify the process of teaching the academic discipline "Probability Theory and Mathematical Statistics" the following educational technologies are applied: problem-based lectures, minilectures, work in small groups, discussions, brainstorms, moderations, presentations, computer simulation (games), Delphi's method, the method of scenarios, banks of visual support (Table 9.1).

Table 9.1

Distribution of forms and methods of intensification of the educational process according to the themes of the academic discipline

Theme	Practical application of educational technologies
1	2
Thematic module 1 Probability theory	
Theme 1. The empirical and logical bases of probability theory	A minilecture on the theme: "The formation of probability theory as a science". Work in small groups with further discussion of the results of laboratory work
Theme 2. The basic theorems of probability theory, their economic meaning	A minilecture on the theme: "The connection between theory of sets and Kolmogorov's axiomatics". Work in small groups with further discussion of the results of laboratory work
Theme 3. The scheme of independent trials	A minilecture on the theme: "Bernoulli and a classical definition of a probability"
Theme 4. Random variables and their economic meaning	A minilecture on the theme: "Making managerial decisions under risk conditions"
Theme 5. Distribution laws and numerical characteristics of discrete random variables	A problem-based lecture on the theme: "Modern trends in the definition of numerical characteristics of random variables". A computer simulation on the theme: "Investigation of the influence of the distribution parameters of a discrete random variable on its polygon distribution"
Theme 6. The basic distribution *laws* of a continuous random variable	A minilecture on the theme: "Normal distribution law and error theory". Work in small groups with further discussion of the results of laboratory work
Theme 7. Multidimensional random variables	Using a visual support for illustration of conditional distribution laws based on the example of a two-dimentional random variable. Presentation of independent creative work
Theme 8. The functions of the random argument	A problem-based lecture on the theme: "Non-random function of a random argument". Work in small groups with further discussion of the results of laboratory work

Table 9.1 (the end)

1	2
Theme 9. The elements of the theory of random processes and the theory of queuing problems	A minilecture on the theme: "A tree of states". Brainstorming on the theme: "Uncertainty as the primary cause of business risk". Presentation of independent creative work
Thematic module 2 **Mathematical statistics**	
Theme 10. Limit theorems of probability theory	A minilecture on the theme: "The relationship between probability theory and mathematical statistics". Work in small groups with further discussion of the results of laboratory work
Theme 11. Preprocessing of statistical data	A problem-based lecture on the theme: "Some differences between the theoretical definition of distribution characteristics and the realization of calculation using the software"
Theme 12. Statistical estimation of the parameters of a distribution	A minilecture on the theme: "A comparative overview of applied programs for processing statistical data". Presentation of independent creative work
Theme 13. Checking the statistical hypothesis	A minilecture on the theme: "Parametric and nonparametric statistical tests". Work in small groups with further discussion of the results of laboratory work
Theme 14. The elements of the correlation theory	Using visual support for construction of an empirical distribution of a two-dimensional random variable. Presentation of independent creative work
Theme 15. The elements of the variance analysis	Using visual support for illustration of the features of the variance analysis with the help of comparison of the correlation analysis
Theme 16. The elements of the regression theory	A minilecture on the theme: "Application of multifactor regression models to the investigation of economic processes". Work in small groups with further discussion of the results of laboratory work

The basic difference of active and interactive methods of education from traditional ones is not only defined by the methods and techniques of teaching, but also by high efficiency of the educational process which reveals itself in the high motivation of students; consolidation of theoretical knowledge in practice; raised students' consciousness; forming the ability to make independent decisions; forming the ability to approve collective

decisions; forming the ability for social integration; getting skills in resolving conflicts; development of the ability to reach compromises.

Problem-based lectures are designed to develop students' logical thinking. The theme is confined to two or three key issues, students' attention is concentrated on the material which has not been represented in textbooks, the experience of foreign educational universities is used with handing out printed materials to students during the lecture and drawing basic conclusions as to the issues considered. In the course of lectures, students are asked questions for independent reflection which the lecturer answers himself, without waiting for students' answers. This kind of system makes students concentrate and begin to actively think in search of a correct answer.

Minilectures provide for the delivery of the educational material during a short-length segment of time and they are characterized by a significant content, complexity of logical constructions, forms, proofs and generalizations. They are conducted, as a rule, as a part of a study-investigation. Minilectures differ from full-size lectures by a shorter duration. Usually, they last no more than 10 – 15 minutes and they are used to briefly give new information to all students. Minilectures are often used as parts of a whole theme, which it is desirable to deliver as a full-size lecture in order to avoid the audience's getting tired. Then the information is given by turn as several particular fragments, with other forms and methods of study used between them.

Seminar-discussions provide for exchange of thoughts and ideas of students on the given theme and develop thinking, help to form ideas and beliefs, produce skills in formulating thoughts and expressing them, teach to assess other people's proposals, critically come to personal ideas.

Banks of visual support help to intensify the educational process of studying the themes of the academic discipline with the help of vizualization.

The method of scenarios is used for designing probabilistic models of behavior and development of concrete events in the long term.

Work in small groups gives an opportunity to structure practical studies in the form and content, gives a possibility for each student's participation in the work on the theme under study, stimulates forming personal qualities and experience of social communication.

Brainstorming is a method of solving urgent tasks; its core lies in expressing as many ideas as possible in a short period of time, discussing and selecting them.

Presentations are speeches to students with a view to presenting certain achievements, group work results, reports on the fulfillment of individual tasks, instruction, demonstration of new goods and services.

Moderation is a way to conduct a discussion, which quickly leads to concrete results, gives a possibility for all present students to take part in the process of searching for a solution to a problem and take full responsibility for the result. The function of the moderator is to see to it that the rules of the discussion are observed, which gives a possibility to simplify searching for a solution without interfering in its essence.

The Delphi method is used for the purpose of reaching a consensus in expert judgements. It gives a possibility for students to express their thoughts to a group of experts, which work individually in different places. To choose a management decision according to this method, the academic group is divided, for example, into five small groups. Four groups work, develop and make a management decision, and the fifth group is the expert team. This group carries out analysis of the variants of management decisions, which are proposed by the working groups, and assesses these variants. Within the expert group the distribution of its members according to specialities is fulfilled.

A business game is a method of imitation of making administrative decisions in various situations by means of playing according to the rules which have been worked out or are worked out by the members themselves. This method is realized through students' independent solving the set problem provided a shortage of the necessary knowledge when students themselves are forced to master the new content or search new connections in the learnt material.

Computer simulation (game) is an educational method, which is based on the use of a specific computer program in order to get visual modelling of a process. Students can change the parameters and data, decisions and analyze the results of such decisions. The purpose of using this method is the development of systematic thinking of students, their ability to plan, form skills in identifying and analyzing problems, compare and estimate alternatives, make optimal decisions and work under the conditions of limited time.

The interactive distant education is a set of educational technologies based on the principles of contact in the information educational space. They serve to organize the education of users distributed in the space and time.

10. Methods of control

The system of assessment of competences which were formulated for a student during the learning of the academic discipline (Table 2.1), takes into consideration the forms of studies which according to the syllabus of the academic discipline provide lectures, practical studies, laboratory work, fulfillment of students' independent work. The assessment of the formed competences of students is carried out on the accumulative 100-point system. According to the temporary provision "About the Order of Assessment of Students' Academic Performance on the Accumulative Point Rating System" of Simon Kuznets Kharkiv National University of Economics control ways include:

current control which is carried out within the term during lectures, practical studies and laboratory work and is assessed as a sum of accumulated points (the maximum equals 60 points; the minimum which makes it possible for a student to pass an exam, equals 35 points);

module control which is carried out in the form of a colloquium with taking into account the current control according to a corresponding thematic module, provides an integral assessment of student's results after learning the material of a logically completed part of the discipline (or a thematic module);

final/term control, which is carried out as a terminal exam, according to the schedule of the educational process.

Current control on the given academic discipline is carried out in the following forms:

active in-class work (lecture);

active in-class work (practical study);

active in-class work (laboratory study);

homework;

competence-oriented tasks (defence of laboratory work);

an independent test;

a written test;

independent creative work.

A colloquium is a form of reviewing and assessment of students' knowledge in the system of institutes of higher education. *The purpose* of carrying out a colloquium is to clarify the theoretical and practical knowledge

obtained by a student as a result of listening to lectures, attendance of practical and laboratory studies and independent learning of the material. Within the bounds of the assigned purpose the following tasks are fulfilled: evaluation of the quality and degree of student's understanding of the lecture material; the development and fixing of the skills in expressing thoughts; the development of student's ability for independent single-minded training; the development of skills in the generalization of different literary sources; giving a possibility for a student to compare different points of view on a given question.

A colloquium is conducted as an intermediate mini-exam on the initiative of the lecturer and includes theoretical questions and practical tasks on the academic discipline. The list of questions, which are included into a colloquium on the themes of the thematic module, contains questions for self-diagnostics.

Final/term control is conducted in the form of a term exam. **Term exams** are a form of assessment of students' final mastery of the theoretical and practical material of a particular module of the academic discipline or the academic discipline on the whole, which are conducted as tests.

The order of conducting the current assessment of students' knowledge. Assessment of student's knowledge during practical studies and carrying out laboratory work is conducted on the accumulative system according to the following criteria:

understanding, the degree of the mastery of the theory and methodology of the problems which are considered;

the degree of the mastery of the factual material of the academic discipline; familiarizing with the recommended literary sources and modern literature on the questions which are considered;

the ability to connect theory and practice in the consideration of particular examples, solving problems, carrying out laboratory work, carrying out calculations in the process of doing homework and tasks which are considered in class;

the logic, structure, style of presenting the material in written papers and oral answers in class, the ability to ground one's own position, carry out generalization of the information and draw conclusions.

The maximum possible points which correspond to a particular task, are given on the condition of the correspondence of the solved task or the

oral answer of a student to all the defined criteria. Lack of one or another component decreases the number of accumulated points. In the assessment of tasks set for independent work in laboratory and practical studies, the quality of fulfillment is also considered. Besides, handing in the performed task to the lecturer in accordance with the period defined by the schedule of the educational process plays an important role. If one of these conditions is not satisfied, the points are decreased.

A written test is carried out 2 times during a term and it includes practical tasks of different level of difficulty (complexity) according to the themes of the thematic module.

The criteria for assessment of the written test are as follows:

6 points, if deep knowledge of the syllabus material has been demonstrated, a sequential, complete and logical answer has been given, a correct decision has been made, the mastery of different methods and techniques in carrying out practical tasks has been demonstrated;

5 points, if knowledge of the syllabus material has been demonstrated, an answer without essential inaccuracies has been given, mastery of the necessary methods in carrying out practical tasks has been demonstrated;

4 points, if knowledge of the basic material has been demonstrated, an answer with inaccuracies has been given, mastery of the necessary methods in carrying out practical tasks has been demonstrated;

3 points, if knowledge of the basic material has been demonstrated, an answer with inaccuracies and quite incorrect formulations has been given, mistakes have been made in the use of the necessary methods in carrying out practical tasks;

2 points, if knowledge of the basic material has not been demonstrated, an answer with essential mistakes and incorrect formulations has been given, lack of skills in the use of the necessary methods in carrying out practical tasks has been demonstrated;

1 point, if an incorrect solution has been given, the fulfillment of the colloquium practical tasks has not been begun, but some particular correct thinking has been shown;

0 point, if the task is unavailable.

Revision and marking of the competence-oriented tasks (defence of laboratory work on the themes which are combined into a corresponding thematic module) is carried out twice during a term in the form of work in

small groups. Besides, the quality of fulfillment of the tasks for laboratory work, the ability to present the results of investigations, give reasonable answers to the questions of opponents, think critically, assess the results of the work of other participants must be assessed.

A colloquium is carried out twice during a term, in the written form or in the form of an oral test for controlling students' knowledge of the theoretical material and the mastery of the categorical apparatus.

The criteria for assessment of a colloquium:

5 points, if the test has been carried out without mistakes and deficiencies, all the tasks contain the necessary explanations, illustrations, analysis of the results and conclusions;

4 points, if the test has been carried out, but there are no more than one mistake and no more than one deficiency or no more than three deficiencies;

3 points, if no less than 2/3 of the test have been carried out, there are no more than two mistakes and no more than two deficiencies;

2 points, if less than 2/3 of the test have been performed and the number of mistakes and deficiencies exceeds the norm for the mark of three points;

1 point, if the fulfillment of the tasks has not been begun, but there is a particular correct thinking;

0 point, if the task is unavailable.

The criteria for assessment of independent work of students

The general criteria for the assessment of independent work of students are profound and deep knowledge, the level of thinking, skills in the systematization of knowledge on particular themes, skills in drawing conclusions, attainments and techniques of carrying out practical tasks, the ability to find necessary information, carry out its classification and processing, self-realization in practical and laboratory studies.

The criteria for assessment of independent creative work and independent tests are:

the ability to carry out a critical independent estimation of the defined problems;

skills in the explanation of alternative views and availability of a students' own point of view, position on the defined problem;

using the analytical approach; the quality and accuracy of expressing the thought;

the logic, structure and explanation of conclusions about a particular problem;

independence of carrying out the work;

grammatical correctness of the presentation of the material; using the methods of comparison, generalization of the concepts and facts;

the design of the work;

the quality of presentation.

The order of final control on the academic discipline

The final control of knowledge and competences of students on the academic discipline is carried out on the basis of the term exam. The examination paper includes the syllabus of the discipline and provides for assessment of the knowledge level and degree of the mastery of corresponding competences of students (Table 2.1).

The purpose of the exam is to test student's understanding of the syllabus material on the whole, the logic and relations between its particular parts, the skills in the creative use of the gained knowledge, the ability to formulate their attitude to a particular problem of the academic discipline and so on. The competent approach to the assessment of the exam implies measuring the level of the student's mastery of the competences provided by the qualifying requirements.

Each examination paper contains 5 practical tasks, including two first-level (diagnostic) tasks, two second-level (situational) tasks and one third-level (diagnostic and heuristic) task.

The structure of the examination paper is given in Table 10.1.

The examination paper was formed according to the form No. H-5.05, "About the Approval of the Forms of Documents for Personnel Training at Higher Educational Establishments of the I – IV Levels of Accreditation" which was approved by the Ministry of Education and Science. A sample examination paper is given below.

Table 10.1

The structure of the examination paper

Task level	Content of tasks by themes
First	A classical definition of a probability and its calculation. Basic formulas of combinatorics. Venn – Euler diagram. Addition theorems of probabilities. Dependent and independent events. A conditional probability. Joint (compatible) and disjoint (incompatible) events. Multiplication theorems of probabilities. A complete group of events. Complementary events. The probability of at least one event. The probability that an event will occur at least once. The total probability formula. Bayes formula (the theorem of hypothesis). A scheme of repeated independent trials. Bernoulli's formulas. The local theorem of Moivre – Laplace. The integral theorem of Moivre – Laplace. Poisson's theorem
Second	Discrete and continuous random variables. Distribution laws of probabilities for a random variable. The distribution function of probabilities. Basic numerical characteristics of a random variable: mathematical expectation, a variance and a root-mean-square deviation. Distribution laws of a discrete random variable which are often used to discribe social and economic phenomena: a binomial distribution, a geometrical distribution, a hypergeometrical distribution. Distribution laws of a continuous random variable which are often used to discribe social and economic phenomena: a uniform distribution, a normal distribution and an exponential distribution. Discrete and interval variational series. A polygon and a histogram. Basic sampling numerical characteristics. Point and interval estimations. The confidence interval for mathematical expectation of a normal population. Checking the statistical hypothesis about defining the distribution law for a population using the results of investigation of a sample
Third	The sampling coefficient of a correlation. The coefficient of determination. The correlation ratio. The correlation dependence. The correlation table. Empirical lines of a regression. Estimation of parameters of a pair regression equation using the least-squares method. Point estimations. Checking the significance of parameters of a pair regression equation. The confidence interval for a line of a pair regression

The assessment of the exam is carried out according to the temporary provision "About the Order of Assessment of Students' Academic Performance on the Accumulative Point Rating System" of Simon Kuznets Kharkiv National University of Economics.

In the case of irreproachable fulfillment of all the examination tasks with the demonstration of deep knowledge of the academic discipline, skills in the practical use of the formed competences which are based on the ability to analyze and solve a wide range of problems, a high level of completing the written work, the student obtains 40 points.

The assessment of a particular task is carried out according to the following criteria.

The task of the first level is assessed as follows:

7 points, in the case of the exact use of the scientific terminology and symbols in the necessary logical sequence; a creative approach to solving original problems which require a high level of knowledge;

6 points, in the case of the exact use of the mathematical terminology and symbols; irreproachable mastery of mathematical tools; correct use of mathematical methods, facts, formulas and relations for solving the task of the third level;

5 points, if a logically right sequence of steps of solution has been made. All the key points of the solution have been grounded. 1 – 2 slight mistakes or slips are possible in the calculations which don't influence the correctness of the further solution;

4 points, if mastery of a small part of obligatory skills and attainments required by the syllabus of the academic discipline has been demonstrated in solving the tasks; the conclusions, reproduction of the syllabus material of the discipline has not always been shown;

3 points, if the task has been solved only partially with initial right considerations, but there are mistakes which considerably influenced the process of the right solution of the task;

2 points, if the task fulfillment has been begun, there are separate correct considerations, but a logical mistake has been made which resulted in an incorrect solution.

1 point, if the condition has been written;

0 point, if no task has been fulfilled.

The task of the second level is assessed as follows:

8 points, if solving the assigned tasks is characterized by a creative use of the theoretical instrument, logical correctness, precision, explanation of conclusions, rationality or using original approaches to solving the tasks;

7 points, if perfect mastery of the skill in the use of mathematical tools with application of information from other educational courses and disciplines has been demonstrated; one slight mistake has been made; a high level of standards of carrying out the tasks has been shown;

6 points, if a logically right sequence of steps of solution has been chosen. All the key points of solution have been grounded. 1 – 2 slight mistakes or slips are possible in the calculations which don't influence the correctness of the further solution;

5 points, in the case of correct use of the terminology of the discipline and the basic methods for solving standard problems; showing the ability to use theoretical knowledge for solving standard (multistep) problems, some mistakes or deficiencies on the calculating stage of presentation of the solution; the ability to conclude;

4 points, in the case of more than one mistake and one or two deficiencies in the calculations, graphs, the choice of the method of solution, which have caused a wrong final result in some cases;

3 points, if the task fulfillment has been begun, there are separate correct considerations, but a logical mistake has been made which resulted in an incorrect solution;

2 points, if gross numerical mistakes have been made in the process of using the concepts of the discipline in the formulas which prove the absence of a minimum necessary part of the compulsory skills and the practical attainments provided by the discipline syllabus;

1 point, if no task fulfillment has been begun, but the condition has been written;

0 point, if no task fulfillment has been begun.

The task of the third level is assessed as follows:

10 points, if the ability for scientific investigative developments on the problems of the discipline has been shown; perfect skills in the use of mathematical tools and modern scientific theoretical approaches, a high level of standards of carrying out tasks have been demonstrated;

9 points, in the case of using scientific terminology and symbols in the necessary logical sequence; solving the assigned tasks characterized by precision, explanation; a creative approach; rationality of the choice of the method of solution; correct necessary calculations and transformations;

8 points, if systematic, deep and full knowledge of all the parts of the academic discipline and the basic questions which go beyond the discipline has been shown; a high level of standards of carrying out the tasks has been demonstrated;

7 points, in the case of sporadic slight deficiencies which don't influence the final result; correct use of mathematical methods, facts, formulas and relations for solving the task of different level of complexity;

6 points, if the ability to conclude and compare the theoretical and practical material has been demonstrated; correct (but not always rational) use of mathematical methods of solution, facts, formulas and relations has been shown;

5 points, if half of the tasks have been done, the interpretation of the obtained results is unavailable; the level of the standards of carrying out the tasks is acceptable;

4 points, if the tasks have been carried out without any logical relationship of the mathematical concepts and practical solutions have not been given sufficient theoretical explanation;

3 points, if an acceptable volume of knowledge has been shown within the educational standard; the use of mathematical symbols and terminology has been insufficient and inexact, the knowledge of the basic formulas and concepts on the discipline has not been demonstrated;

2 points, in the case of solving the tasks with the theoretical material used only on the level of concepts; the inability to understand the connection of the theoretical material with the practical tasks;

1 point, if the condition has been written;

0 point, if no task fulfillment has been begun.

A student, who for a valid reason, attested documentally, hasn't had a possibility to take part in the forms of current control, that is, hasn't passed the thematic module, has the right to complete it during two weeks after coming back to studies according to the notice of the dean of the department subject to a given period.

A sample examination paper

Form No. H-5.05

SIMON KUZNETS KHARKIV NATIONAL UNIVERSITY OF ECONOMICS

Educational qualification degree: bachelor
Speciality: 073 "Management"
Term 1
Academic discipline: "Probability Theory and Mathematical Statistics"

Examination paper

Task 1 (diagnostic). At an assembly plant, three machines make 25 %, 45 % and 30 % of the products, respectively. It is known from past experience that 4 %, 2.5 % and 3 % of the products made by each machine, respectively, are defective. Now, suppose that a finished product is randomly selected.

a) What is the probability that it is standard?

b) What is the probability that this standard product was made by the third machine?

c) What is the probability that it is defective?

d) What is the probability that this defective product was made by the second machine?

Task 2 (diagnostic). The probability of finding a mistake on a book page is equal to 0.005. 1000 pages are checked. Find the probability that there is a mistake

a) on 4 pages;

b) on pages from 4 to 6.

c) Find the most probable number of pages with mistakes.

d) Find the most probable number of pages without mistakes.

Task 3 (situational). The continuous statistical series is given in the table:

$[x_i, x_{i+1}]$	2 − 12	12 − 22	22 − 32	32 − 42	42 − 52
m_i	13	20	36	19	12

a) Find the mean ($\bar{x}_s$) for this sample.

b) Find the variance (S_x^2) for this sample.

c) Find the root-mean-square deviation (S_x) for this sample.

d) Plot the histogram and make the assumption about the distribution law.

e) Find the confidence interval for the population mean with the probability $P = 0.93$.

f) Find the confidence interval for the population root-mean-square deviation with the probability $P = 0.999$.

g) Analyze the obtained values.

Task 4 (situational). Measurements of scientific systems are always subject to variation, some more than others. There are many structures for measurement errors and statisticians spend a great deal of time modeling these errors. Suppose the measurement error X of a certain physical quantity is determinated by the density function

$$f(x) = \begin{cases} 0, & x \le -1 \\ c\left(3 - x^2\right), & -1 < x \le 1. \\ 0, & x > 1 \end{cases}$$

Determine c that renders $f(x)$ a valid density function. Find the probability that a random error in measurement is less than 1/2. For this particular measurement, it is undesirable that the magnitude of the error exceed 0.8. What is the probability that this occurs?

Task 5 (heuristic). Analyze the dependence between *x* and *y* if: $n = 100;\quad r_{xy} = 0.68;\quad \sum x_i \cdot y_i = 1273;\quad \sum x_i = 229.6;\quad \sum y_i = 269;$ $s_x^2 = 7.45;\ s_y^2 = 10.208$. Form a regression equation on the assumption of a linear correlation between factors, check its significance with the help of Fisher − Snedeker and give an economic interpretation.

Approved at the meeting of the Department of Higher Mathematics and Economic Mathematical Methods.

Protocol No. 1 of August 28, 2017.
The head of the department L. Malyarets
The lecturer Ie. Misiura

A student can't be allowed to take the exam if the number of points obtained during the current and module control according to the thematic module during the term does not make 35.

After the examination period the dean of the department gives a notice about sitting the failed exams. In a given period the student adds the required points.

The final mark on the academic discipline is calculated according to the points obtained during the exam and the points obtained during current control on the accumulative system.

The student's progress is assessed if the number of points obtained as a result of all forms of control equals or exceeds 60.

Accordingly the minimum possible number of points in the current and module control during the term equals 35 and the minimum possible number of points obtained in the exam equals 25.

The result of the terminal exam is assessed in points (the maximum is 40 points, the minimum possible number is 25 points) entered into a corresponding column of the *Examination Record List*.

The final mark on the academic discipline is calculated according to the points obtained during the exam and points obtained during the current control on the accumulative system.

The total result in points during the term is: *60 and more points mean "passed", 59 and fewer points mean "failed"* and it is entered into the *Examination Record List* on the academic discipline.

11. The distribution of points which students obtain

An example of a technological chart of accumulative rating points and the system of assessment of the level of formed professional competences which a student of the day-time form of studies has to get, is given in Table 11.1 according to the forms of study and methods of control which are used in teaching the academic discipline.

Table 11.1

The system of assessment of the professional competences formed

Profes-sional compe-tences		Educational week	Hours		Forms of study		Assessment of the level of the formed competences	
							Forms of control	Maxi-mal points
1		2	3		4		5	6
Thematic module 1 Probability theory								**26.2**
PT & M1	The ability to choose the methods of probability theory for prognosis of probabilistic random events	1	Class	2	**Lecture**	*Theme 1. The empirical and logical bases of probability theory*	Active class work	0.3
				2	**Practical study**	Learning the probability of a random event using the elements of combinatorics, using addition and multiplication theorems, the total probability and Bayes' formula	Active class work	0.3
			IWS	4	**Preparation for studies**	Search, choice and looking through literary sources on the themes of the academic discipline. Learning the lecture material and preparation for practical studies	There is no control of independent work	–
		2	Class	2	**Lecture**	*Theme 2. The basic theorems of probability theory, their economic meaning*	Active class work	0.3
				2	**Laboratory study**	The functions of MS Excel for making up the data. Calculation of the probability of random events	Active class work	0.3
			IWS	4	**Preparation for studies**	Learning the lecture material. Carrying out practical homework and the independent test. Search of the material for the independent creative task	–	–

Table 11.1 (continuation)

1		2	3		4		5	6
PT & MS1	The ability to choose the methods of probability theory for prognosis of probabilistic random events, define the laws of the distribution of discrete and continuous (one-dimensional) random variables, calculate their basic numerical characteristics	3	Class	2	**Lecture**	*Theme 3. The scheme of independent trials*. *Theme 4. Random variables and their economic meaning*	Active class work	0.3
				2	**Practical study**	Solving problems using the formulas of Bernoulli, Moivre − Laplace, Poisson	Active class work	0.3
			IWS	4	**Preparation for studies**	Search, choice and looking through literary sources on the theme. Carrying out practical homework and the independent test	Homework	0.4
		4	Class	2	**Lecture**	*Theme 5. Distribution laws and numerical characteristics of discrete random variables*	Active class work	0.3
				2	**Laboratory study**	Calculation of basic and additional numerical characteristics of a discrete random variable	Active class work	0.3
			IWS	5	**Preparation for studies**	Learning the lecture material and preparation for practical studies. Carrying out practical homework and the independent test	–	–
		5	Class	2	**Lecture**	*Theme 6. The basic distribution laws of a continuous random variable*	Active class work	0.3
				2	**Practical study**	Solving problems relative to a random variable, distributed by the normal law	Active class work	0.3
			IWS	4	**Preparation for studies**	Learning the lecture material and preparation for practical studies. Carrying out practical homework and the independent test	Homework. Independent test	0.5 + 4

Table 11.1 (continuation)

1		2	3		4		5	6
PT & MS1	The ability to choose the methods of probability theory for prognosis of probabilistic random events, define the laws of the distribution of discrete and continuous random variables, calculate their basic numerical characteristics	6	Class	2	**Lecture**	*Theme 7. Multi-dimensional random variables*	Active class work	0.3
				2	**Laboratory study**	Construction of a binomial distribution law on the basis of the model of repeated trials by Bernuolli's scheme. Calculation of basic numerical characteristics	Active class work. Competence-oriented task	0.3 + 5
			IWS	4	**Preparation for studies**	Search, choice and looking through literary sources on the theme. Carrying out practical homework and the independent test	–	–
		7	Class	2	**Lecture**	*Theme 8. The functions of the random argument*	Active class work	0.3
				2	**Practical study**	Calculation of numerical characteristics of the functions of a random argument	Active class work Written test	0.3 + 6
			IWS	5	**Preparation for studies**	Learning the lecture material. Carrying out practical homework. Preparation for the colloquium. Preparation for a written test	Homework	0.5

Table 11.1 (continuation)

1		2	3		4		5	6
PT & MS1	The ability to use the theory of random processes and the theory of queueing problems for modeling of economic processes	8	Class	2	**Lecture**	*Theme 9. The elements of the theory of random processes and the theory of queuing problems*	Active class work. Colloquium	0.3 + 5
				2	**Laboratory study**	Construction of a theoretical law of a multidimentional random variable	Active class work	0.3
			IWS	5	**Preparation for studies**	Learning the lecture material. Carrying out the practical homework and the independent test. Preparation for the written test	–	–
Thematic module 2 Mathematical statistics								**33.8**
PT & MS2	The ability to identify quantitative characteristics of economic processes with the help of the sampling method	9	Class	2	**Lecture**	*Theme 10. Limit theorems of probability theory*. *Theme 11. Preprocessing of statistical data*	Active class work	0.3
				2	**Practical study**	Solving problems using the theorems of Bernoulli, Chebishev and Lyapunov	Active class work	0.3
			IWS	5	**Preparation for studies**	Search, choice and looking through literary sources on the theme. Carrying out the practical homework and tasks of the independent test	Homework	0.5

Table 11.1 (continuation)

1		2	3		4		5	6
PT & MS2	An ability to understand the possibilities and restrictions of using the instruments of mathematical statistics for solving real economic problems	10	Class	2	**Lecture**	*Theme 12. Statistical estimation of the parameters of a distribution*	Active class work	0.3
				2	**Laboratory study**	Construction of a variational series. Finding the point and interval estimations of basic numerical characteristics	Active class work	0.3
			IWS	5	**Preparation for studies**	Search, choice and looking through literary sources on the theme. Carrying out the practical homework and tasks of the independent test	–	–
		11	Class	2	**Lecture**	*Theme 13. Checking the statistical hypothesis*	Active class work	0.3
				2	**Practical study**	Finding the point and interval estimations of basic and additional numerical characteristics. Checking the statistical hypothesis about the equality of two population means on the assumption of the normal distribution law and variances	Active class work. Independent test	0.3 + 4
			IWS	4	**Preparation for studies**	Search, choice and looking through literary sources on the theme	Homework	0.5
		12	Class	2	**Lecture**	*Theme 13. Checking the statistical hypothesis* (the end)	Active class work	0.3
				2	**Laboratory study**	Checking the statistical hypothesis about defining the distribution law in a population	Active class work	0.3
			IWS	4	**Preparation for studies**	Learning the lecture material. Carrying out practical homework and the independent test	–	–

Table 11.1 (continuation)

1		2	3		4		5	6
PT & MS3	The ability to use the elements of the correlation theory for solving real economic problems	13	Class	2	**Lecture**	*Theme 14. The elements of the correlation theory*	Active class work	0.3
				2	**Practical study**	Calculation of the sample correlation coefficient. Using the single-factor analysis for checking the hypothesis of the lack of the factor influence on the feature under investigation	Active class work. Written test	0.3 + 6
			IWS	4	**Preparation for studies**	Learning the lecture material, preparation for practical studies. Carrying out practical homework and tasks of the independent test	Homework	0.5
	The ability to use the elements of the variance and regression analysis for solving real economic problems	14	Class	2	**Lecture**	*Theme 15. The elements of the variance analysis*	Active class work	0.3
				2	**Laboratory study**	Calculation of basic numerical characteristics of the distribution of a two-dimensional random variable according to the definition and with the help of the built-in functions. Construction of an empirical regression equation	Active class work. Competence-oriented task	0.3 + 5
			IWS	5	**Preparation for studies**	Search, choice and looking through literary sources on the theme	–	–
		15	Class	2	**Lecture**	*Theme 16. The elements of the regression theory*	Active class work. Colloquium	0.3 + 5
				2	**Practical study**	Checking the significance of the correlation between two random variables and the significance of the model	Active class work	0.3
			IWS	5	**Preparation for studies**	Search, choice and looking through literary sources on the theme. Carrying out practical homework	Homework	0.5

Table 11.1 (the end)

1		2	3		4		5	6
PT & MS3	The ability to use the elements of regression analysis for solving real economic problems	16	Class	2	**Lecture**	*Theme 16. The elements of the regression theory* (the end)	Active class work. Creative task	0.3 + 7
				2	**Laboratory study**	Construction of the confidence interval of a regression line with the help of the built-in functions of MS Excel	Active class work	0.3
			IWS	5	**Preparation for studies**	Search, choice and looking through literary sources on the theme	–	–
		17	IWS	3	**Preparation for studies**	Search, choice and looking through literary sources on the theme	–	–
Examination period			Class	2	**Consultation for the exam**	Solving practical tasks according to the themes which are included in the final control	**Total control**	40
				2	**Exam**	Carrying out the tasks of the examination paper		
			IWS	7	**Preparation for the exam**	Review of the material of thematic modules		
Total sum of hours				150	**Total maximal number of points for the discipline**			**100**
including								
class				**60**	40 %		*current control*	**60**
independent work				**90**	60 %		*total control*	**40**

The distribution of points according to the themes of thematic modules is given in Table 11.2.

The maximum number of points which a student can accumulate during a week according to the forms and methods of study and control is given in Table 11.3.

The final mark on the academic discipline is defined according to the temporary provision "About the Order of Assessment of Students' Academic Performance on the Accumulative Point Rating System" of Simon Kuznets Kharkiv National University of Economics (Table 11.4). Marks according to this scale are entered in the Examination Record List, the individual educational plan of a student and other academic documents.

Table 11.2

The distribution of points according to the themes

<table>
<tr><td colspan="16">Current testing and independent work</td><td>Final test (exam)</td><td>Sum total</td></tr>
<tr><td colspan="9">Thematic module 1</td><td colspan="7">Thematic module 2</td><td>40</td><td>100</td></tr>
<tr><td>T1</td><td>T2</td><td>T3</td><td>T4</td><td>T5</td><td>T6</td><td>T7</td><td>T8</td><td>T9</td><td>T10</td><td>T11</td><td>T12</td><td>T13</td><td>T14</td><td>T15</td><td>T16</td><td rowspan="11"></td><td rowspan="11"></td></tr>
<tr><td>0.6</td><td>0.6</td><td>1.0</td><td>0.6</td><td>1.1</td><td>0.6</td><td>1.1</td><td>0.6</td><td>1.1</td><td>0.6</td><td>1.1</td><td>0.6</td><td>1.1</td><td>0.6</td><td>1.1</td><td>0.6</td></tr>
<tr><td colspan="9">Written test</td><td colspan="7">Written test</td></tr>
<tr><td colspan="9">6</td><td colspan="7">6</td></tr>
<tr><td colspan="9">Independent test</td><td colspan="7">Independent test</td></tr>
<tr><td colspan="9">4</td><td colspan="7">4</td></tr>
<tr><td colspan="9">Competence-oriented task</td><td colspan="7">Competence-oriented task</td></tr>
<tr><td colspan="9">5</td><td colspan="7">5</td></tr>
<tr><td colspan="9">Colloquium</td><td colspan="7">Colloquium</td></tr>
<tr><td colspan="9">5</td><td colspan="7">5</td></tr>
<tr><td colspan="16">Independent creative work</td></tr>
<tr><td colspan="16">7</td></tr>
</table>

Note. T1, T2, ..., T16 are themes of thematic modules.

Table 11.3

The distribution of points within the weeks

Themes of the thematic module			Lectures	Practical study	Laboratory study	Homework	Competence-oriented task	Independent test	Written test	Independent creative work	Colloquium	Total
1	2	3	4	5	6	7	8	9	10	11	12	13
Thematic module 1 Probability theory	Theme 1	week 1	0.3	0.3	–	–						0.6
	Theme 2	week 2	0.3	–	0.3	–						0.6
	Theme 3	week 3	0.3	0.3	–	0.4						1.0
	Theme 4	week 4	0.3	–	0.3	–						0.6
	Theme 5	week 5	0.3	0.3	–	0.5		4				5.1
	Theme 6	week 6	0.3	–	0.3	–	5					5.6
	Theme 7	week 7	0.3	0.3	–	0.5			6			7.1

Table 11.3 (the end)

1	2	3	4	5	6	7	8	9	10	11	12	13
Thematic module 2 Mathematical statistics	Theme 8	week 8	0.3	–	0.3	–					5	5.6
	Theme 9	week 9	0.3	0.3	–	0.5						1.1
	Theme 10	week 10	0.3	–	0.3	–						0.6
	Theme 11	week 11	0.3	0.3	–	0.5		4				5.1
	Theme 12	week 12	0.3	–	0.3	–						0.6
	Theme 13	week 13	0.3	0.3	–	0.5			6			7.1
	Theme 14	week 14	0.3	–	0.3	–	5					5.6
	Theme 15	week 15	0.3	0.3	–	0.5					5	6.1
	Theme 16	week 16	0.3	–	0.3	–				7		7.6
Total			**4.8**	**2.4**	**2.4**	**3.4**	**10**	**8**	**12**	**7**	**10**	**60**

Table 11.4

The scales of assessment: national and ECTS

Sum of points including all forms of study	Mark on the ECTS scale	Mark on the national scale	
		for an exam, a term paper, practice	for a test
90 – 100	A	excellent	passed
82 – 89	B	very good	
74 – 81	C	good	
64 – 73	D	satisfactory	
60 – 63	E		
35 – 59	FX	unsatisfactory	failed
1 – 34	F		

12. Recommended reading

12.1. Main

1. Лабораторний практикум із розділу "Теорія ймовірностей та математична статистика" навчальної дисципліни "Математика для економістів" : навч.-практ. посіб. / І. Л. Лебедєва, О. О. Єгоршин, Е. Ю. Железнякова та ін. – Харків : Вид. ХНЕУ, 2009. – 116 с.

2. Малярець Л. М. Математика для економістів. Теорія ймовірностей та математична статистика : навч. посіб. У 3-х ч. Ч. 3 / Л. М. Малярець, І. Л. Лебедєва, Л. Д. Широкорад. – Харків : Вид. ХНЕУ, 2011. – 568 с.

3. Малярець Л. М. Практикум з теорії ймовірностей та математичної статистики в Excel : навч.-практ. посіб. / Л. М. Малярець, І. Л. Лебедєва, Е. Ю Железнякова. – Харків : Вид. ХНЕУ, 2007. – 160 с.

4. Малярець Л. М. Теорія ймовірностей та математична статистика : навч. посіб. / Л. М. Малярець, І. Л. Лебедєва, Е. Ю. Железнякова та ін. – Харків : Вид. ХНЕУ, 2010. – 404 с.

5. Малярець Л. М. Теория вероятностей и математическая статистика в примерах и задачах : учебное пособие для студентов-иностранцев отрасли знаний 0305 "Экономика и предпринимательство" / Л. М. Малярец, Е. Ю. Железнякова, А. В. Игначкова. – Харків : ХНЕУ, 2012. – 124 с.

6. Місюра Є. Ю. Теорія ймовірностей : конспект лекцій / Є. Ю. Місюра. – Харків : Вид. ХНЕУ, 2013. – 95 с. (Англ. мов.)

12.2. Additional

7. Барковський В. В. Теорія ймовірностей та математична статистика / В. В. Барковський, Н. В. Барковська, О. К. Лопатін. – 5-е вид. – Київ : Центр учбової літератури, 2010. – 424 с.

8. Валєєв К. Г. Збірник задач з теорії ймовірностей та математичної статистики / К. Г. Валєєв, І. А. Джалладова. – Київ : КНЕУ, 2005. – 340 с.

9. Гмурман В. Е. Руководство по решению задач по теории вероятностей и математической статистике / В. Е. Гмурман. – Москва : Высшая школа, 2001. – 576 с.

10. Гмурман В. Е. Теория вероятностей и математическая статистика : учеб. пособ. для вузов / В. Е. Гмурман. – 6-е изд. – Москва : Высшая школа, 1998. – 480 с.

11. Дороговцев А. Я. Теорія ймовірностей : збірник задач / А. Я. Дороговцев ; під ред. А. В. Скорохода. – Київ : Вища школа, 1976. – 384 с.

12. Елисеева И. И. Теория статистики с основами теории вероятностей : учеб. пособ. / И. И. Елисеева, В. С. Князевский. – Москва : ЮНИТИ-ДАНА, 2001. – 446 с.

13. Жлуктенко В. І. Теорія ймовірностей і математична статистика : навч.-метод. посіб. У 2 ч. Ч. І. Теорія ймовірностей / В. І. Жлуктенко, С. І. Наконечний. – Київ : КНЕУ, 2000. – 304 с.

14. Жлуктенко В. І. Теорія ймовірностей і математична статистика : навч.-метод. посіб. У 2 ч. Ч. ІІ. Математична статистика / В. І. Жлуктенко, С. І. Наконечний, С. С. Савіна. – Київ : КНЕУ, 2001. – 336 с.

15. Збірник вправ з розділу "Теорія ймовірностей та математична статистика" навчальної дисципліни "Математика для економістів" для студентів галузі знань "Економіка і підприємництво" усіх форм навчання / уклад. Е. Ю. Железнякова, А. В. Ігначкова, З. Г. Попова та ін. – Харків : Вид. ХНЕУ, 2009. – 116 с.

16. Кремер Н. Ш. Теория вероятностей и математическая статистика / Н. Ш. Кремер. – Москва : ЮНИТИ-ДАНА, 2000. – 544 с.

17. Ross S. Introduction to probability and mathematical statistics / Sheldon Ross. – San Diego : Elsevier Academic Press, 2009. – 641 p.

12.3. Methodological support

18. Регіони України : статистичний збірник / за ред. О. Г. Осауленка. – Київ : Державна служба статистики України, 2013. – 783 с.

19. Доклад ЮНЕСКО по науке: на пути к 2030 году [Электронный ресурс]. – Режим доступа : http://unesdoc.unesco.org/images/0023/002354/235407r.pdf.

Appendices

Appendix A
Table A.1

The structure of components of professional competences formed on mastering the academic discipline "Probability Theory and Mathematical Statistics" according to Ukraine's National Framework of Qualifications

Competence formed within the theme	Minimal experience	Knowledge	Skills and abilities	Communication	Autonomy and responsibility
1	2	3	4	5	6
Theme 1. The empirical and logical bases of probability theory					
Understanding the content of axioms on which the probability theory is based	Basic definitions and theorems of probability theory. Types of events and operations with events. Calculation of their probabilities	Revision of the material of the course. Knowledge of mathematical symbols and basic definitions and theorems on the theme. The ability to define the content of random events and calculate their probabilities	Complementing the course of mathematics by basic notions. Skills in the use the Venn – Euler diagrams as an illustration of the structure of an event	Understanding the role and the place of probability theory and mathematical statistics in modern scientific research and their significance for further solving of professional problems. The ability to present research results. The development of creative thinking	A student **must** 1) **know** the basic theorems and statements of the theme; 2) **give** examples of the use of the basic notions of probability theory for solving different problems

Appendix A (continuation)
Table A.1 (continuation)

1	2	3	4	5	6
Theme 2. The basic theorems of probability theory, their economic meaning					
Skills in the independent calculation basic notions of probability theory, combinatorial analysis and mathematical statistics. Independent solving and analysis of the given problem	Availability of the basic knowledge of the course gained at a secondary educational institution or an institution of the first and the second level of accreditation. Understanding the notions: a probability, a probability of dependent or independent events; a definition of a complete group of events; the notion of a conditional probability	Knowledge of mathematical symbols and basic definitions and theorems on the theme. The formula of the total probability, the formula of Bayes, the notion of Bayes' approach	The ability to define the type of the problem, to carry out the task with the help of the basic theorems of probability theory independently. The ability to solve problems with the help of the formula of the total probability and the formula of Bayes, to distinguish between dependent and independent events. The ability to define the type of the problem, to reasonably use the corresponding theorem, to independently explain and analyze the obtained solution	The ability to present the research results using the built-in functions of the MS Excel	A student **must** 1) **know** the basic theorems and statements of the theme; 2) **give** examples of the use of the basic notions of probability theory for solving different problems; 3) **develop** and improve the ability to reasonably construct an optimal analytic solution to a problem

1	2	3	4	5	6
Theme 3. The scheme of independent trials					
The definition of the type or class. Forming possible ways of solving a problem on the basis of modern scientific attainments. Using basic methods of probability theory for solving a composite problem and reducing it to simple problems	Carrying out simplest mathematical calculations; solving simplest combinatoric problems. Understanding the notions: a probability, a probability of dependent or independent events; the definition of a complete group of events; the notion of a conditional probability. Understanding the notions of repeated trials, an independent event, revision of formulas of combinatorial theory	The theorem of Bernoulli, the theorems of Moivre – Laplace, Poisson, the limits and conditions of the use of them. The ability to carry out calculations using Bernoulli's formula and the tables of the values of Gauss and Laplace functions for calculation of probabilities by asymptotic theorems	The ability to solve problems with the help of the formula of the total probability and the formula of Bayes; the ability to distinguish between dependent and independent events. The ability to define the type of the problem, to reasonably use the corresponding theorem, to independently explain and analyze the obtained solution	The ability to explain the choice of the calculation scheme of the probability that a random event will occur a certain number of times in Bernoulli scheme and present the research results	A student **must** 1) **know** the basic theorems and statements of the theme; 2) **give** examples of the use of the basic notions of probability theory for solving different problems; 3) **develop** and improve the ability to reasonably construct an analytic optimal solution to a problem

Appendix A (continuation)
Table A.1 (continuation)

1	2	3	4	5	6
Theme 4. Random variables and their economic meaning					
Using the laws of distribution of discrete random variables	Understanding the notion of a discrete random variable, a series of distribution, a mean (a mathematical expectation), a variance, a root-mean-square deviation	The basic notions: series of distribution, the numerical characteristics, the relationship between them, the graphic interpretation	The ability to calculate the numerical characteristics of a discrete random variable and analyze the obtained result	Solving economic problems with the use of the apparatus of probability theory and random variables	A student **must be able** to find the numerical characteristics of a random variable by the given law of distribution
Theme 5. Distribution laws and numerical characteristics of discrete random variables					
Reasoning the suitability and necessity of using random variables; prediction of processes which might occur in real economic models on the basis of knowledge of the theme	Understanding the notion of a discrete random variable, the function and the density of a distribution	The basic notions: the function of distribution, the density of distribution, the relationship between them, the graphic interpretation	Skills in the use of MS Excel software to carry out calculations of the numerical characteristics of a discrete random variable	Forming the ability to analyze the results and present them for discussion	A student **must be able** to plot a polygon, a histogram, graphs of the integral function and the density of a probability

1	2	3	4	5	6
Theme 6. The basic distribution laws of a continuous random variable					
Using the laws of distribution of continuous random variables	Understanding the notion of a continuous random variable, a series of distribution, a mean (a mathematical expectation), a variance, a root-mean-square deviation	The basic notions: series of distribution, the numerical characteristics, the relationship between them, the graphic interpretation	The ability to calculate the numerical characteristics of a continuous random variable and analyze the obtained result	Solving economic problems with the use of the apparatus of probability theory and random variables	A student **must be able** to find the numerical characteristics of a continuous random variable by the given law of distribution
Theme 7. Multidimensional random variables					
Reasoning the suitability and necessity of using random variables; prediction of processes which might occur in real economic models on the basis of knowledge of the theme	Understanding the notion of a multidimensional random variable, a function and the density of distribution	The basic notions: the function of distribution, the density of distribution, the relationship between them, the graphic interpretation	Skills in the use of MS Excel software to carry out calculations of the numerical characteristics of a multidimensional random variable	Forming the ability to analyze the results and present them for discussion	A student **must be able** to find the numerical characteristics of a multidimensional random variable by the given law of distribution

Appendix A (continuation)
Table A.1 (continuation)

1	2	3	4	5	6
Theme 8. The functions of the random argument					
Skills and abilities to understand the meaning of a random argument function	Understanding the notion of the function of a random argument	Knowledge of the features of constructing the distributions of the function of one random variable if its argument is a discrete or continuous random variable, determining the basic numerical characteristics of such functions	The ability to construct the distribution law of the function of one random argument in the known distribution of its argument and determine the basic numerical characteristics of this function	Solving economic problems with the use of the apparatus of random variables	The ability to independently solve problems using the function of a random argument and its numerical characteristics
Theme 9. The elements of the theory of random processes and the theory of queuing problems					
1) Forming skills in the use of the instrument of operations research for solving economic problems; 2) the ability to form, solve and analyze queuing problems	Skills in the analysis of costs, calculation of parameters of a queuing system	Mastery of the basic methods of finding a solution to queuing problems	The ability to form, solve and analyze queuing problems	Forming the ability to solve economic problems using the theory of queuing problems	A student **must** 1) **use** the obtained knowledge for fundamental analysis of the obtained problem; 2) **form**, **solve** and **analyze** queuing problems

1	2	3	4	5	6
Theme 10. Limit theorems of probability theory					
Prediction and analysis of processes which might occur in real economic models	Basic knowledge of the theme: definitions, theorems, mathematical recording	Skills in the use of the law of large numbers for real processes	The ability to use the law of large numbers and analyze the suitability of using it in each individual case	Understanding and solving economic problems using the laws of large numbers and the central limiting theorem	A student **must** 1) **use** the acquired knowledge for fundamental analysis of the obtained problem; 2) **carry out** detailed analysis and corresponding mathematical calculations
Theme 11. Preprocessing of statistical data					
Knowledge of theoretical and probabilistic fundamentals of mathematical statistics, solving statistical problems	Availability of the basic knowledge of probability theory	The fundamentals of the sampling method, the distribution, statistical estimations and their properties	Knowledge of the fundamentals of the sampling method and the basic methods of modern statistical research	Understanding the role and the place of mathematical statistics in modern scientific research and its significance for further solving of professional problems	A student **must be able to** systematize the obtained data in the form of a table or presentation of data

1	2	3	4	5	6
Theme 12. Statistical estimation of the parameters of a distribution					
Knowledge of theoretical and probabilistic fundamentals of mathematical statistics, understanding the role of the basic mathematical assumptions in posing a question and solving statistical problems	The availability of basic knowledge of probability theory	The fundamentals of the sampling method, distribution, statistical estimations and their properties	Knowledge of the fundamentals of the sampling method and the basic methods of modern statistical research	Understanding the role and the place of mathematical statistics in modern scientific research and its significance for further solving of professional problems	A student **must know** the basic notions, definitions, types of basic mathematical and statistical calculations, basic models and methods of statistical research
Theme 13. Checking the statistical hypothesis					
Skills and abilities to apply the fitting tests for checking statistical hypotheses relative to the homogeneity of samples and the correspondence of the characteristics of a sample to characteristics of the population	The ability to make a presentation about the types of statistical hypotheses and the fitting test for their verification	Knowledge of the basic types of statistical hypotheses, understanding the principles of constructing statistical tests and determining their critical points, the methods of checking statistical hypotheses	The ability to check the correspondence between the numerical characteristics and the distribution law of random variables in the population and their estimations based on the results of the population	The ability to present the justification of the statistical test chosen to check the statistical hypothesis according to its content	Formation of responsibility for the choice of statistical tests which are used to solve problems of economic meaning

1	2	3	4	5	6
Theme 14. The elements of the correlation theory					
Mastery of skills in the use of correlation analysis for defining the influence of the factor-argument on the functional factor	The availability of the basic knowledge of correlation analysis. The ability to make a presentation about the problems of correlation analysis, to determine the correlation coefficient and the method of estimation of this coefficient	Knowledge of the methods for determining the point estimations of the basic numerical characteristics of a two-dimensional random variable according to the sample data	The ability to use the built-in functions and the superstructure of MS Excel for estimation of numerical characteristics of a two-dimensional random variable	Understanding the role and the place of the correlation analysis in modern scientific research and its significance for further solving of professional problems	A student **must** 1) **know** the basic notions, definitions, types of correlation analysis; 2) **use** the acquired experience for investigation of the real economic processes
Theme 15. The elements of the variance analysis					
Knowledge, skills and abilities in applying the variance analysis to the investigation of economic processes	Knowledge of the problems of variance analysis	Calculation of the determination coefficient, its properties and the methods of calculation, the correlation ratio, its properties	The ability to check the density of the statistical relation between the components of a two-dimensional random variable using the variance analysis	Solving economic problems with the use of the apparatus of the variance analysis	A student **must** use the acquired experience for investigation of the real economic processes with the help of the variance analysis

Appendix A (the end)
Table A.1 (the end)

1	2	3	4	5	6
Theme 16. The elements of the regression theory					
1. Mastery of skills in the use of regression analysis for defining the influence of the factor-argument on the functional factor. 2. Mastery of skills in the use of the correlation analysis and regression analysis for construction of economic models and making predictions	Knowledge of the problems of regression analysis, definition of the sample coefficient of correlation and the method of estimation. Understanding of the meaning of economic values which are included in the pair regression model	Knowledge of the methods of defining point estimations of basic numerical characteristics of a two-dimensional random variable using the sampling data. Knowledge of the properties of construction of a pair regression model, using the methods of checking the significance of parameters of a regression model and estimating the adequacy of the model	The ability to use built-in functions and the superstructure of MS Excel for estimation of numerical characteristics of a two-dimensional random variable and for construction of an equation of a pair linear regression, carry out a prognosis using this equation and define its accuracy	Presentation and discussion of the results of statistical investigations relative to the strength of the correlation. The ability to present and reasonably give the results of the application of regression analysis to the construction of a pair regression model	The ability to use the acquired experience for investigation of the real economic processes. The choice of the function which is used as an approximation for investigation of economic processes

Contents

НАВЧАЛЬНЕ ВИДАННЯ

ТЕОРІЯ ЙМОВІРНОСТЕЙ ТА МАТЕМАТИЧНА СТАТИСТИКА

Робоча програма
для студентів усіх спеціальностей першого (бакалаврського) рівня

(англ. мовою)

Самостійне електронне текстове мережеве видання

Укладачі: **Місюра** Євгенія Юріївна
Норік Лариса Олексіївна

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Редактор *З. В. Зобова*

Коректор *З. В. Зобова*

Подано тематичний план навчальної дисципліни та її зміст за модулями й темами. Вміщено плани лекцій і практичних занять, матеріал для закріплення знань студентів (контрольні запитання, завдання для самостійної роботи), а також методику оцінювання знань студентів відповідно до вимог кредитно-трансферної системи процесу навчання.

Рекомендовано для студентів усіх спеціальностей першого (бакалаврського) рівня.

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