MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE

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ENGLISH FOR BUSINESS ANALYSTS

Textbook
In 3 parts

PART 1. MATHEMATICAL FUNDAMENTALS

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В 78

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The material on mastering the English language for business analysts is offered. The tasks promote formation of the professional communicative competence of the mathematical orientation. The structure of the textbook corresponds to the modern system of organization of learning English for professional communication. The textbook can be used both for training in groups and independent learning.

For students of speciality 051 "Economics", lecturers, as well as those who learn and use English in the professional activity connected with the application of mathematical methods in economics.

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Introduction

Fluency in English is an indisputable condition for success in all areas of life, including business. This textbook aims to provide an opportunity for future specialists in business analytics to master English of the economic and mathematical area of focus and develop communicative competences (linguistic and pragmatic) for general and professional purposes to ensure effective communication in the academic and professional environment. Step by step, students learn the basics of analytical activities in the area of business in English while simultaneously improving communicative and language skills.

The textbook consists of three parts, each of which is an independent edition. The first part deals with the mathematical fundamentals of business analysis and contains six sections that correspond to the main sections of mathematics studied by students of economics with a view to mastering the mathematical basics of business analysis such as: general introduction to mathematics, arithmetic, discrete mathematics, calculus, algebra, probability theory, and statistics.

The second part is focused on the disciplines of mathematical fields of economics that are the basis of business analysis. In particular, the topics related to operations research, econometrics, game theory and decision theory are considered. These topics provide insight into different economic and mathematical aspects of business analysis.

The third part deals with tools, application of methods and models of business analysis such as: methods of Data Mining, its software support, using large amounts of data in business analysis, etc.

All parts have the same framework. All units are identical in structure and consist of the basic text with comprehension exercises, including semantization of new lexical items and improving the grammatical competence of students and speaking tasks, promoting more efficient assimilation of new material. The textbook is based on the gradual complication of professional material.

The content of the authentic texts selected for the textbook meets the academic and professional purposes. Language skills that are necessary to perform the communicative tasks are connected with learning economic and mathematical methods used in business analytics. The vocabulary selected according to the requirements to the educational level of graduates is topically introduced and drilled in various tasks. Communicative integrated skills promote

English study efficiency. The textbook materials aim to develop students' professional communicative competences.

The structure of the textbook meets the requirements of the credit-transfer system of learning English, the syllabus of English for professional purposes and the Common European Framework of Reference for Languages.

The publication contains different materials for self-study and development of language and communicative skills.

The textbook can be recommended to students studying economics with the focus on economic and mathematical methods; lecturers, postgraduate students, and all English language learners who use it for business and analytics purposes.

Unit 1. What Is Mathematics

Task 1. Answer the questions.

1. What is mathematics for you? Give five associations with this word.

2. What do you know about mathematics and mathematicians?

Match the names with the dates and phrases.

1. Galileo Galilei (1564 – 1642)	a) called mathematics "the science that draws necessary conclusions"
2. Carl Friedrich Gauss (1777 – 1855)	b) said of mathematics: "We are not speaking here of arbitrariness in any sense. Mathematics is not like a game whose tasks are determined by arbitrarily stipulated rules. Rather, it is a conceptual system possessing internal necessity that can only be so and by no means otherwise"
3. Benjamin Peirce (1809 – 1880)	c) said: "The universe cannot be read until we have learned the language and become familiar with the characters in which it is written. It is written in mathematical language, and the letters are triangles, circles and other geometrical figures, without which means it is humanly impossible to comprehend a single word. Without these, one is wandering about in a dark labyrinth"
4. Albert Einstein (1879 – 1955)	d) referred to mathematics as "the Queen of the Sciences"
5. David Hilbert (1862 – 1943)	e) stated that "as far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality"

Task 2. Read the text and answer the questions.

1. What does mathematics study?

2. What are the main functions of mathematics?

3. Where did mathematics appear?

4. What fields is mathematics used in?

Mathematics (from the Greek *μάθημα*, "knowledge, study, learning") is the study of topics such as quantity (numbers), structure, space, and change. There is a range of views among mathematicians and philosophers as to the exact scope and definition of mathematics.

Mathematicians seek out patterns and use them to formulate new conjectures. Mathematicians resolve the truth or falsity of conjectures by mathematical proof. When mathematical structures are good models of real phenomena, then mathematical reasoning can provide insight or predictions about nature. Through the use of abstraction and logic, mathematics developed from counting, calculation, measurement, and the systematic study of the shapes and motions of physical objects. Practical mathematics has been a human activity for as far back as written records exist. The research required to solve mathematical problems can take years or even centuries of sustained inquiry.

Rigorous arguments first appeared in Greek mathematics, most notably in Euclid's *Elements*. Since the pioneering work of Giuseppe Peano (1858 – 1932), David Hilbert (1862 – 1943), and others on axiomatic systems in the late 19th century, it has become customary to view mathematical research as establishing truth by rigorous deduction from appropriately chosen axioms and definitions. Mathematics developed at a relatively slow pace until the Renaissance, when mathematical innovations interacting with new scientific discoveries led to a rapid increase in the rate of mathematical discovery that has continued to the present day.

Mathematics is used throughout the world as an essential tool in many fields, including natural science, engineering, medicine, finance and the social sciences. Applied mathematics, the branch of mathematics concerned with application of mathematical knowledge to other fields, inspires and makes use of new mathematical discoveries, which has led to the development of entirely new mathematical disciplines, such as statistics and game theory. Mathematicians also engage in pure mathematics, or mathematics for its own sake, without having any application in mind. There is no clear line separating pure and applied mathematics, and practical applications for what began as pure mathematics are often discovered.

(Adapted from [10])

Focus on Reading

Task 3. These are the key terms to the text under study. Read them carefully and find the best explanations. Use a dictionary for help [6].

1) acquire knowledge	a) any rational number that can be expressed as the sum or difference of a finite number of units, being a member of the set ... –3, –2, –1, 0, 1, 2, 3, ...; an individual entity or whole unit
2) general knowledge	b) numbers which cannot be expressed as a fraction (e.g. the square root of 2)
3) fractions	c) any mathematical system of calculation involving the use of symbols

4) irrational numbers	d) common knowledge
5) calculus	e) a ratio of two expressions or numbers other than zero
6) differential equation	f) conclude (that)
7) integer	g) get or gain knowledge
8) axiom	h) a term reserved for an established fact of some consequence
9) equality	i) a relation between two sets that associates a unique element (the value) of the second (the range) with each element (the argument) of the first (the domain): a many-one relation
10) theorems	j) a statement, for which no justification is sought, that is used to define a system
11) deduce	k) an equation containing differentials or derivatives of a function of one independent variable
12) function	l) a statement, usually an equation, indicating that quantities or expressions on either side of an equal sign are equal in value

Task 4. Some of the key terms given in Task 3 are in the text (Task 2). Look through the text and underline them.

Task 5. Read the text (Task 6) and choose the best heading to it.

A. Geometry.

B. Mathematics of Number.

C. Mathematics as a Science.

D. The Pythagorean School.

Task 6. Read the text carefully once more. Say whether the statements below are true or false.

1. Mathematics is a Latin word.
2. The incidental members were called "mathematicians"; the regular members were named "auditors".
3. The largest branch of Mathematics is called the real number system.
4. Geometry, the study of functions, the calculus, differential equations, and various other subjects which follow the calculus in logical order are all developments of the real number system.
5. Each branch does not have the same logical structure.
6. The concepts, axioms and theorems are the essential components of any branches of mathematics.

7. If Mathematics is isolated from other provinces, it does not lose importance.

...

The students of mathematics may wonder where the word "mathematics" comes from. Mathematics is a Greek word, and, by origin or etymologically, it means "something that must be learnt or understood", perhaps "acquired knowledge" or "knowledge acquirable by learning" or "general knowledge". The word "mathematics" is a contraction of all these phrases. The celebrated Pythagorean School in ancient Greece had both regular and incidental members. The incidental members were called "auditors"; the regular members were named "mathematicians" as a general class and not because they specialized in mathematics; for them mathematics was a mental discipline of science learning.

Mathematics as a science, viewed as a whole, is a collection of branches. The largest branch is that which builds on the ordinary whole numbers, fractions, and irrational numbers, or what, collectively, is called the real number system. Arithmetic, algebra, the study of functions, the calculus, differential equations, and various other subjects which follow the calculus in logical order are all developments of the real number system. This part of mathematics is termed the mathematics of number. A second branch is geometry consisting of several geometries. Mathematics contains many more divisions. Each branch has the same logical structure: it begins with certain concepts, such as the whole numbers or integers in the mathematics of number, and such as point, line and triangle in geometry. These concepts must verify explicitly stated axioms. Some of the axioms of the mathematics of number are the associative, commutative, and distributive properties and the axioms about equalities. Some of the axioms of geometry are that two points determine a line, all right angles are equal, etc. From the concepts and axioms theorems are deduced. Hence, from the standpoint of structure, the concepts, axioms and theorems are the essential components of any branch of mathematics.

We must break down mathematics into separately taught subjects, but this division taken as a necessity, must be compensated for as much as possible. Students must see the interrelationships of the various areas and the importance of mathematics for other domains. Knowledge is not additive but an organic whole and mathematics is an inseparable part of that whole.

The full significance of mathematics can be seen and taught only in terms of its intimate relationships to other fields of knowledge. If mathematics is isolated from other provinces, it loses importance.

(Adapted from [2])

Task 7. Now read the text carefully and choose the best answer to each question according to the information given in the text.

1. Where does the word "mathematics" come from?

a) Greece;
b) England;
c) Italy;
d) Alexandria.

2. What does the word "mathematics" mean by origin or etymologically?

a) acquired knowledge;
b) logical construction;
c) scientific knowledge;
d) knowledge about nature.

3. What is mathematics as a science?

a) a real number system;
b) a collection of branches;
c) a calculus in logical order;
d) a calculus, differential equations, and functions.

4. What is the largest branch of mathematics?

a) geometry;
b) differential equations;
c) the whole number system;
d) the real number system.

5. What is the certain concept of mathematics of number?

a) whole numbers or integers;
b) points, lines, and triangles;
c) differential equations;
d) fractions and irrational numbers.

6. What is deduced from the concepts and axioms?

a) structures;

b) theorems;

c) calculus;

d) equations.

Task 8. Read the text (task 6) and think over answers to the following questions. Do it with your partner or in a group of three or four.

1. What does it mean: "something that must be learnt or understood"?
2. What did the celebrated Pythagorean School in ancient Greece have?
3. What main branches does mathematics consist of?
4. What is the difference between them?
5. What logical structure does each branch have?
6. What are stated axioms?
7. What are some of the axioms of the mathematics of number?
8. What are some of the axioms of geometry?
9. Are theorems deduced from the concepts and axioms?
10. What are the essential components of any branch of mathematics?
11. What should students know about mathematics?

Task 9. Translate the paragraph "Mathematics as a Science" into your own language. Let your friend check your work.

Focus on Vocabulary

Task 10. Match the words and phrases with their equivalents.

1) abstraction	a) множення / умножение
2) concept	b) додавання / сложение
3) irrational number	c) кількісна величина / количественная величина
4) negative number	d) концепція / концепция
5) notion	e) ірраціональне число / иррациональное число
6) addition	f) від'ємне число / отрицательное число
7) multiplication	g) поняття / понятие
8) variable	h) абстракція / абстракция
9) function	i) змінна / переменная
10) quantitative values	j) функція / функция

Task 11. Complete the text with the following words and phrases.

Branches, calculate, methods, sciences, language.

Mathematics

Mathematics is the oldest of the **1** … . It began with man's need to count objects and to measure distances. A mathematician uses numbers and signs to **2** … fixed quantities or to compute variable quantities.

Mathematics is known as the most exact of all the sciences, since the proper use of its **3** … can provide only one correct answer to a specific problem. It is the **4** … used by all the other sciences. It is the basis for precision in such **5** … as astronomy, chemistry, and physics.

Task 12. Choose the best option to complete each sentence.

1. The basic concepts of the main branches of mathematics are … from experience.

a) abstractions;
b) notions;
c) concepts;
d) functions.

2. Irrational numbers, …, and so forth are not wholly abstracted from the physical experience.

a) functions;
b) negative numbers;
c) quantitative values;
d) abstractions.

3. The notion of … that represents the quantitative values of some changing physical phenomena, such as temperature and time, is also at least one mental step beyond the mere observation of change.

a) a function;
b) an abstraction;
c) a variable;
d) an addition.

4. The concepts of a ..., or relationship between variables, is almost totally a mental creation.

a) irrational number;
b) negative number;
c) multiplication;
d) function.

5. The gradual introduction of new ... in any field enables mathematics to grow rapidly.

a) notions;
b) concepts;
c) functions;
d) quantitative values.

Task 13. Match the words and phrases with the definitions.

What is Research Mathematics

1. Algebra	a) deals with the rate of change of a variable and is a means for finding tangents to curves
2. Arithmetic	b) is the branch of mathematics which deals with the measurements and relationships of lines and angles
3. Calculus	c) is concerned with the limiting values of differentials and is a means of determining length, volume, or area
4. Differential calculus	d) is the study of relationships between numbers as they are represented by symbols. Its processes consist of multiplication, division, raising to a power, and extracting a root
5. Integral calculus	e) is the system of mathematics used to figure the rate of change of a function. It includes other specialized methods of treating problems which utilize algebra
6. Geometry	f) is the science of computation by the use of numbers. Addition, subtraction, multiplication, and division are the basic processes of arithmetic
7. Analytical geometry	g) includes the collection of numerical facts, together with the processes of tabulation and interpretation. It is the science of reaching conclusions from materials that are variable and of predicting results in terms of probability
8. Descriptive geometry	h) deals with symmetrical forms that are not true circles
9. Differential geometry	i) deals with mathematical proofs used in statistical methods
10. Elliptic geometry	j) is used to solve problems dealing with the space relationships of geometric forms which comprise an object
11. Plane geometry	k) deals with figures of three dimensions such as cubes, cylinders, and spheres

12. Trigonometry	l) is the application of algebraic results to geometry
13. Spherical trigonometry	m) is the study of symbols, terms, and methods of mathematics for the purpose of establishing consistency
14. Solid geometry	n) is restricted to magnitudes of two dimensions in a single plane. It is concerned with polygons, triangles and circles that can be drawn with a ruler and compass
15. Topology	o) is concerned with the collecting and tabulating of data and the summarization of processed data
16. Metamathematics	p) involves triangles inscribed in circles
17. Statistics	q) is the geometric treatment of triangles
18. Descriptive statistics	r) is the geometry of distorted geometric forms
19. Mathematical statistics	s) is the application of calculus to the study of flat surfaces and curves

Task 14. Complete the text with the following words and phrases.

Auditing, actuary studies, statistics, engineering.

Applied Mathematics

1. … involve the calculation of risk and the establishment of premiums and dividends for insurance companies.

2. … is the analysis of the financial records used in business and industry and the preparation of statements and reports based on these records.

3. … of all kinds requires the use of many branches of the science of mathematics.

4. … involves collecting, tabulating, and analyzing data to discover relationships between variable happenings so as to predict the probable outcomes under known conditions.

Task 15. Complete the table.

Noun	**Adjective**	**Verb**
development		
		determine
	variable	
		measure
		involve
		calculate
		require
		count

Focus on Grammar

Task 16. Read the text and match the headings with the paragraphs.

A. Arithmetic and Algebra.

B. Geometry and Trigonometry.

C. Graphs and Analytic Geometry.

D. Mathematics.

Mathematics – the Study of Number and Quantities

1. ...

The ability to count and measure and to deal with numbers and symbols is of great importance to civilized men. By the dawn of historic times, about 5,000 years ago, the fundamentals of arithmetic and geometry *is known/were known*. Since then, men have learned much more about mathematics, and many new topics *had been developed/have been developed*.

2. ...

Arithmetic *is based/was based* on a numeration system made up of symbols that are used to express numerical quantities. Many different numeration systems *had been devised/have been devised*. Today, however, most civilized peoples use the decimal system. The numbers *were represented/ are represented* by such symbols as 1, 5, 25, and 150.

Arithmetic is the science of operating with numbers to obtain solutions to problems. Addition and multiplication are examples of arithmetical operations.

Sometimes, facts about a number or quantity are known, but the value of the number or quantity must be found. Procedures used to solve such problems constitute algebra. For example, to find the number which when tripled and added to four is ten, solve the equation: $3x + 4 = 10$.

3. ...

Geometry deals with the properties of plane figures and solid objects. Plane geometry deals with problems that can be worked out on flat surfaces. Solid geometry deals with objects in space such as cubes and spheres.

A special branch of geometry called trigonometry deals with the properties of triangles. If two angles of a triangle and the length of the side between them are known, the value of the remaining angle and sides can be calculated by using trigonometry.

Trigonometry applied to flat surfaces, such as the land within a county, *is called/was called* plane trigonometry. Spherical trigonometry provides methods for dealing with problems on the surface of a sphere, such as the earth or what appears to be the "dome" of the sky. These methods can be used for mapping large areas such as continents, for navigation of ships and airplanes, and for astronomical observations.

4. ...

One of the simplest ways to present many kinds of statistical or other facts is by the use of graphs. Graphs are a visual presentation of mathematical data.

Analytic geometry is based upon the fact that any geometric figure can be expressed in an algebraic equation. For example, the area of a square (y) or the volume of a cube (z) having x as one edge can be expressed as $y = x^2$ or $z = x^3$. Thus algebraic computation can be used instead of complicated geometric figures.

(Adapted from [3])

Task 17. Read the text again and choose the correct form of the word or phrase in italics.

Task 18. Discuss in pairs. What is mathematics in the modern sense of the term, its implications and connotations?

Useful phrases

My view is …, because …

Surely the main point is …

The fact is …

On the one hand …, on the other hand …

Another idea is that …

I think that / In my opinion …

As far as I'm concerned / In my experience / In my view / From my experience / From my perspective / Personally speaking / In my estimation ...

Unit 2. Arithmetic

Task 1. Answer the questions.
1. When do you think the use of numbers began?
2. How did the use of numbers begin?
3. What do you know about the systems of numbers?
4. What do you think about the information below?

Primitive man knew only ten number-sounds. The reason was that he counted in the way a small child counts today, one by one, making use of his fingers. The needs and possessions of primitive man were few: he required no large numbers. When he wished to express a number greater than ten he simply combined certain of the ten sounds connected with his fingers. Thus, if he wished to express "one more than ten" he said "one-ten" and so on.

Task 2. Match the words and phrases with the definitions.

1) acquire	a) a very tall modern city building
2) advance	b) to get or gain something, to obtain
3) ancestors	c) move forward
4) skyscrapers	d) a member of your family who lived a long time ago

Task 3. Read the text and answer the questions.
1. When did people begin to count?
2. For what purposes do we use numbers?
3. Why are mathematics and numbers important?
4. What letters did the Romans use to represent numbers?

How the Use of Numbers Began

Many thousands of years ago this was a world without numbers. Nobody missed them. Everyone knew just what belonged to him and what not. If a cow was missing, the owner knew it was gone, not by counting cows, but for the same reason your mother would know if you did not come home for dinner.

But some people acquired more and more property. They would count one cow, two cows, three cows; one vase, two vases, three vases; always one, two, three or more of something they owned or saw.

How far we have advanced from the time of our ancestors! Today, using numbers, numerals and mathematics, man builds bridges, skyscrapers, flies off the earth like a bird, even measures the distance to the moon and the brightness of the light given off by the firefly. But just as important though not so exciting, is that he can tell the time, pay the grocer, count the runs in a baseball game and use the same numbers in many different ways in every-day life.

So you see, mathematics and numbers, from simple arithmetic to complex algebraic and geometric calculations, are important to life in our time.

Roman Numerals. The Romans used seven capital letters to represent numbers. They mixed them together to form many different combinations.

The Roman system of numbers is based upon the letters, I, V, X, C, D and M. This is what each letter represents:

Roman numeral	I	V	X	L	C	D	M
Hindu-Arabic numeral	1	5	10	50	100	500	1000

(Adapted from [1])

Task 4. Complete the table.

Verb	Noun	Adjective
count		
advance		
use		
build		
pay		
represent		
determine		
contain		

Task 5. Complete the text with the following words and phrases.

Limited, resistance, translated, decimal, computation, system, replaced.

Hindu-Arabic and Roman Numerical Systems

The ancient Hindus are credited with discovering the **1** … system of numeration we use today. This system was **2** … into Arabic prior to its introduction into Europe by travelling merchants around the 13th century. Hence it is also known as the Hindu-Arabic **3** … .

Adoption of the Hindu-Arabic system met **4** … due to the widespread use of the Roman numeral system during this period. Gradually, however, the superior Hindu-Arabic system was learned by the Europeans, and eventually it **5** … the Roman system.

The Roman numerals are still used to a **6** … extent – on clock faces, for instance, and in books for numbering introductory pages and chapters.

The Roman system, like others that are not based on the principle of position, does not provide an efficient and easy method of **7** … .

(Adapted from [3])

Focus on Reading

Task 6. Complete the text with the following words and phrases.

Proper, contain, characters, suffice, invention.

Our present-day number-symbols are Hindu **1** … . It is important to notice that no symbols for zero occur in any of these early Hindu number systems. They **2** … symbols for numbers like twenty, forty, and so on. A symbol for zero had been invented in India. The **3** … of this symbol for zero was very important, because its use enabled the nine Hindu symbols 1, 2, 3, 4, 5, 6, 7, 8 and 9 to **4** … for the representation of any number, no matter how great. The work of a zero is to keep the other nine symbols in their **5** … place.

Task 7. Match the words and phrases with the definitions.

1) separate	a) not equal (the same) in number, amount, or level
2) average	b) length of time
3) digit	c) the importance or usefulness of something
4) unequal	d) a system based on the number 10
5) period	e) the amount calculated by adding together several quantities, and then dividing this amount by the total number of quantities
6) decimal system	f) recognize difference, divide
7) value	g) one of the written signs that represent the numbers 0 to 9

Task 8. Read the text and answer the questions.

1. Why do we separate the figures of the numbers by commas?

2. What is each group of three figures called?
3. What is the system of numbers we use called?
4. How many digits does a period of a number contain?
5. How do we find the average of unequal numbers?

How We Read and Write Numbers

To make it easier to read large numbers, we separate the figures of the numbers by commas into groups of three, counting from right to left. Each group is called a period and has its own name.

The system of numbers we use, called the Arabic system, is a decimal system: that is, it is based on tens. In this system, the value a digit represents is determined by the place it has in the number; if a digit is moved to the left one place, the value it represents becomes ten times as great.

Zero in the decimal system is a "place-holder"; in the number 30, the zero shows that 3 has been moved to the left one place, thus counting tens instead of ones. The place value in numbers is shown below:

682,000,000,000	847,000,000	136,000	592
Billions	Millions	Thousands	Ones

These numbers are read: six hundred and eighty-two billion, eight hundred and forty-seven million, one hundred thirty-six thousand, five hundred ninety-two.

682,000,000,000	847,000,000	136,000	592
Billions	Millions	Thousands	Ones or units
4 periods	3 periods	2 periods	1 period

Notes:

1. Rules to remember: a) all periods of a number contain three digits, or places (the first period on the left may or may not); b) zero is used as a place-holder.

2. Average. When we want to find a single number that will represent all the numbers in a group of unequal numbers or quantities we find the average (or arithmetic mean).

To find the average of a group of unequal numbers, we add the numbers and then divide their sum by the number of addends.

(Adapted from [1])

Task 9. Match the sentence halves.

Signs of Operations Used in Arithmetic

1. The signs most used in arithmetic to indicate operations with numbers are	a) the sum, difference, product, or quotient of the two numbers is to be found
2. When either of these is placed between any two numbers it indicates respectively that	b) written before it (on the left) produces the result or number written after it
3. The equality sign (=) shows that any indicated operation or combination of numbers	c) plus (+), minus (–), multiplication (×), and division (:) signs

Task 10. Read the text and answer the questions.

1. What is the result of addition called?
2. What do we do while adding a series of numbers?
3. Why do we sometimes make mistakes in adding numbers?
4. What is the result of subtracting whole numbers called?
5. How do we check a subtraction example?
6. What is the result of multiplication called?
7. What is the result of division called?

Adding, Subtracting, Multiplying and Dividing the Whole Numbers

The result of additions of numbers is called the sum or total of the numbers. The numbers to be added are called the addends. In adding a series of numbers, begin with the column at the right. If the sum of a column of digits is ten or larger, carry the ten's digit and add it to the sum of the digits in the next column to the left. Careless mistakes are sometimes made because the work was not checked. It is always wise therefore to check your answer.

In subtracting whole numbers, the number which is to be made smaller or diminished is called the minuend; the number "taken away" or subtracted is called subtrahend. The answer is the difference between the minuend and the subtrahend and it is called the remainder, or difference. In checking a subtraction example, add the remainder and the subtrahend. If your answer is correct, the result obtained by addition equals the minuend.

In multiplication, the number by which you multiply is called the multiplier, the number being multiplied is called the multiplicand. The number resulting from the multiplication is called the product. Multiplication can be checked by interchanging the multiplier and multiplicand and multiplying again.

Remember that the product of any number multiplied by zero is zero. The product of any number multiplied by one is the same number. The order in which numbers are multiplied does not change the product.

In division, the number that is to be divided is called the dividend. The number by which the dividend is to be divided is called the divisor. The answer is called the quotient. The remainder is what is left over after the dividend has been divided into equal parts. If there is a remainder, it may be written over the divisor and expressed as a fraction in the quotient.

(Adapted from [1])

Task 11. Match the words and phrases with their equivalents.

1) divisor	a) додавати/прибавлять, добавлять; збільшувати/увеличивать
2) minuend	b) другий додаток/второе слагаемое
3) quotient	c) додавання/сложение
4) to add	d) віднімати/вычитать
5) to multiply	e) від'ємник/вычитаемое
6) adding	f) зменшуване/уменьшаемое
7) addend	g) залишок/остаток
8) subtrahend	h) множити й помножати/умножать; збільшувати/увеличивать в числе
9) to subtract	i) ділене/делимое
10) remainder	j) дільник/делитель
11) dividend	k) 1) частка/частное; 2) коефіцієнт/коэффициент; показник/показатель

Task 12. Complete the text with the following words.

Division, subtraction, addition, multiplication.

Arithmetic Operations

1. … . The concept of adding stems from such fundamental facts that it does not require a definition and cannot be defined in formal fashion. We can use synonymous expressions, if we so much desire, like saying it is the process of combining.

Notation: 8 + 3 = 11; 8 and 3 are the *addends*, 11 is the *sum*.

2. … . When one number is subtracted from another the result is called the *difference* or *remainder.* The number subtracted is termed the *subtrahend*, and the number from which the subtrahend is subtracted is called the *minuend*.

Notation: 15 – 7 = 8; 15 is the *subtrahend*, 7 is the *minuend* and 8 is the *remainder*. Subtraction may be checked by addition: 8 + 7 = 15.

3. … . is the process of taking one number (called the *multiplicand*) a given number of times (this is the *multiplier*, which tells us how many times the multiplicand is to be taken). The result is called the *product.* The numbers multiplied together are called *the factors of the product*.

Notation: 12 × 5 = 60 or 12 · 5 = 60; 12 is the *multiplicand*, 5 is the *multiplier* and 60 is the *product* (here, 12 and 5 are *the factors of the product*).

4. … . is the process of finding one of two factors from the product and the other factor. It is the process of determining how many times one number is contained in another. The number divided by another is called the *dividend*. The number divided into the dividend is called the *divisor,* and the answer obtained by division is called the *quotient*.

Notation: 48 : 6 = 8; 48 is the *dividend*, 6 is the *divisor* and 8 is the *quotient*. Division may be checked by multiplication.

(Adapted from [7])

Task 13. Match the numbers with words and phrases.

1) 7 × 2 + 6	a) divide ten by two, then add six
2) 5 × 8 = 40	b) multiply three by four, then square the answer
3) (6 + 9) × 2	c) square 5, then add nine
4) 10/2 + 6	d) add nine to five, then divide by two
5) (6 + 10)/2	e) add six to nine, then multiply by two
6) $5^2 + 9$	f) add six to ten, then square the answer
7) (5 + 9)/2	g) five times eight are forty
8) $(10 + 6)^2$	h) add six to ten, then divide by two
9) $(3 \times 4)^2$	i) multiply seven by two, then add six

Task 14. Match the signs with the sentences.

1) $\frac{5}{\ }$	a) the horizontal line separating the two numbers in each fraction is called the fraction line
2) $\frac{\ }{8}$	b) the number above the fraction line is the numerator
3) $\frac{5}{8}$	c) the number below the fraction line is the denominator of the fraction
4) –	d) the denominator names the fractional unit and the numerator indicates the number of those units contained in the fraction

Task 15. Read the text and answer the questions.

1. What does a fraction represent?
2. What do we call "the terms of fractions"?
3. What is the numerator? (denominator?)
4. What does a fraction indicate?
5. When is the fraction equal to 1?

Fractions and Their Meaning

A fraction represents a part of one whole thing. A fraction indicates that something has been cut or divided into a number of equal parts. For example, a pie has been divided into four equal parts. If you eat one piece of the pie, you have taken one part out of four parts. This part of the pie can be represented by the fraction 1/4. The remaining portion of the pie, which consists of three of the four equal parts of the pie, is represented by the fraction 3/4.

In a fraction the upper and lower numbers are called the terms of the fraction. The horizontal line separating the two numbers in each fraction is called the fraction line. The top term of a fraction or the term above the fraction line is called the numerator; the bottom term or the term below the fraction line is called the denominator.

A fraction may stand for part of a group. There is a group of 5 apples. Each is 1/5 (one fifth) of the group. If we take away 2 apples, we say that we are removing 2/5 of the number of apples present. If we take away 3 apples, we are removing 3/5 of the apples present. In this instance, a fraction is being used to stand for a part of a group.

A fraction also indicates division. For example: one apple was divided into eight parts and the man has eaten one part. Therefore he has eaten 1/8 of the apple. How much of the apple is left? How many eighths are in the whole apple?

Principle to remember. If in any fraction the numerator and denominator are equal, the fraction is equal to 1.

(Adapted from [1])

Task 16. Read the text and answer the questions.

1. What do you get if you multiply a whole number by 1?
2. Do you change the fraction when you multiply 1/2 by 2/2?
3. What division do you use to change 6/8 to lower terms?

Fractions

Every fraction has a numerator and a denominator. The denominator tells you the number of parts of equal size into which some quantity is divided. The numerator tells you how many of these parts are to be taken.

Fractions representing values less than 1, like $\frac{2}{3}$ for example, are called proper fractions. Fractions which name a number equal to or greater than 1, like $\frac{2}{2}$ or $\frac{3}{2}$, are called improper fractions. There are numerals like $1\frac{1}{2}$, which name a whole number and a fractional number. Such numerals are called mixed fractions. Fractions which represent the same fractional number like $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}$ and so on, are called equivalent fractions.

We have already seen that if we multiply a whole number by 1 we leave the number unchanged. The same is true of fractions when we multiply both integers in a fraction by the same number. For example, $1\times\frac{1}{2}=\frac{1}{2}$. We can also use the idea that 1 can be expressed as a fraction in various ways $\frac{2}{2}$, $\frac{3}{3}, \frac{4}{4}$ and so on.

Now see what happens when you multiply $\frac{1}{2}$ by $\frac{2}{3}$. You will have

$$\frac{1}{2}=1\times\frac{1}{2}=\frac{2}{2}\times\frac{1}{2}=\frac{2\times1}{2\times2}=\frac{2}{4}.$$

As a matter of fact in the above operation you have changed the fraction to its higher terms.

Now look at this: $\frac{6}{8}:1=\frac{6}{8}:\frac{2}{2}=\frac{6\times2}{8\times2}=\frac{3}{4}$. In both of the above operations the number you have chosen for 1 is $\frac{2}{2}$.

In the second example you have used division to change $\frac{6}{8}$ to lower terms, that is to $\frac{3}{4}$. The numerator and denominator in this fraction are prime and accordingly we call such a fraction the simplest fraction for the given rational number.

(Adapted from [7])

Focus on Speaking

Task 17. Say these expressions.

1. Fractions:

$\frac{1}{2}, \frac{1}{4}, \frac{1}{3}, \frac{2}{3}, \frac{3}{4}, \frac{1}{8}, \frac{3}{16}.$

2. Equations:

a) $x = \frac{a+b}{c}$;

b) $x + y = \frac{\Delta}{a-b}$;

c) $l = a + (n-1)d$;

d) $V = IR$;

e) $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$;

f) $v = u + at$.

Task 18. Read the text and answer the questions.

1. What should a ratio be thought of as?
2. How is the statement read when we write $a : b = c : d$ or $\frac{a}{b} = \frac{c}{d}$?
3. How are proportions treated?

Ratio and Proportion

A ratio is an indicated division. It should be thought of as a fraction. The language used is: "the ratio of *a* to *b*" which means $a : b$ or $\frac{a}{b}$ and the symbol is $a : b$. In this notation *a* is the first term or the antecedent, and *b* is the second term or the consequent. It is important to remember that we treat the ratio as a fraction. A proportion is a statement that two ratios are equal. Symbolically we write: $a : b = c : d$ or $\frac{a}{b} = \frac{c}{d}$.

The statement is read "*a* is to *b* as *c* is to *d*" and we call *a* and *d* the extremes, *b* and *c* the means, and *d* the fourth proportional. Proportions are treated as equations involving fractions. We may perform all the operations on them that we do on equations, and many of the resulting properties may already have been met in geometry.

(Adapted from [7])

Task 19. Match the types of fractions with the explanations.

1. Common fraction	a) if the numerator of a fraction is equal to or larger than the denominator, the fraction is called an improper fraction. The value of an improper fraction is equal to or larger than 1. 5/3, 3/2, 8/8 are improper fractions
2. Proper fraction	b) a number which consists of a whole number and a fraction is called a mixed number. 2 1/9, 5 1/4, 9 1/4 are fixed numbers
3. Improper fraction	c) a common fraction is a number that has the numerator and the denominator represented by numbers placed the one above, and the other below a horizontal line. 3/7 is a common fraction
4. Mixed number	d) for convenience and clarity a fraction must always be expressed in its simplest form. That is, it must be reduced to its lowest terms. To reduce a fraction to its lowest terms, divide the numerator and the denominator by the largest number that will divide into both of them evenly. The process of crossing all common factors out of numerator and denominator is called the reduction of a fraction to its lowest terms. The greatest (largest) quantity which is a common divisor of two or more quantities is called a greatest common divisor of these quantities. It is written G.C.D.
5. Reducing a fraction to lower terms	e) if the numerator of a fraction is less than the denominator, the fraction is called a proper fraction. The value of a proper fraction is always less than 1. 6/7, 1/5 and 9/10 are proper fractions

Task 20. Answer the questions.

1. What is a common fraction called?
2. What is a proper fraction called?
3. Is the value of a proper fraction more or less than 1?

4. What do we call mixed numbers?

5. How do you reduce a fraction to its lower terms?

Task 21. Answer the questions.

1. What should you do when denominators and numerators of different fractions are both different?

2. What is the general principles of fractions?

Task 22. Read the text and answer the questions above.

When denominators and numerators of different fractions are both different, the values of the fractions cannot be compared until they are converted so as to have the same denominators.

Since fractions indicate division, all changes in the terms of a fraction (numerator and denominator) will affect its value (quotient) according to the general principles of division. These relations constitute the general principles of fractions.

Task 23. Read the text and answer the questions.

1. What is an equivalent fraction?

2. How do you change a mixed number to an improper fraction?

3. How do you change an improper fraction to a whole number or mixed number?

4. How do you change a whole number to an improper fraction with a specific denominator?

5. What must you do to compare unlike fractions?

6. How do you compare fractions?

Changing Fractions

The numerator and denominator of a fraction may be multiplied by the same number without changing the value of the fraction. The resulting equivalent fraction is actually the same fraction expressed in higher terms.

To change a mixed number to an improper fraction we must: 1) multiply the denominator of the fraction by the whole number; 2) add the numerator of the fraction to the product of the multiplication; 3) write the result over the denominator.

To change an improper fraction to a whole or a mixed number we must divide the numerator by the denominator. If there should be a remainder, write it over the denominator. The resulting fraction should then be reduced to its lowest terms.

To change a whole number to an improper fraction with a specific denominator: 1) multiply the specific denominator and the whole number; 2) write the result over the specific denominator.

Fractions can be compared. To compare unlike fractions we must change them to equivalent fractions so that all have like denominators.

When fractions have different numerators but the same denominator, the fraction having the largest numerator has the greatest value.

When fractions have different denominators but the same numerator, the fraction having the largest denominator has the smallest value.

(Adapted from [1])

Task 24. Read the text again and explain the following changes: 5/10, 2/6.

Task 25. Match the phrases with the numbers.

1) 0.4 and 0.07 ("four tenths" and "seven hundredths")	a) are called common fractions, or simply fractions
2) 4/10 and 7/100 ("four tenths" and "seven hundredths")	b) are called decimal fractions, or simply decimals

Task 26. Read the text and answer the questions.

1. What is the decimal fraction?
2. How do we write decimal fractions?
3. How do you round off a decimal to a particular place?
4. How do you compare decimal fractions?
5. How do you change decimal fractions?
6. How do you change a decimal fraction to a common fraction?
7. What is the difference between decimal and common fractions?

Decimal Fractions

A decimal fraction is a special type of fraction written without a denominator (which is 10 or a power of 10) but in which the number of figures on the right-hand side of a dot, called the decimal point, indicates whether the denominator is 10 or a higher power of 10; e.g. 2/10 is written as a decimal in the form 0.2, 23/100 as 0.23, and 23/1000 as 0.023.

If any figure of the number is moved one place to the left, the value of the number is multiplied by 10.

To round off a decimal to a particular place, inspect the figure to the right of the required place: if it is 5 or over, change the last required digit to the next higher figure and drop all decimals to the right of the required figures, if it is less than 5, drop all decimals to the right of the required figure.

To compare decimal fractions, annex zeroes so that the decimals have the same number of places.

A decimal fraction may be changed to a common fraction by: leaving out the decimal point, writing the decimal number as the numerator and the number shown by the name of the last decimal place as the denominator.

Since fractional parts of thing may be written either as common fractions or as decimal fractions, we should be able to change a decimal fraction to a common fraction.

To change a decimal fraction to a common fraction take out the decimal point and write the decimal number as the numerator of the fraction. For the denominator write the number as shown by the name of the last decimal place. Reduce the common fraction to lowest terms.

Common fractions have been used for a longer time than have decimal fractions. In fact, decimal fractions did not appear until the latter part of the 16th century. Decimals are easier to write and to print than common fractions are. It is also easier to compute with them. For these reasons decimals have come to be widely used in business, science, and statistics.

(Adapted from [1])

Task 27. Complete the table.

Noun	**Adjective**	**Verb**
		place
determination	determinant	determine
		indicate
		move
		inspect
		require
		compare

Task 28. Read the text and answer the questions.

1. What methods are there for writing fractional parts?

2. Where do we put the per cent sign?
3. What does the sign % actually refer to?
4. How do we change a per cent to a decimal fraction?

What is Percent?

We have already learned two ways of writing fractional parts: common fractions and decimal fractions. Another method is by using per cents.

Per cent tells the number of parts in every hundred. This number is followed by the per cent sign (%). The word "per cent" and sign % actually refer to the denominator of a fraction expressed as hundredths.

When working with per cent, we do not write the word, but use the sign %, 20 per cent is written 20 % and so on.

In working with problems involving percentage we must be able to change per cents to decimals and decimals to per cents.

We can change a per cent to a decimal by dropping the per cent sign and moving the decimal point two places to the left. We can change a per cent to a common fraction with the given number as the numerator and 100 as a denominator.

One hundred per cent of quantity is the entire quantity. To find a per cent of a number, change the per cent to the equivalent decimal fraction or common fraction and multiply the number by the fraction. To find the per cent of one number from a second number, form a fraction in which the first number is the numerator and the second number is the denominator. Divide the numerator into the denominator and change the decimal fraction to a per cent. To find a number when a per cent of it is known, change the per cent to an equivalent decimal fraction or common fraction, divide the given number by this fraction.

(Adapted from [1])

Follow up

Task 29. Match the words with the definitions then find them in the text below.

1) subtrahend	a) the value of a number or quantity raised to some exponent
2) annex	b) a number that divides another without a remainder
3) arrange	c) to remove (a part of a thing, quantity, etc.) from the whole

4) power	d) the number to be subtracted from another number (the minuend)
5) subtract	e) put into the required order; classify
6) divisor	f) to join or add, esp. to something larger; attach

Task 30. Read the text and answer the questions.

1. How are decimal fractions added?
2. How do we write decimal fractions when we want to subtract them?
3. How do we check the answer?
4. How do we multiply (divide) decimal fractions?
5. How do we arrange numbers in adding decimal fractions?

Adding, Subtracting, Multiplying and Dividing Decimal Fractions

Decimal fractions are added in the same way that the whole numbers are added. Since only like decimal fractions can be added, that is hundredths to hundredths, and tenths to tenths, the addends are arranged in a vertical column with the decimal points directly below one another, all the way down to the answer.

Test: Find the sum of the following numbers: 2.23; 4.8; 9 and 0.067.

Annex zeroes and arrange numbers in columns:

$$\begin{array}{r} 2.230 \\ 4.800 \\ 9.000 \\ \underline{0.067} \\ 16.097 \end{array} \quad \text{answer.}$$

In subtracting decimal fractions we must write decimal fractions so that the decimal point of the minuend, subtrahend and remainder are below each other. Zeroes should be annexed so that both minuend and subtrahend are carried out to the same number of places. Check the answer the same way that you check the subtraction of whole numbers.

In multiplying a decimal fraction or mixed decimals multiply as you do whole numbers. Then, starting at the right, mark off as many decimal places in the product as there are in the multiplier and multiplicand together.

To divide a number by 10 or any power of ten, move the decimal point in the dividend as many places to the left as there are zeroes in the divisor. Add zeroes when needed.

(Adapted from [1])

Focus on Speaking

Task 31. Say the numbers: 8.4567; 7.94; 0.34; 3.2; 10.98; 15.361

Focus on Reading

Task 32. Match the words with the definitions then find them in the text below.

1) blueprint	a) the outward form of an object defined by outline
2) reduce	b) very small in amount, relating to fractions
3) scale	c) to pieces or in pieces
4) shape	d) a photographic print of plans, technical drawings, etc., consisting of white lines on a blue background
5) apart	e) the ratio between units in a numerical system (decimal scale)
6) fractional	f) to make or become smaller in size, number, extent, degree, intensity, etc.

Task 33. Read the text. Are the statements true or false?

1. An architect's drawing is much bigger than the actual house.
2. The reduced drawing is opposite to a scale drawing.
3. In a scale drawing each line is not a definite fractional part of the line it represents.
4. A scale drawing has the same shape and the same size as the original.
5. The scale does not depend on the size of the original object and how much it must be reduced.

Scale Drawing

When an architect makes plan or blueprint of a house, his drawing is much smaller than the actual house. The plan is reduced in size to fit the paper he is using. This process of reducing in size is called drawing to scale. The reduced drawing is known as a scale drawing.

In a scale drawing each line is a definite fractional part of the line it represents. A line in the scale drawing may be one-half of the line it represents, one-fourth of it, one-hundredth, one-thousandth, or any other definite part of it. In a scale drawing the scale may be written: $\frac{1''}{4} = 1'$. This means that every

1/4 inch length on the scale drawing represents a1-foot length of the original object.

A scale drawing has the same shape as the original, but not the same size. Thus, we say that scale drawing is similar to the actual object.

The maps printed in history or geography books are also scale drawings with all distances in the same ratio to the corresponding distances in the original. For example, on a map the scale may be given as $\frac{1''}{2} = 50$ miles. On another map the scale may be $1'' - 1\ 000$ miles. This means that the actual distance between two points which are $1\frac{1''}{2}$ apart on a map whose scale reading is $\frac{1''}{2} = 50$ miles, is 150 miles.

In choosing a scale, we always pick one that is convenient to work with – not too large for the paper we are drawing, nor too small to measure. The scale depends upon the size of the original object and how much it must be reduced.

Once a scale is chosen, the same scale must be used in drawing all parts of the same object. Whenever a scale drawing is made, the scale being used must be stated. It is usually written at the bottom of the drawing.

(Adapted from [1])

Focus on Speaking

Task 34. Read the numbers (see the text):

$\frac{1''}{5} = 2'$;

$\frac{1''}{5} = 100$.

Task 35. Read the text and answer the questions.

1. What is an inequality in mathematics?
2. What does the following mean: $a > b$?
3. Which symbol do we use to signify the expression "is either greater than or equal to"?
4. When are two inequalities like or unlike in sense?
5. Do you know any kinds of inequality?

Inequalities

An inequality is simply a statement that one expression *is greater than or less than another*. We have seen the symbol $a > b$, which reads "a is greater than b" and $a < b$, which reads "a is less than b". There are many ways to make these statements. For example, there are three ways of expressing the statement "a is greater than b":

- $a > b$ or $b < a$;
- $a - b > 0$; $a - b$ is a positive number;
- $a - b = n$; n is a positive number.

If an expression *is either greater than or equal to*, we use the symbol ≥, and similarly, ≤ states *is less than or equal to.* Two inequalities are *alike in sense*, or of the *same sense*, if their symbols for inequality point in the same direction. Similarly, they are *unalike*, or *opposite in sense*, if the symbols point in opposite directions.

In discussing inequalities of algebraic expressions we see that we can have two classes of them:

If the sense of inequality is the same for all values of the symbols for which its members are defined, the inequality is called an absolute or unconditional inequality.

Illustrations: $x^2 + y^2 > 0$, $x \neq 0$ or $y \neq 0$, $\pi < 4$.

If the sense of inequality holds only for certain values of the symbols involved, the inequality is called a conditional inequality.

Illustrations: $x + 3 < 7$, true only for values of x less than 4;
$x^2 + 6 < 5x$, true only for x between 2 and 3.

The inequality symbols are frequently used to denote the values of a variable between given limits. Thus, $1 \leq x < 4$, states "values of x from 1, including 1, to 4 but not including 4", i.e., x may assume the value 1 and from 1 to 4 but no others. This is also called "defining the range of values":

$x^2 + 6 < 5x$ for $2 < x < 3$.

Properties:

The sense of an inequality is not changed if both members are increased or decreased by the same number.

If $a > b$, then $a + x > b + x$ and $a - x > b - x$.

If $a > b$ and $x > 0$, then: $ax > bx$ and $\frac{a}{x} > \frac{b}{x}$.

If $a > b$ and $x < 0$, then: $ax < bx$ and $\frac{a}{x} < \frac{b}{x}$.

If a, b and n are positive numbers and $a > b$, then: $a^n > b^n$ and $\sqrt[n]{a} > \sqrt[n]{b}$.

If $x > 0$, $a > b$ and a, b have like signs, then: $\frac{x}{a} < \frac{x}{b}$.

We can illustrate these properties by using numbers. Illustrations:

a) since 4 > 3, we have 4 + 2 > 3 + 2 as 6 > 5;

b) since 4 > 3, we have 4 × 2 > 3 × 2 as 8 > 6;

c) since 4 > 3, we have 4 × (–2) < 3 × (–2) as –8 < –6;

d) since 16 > 9, we have $\sqrt{16} > \sqrt{9}$ as 4 > 3;

e) since 4 > 3, we have $\frac{2}{4} < \frac{2}{3}$ as $\frac{1}{2} < \frac{2}{3}$.

The solutions to inequalities are obtained in a manner very similar to that of obtaining solutions to equations. The main difference is that we are now finding a *range* of values of the unknown such that the inequality is satisfied. Furthermore, we must pay strict attention to the properties so that in performing operations we do not change the sense of inequality without knowing it.

(Adapted from [7])

Task 36. Work in pairs. Illustrate 5 properties of inequality by using numbers.

a) since 4 > 3, we have 4 + 2 > … ;

b) since 4 > 3, we have … ;

c) since 4 > 3, we have … ;

d) since 16 > 9, we have … ;

e) since 4 > 3, we have … .

Task 37. State the expression "*a* is greater than *b*" in different ways.

Task 38. Discuss in groups the following problem.

What is the difference (distinction) between two math terms: "natural numbers" and "cardinal numbers". Is the number 5 natural or cardinal?

Task 39. Read the text and answer the questions.

1. How many axioms did the Italian mathematician G. Peano give? What were they?

2. Which axiom is the most important? Why?

3. What does G. Peano's theory state in essence?

4. What can we state from G. Peano's five rules?
5. Who developed these axioms? What did he do?
6. How useful is Freund's system of 12 postulates?

J. E. Freund's System of Postulates for Natural Numbers

Modern mathematicians are accustomed to deriving properties of natural numbers from a set of axioms or postulates (i.e., undefined and unproven statements that disclose the meaning of the abstract concepts).

The well-known system of 5 axioms of the Italian mathematician, G. Peano provides the description of natural numbers. These axioms are:

First: 1 is a natural number.

Second: Any number which is a successor (follower) of a natural number is itself a natural number.

Third: No two natural numbers have the same follower.

Fourth: The natural number 1 is not the follower of any other natural number.

Fifth: If a series of natural numbers includes both the number 1 and the follower of every natural number, then the series contains all natural numbers.

The fifth axiom is the principle (law) of math induction.

From the axioms it follows that there must be infinitely many natural numbers since the series cannot stop. It cannot circle back to its starting point either because 1 is not the immediate follower of any natural number. In essence, Peano's theory states that the series of natural numbers is well-ordered and presents a general problem of quantification. It places the natural numbers in an ordinal relation and the commonest example of ordination is the counting of things. The domain of applications of Peano's theory is much wider than the series of natural numbers alone e.g., the relational fractions 1,1/2,1/3,1/4 and so on, satisfy the axioms similarly. From Peano's five rules we can state and enumerate all the familiar characteristics and properties of natural numbers. Other mathematicians define these properties in terms of 8 or even 12 axioms (J. E. Freund) and these systems characterize properties of natural numbers much more comprehensively and they specify the notion of operations both arithmetical and logical.

Note that sums and products of natural numbers are written as $a + b$ and $a \cdot b$ or ab, respectively.

Postulate No. 1: For every pair of natural numbers, a and b, in that order, there is a unique (one and only one) natural number called the sum of a and b.

Postulate No. 2: If a and b are natural numbers, then $a + b = b + a$.

Postulate No. 3: If a, b and c are natural numbers, then $(a + b) + c = a + (b + c)$.

Postulate No. 4: For every pair of natural numbers, a and b, in that order, there is a unique (one and only one) natural number called the product.

Postulate No. 5: If a and b are natural numbers, then $a \times b = b \times a$.

Postulate No. 6: If a, b and c are natural numbers, then $(a \times b) \times c = a \times (b \times c)$.

Postulate No. 7: If a, b and c are natural numbers, then $a \times (b + c) = a \times b + a \times c$.

Postulate No. 8: There is a natural number called "one" and written 1 so that if a is an arbitrary natural number, then $a \times 1 = a$.

Postulate No. 9: If a, b and c are natural numbers, and if $a \times c = b \times c$, then $a = b$.

Postulate No. 10: If a, b and c are natural numbers, and if $a + c = b + c$, then $a = b$.

Postulate No. 11: Any set of natural numbers which (1) includes the number 1 and which (2) includes $a + 1$ whenever it includes the natural number a, includes every natural number.

Postulate No. 12: For any pair of natural numbers, a and b, one and only one of the following alternatives must hold: either $a = b$, or there is a natural number x such that $a + x = b$, or there is a natural number y such that $b + y = a$.

Freund's system of 12 postulates provides the possibility of characterizing natural numbers when we explain how they behave and what math rules they must obey. To conclude the definition of natural numbers we can say that they must be interpreted either as standing for the whole number or else for math objects which share all their math properties. Using these postulates mathematicians are able to prove all other rules about natural numbers with which people have long been familiar.

(Adapted from [7])

Task 40. Work in pairs. Complete the formulae written by Freund's system of 12 postulates.

If a, b, c are natural numbers:

$a + b = \ldots$	$a(b + c) = \ldots$
$(a + b) + c = \ldots$	$a \cdot 1 = \ldots$
$ab = \ldots$	$ac = bc \Rightarrow \ldots$
$(ab)c = \ldots$	$a + c = b + c \Rightarrow \ldots$

Task 41. Read the text. Are the statements true or false?

1. A geometric progression (G.P.) is not a sequence of numbers obtained by repeated multiplication.

2. The number of times r is used as a multiplier is 1 time less than the number of the term.

3. A G.P. with a negative first term in which the common ratio is a number less than 1 is said to be a decreasing sequence.

4. The formula for the last term in a G.P. can be evaluated for any letter in it.

5. If you want to find the value of r or of n, it will not be well to apply it in the form $r^{n-1} = \frac{b_n}{a}$.

Sequences Obtained by Repeated Multiplication

A geometric progression (G.P.) is a sequence of numbers obtained by repeated multiplication. If a, b and c are three numbers in a G.P., there is $\frac{b}{a} = \frac{c}{b}$. Consider the first three terms of a geometric sequence. Let a represent the first term, and let r represent the common ratio.

First term: $a = a \times r^0$.

Second term: $a \times r^1$.

Third term: $a \times r \times r = a \times r^2$.

For each term, the number of times r is used as a multiplier is 1 less than the number of the term. If the total number of terms in a G.P. is n, then to find the n-th or last term, r will have to be used as a multiplier (n – 1) times. That is, $b_n = a \times r^{n-1}$. On the chessboard G.P. 1, 2, 4, 8, ..., the value of a is 1 and r is 2. Since there are 64 squares on a chessboard, n = 64. Then $b_{64} = 1 \times \times 2^{64-1}$ or accordingly, $b_n = 2^{63}$. You can readily find the value of b_{64} by making use of logarithms; in standard form it is about 9.2×10^{18}. The chessboard G.P. is clearly understood to be an increasing progression. A G.P. with a positive first term in which the common ratio is a number less than 1 is said to be a decreasing sequence. The common ratio may be negative. If this is the case and the terms are alternatively positive and negative as in +1, –2, +4, –8, +16, ... the sequence will move back and forth or oscillate from positive to negative, or from negative to positive. Such a G.P. is an oscillating sequence. The formula for the last term in a G.P. can, like any formula, be evaluated for any letter in it. If you wish to find the value of a, it will be convenient to apply the formula in the form $a = \frac{b_n}{r^{n-1}}$. If you want to find the value of r or of n, it will be well to apply it in the form $r^{n-1} = \frac{b_n}{a}$. Logarithms may prove helpful, or else, you may be able to apply the laws of exponents.

(Adapted from [7])

Task 42. Work in pairs. Choose the correct option.

1. If a, b, and c are three numbers in a G.P., there *is/was* $\frac{b}{a} = \frac{c}{b}$.

2. If the terms are positive and negative, the sequence *will/would* move back and forth from positive to negative or from negative to positive.

3. If you want to find the value of r or of n, it *will/would* be well to apply it in the form $r^{n-1} = \frac{b_n}{a}$.

Focus on Reading

Task 43. Match the words with the definitions then find them in the text below.

1) graph	a) a visual image or display, a picture
2) pictograph	b) a map designed to aid navigation by sea or air
3) circle	c) a structure represented by a diagram consisting of points (vertices) joined by lines (edges)
4) visual	d) a thing or unit, esp. included in a list or collection
5) item	e) a closed plane curve every point of which is equidistant from a given fixed point, the centre
6) chart	f) a chart on which symbols are used to represent values, such as population levels or consumption

Task 44. Read the text. Answer the questions.

1. Where are graphs used?
2. What does a graph present?
3. What can we do by using a graph?
4. What are the most commonly used graphs?
5. What is the difference between a pictograph and the bar graph?
6. How are the bars of a vertical (horizontal) graph drawn?
7. What do we call a circle or a line graph?

Graphs

Graphs are used very frequently in newspapers, magazines, textbooks and reference books. Graphs picture facts and figures so clearly that one can understand them at a glance. Graph is the picture of mathematical equation. It is a method of showing on squared paper the changes in value of an expression containing unknown quantities when one of the unknown quantities is given various definite values. Any other unknown quantity is dependent in some way on the value of the first unknown quantity, which is called the independent value.

A graph represents numerical relationship in visual form. By use of a graph we can show the relation between certain sets of numbers in an interesting, pictorial manner so that they can actually be seen.

The most commonly used graphs are: the pictograph, the bar graph, the line graph and the circle graph. In a pictograph, each picture or symbol represents a definite quantity. In a pictograph we use pictures of objects to represent numbers. The length of bars in a bar graph represents numerical facts. The bars are of varying length but of the same width. They are usually used to show size or amount of different items or size or amount of the same item at different times. The bars of a vertical bar graph are drawn straight up and down, that is at right angles with the horizontal base line of the graph. The bars of a horizontal bar graph are drawn across the page.

The line graph shows the changes in a quantity by the rising or falling of a line. The position of the line with relation to the horizontal and vertical scales represents numerical facts. The line connects a number of points.

An apportionment or distribution graph shows the relationship of all parts of a particular whole. The whole graph represents 100 %. A chart which consists of a circle broken down into subdivisions is called a circle graph. A circle graph is used to show how all the parts are related to the whole. The entire circle, which equals 360°, represents the entire thing.

(Adapted from [1])

Focus on Vocabulary

Task 45. Match the words with similar meaning.

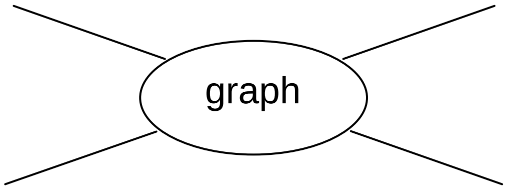
chart	alter
visual	spread about
equal	several different

change	graph
distribute	equivalent
various	optical

Task 46. Match the words with opposite meaning.

horizontal	dependent
rising	not often
straight up	the same
different	straight down
independent	vertical
frequent	falling

Task 47. Read the text again and complete a spider graph. Why do you use different kind of graphs?



Task 48. Put the words and phrases into the correct column.

Cut, drop, fall, lower, put up, raise, rise, reduce, push down, rocket.

increase	decrease

Task 49. Complete the table.

Verb	**Noun**
to cut	a cut
	a rise
to drop	
	a fall
to fluctuate	
	an improvement
to increase	
	a decrease
to reduce	
	a recovery

Task 50. Draw the graphs according to their descriptions.

1. The company share price over six month of the year has first increased from 85 % in January to 105 % in June.

2. Changes in the price of oil during the last year fluctuated between 20 and 30 dollars a barrel.

3. The average cost of renting an apartment between 2010 and 2015 reached a peak of 100 dollars a square meter in 2012. More recently, the cost fell.

4. The average cost of travel between London and Paris over a five-year period fell to a low point in 2013. But in the last year, it went up.

5. The rate of inflation over the last three years has remained steady at 3.5 per cent during this period.

6. As you can see from the bar graph, prices fell to a low point in June and then recovered and reached a pick of $170 a barrel.

7. If we look at the pie chart, the largest segment shows that fresh food products accounted for over 50 % of the supermarket's total revenues.

8. The table of figures shows that the price fluctuated between €1.55 and €1.77, depending on the country.

9. The graph shows that in the first quarter, prices increased from an average of $180,000 to $195,000. They remained steady in July and then started to fall again.

Focus on Grammar

Task 51. Tick the correct sentences and change the incorrect ones.

1. Two years ago, we have seen big increases in the price of oil.

2. The price fell in the last two days and it is now 5 per cent lower than before.

3. Our competitors reduced their prices in June.

4. We did not raise our prices in recent weeks: they are still at the same level as they were two months ago.

5. We have reviewed our prices in October.

Task 52. Complete the sentences with the past simple or present perfect from the verb in brackets.

1. Yesterday the exchange rate was €30.50. Today it is €30.30. It … (go down) by 20 percent.

2. In 2014 the price per litre was $2.50. In 2015 it was $3.50. It … (increase) by $1.00.

3. Last year it cost $400 and this year it still costs $400. The price … (not change).

4. At the start of the week share prices in London rose by 3.5 %. They are not rising now. They … (remain) steady.

5. Prices started to fall last year. This year they are still falling very quickly. They … (drop) to the lowest point in 5 years.

6. The price was the same in January 2015 as it was in December 2015. Prices … (not rise) in 2015.

Focus on Speaking

Task 53. Discuss in pairs the prices of the items below. Have these prices increased, decreased or stayed at the same level in your country in recent months?

Petrol, train fares, houses and flats, food, cars.

Useful phrases

The fact is …

On the one hand …, on the other hand …

I think that …

In my opinion …

Unit 3. Discrete Mathematics

Task 1. Answer the questions.
What is logic for you? Give three associations to this word.
What do you know about this phenomena?

Task 2. These are the key terms to the text under study. Read them carefully and find the best explanations. Use a dictionary for help [6].

1) unbearably	a) making the validity of the successive steps completely explicit
2) inescapable	b) credible or plausible
3) illogical	c) a process of deductive or inductive reasoning that purports to show its conclusion to be true
4) validate	d) unavoidable; sure to happen
5) rational	e) a piece of deductive reasoning from the general to the particular
6) argument	f) a system of rules to aid the memory
7) rigorous	g) very bad
8) premise	h) characterized by lack of logic; senseless or unreasonable
9) syllogism	i) a statement that is assumed to be true for the purpose of an argument from which a conclusion is drawn
10) inevitable	j) to give legal force or official confirmation to; declare legally valid
11) plainly	k) latent, semi-conscious (a lurking suspicion)
12) quantifier	l) not complicated; clear
13) lurking	m) using reason or logic in thinking out a problem
14) convincing	n) that cannot be escaped or avoided
15) mnemonics	o) a symbol including a variable that indicates the degree of generality of the expression in which that variable occurs

Task 3. Some of the key terms given in Task 2 are used in the text (Task 5). Look through the the text and underline them.

Task 4. Read the text below (Task 5) and choose the best heading to its parts.

A. Other Logics.
B. Propositional Logic.
C. Two Premises and a Conclusion.

Task 5. Read the text again. Say whether the statements below are true or false.

1. Logic can help us decide whether some argument is valid or not – for it concerns the rigorous checking of reasoning.

2. Aristotle's approach wasn't based on the different forms of the syllogism, a style of argument based on three statements: two premises and a conclusion.

3. A variety of syllogisms are possible if we vary the quantifiers such as "All", "Some" and "No" (as in "No As are Bs").

4. Aristotle's logic – his theory of the syllogism – was thought to be a perfect science well into the 18th century.

5. Fuzzy set theory deals with what appear to be imprecisely defined sets.

6. Logic is a dry subject.

Logic

"If there are fewer cars on the roads the pollution will be acceptable. Either we have fewer cars on the road or there should be road pricing, or both. If there is road pricing, the summer will be unbearably hot. The summer is actually turning out to be quite cool. The conclusion is inescapable: pollution is acceptable."

Is this argument from the leader of a daily newspaper "valid" or is it illogical? We are not interested in whether it makes sense as a policy for road traffic or whether it makes good journalism. We are only interested in its validity as a rational argument. Logic can help us decide this question – for it concerns the rigorous checking of reasoning.

1. ...

As it stands the newspaper passage is quite complicated. Let's look at some simpler arguments first, going all the way back to the Greek philosopher Aristotle of Stagira who is regarded as the founder of the science of logic. His approach was based on the different forms of the syllogism, a style of argument based on three statements: two premises and a conclusion. An example is:

All spaniels are dogs.
All dogs are animals.
____
All spaniels are animals.

Above the line we have the premises, and below it, the conclusion. In this example, the conclusion has a certain inevitability about it whatever meaning we attach to the words "spaniels", "dogs" and "animals". The same syllogism, but using different words is:

All apples are oranges.
All oranges are bananas.

All apples are bananas.

In this case, the individual statements are plainly nonsensical if we are using the usual connotations of the words. Yet both instances of the syllogism have the same structure and it is the structure which makes this syllogism valid. It is simply not possible to find an instance of As, Bs and Cs with this structure where the premises are true but the conclusion is false. This is what makes a valid argument useful.

All As are Bs.
All Bs are Cs.

All As are Cs.

A variety of syllogisms are possible if we vary the quantifiers such as "All", "Some" and "No" (as in "No As are Bs"). For example, another might be

Some As are Bs.
Some Bs are Cs.

Some As are Cs.

Is this a valid argument? Does it apply to all cases of As, Bs and Cs, or is there a counterexample lurking, an instance where the premises are true but the conclusion is false? What about making A spaniels, B brown objects, and C tables? Is the following instance convincing?

Some spaniels are brown.
Some brown objects are tables.

Some spaniels are tables.

Our counterexample shows that this syllogism is *not* valid. There were so many different types of syllogism that medieval scholars invented mnemonics to help remember them. Our first example was known as B<u>A</u>RB<u>A</u>R<u>A</u> because it contains three uses of "All". These methods of analyzing arguments lasted for more than 2000 years and held an important place in undergraduate studies in medieval universities. Aristotle's logic – his theory of the syllogism – was thought to be a perfect science well into the 19th century.

2. ...

Another type of logic goes further than syllogisms. It deals with propositions or simple statements and the combination of them. To analyze the newspaper leader we'll need some knowledge of this "propositional logic". It used to be called the "algebra of logic", which gives us a clue to its structure, since George Boole realized that it could be treated as a new sort of algebra. In the 1840s there was a great deal of work done in logic by such mathematicians as Boole and Augustus De Morgan.

Let's try it out and consider a proposition *a*, where *a* stands for "Freddy is a spaniel". The proposition *a* may be true or false. If I am thinking of my dog named Freddy who is indeed a spaniel, then the statement is true (T) but if I am thinking that this statement is being applied to my cousin whose name is also Freddy, then the statement is false (F). The truth or falsity of a proposition depends on its reference.

If we have another proposition *b* such as "Ethel is a cat", then we can combine these two propositions in several ways. One combination is written as $a \vee b$. The connective $\vee$ corresponds to "or" but its use in logic is slightly different from "or" in everyday language. In logic, $a \vee b$ is true if *either* "Freddy is a spaniel" is true or "Ethel is a cat" is true, *or* if both are true, and it is only false when *both* *a* and *b* are false. This conjunction of propositions can be summarized in an *or*-truth table (Fig. 3.1).

a	b	$a \vee b$
T	T	T
T	F	T
F	T	T
F	F	F

Fig. 3.1. **An *or*-truth table**

We can also combine propositions using "and", written as $a \wedge b$, and "not", written as $\neg a$ (Fig. 3.2 and 3.3). The algebra of logic becomes clear when we combine these propositions using a mixture of the connectives with a, b and c like $a \wedge (b \vee c)$. We can obtain an equation we call an identity: $a \wedge (b \vee c) = (a \wedge b) \vee (a \vee c)$. The symbol ≡ means equivalence between logical statements where both sides of the equivalence have the same truth table. There is a parallel between the algebra of logic and ordinary algebra because the symbols $\wedge$ and $\vee$ act similarly to $\times$ and $+$ in ordinary algebra, where we have $x \times (y + z) = (x \times y) + (x \times z)$. However, the parallel is not exact and there are exceptions.

a	b	$a \wedge b$
T	T	T
T	F	F
F	T	F
F	F	F

Fig. 3.2. **An *and*-truth table**

a	$\neg a$
T	F
F	T

Fig. 3.3. **A *not*-truth table**

Other logical connectives may be defined in terms of these basic ones. A useful one is the "implication" connective $a \rightarrow b$ which is defined to be equivalent to $\neg a \wedge b$ and has the truth table shown in Fig. 3.4.

a	b	$a \rightarrow b$
T	T	T
T	F	F
F	T	T
F	F	T

Fig. 3.4. **An *implies*-truth table**

Now if we look again at the newspaper leader, we can write it in symbolic form to give the argument in the margin:

C = fewer Cars on the road.

P = Pollution will be acceptable.

S = there is a road pricing Scheme.

H = summer will be unbearably Hot.

$$C \to P$$
$$C \vee S$$
$$S \to H$$
$$\neg H$$

$$P$$

Is the argument valid or not? Let's assume the conclusion P is false, but that *all* the premises are true. If we can show this forces a contradiction, it means the argument must be valid. It will then be impossible to have the premises true but the conclusion false. If P is false, then from the first premise $C \to P$, C must be false. As $C \vee S$ is true, the fact that C is false means that S is true. From the third premise $S \to H$ this means that H is true. That is, $\neg H$ is false. This contradicts the fact that $\neg H$**,** the last premise, was assumed to be true. The content of the statements in the newspaper leader may still be disputed, but the structure of the argument is valid.

3. ...

Gottlob Frege, C. S. Peirce, and Ernst Schröder introduced quantification to propositional logic and constructed a "first-order predicate logic" (because it is predicated on variables). This uses the universal quantifier, $\forall$, to mean "for all", and the existential quantifier, $\exists$, to mean "there exists".

Another new development in logic is the idea of fuzzy logic. This suggests confused thinking, but it is really about a widening of the traditional boundaries of logic. Traditional logic is based on collections or sets. So we had the set of spaniels, the set of dogs, and the set of brown objects. We are sure what is included in the set and what is not in the set. If we meet a pure bred "Rhodesian ridgeback" in the park we are pretty sure it is not a member of the set of spaniels.

Fuzzy set theory deals with what appear to be imprecisely defined sets. What if we had the set of heavy spaniels. How heavy does a spaniel have to be to be included in the set? With fuzzy sets there is a *gradation* of membership and the boundary as to what is in and what is out is left fuzzy. Mathematics allows us to be precise about fuzziness. Logic is far from being a dry subject. It has moved on from Aristotle and is now an active area of modern research and application.

Note. Conventional signs:

$\vee$ or

$\wedge$ and

$\neg$ not

$\rightarrow$ implies

$\forall$ for all

$\exists$ there exists

(Adapted from [5])

Task 6. Translate the second (2) paragraph into your own language.

Focus on Vocabulary

Task 7. Choose words from the list to fill in the gaps.

Assume, inescapable, connotations, validity, active, conclusion, argument, syllogism, makes, summer, question.

1. The … will be unbearably hot.
2. The conclusion is … .
3. It … sense.
4. Its … as a rational argument.
5. Decide this … .
6. Two premises and a … .
7. The usual … of the words.
8. A valid … .
9. Different types of … .
10. … the conclusion.
11. An … area of modern research and application.

Task 8. Complete the table.

Verb	Noun	Adjective
accept		
invent		
contain		
consider		
apply		

Focus on Grammar

Task 9. Read the text and choose the correct word or phrase in brackets.

Mathematical Logic

(However/In order) to communicate effectively, we must agree on the precise meaning of the terms which we use. It's necessary to define all terms to be used. (However/In order), it is impossible to do this since to define a word we must use others words and thus circularity cannot be avoided. In mathematics, we choose certain terms as undefined and define the others by using these terms. (However/Similarly), as we are unable to define all terms, we cannot prove the truth of all statements. (Thus/In order), we must begin by assuming the truth of some statements without proof. (So/Such) statements which are assumed to be true without proof are called *axioms.* Sentences which are proved to be laws are called *theorems.* The work of a mathematician consists of proving that certain sentences are (or are not) theorems. To do this he must use only the axioms, undefined and defined terms, theorems already proved, and some laws of logic which have been carefully laid down.

(Adapted from [7])

Focus on Speaking

Task 10. Discuss in pairs. What is logic in the modern sense of the term, its implications and connotations?

Task 11. Read this phrase. What does it mean? Discuss it with your partner.

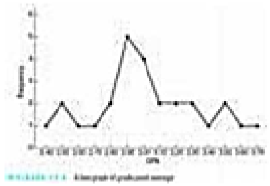
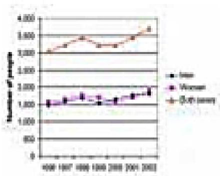
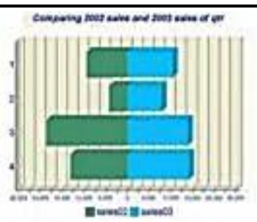
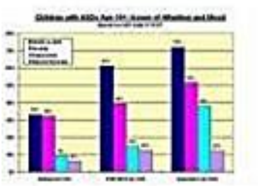
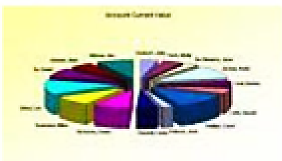
A lie begets lie.

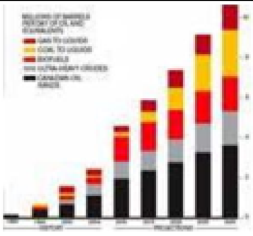
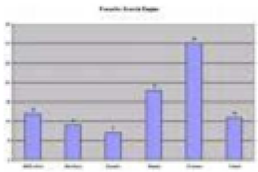
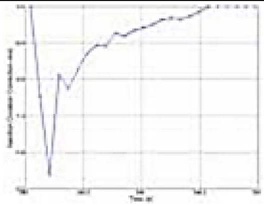
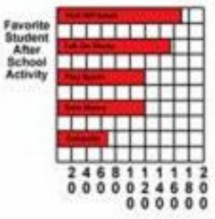
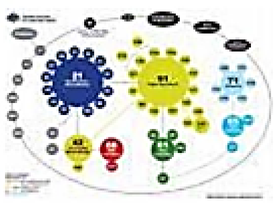
Useful phrases

My view is …, because …
Surely the main point is …
The fact is …
On the one hand …, on the other hand **...**
Another idea is that …
I think that / In my opinion …
As far as I'm concerned / In my experience / In my view …
From my experience / From my perspective / Personally speaking …
In my estimation …

Graphs

Task 12. Match the charts, graphs and diagrams with their names.

1. 	a) stacked bar chart
2. 	b) compared bar chart
3. 	c) flow chart
4. 	d) single line graph
5. 	e) pie chart

6. 	f) bargraph
7. 	g) multiple line graph
8. 	h) bubble diagram
9. 	i) diagram
10. 	j) percentage bar chart
11. 	k) single bar chart
12. 	l) population chart

Focus on Reading

Task 13. Read the text and answer the questions.

1. Does the report have a suitable structure?
2. Does it have an introduction, body and conclusion?
3. Does it include connective words to make the writing cohesive within sentences and paragraphs?
4. Does the report use suitable grammar and vocabulary?
5. Does it include a variety of sentence structures?
6. Does it include a range of appropriate vocabulary?
7. Does the report meet the requirements of the task?
8. Does it meet the word limit requirements?
9. Does it describe the whole graph adequately?
10. Does it focus on the important trends presented in the graphic information?

Writing about Graphs

Before You Begin.

Underline key words. Write related words – turn nouns into verbs, verbs into nouns, adjectives into adverbs, etc. Write opposite words, similar words, synonyms, etc.

Circle and highlight the graph. Use arrows. Make notes. Circle the biggest, the smallest, stable or unchanging parts, sudden increases, etc.

Identify trends. A trend is the overall idea of the graph:

• what is happening/what happened;
• the main change over time;
• the most noticeable thing about the graph;
• the pattern over time;
• the pattern for different places or groups or people.

Most graphs will have two trends, or there will be two graphs with a trend in each. You could tell about the two trends in two separate paragraphs. Make sure you have identified the trends in the graph.

Introduction.

First sentence: Describe the graph. You can use some slightly different words or word forms from those on the question paper, but be careful to give the full information. Start with "The graph shows".

Second sentence: This gives the trend or trends. You can put two trends in this sentence or only one – you could keep the other one for the conclusion. Start with "Overall, ...".

Paragraph 1: Trend 1.

Start with a sentence with no number. "City size increased sharply over the period". "The most obvious trend in the graph is that women are having fewer babies". "Oil production has increased slightly in all the countries in the graph".

Follow this sentence with an example (a sentence with a number) and perhaps another example (another sentence with a number). Keep alternating.

Paragraph 2: Trend 2.

Start with a sentence with no number. "City size increased sharply over the period".

Give an example (a sentence with a number) and perhaps another example.

Conclusion.

Finish by repeating the main trends, or identify a second trend. Use different vocabulary.

Don't have any numbers in the conclusion (you could use words like "most", "the majority", "a minority", "a small number").

Don't give an opinion.

While You Write: Some Don'ts.

Don't describe the X and Y axis. Give the information.

Don't write about everything on the graph. Pick the biggest, the smallest, the main points, the main trends. Group similar things together.

Don't write about the line or the bar: "The line went up"; "The bar went down". Instead, write about the idea. "The number of people going to work by train increased gradually". "Oil production shot up in 1965".

Make sure you write about the idea. Don't use shorthand: "Men went up"; "Women went down". Instead, write about the real data: "The number of men at university fell dramatically"; "The percentage of female students getting a degree rose suddenly".

Don't use "I feel", "as I have written", "as you can see", etc. Keep it academic.

Don't start sentences with "But", "So", "Also", "And", "For", "Since", "Because", "Although".

We use words to express approximation when the point we are trying to describe is between milestones on the graph (e.g. just under, well under, just over, well over, roughly, nearly, approximately, around, about).

Word Length. Make sure you have 150 words. You should have some short sentences (about 6 – 10 words) and some long ones (12 – 18) words, but your average should be about 12 or 13 words per sentence. A sentence without a number will usually be short. Use a mix – a sentence without a number followed by a sentence or two with a number.

(Adapted from [9])

Task 14. You should spend about 20 minutes on this task. Write a report for a university lecturer describing the information in the graph below. You should write at least 150 words.

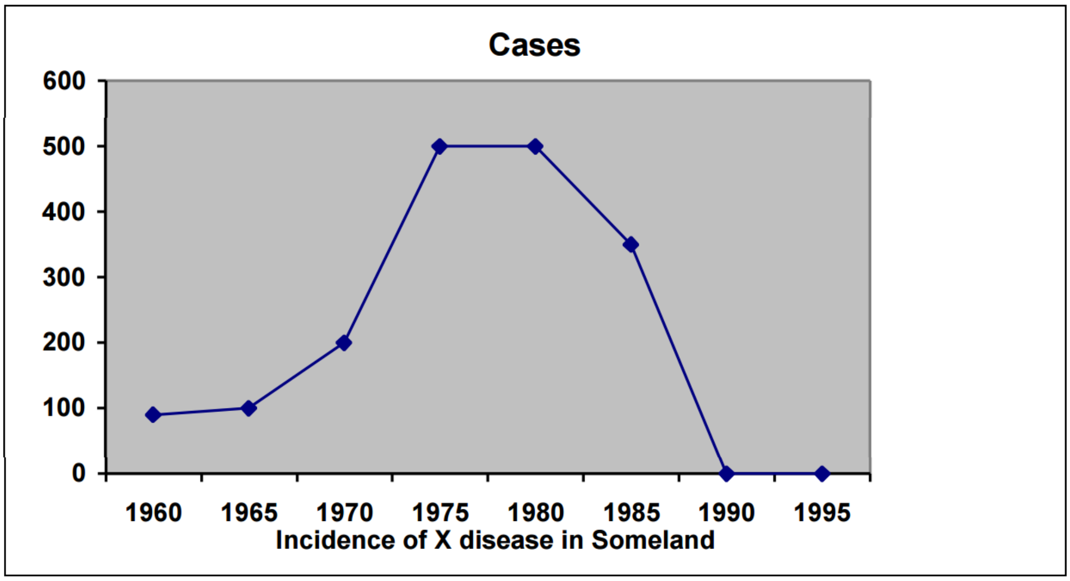
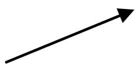
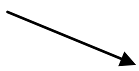
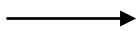


Fig. 3.5. **The incidence of *x* disease in Someland** (Adapted from [9])

Focus on Vocabulary

Task 15. Complete the table.

Direction	**Verb**	**Noun**
	rose (to)	a rise
		
		

A decrease, rose (to), reached a peak (of), peaked (at), plateaued (at), increased (to), declined (to), decreased (to), went up (to), a rise, an increase, growth, an upward, reached a plateau (at), a boom (a dramatic rise), fell (to), dipped (to), climbed (to), boomed, dropped (to), went down (to), a decline, a fall, a drop, a slump (a dramatic fall), a reduction, levelled out (at), did not change, stayed constant (at), maintained the same level, a levelling out, no change, slumped (to), reduced (to), remained stable (at), remained steady (at), stood at, a fluctuation, fluctuated (around).

Task 16. Complete the table.

Adjective	Adverb
Describing the degree of change	
dramatic	
sharp	
huge	
enormous	
steep	
substantial	
considerable	
significant	
marked	
slight	
small	
minimal	
Describing the speed of change	
rapid	
quick	
swift	
sudden	
steady	
gradual	
slow	

Task 17. Complete the text with the correct words.

A low point, declined, doubled, drop, from, increased slightly, recovered, remained, rising sharply, sudden, to, were.

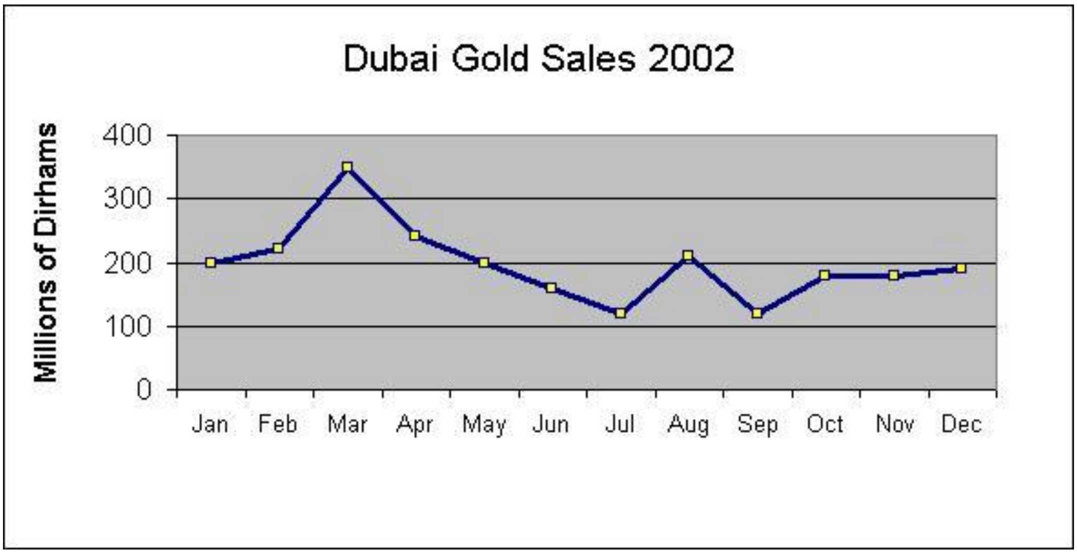


Fig. 3.6. **Dubai Gold Sales 2002** (Adapted from [9])

In January, gold sales … about 200 million dirhams per month. In February they … to Dhs 220 million, … to a peak of 350 million dirhams in March. Over the next four months, sales … steadily, reaching … of 120 million dirhams in July.

In August, there was a … increase. Sales almost … , rising … .

From September to October, sales … from Dhs 120 million to Dhs 180 million. In October and November, sales … steady, and there was a small increase in December … 190 million dirhams.

(Adapted from [9])

Focus on Grammar

Task 18. Connect these sentences using "but".

e.g. It rose to 35,000 by 2012 but later fell to 12,000 by 2015.

1. It fluctuated around 100 in 2013. Then it levelled out in the year 2,000.
2. It went down to 15,000 in 2014. Then it climbed back to 2,000 in 2015.

Task 19. Connect these sentences using "which was followed by", "which led to", "which preceded".

e.g. There was a fall to 6,000 by 2014 **which preceded** an increase to 8,000 by 2015.

1. There was a sharp rise to 900 in 2014. Then there was a gradual decline to 800 in 2015.

2. There was a slight drop to 90. Then there was a more marked decline to 50.

3. It reached a peak at Christmas. Then it dropped back to the November levels of 500.

Task 20. Complete the table. Write the comparative and superlative forms of these adjectives.

Adjective	Comparative	Superlative
accurate		
certain		
convenient		
correct		
dangerous		
happy		
likely		
modern		
new		
possible		
probable		
up-to-date		

Focus on Speaking

Task 21. You should spend about 20 minutes on this task. Write a report for a university lecturer describing the information in the graph below. You should write at least 150 words.

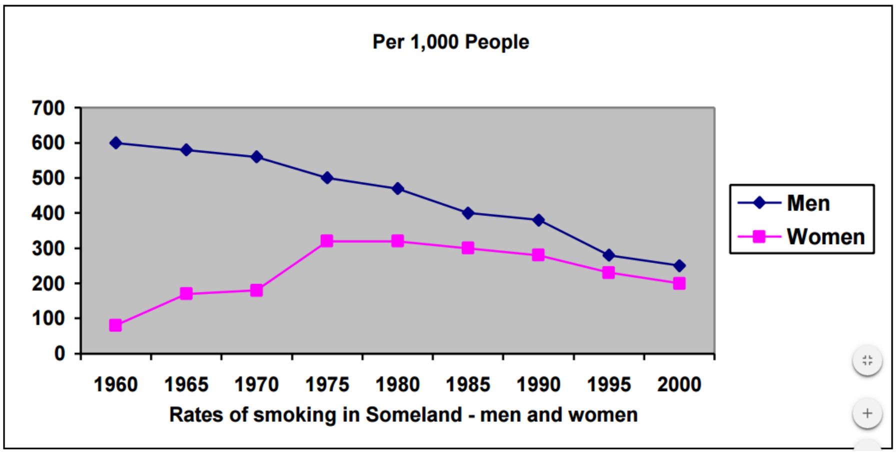


Fig. 3.7. **Rates of smoking in Someland – men and women**

(Adapted from [9])

Useful phrases

Introducing visuals

I'd like you to look at this graph ...
Let me show you this pie chart ...
Let's have a look at this model ...
Let's turn to this map ...
To illustrate my point let's look at some diagrams ...
As you can see from these figures ...
If you look at these photographs, you'll see ...
If you look at this bar chart, you'll notice ...

Explaining diagrams

Sales rose slightly in the final quarter.
Profits fell a little last year.
Demand increased gently.
Turnover decreased steadily.
Turnover dropped suddenly.
Turnover decreased quickly.
Demand increased rapidly.
Profits fell dramatically.
At the beginning of this year sales stagnated.
In the middle of August profits slumped.
At the end of last year demand peaked.
In the first quarter of 2014 sales plummeted.
In the second quarter of 2015 sales flattened out.
In the third quarter of 2013 sales leveled off.
In the last quarter of 2012 sales remained steady.

Unit 4. Calculus

Task 1. Answer the questions.

1. What do you know about the calculus? Give real-world examples of calculus.

2. What is calculus?

Task 2. Read the text and find information to answer the questions (Task 1).

Real-World Examples of Calculus

So, with regular math you can do the straight incline problem; with calculus you can do the curving incline problem. With regular math you can determine the length of a buried cable that runs diagonally from one corner of a park to the other (remember the Pythagorean Theorem?). With calculus you can determine the length of a cable hung between two towers that has the shape of a catenary (which is different, by the way, from a simple circular arc or a parabola). Knowing the exact length is of obvious importance to a power company planning hundreds of miles of new electric cable.

You can calculate the area of the flat roof of a home with regular math. With calculus you can compute the area of a complicated, nonspherical shape like the dome of the Minneapolis Metrodome. Architects need to know the dome's area to determine the cost of materials and to figure the weight of the dome (with and without snow on it). The weight, of course, is needed for planning the strength of the supporting structure.

With regular math and simple physics, you can calculate how much a quarterback must lead a pass receiver if the receiver runs in a *straight* line and at a *constant* speed. But when NASA, in 1975, calculated the necessary "lead" for aiming the Viking I at Mars, it needed calculus because both the Earth and Mars travel on *elliptical* orbits, and the speeds of both are *constantly changing* – not to mention the fact that on its way to Mars, the spacecraft was affected by the different and *constantly changing* gravitational pulls of the Earth, the Moon, the Mars, the Sun (Fig. 4.1).

(Adapted from [8])

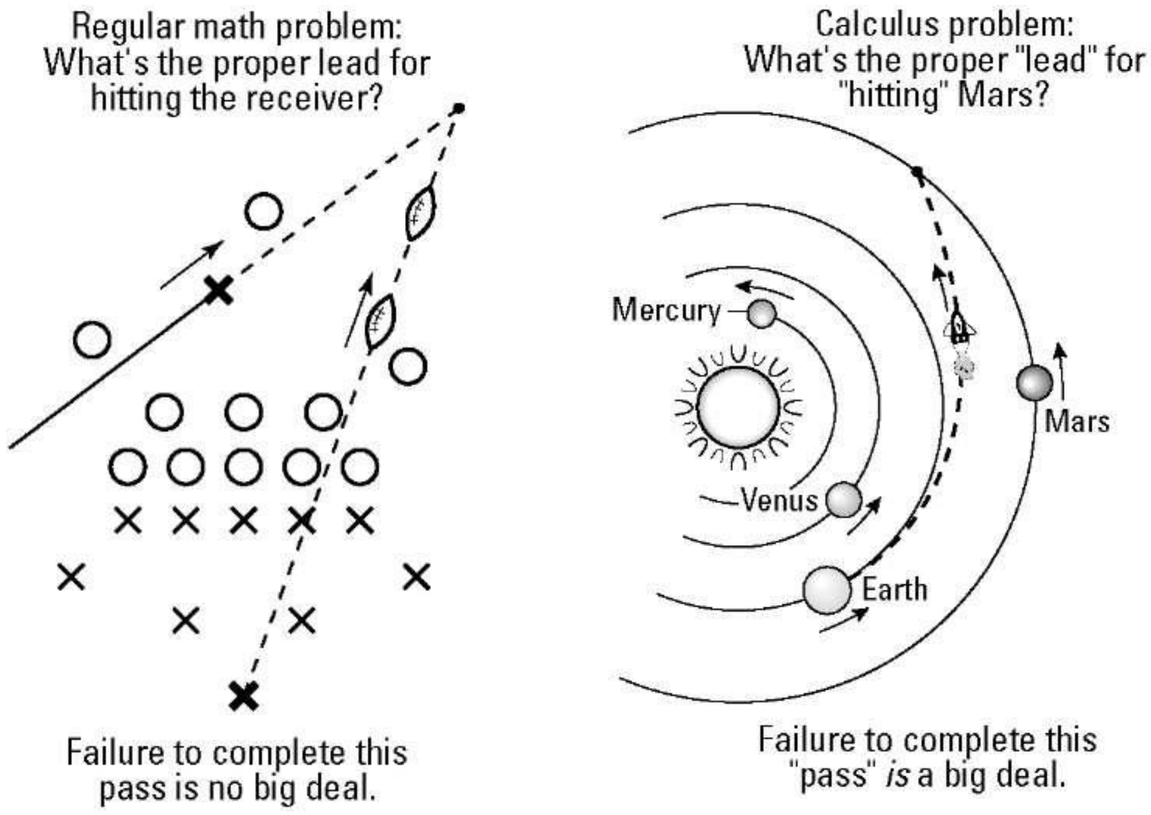


Fig. 4.1. **B.C.E. (Before the Calculus Era) and C.E. (the Calculus Era)**

Focus on Reading

Task 3. These are the key terms to the text under study. Read them carefully and find the best explanations. Use a dictionary for help [6].

1) tweak	a) smooth in shape or movement; flowing
2) incline	b) a considerable amount
3) crate	c) having an unlimited number of digits, factors, members, etc.
4) fluid	d) to twist, jerk, or pinch with a sharp or sudden movement
5) evolve	e) slope or slant
6) chunk	f) to yield, emit, or give off (heat, gas, etc.)
7) infinitely	g) a fairly large container, usually made of wooden slats or wickerwork, used for packing, storing, or transporting goods

Task 4. Read the text. Are these statements true or false?

1. Calculus is a new subject.

2. The man is pushing the same crate up a curving incline.

3. For the straight incline, the man pushes with an unchanging force, and the crate goes up the incline at an unchanging speed.

4. Calculus doesn't take the regular rules of math and applies them to fluid, evolving problems.

5. Each small chunk can't be solved the same way, and then you just add up all the chunks.

6. Everything in calculus involves infinity in one way or another.

So What Is Calculus Already?

(1) Calculus is basically just very advanced algebra and geometry. In one sense, it's not even a new subject – it takes the ordinary rules of algebra and geometry and tweaks them so that they can be used on more complicated problems.

(2) Look at Fig. 4.2. On the left is a man pushing a crate up a straight incline. On the right, the man is pushing the same crate up a curving incline. The problem, in both cases, is to determine the amount of energy required to push the crate to the top. You can do the problem on the left with regular math. For the one on the right, you need calculus (if you don't know the physics shortcuts).

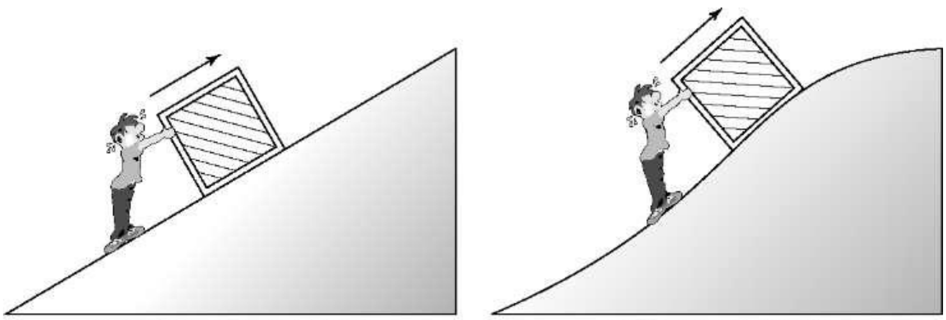


Fig. 4.2. **The difference between regular math and calculus: in a word, it's the curve** [10]

(3) For the straight incline, the man pushes with an unchanging force, and the crate goes up the incline at an unchanging speed. With some simple physics formulas and regular math (including algebra and trig), you can compute how many calories of energy are required to push the crate up the incline. Note that the amount of energy expended each second remains the same.

(4) For the curving incline, on the other hand, things are constantly changing. The steepness of the incline is changing – and it's not like it's one

steepness for the first 3 feet and then a different steepness for the next 3 – it's constantly changing. And the man pushes with a constantly changing force – the steeper the incline, the harder the push. As a result, the amount of energy expended is also changing, not just every second or thousandth of a second, but constantly, from one moment to the next. That's what makes it a calculus problem. It should come as no surprise to you, then, that calculus is called "the mathematics of change". Calculus takes the regular rules of math and applies them to fluid, evolving problems.

(5) For the curving incline problem, the physics formulas remain the same, and the algebra and trig you use stay the same. The difference is that – in contrast to the straight incline problem, which you can sort of do in a single shot – you've got to break up the curving incline problem into small chunks and do each chunk separately. Fig. 4.3 shows a small portion of the curving incline blown up to several times its size.

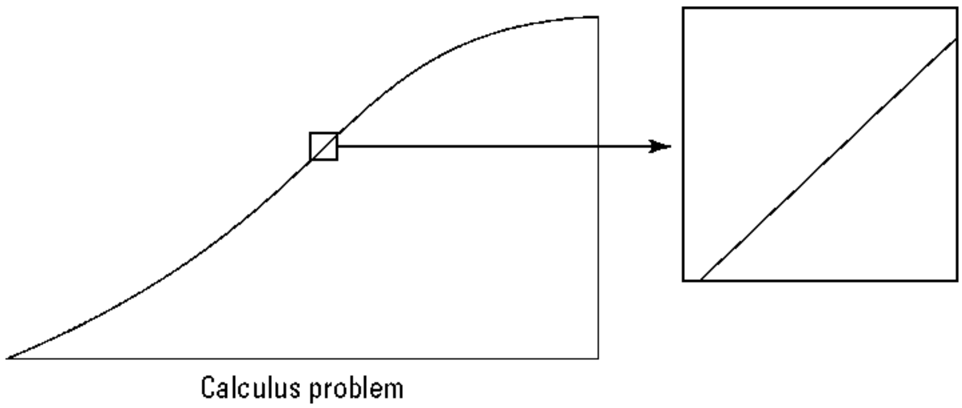


Fig. 4.3. **Zooming in on the curve – voila, it's straight (almost)** [8]

(6) When you zoom in far enough, the small length of the curving incline becomes practically straight. Then you can solve that small chunk just like the straight incline problem. Each small chunk can be solved the same way, and then you just add up all the chunks.

(7) That's calculus in a nutshell. It takes a problem that can't be done with regular math because things are constantly changing – the changing quantities show up on a graph as curves – it zooms in on the curve till it becomes straight, and then it finishes off the problem with regular math.

(8) What makes calculus such a fantastic achievement is that it does what seems impossible: it zooms in infinitely. As a matter of fact, everything in calculus involves infinity in one way or another, because if something

is constantly changing, it's changing infinitely often from each infinitesimal moment to the next.

(Adapted from [8])

Task 5. Translate paragraphs 6, 7, 8 into your own language.

Focus on Vocabulary

Task 6. Choose words from the list to fill in the gaps.

Incline, rules, curving, unchanging, crate, energy, problem, chunks, new, constantly.

1. A … subject.
2. Ordinary … of algebra and geometry.
3. A straight … .
4. A … incline.
5. Push the … to the top.
6. An … force.
7. The amount of … .
8. … changing.
9. A calculus … .
10. Small … .
11. Changing … .

Focus on Reading

Task 7. Visualize your best friend. What is accompanying this image? Why?

Task 8. Complete the text with the following words and phrases.

Intuitive, characterizes, resulting, concepts, pairing, mapping, object, mathematical, proceeding, associating.

Mappings

Now, we shall concern ourselves with another of the important **1** … of mathematics – the notion of a mapping. But first let us try an experiment designed to yield some information about our mental habits. Visualize your best friend. Of course, the image of a certain individual forms in your mind.

But did you notice that accompanying this image is a name – the name of your friend? Not only you did "see" your friend, but you also thought of his name. In fact, is it possible for you to visualize any individual without his name immediately emerging in your memory? Try! Furthermore, is it possible for you to think of the name of an individual, at the same time, visualizing that individual? The point of the **2** … experiment is to demonstrate that we habitually link together a person and his name; we seldom think of one without the other. Let us see what there is of **3** … value in the above observation. First, let us state the essentials of the situation. On the one hand, we have a set of persons; on the other hand, the set of names of these persons.

With each member of the first set we associate, in a natural way a member of the second set. It is in the process of **4** … members of one set with the members of another set that something new has been created. Let us analyze the situation mathematically. Denote the set of persons by P and the set of names by N. We want to associate with each member of P an appropriately chosen member of N; in fact, we want to create a mathematical **5** … which will characterize this association of members of N with members P. We rely on one simple observation: there is no better way of indicating that two objects are linked together than by actually writing down the names of the objects, one after the other; i.e., we indicate that two objects are associated by **6** … the objects. Now we see the importance of ordered pairs. The ordered pair (a, b) can be used to indicate that a and b are linked together.

Now we know how to characterize associating members of N with members of P: construct the subset $P \times N$ obtained by pairing with each person his name. The **7** … set of ordered pairs express mathematically the associating process described above, since the person and the name that belong together appear in the same ordered pair.

Now a definition. Let A and B denote any nonempty set. A subset of $A \times B$, say p, is said to be a **8** … of A into B iff (iff = if and only if) each member of A is a first term of exactly one ordered pair in μ. Moreover, we shall say that the mapping μ associates with a given member of A, say a, the member of B paired with a. Thus, iff $(a, b) \in \mu$ we shall say that b is associated with a under the mapping μ, b is also called the image of a under the mapping. Note that the subset $P \times N$ constructed above is a mapping of P into N. Thus our notion of a mapping of A into N permits us to characterize mathematically the **9** … idea of associating a member of B with each member of A.

Intuitive idea   Mathematical representation

Under the intuitive idea, *b* is associated with *a*; this is represented by the mathematical assertion $(a, b) \in \mu$. In short, the set μ **10** … the intuitive idea of associating a member of *B* with a member of *A*. If μ is a mapping of *A* into *B* such that each member of *B* is a second term of at least one member of μ, then we shall say that *p* is a mapping of *A* into *B*. Furthermore, if a mapping of *A* into *B* such that no member of *B* is a second term of two ordered pairs in the mapping, then we shall say that this subset of $A \times B$ is one to one mapping of *A* into *B*. For example: {(1, 3), (2, 4), (3, 5), (4, 6)} is one to one mapping of {1, 2, 3, 4} into {1, 2, 3, 4, 5, 6, 7}. If μ is both a one to one mapping of *A* into *B* and a mapping of *A* onto *B*, then μ is said to be a one to one mapping of *A* onto *B*.

(Adapted from [7])

Task 9. Read the text again and answer the questions.

1. Which experiment in this text is designed to yield some information about our mental habit?
2. Does the image of a man usually accompany his name?
3. Does one necessarily visualize a man when hearing his name?
4. Why do we habitually link together a person and his name?
5. What do we show by pairing objects?
6. How do we characterize associating members of *N* with the members of *B*?
7. When do we say that *p* is a mapping of *A* into *B*?
8. Under what condition is a subset of $A \times B$ said to be a mapping of *A* into *B*?
9. What do we mean by saying that the subset of $A \times B$ is a one to one mapping of *A* into *B*?

Task 10. Discuss the following questions.

1. Each mapping of *A* into *B* is also a mapping of *B* into *A*, isn't it?
2. Given that *B* is a subset of *C*, show that each mapping of *A* into *B* is also a mapping of *A* into *C*.

Task 11. These are the key terms to the text under study. Read them carefully and find the best explanations. Use a dictionary for help [6].

1) finite	a) having an unlimited number of digits, factors, terms, members
2) correspondence	b) a straight line from any point drawn parallel to one coordinate axis and meeting the other, usually a coordinate measured parallel to the vertical
3) infinitely	c) having a number of elements that is a natural number; able to be counted using the natural numbers less than some natural number
4) encounter	d) to see; perceive; notice
5) argument	e) similarity or analogy
6) ordinate	f) to be a sign, symbol, or symptom of; indicate or designate
7) denote	g) a process of deductive or inductive reasoning that purports to show its conclusion to be true
8) observe	h) having or relating to right angles
9) rectangular	i) meet by chance or unexpectedly

Task 12. Read the text and answer the questions.

1. Why is the concept of a function fundamental in analysis?
2. Is it easy to give a precise definition of a function?
3. What does a function deal with?
4. How many numbers does the set of values of the functions (1) and (2) contain?
5. Is this set finite or infinite?
6. What is the essential feature of the definition of a function?
7. What kinds of correspondence do you know?

Functions

The concept of a function is fundamental in analysis, but it is not easy to give a precise definition of it.

Clearly it deals with the set of values of a function when another variable x takes certain values.

Consider, for example, the two functions:

1) the function whose value is 1 when $x \geq 0$ and 0 when $x < 0$;

2) the function whose value is 1 when x is rational and 0 when x is irrational.

The set of values of each of these functions is the finite set containing the two numbers 0 and 1; but the two functions are quite different from each other. The sets of values of the two functions x^3 and x^5 are identical (in this case the set of all real numbers), but the functions are not the same.

The essential feature of the definition of a function is the concept of a "correspondence" or "relationship" between the individual members of two sets. This correspondence is known as "many-one", that is if *x* denotes any member of one set and *y* any member of the other, then to one value of *y* there may correspond one, or several, or even infinitely many values of *x*. The student may have encountered "one-one" correspondence in geometry; this is a special case of many-one correspondence.

Definition: If to each member *n* of a certain set *M* there corresponds one value of a variable *y*, then *y* is said to be a function of the variable *x*. The variable *x* is called the argument of the function, and the set *M* the domain of the function. The set of all the values taken by the variable *y* is called the ordinate set. Both the domain and the ordinate set may be either finite or infinite, bounded or unbounded. A function is frequently denoted by a symbol such as *f(x)*. If *a* is a particular member of the domain of *f(x)*, the corresponding value of *y* is denoted by *f(a).*

It is important to observe that it is not implicit in the definition of a function that there should exist an algebraic equation connecting *x* and *y*. If *y* and *x* are related so that *y* is a function of *x*, it does not necessarily follow that *x* is a function of *y*, although this may sometimes be true: For let *X* be the domain of this function and *Y* the ordinate set. Then if *x* is any member of *X*, we know that there is just one member *y* of *Y* which corresponds to it.

But if *y* is a member of *Y*, there may be more than one value of *x* in *X* which gives rise to a particular number, as the correspondence is many-one. If there are any values of *y* for which this is so, then *x* is not a function of *y* according to the definition.

Functions may be represented geometrically. For this we take a rectangular system of Cartesian coordinates in a plane and associate with each member *x* of the domain of the function the point *P* whose coordinates are (*x*, *y*). The set of points *P* is called the graph of the function. A function defined by means of a formula may have its domain restricted by the character of the formula itself.

(Adapted from [4])

Task 13. Discuss the following questions.

1. In what mathematical sciences may you have encountered one-to-one correspondence?

2. What kind of correspondence is "one-one" correspondence a special case of?

3. On what condition is *y* said to be a function of the variable *x*?

4. What is the variable *x* called?

5. What is the set *M* called?

6. May the domain and the ordinate set be either finite or infinite, bounded or unbounded?

7. What symbol is a function frequently denoted by?

8. Does it always necessarily follow that *x* is a function of *y*?

9. How may functions be represented?

10. What may a function defined by means of a formula have its domain restricted by?

Task 14. Translate the sentences into your own language.

1. It is always very important. We try to give precise definitions.

2. It is desirable now. We consider a special case of many-one correspondence.

3. It is necessary and sufficient for the definition of a function. To each member *x* of a certain set *M* there corresponds just one value of a variable.

4. It is required in such cases. The function *f(x)* tends to infinity.

5. It is recommended now. This correspondence is of a particular kind known as "many-one".

6. It is important in the definition of a function. The essential feature of this definition is the concept of a "correspondence" or "relationship" between two individual members of two sets.

7. It is always recommended. We often limit the domain of a particular function.

8. It is frequently desirable. We denote a function by a symbol *f(x)*.

9. It is desirable. This fact is explicit in the definition.

10. It is important. This value of *x* in *X* gives rise to a particular number.

Task 15. Complete the table.

1) precise – precision – precisely;

2) rational – irrational – rationalize – rationalization;

3) vary – variable – variability;
4) argue – argument;
5) frequent – frequency – frequently;
6) important – importance;
7) concern – concerned;
8) behave – behavior;
9) type – typical;
10) pure – purely – purity;
11) require – requirement;
12) appropriate – appropriation;
13) select – selection.

Noun	**Verb**	**Adjective**	**Adverb**

Task 16. Match the word with the opposites.

satisfactory	implicit
irrational	constant
variable	unsatisfactory
explicit	rational

Task 17. Complete the text with the following words and phrases.

Argument, tending, function, mathematical, connection, between, large, values, unbounded, domain.

Limit Definitions

In this section we shall be concerned with functions whose **1** … is either an interval or a set of all real numbers.

We consider first a number of different ways in which functions behave when the **2** … takes values which are very large. Some of these are illustrated by the following examples:

1. The **3** … of x^2 are themselves large when x is a large number. A function such as this is said to "tend to infinity".

2. The **4** … $f(x) = 3$ has the same value whatever x is, and therefore does not become large for any value of x.

3. The values of the function sin x are all *between* –1 and +1. Moreover, if k is any number **5** … these bounds, it is possible to find very large values of the argument for which sin $x = k$.

4. The values of $1/x$ are very small when x is large. This function is said to "tend to 0".

Consider first functions of which the illustration (1) is typical, whose values are large when x is large. Our object is to express, in terms of precise and well-defined **6** … relationships, the particular feature which distinguishes these functions from all others. We must first explain what we mean by the word "large" in this **7** … . Largeness is a purely relative concept, but we require an absolute standard for deciding whether or not a function should be described as tending to infinity.

Shall we say that, if a function has values between 100 and 1000, for example, then it is large enough to be placed in this category? Clearly, not.

The function $f(x) = 500 + \sin x$ will not be said to tend to infinity.

It is required rather that the function should be able to take values as **8** … as any we care to name, on condition that a large enough value of x is chosen. In particular, we should not wish to include any bounded function in this class.

Taking as an example the function x^2, let a large number be selected: 10 000 000, say. Clearly, the function can take values as large as this, if we take x large enough; in fact, $f(1\ 000) = 1\ 000\ 000$. Moreover, an appropriate value of x could still be found even if the function were required to take far larger values, such as 1 000 000 000 000, or $10^{1\ 000\ 000}$. Although no bounded function can tend to infinity, it is not true that all **9** … functions do so; and there are two reasons for this. Firstly, unbounded functions may also take large negative values. We should not, for example, wish to describe $-x^2$ as **10** … to infinity. Secondly, – and this is a more important factor – it sometimes happens that an unbounded function only takes large values for certain special values of the argument, and that for other values $f(x)$ may be quite small.

An example of such a function is:

$$f(x) = \begin{cases} x^2 & \text{when } x \text{ is an integer} \\ 0 & \text{when } x \text{ is not an integer} \end{cases}.$$

The student should sketch the graph of this function, comparing it with the graph of x^2.

(Adapted from [4])

Task 18. Translate the sentences into your own language.

1. The ordinate set can be the interval [–1, 1], if the domain is the set of all real numbers.
2. The ordinate set can include two numbers –1 and +1, if the domain is the set of integers.
3. The function can be the constant value +1, if the domain is the set of even integers.
4. If k is any number between –1 and +1, we can find very large values of the argument for which $x = k$.
5. Can a function be described as tending to infinity if its value is between 100 and 1 000?
6. The function can be made to take values as large as we please if x is large enough.
7. If there is a number q with the property that $f(x) > 10\ 000$ for all values of x bigger than q, then we can prove this case.
8. If the function is greater than 10 000 for all values of x, for example, $f(x) \geq 100\ 001$ then we can take any number for q.

Task 19. Read the text again and give a short summary of it.

Focus on Grammar

Task 20. Transform the sentences according to the example:

The value of this function is **as great as** that one.

The value of this function **is not so great as** that one.

1. The first class is as wide as the second.
2. This result is as interesting as the first one.
3. This definition is as precise as the previous one.
4. It is as near to it as we please.
5. The value of this function is as small as that one.
6. This function can take values as large as we please.

Task 21. Match the sentences with the correct functions.

1. This function tends to infinity	A. Let the values of this function be between –1 and +1
2. The function has the same value	B. Let this function be tending to infinity
3. The values of this function are all between –1 and +1	C. Let this unbounded function be taking large negative values
4. The values of this function are very small	D. Let the function be tending to 0
5. The function tends to 0	E. Let the function be the same value
6. The function belongs to this class	F. Let the function be assumed to take far larger values
7. The function takes values as large as we please	G. Let the function be belonging to this class
8. The function is assumed to take far larger values	H. Let the function be taking values as large as we please
9. This unbounded function takes large negative values	I. Let the values of this function be very small

Task 22. Read these expressions.

1) $n! = n \times (n - 1)!$;

2) $\sum_{r=1}^{5} r^2$;

3) $f(x)$;

4) $x \to \infty$;

5) $f(x) = 3$;

6) $\sin x = k$;

7) $\frac{1}{x}$;

8) $f(\sqrt{2}) = 2$;

9) $f(\sqrt{2}) > f(1\frac{1}{2})$.

Task 23. Read the text and underline the sentences that can help to describe Fig. 4.4. Then read the graphs of the functions *f*, *g*, *p* and *q*.

Limits and Continuity

Limits are fundamental for both differential and integral calculus. The formal definition of a derivative involves a limit as does the definition of a definite integral.

The limit of a function (if it exists) for some *x*-value, *a*, is the height the function gets closer and closer to as *x* gets closer and closer to *a* from the left and the right.

Let me say that another way. A function has a limit for a given *x*-value if the function zeroes in on some height as *x* gets closer and closer to the given value from the left and the right. It's easier to understand limits through examples than through this sort of mumbo jumbo, so take a look at some.

A continuous function is simply a function with no gaps – a function that you can draw without taking your pencil off the paper. Consider the four functions in Fig. 4.4.

Whether or not a function is continuous is almost always obvious. The first two functions in Fig. 4.4 – *f* and *g* – have no gaps, so they're continuous. The next two – *p* and *q* – have gaps at $x = 3$, so they're not continuous. That's all there is to it. Well, not quite. The two functions with gaps are not continuous everywhere, but because you can draw sections of them without taking your pencil off the paper, you can say that parts of them are continuous. And sometimes a function is continuous everywhere it's defined. Such a function is described as being continuous over its entire domain, which means that its gap or gaps occur at *x*-values where the function is undefined. The function *p* is continuous over its entire domain; *q*, on the other hand, is not continuous over its entire domain because it's not continuous at $x = 3$, which is in the function's domain.

Continuity and limits usually go hand in hand. Look at $x = 3$ on the four functions in Fig. 4.4. Consider whether each function is continuous there and whether a limit exists at that *x*-value. The first two, *f* and *g*, have no gaps at $x = 3$, so they're continuous there. Both functions also have limits at $x = 3$, and in both cases, the limit equals the height of the function at $x = 3$, because as *x* gets closer and closer to 3 from the left and the right, *y* gets closer and closer to $f(3)$ and $g(3)$, respectively.

Functions *p* and *q*, on the other hand, are not continuous at *x* = 3 (or you can say that they're discontinuous there), and neither has a regular, two-sided limit at *x* = 3. For both functions, the gaps at *x* = 3 not only break the continuity, they also cause there to be no limits there because, as you move toward *x* = 3 from the left and the right, you do not zero in on some single *y*-value.

So continuity at an *x*-value means there's a limit for that *x*-value, and discontinuity at an *x*-value means there's no limit there ... except when ... (reason).

(Adapted from [8])

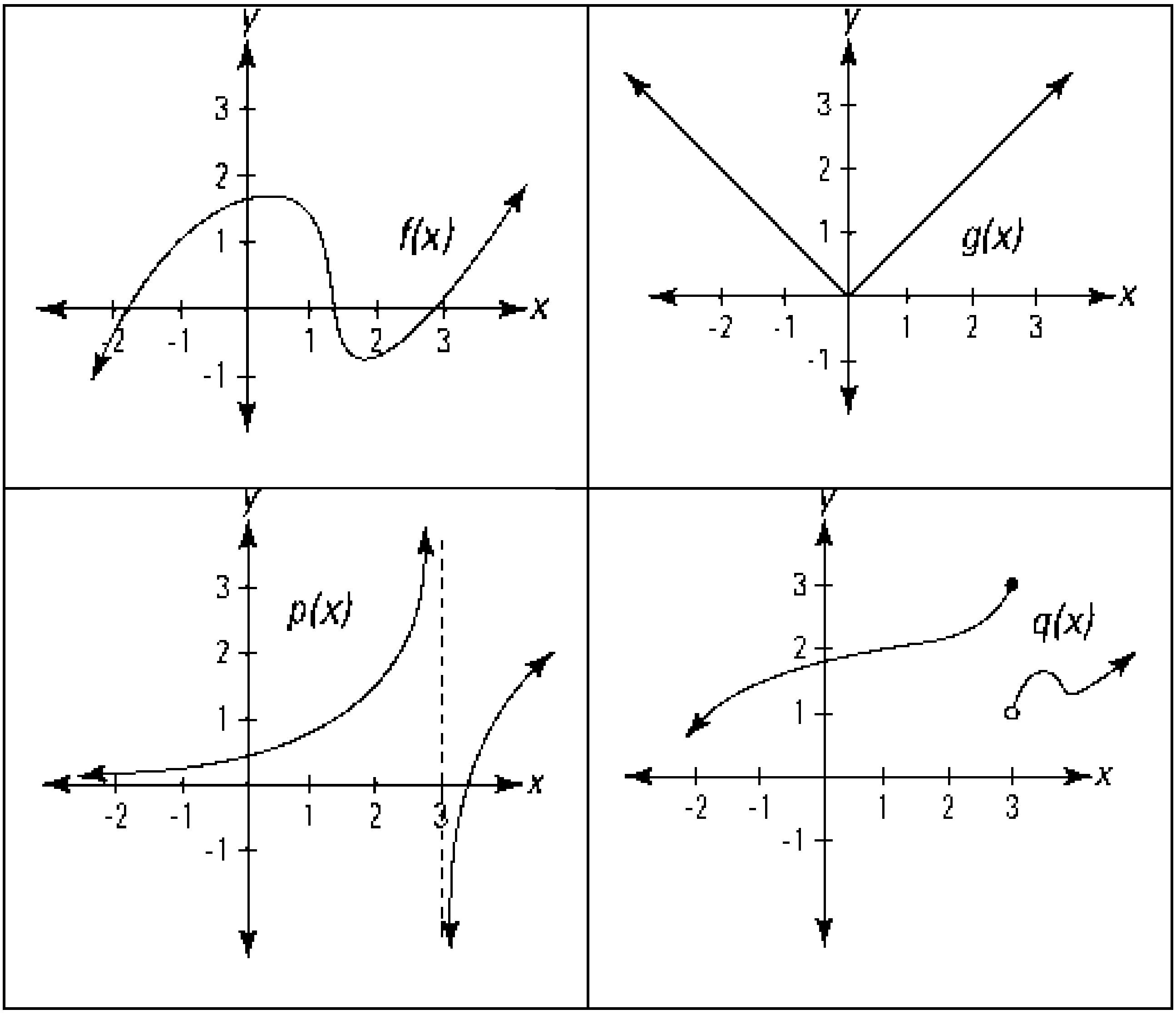


Fig. 4.4. **The graphs of the functions *f*, *g*, *p*, and *q***

Task 24. Read the text and underline the sentences that can help to describe the real-valued functions. Then read the graphs of the functions.

Real-valued Functions

A real-valued function f is one whose codomain is the set of real numbers or a subset thereof. If, in addition, the domain is also a subset of the reals, f is a real-valued function of a real variable. The study of such functions is called real analysis.

Real-valued functions enjoy so-called pointwise operations. That is, given two functions

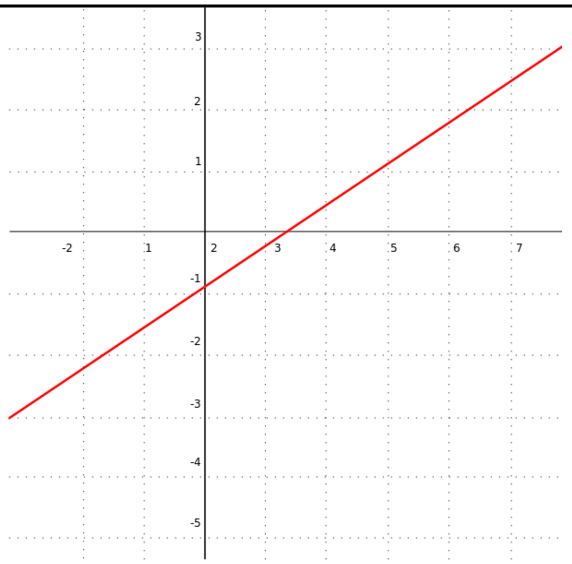
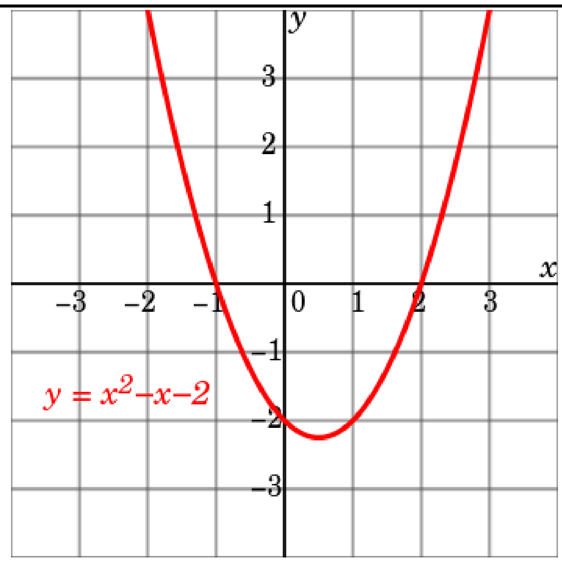
$$f, g: X \to Y,$$

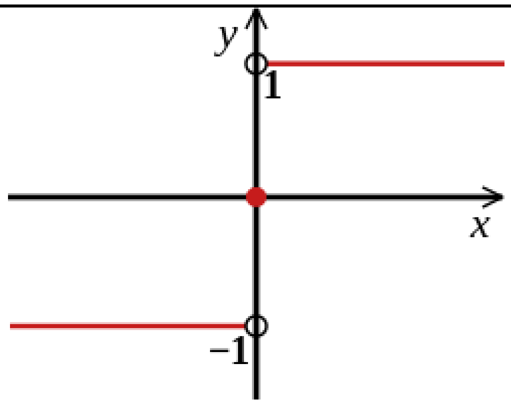
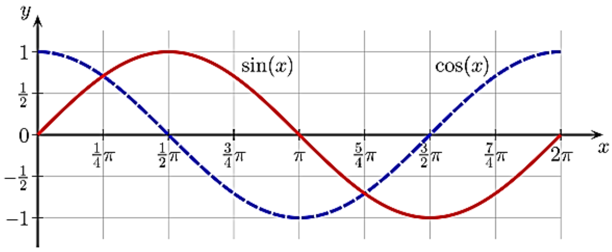
where Y is a subset of the reals (and X is an arbitrary set), their (pointwise) sum $f + g$ and product $f \times g$ are functions with the same domain and codomain. They are defined by the formulas:

$$(f + g)(x) = f(x) + g(x),$$
$$(f + g)(x) = f(x) \times g(x).$$

In a similar vein, complex analysis studies functions whose domain and codomain are both the set of complex numbers. In most situations, the domain and codomain are understood from context, and only the relationship between the input and output is given, but if $f(x) = \sqrt{x}$, then in real variables the domain is limited to non-negative numbers.

The following table contains a few particularly important types of real-valued functions: $f, g: X \to Y$.

Linear function	Quadratic function
	
$f(x) = ax + b$	$f(x) = ax^2 + bx + c$

<table>
<tr><th>Discontinuous function</th><th>Trigonometric functions</th></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td>Roughly speaking, a continuous function is one whose graph can be drawn without lifting the pen</td><td>f(x) = sin(x), solid
f(x) = cos(x), dotted</td></tr>
</table>

(Adapted from [9])

Focus on Reading

Task 25. These are the key terms to the text under study. Read them carefully and find the best explanations. Use a dictionary for help [6].

1) coefficient	a) of, relating to, using, or containing logarithms of a number or variable
2) exponent	b) a change in position or direction of the reference axes in a coordinate system without an alteration in their relative angle
3) except	c) a numerical or constant factor in an algebraic term
4) logarithmic	d) other than; apart from; with the exception of
5) yield	e) an arithmetical operation, defined initially in terms of repeated addition, usually written $a \times b$, $a \cdot b$, or ab, by which the product of two quantities is calculated: to multiply a by positive integral b is to add a to itself b times
6) application	f) a number or variable placed as a superscript to the right of another number or quantity indicating the number of times the number or quantity is to be multiplied by itself
7) multiplication	g) the result, product
8) transformation	h) the process of determining the value of a function for a given argument

Task 26. Read the text and answer the questions.

1. When was the discovery of logarithms made?
2. Why was the discovery of logarithms an important step?
3. What operations can be replaced by logarithms?
4. What logarithmic system is known as the common or Griggs system?
5. What fact does the common logarithmic system make use of?

Logarithms

An important step toward the lessening of the labour of computations was made in the seventeenth century by the discovery of logarithms. Logarithms permit us to replace long process of multiplication with simple addition; the operation of division with that of subtraction; the task of raising to any power with an easy multiplication; and extraction of any root is reduced to a single division.

The logarithm of a given number to a given base is the exponent of the power to which this base must be raised in order to obtain the given number.

Logarithms are exponents.

If $a^x = b$, the exponent is said to be the logarithm of b to the base a, which we write $x = \log_a b$.

The logarithm of a number to a given base is the exponent to which the base must be raised to yield the number.

Any positive number different from unity can be used as the base of a system of real logarithms.

Examples: If $10^3 = 1000$, then $3 = \log_{10}1000$.
If $2^3 = 8$, then $3 = \log_2 8$.
If $5^2 = 25$, then $2 = \log_5 25$.

The logarithmic system using 10 as a base is known as the common or Griggs system and makes use of the fact that every positive number can be expressed as a power of 10. Since our number system uses 10 for a base, it is desirable for us to use 10 for the base of logarithms.

The following table shows the relationships between the exponential and logarithmic forms:

$10^3 = 1000$	$\log_{10}1000 = 3.0000$
$10^2 = 100$	$\log_{10}100 = 2.0000$
$10^1 = 10$	$\log_{10}10 = 1.0000$
$10^0 = 1$	$\log_{10}1 = 0.0000$
$10^{-1} = 0.1$	$\log_{10}0.1 = -1.0000$
$10^{-2} = 0.01$	$\log_{10}0.01 = -2.0000$

From this table it is clear that any number between 100 and 1 000 is a power of 10 for which the exponent is greater than 2 but less than 3, and consequently, its logarithm is between 2 and 3 (2 + *a* decimal). Similarly, the logarithm of 30 is (1 + *a* decimal).

Later, when we use the table, we shall find log 30 = 1.47712 which also means $10^{1.47712} = 30$. The positive decimal part of a logarithm is called the mantissa and the integral part is called the characteristic.

Example. If $\log_{10}300 = 2.47712$, 2 is the characteristic and 47712 is the mantissa.

(Adapted from [1])

Task 27. Translate the text into your own language.

In giving the logarithm of a number, the base must always be specified unless it is understood from the beginning that in any discussion a certain number is to be used as base for all logarithms. Any real number except 1 may be used as base, but we shall see later that in applications of logarithms only two bases are in common use.

Suppose the logarithm of a number in one system is known and it is desired to find the logarithm of the same number in some other system. This means that the logarithm of the number is taken with respect to two bases. It is sometimes important to be able to calculate one logarithms when the other is known.

Focus on Vocabulary

Task 28. Choose words from the list to fill in the gaps.

Multiplication, operation, logarithms, base, exponent, decimal, system, labour.

1. The … of computations.
2. The discovery of … .
3. The … of division.
4. An easy… .
5. A … of real logarithms.
6. The … of logarithms.
7. The … is greater than 2 but less than 3.
8. The positive … part of a logarithm.

Focus on Grammar

Task 29. Complete the sentences with the appropriate form of the verbs in brackets.

Unending Progressions

If a sequence … (has/had) a definite number of terms, it is said to be finite, a word meaning "limited". But a sequence may have an unending number of terms. For example, the integers of the number system go on forever: if you … (count/counted) to one billion you could just as well count to one billion and one. You can easily imagine 10^{21}. It is just as possible to imagine the number $10^{21} + 1$. Such a sequence … (is said/was said) to be infinite; it is unlimited.

It is meaningless to ask for the sum of integers in the number system. It … (will/would) be equally meaningless to ask for the sum of the numbers in any other infinite arithmetic progression (A.P.) if it was an increasing A.P. or decreasing A.P. A G.P. may also be infinite.

Consider the infinite G.P. 1, 1/2, 1/4, … . You could continue writing the terms of this G.P. indefinitely. If you … (did/do) so, the number of terms would increase without limit. This fact is shown in symbol form thus: $n \to \infty$, the arrow meaning approaches, and ∞ being the symbol for infinity. The statement is read "*n* approaches infinity" or "tends to infinity". As $n \to \infty$, what happens to the sum of the G.P.? To answer this question, notice first that the number of terms increases, the value of the *n*-th term b_n … (get/gets) smaller and smaller. It approaches zero; that is, $b_n \to 0$.

If you … (add/added) successive terms, you will add a smaller quantity each time.

If you went on like this, you … (will/would) realize that with each addition, the value of S_n got closer to 2. But no matter how many terms you … (added/add), S_n would never reach 2. If it approaches 2, that is, 2 … (was/is) its limit. As $n \to \infty$ in the G.P. 1, 1/2, 1/4, … $S_n \to 2$. The limit of S_n is an infinite G.P. which is called the sum of the G.P. and is written S_n. Thus for the infinite G.P. 1, 1/2, 1/4, … $S_n \to 2$.

Focus on Reading

Task 30. These are the key terms to the text under study. Read them carefully and find the best explanations. Use a dictionary for help [6].

1) integer	a) (in combination) restricted; confined
2) domain	b) limited or confined
3) restricted	c) usual; in accordance with custom
4) customary	d) to undergo or produce or cause to undergo or produce oscillation
5) isolate	e) the vertical or *y*-coordinate of a point in a two-dimensional system of Cartesian coordinates
6) bound	f) any rational number that can be expressed as the sum or difference of a finite number of units, being a member of the set ...–3, –2, –1, 0, 1, 2, 3
7) ordinate	g) to engage in conversation (with)
8) oscillates	h) to prevent interaction between (circuits, components, etc.); insulate
9) converse	i) the set of values of the independent variable of a function for which the functional value exists

Task 31. Read the text and answer the questions.

1. What functions are frequently called sequences?
2. May the limitation of the domain be implicit in the formula which defines the function?
3. On what condition would $f(x) = x^2$ also be called a function?
4. Which letter is it customary to use to represent a general value of the argument of a sequence?
5. What are the numbers $f(1)$, $f(2)$ called?
6. What symbol is the n-th term of a sequence commonly denoted by?
7. On what condition does $f(n)$ tend to infinity?
8. On what condition does $f(n)$ tend to 1?

Sequences

1. Functions whose domain is the set of positive integers play an important part in analysis. Such functions are frequently called sequences. The limitation of the domain may be implicit in the formula which defines the function, as happens with a function such as $(x – 1)$; but $f(x) = x^2$ would also be called a sequence if the arguments were specifically restricted to positive integral values.

It is customary to use the letter n, rather than x, to represent a general value of the argument of a sequence. The numbers $f(1)$, $f(2)$ etc. are called the terms of the sequence. It is also a common practice to denote the *n-th* term of a sequence by the symbol u_n, rather than *f(n),* especially in connection with infinite series; this notation will be adopted whenever it is found to be more convenient.

2. The graph of a sequence is a set of isolated points.

Definition: If, for any number A it is possible to find a number q (depending on A) such that, for all integers $n > q$, $f(n) > A$, then $f(n)$ tends to infinity.

The definition of $f(n) \to -\infty$ is similar.

Definition: If there is a number l with the property that, for any positive number ε, it is possible to find a number q (depending on ε) such that, for all integers $n > q$,

$|f(n) - l| < \varepsilon$, then $f(n)$ tends to l.

3. We suggest that the bounds of a sequence should be defined, as for other functions, as the bounds of its ordinate set. If, however, a sequence is bounded for all integers $n > k$, it is bounded for all values of its argument, since the integers $n \le k$ only add a finite set of numbers to the ordinate set.

Definition: If $f(n)$ does not tend to a limit or to $\pm\infty$, and if $f(n)$ is bounded, then $f(n)$ oscillates finitely. If it is unbounded, then $f(n)$ oscillates infinitely.

4. Care must be exercised with sequences which are defined by means of a formula, when this formula also has a meaning for nonintegral values of the argument. Consider an example. Regarded as a function of the real variable x, $f(x)$ oscillates finitely as $x \to \infty$. On the other hand, the sequence always has the value 1, so that

$$\mathit{lim} \cos(2\pi n) = 1.$$

Thus it is not true that, if $\lim f(n) = l$, then $\lim f(x) = l$. The converse result is, however, true; for if $f(x)$ lies between $l - \varepsilon$ and $l + \varepsilon$ for all real numbers greater than q, it certainly does so for all integers greater than q.

5. It is occasionally convenient to extend the definition of sequences in one of the two ways:

(a) to allow the argument also to take the value 0. The sequence then has $f(n - 1)$ as its n-th term. The most important use of this is with power series;

(b) to include functions which are undefined for a finite number of values of n. If k is the largest of these, then all the limit definitions will still be applicable provided that we take $q > k$.

(Adapted from [4])

Task 32. Read the text again and answer the questions.

1. What happens if a sequence is bounded for all integers $n > k$?
2. What do the integers $n \leq k$ add to the ordinate set?
3. On what condition does $f(x)$ oscillate finitely?
4. On what condition does $f(n)$ oscillate infinitely?
5. What sequences must care be exercised with?
6. When must care be exercised with sequences which are defined by means of a formula?
7. On what condition does $f(x)$ lie between $1 - \varepsilon$ and $1 + \varepsilon$ for all integers greater than q?

Task 33. Define the following notions: sequence; infinite series; graph of a sequence; bound of a sequence; limit of a sequence.

Focus on Vocabulary

Task 34. Choose words from the list to fill in the gaps.

Domain, positive, represent, values, integral, nonintegral, greater, sequence, argument, numbers, applicable, bounds.

1. The set of … integers.
2. The limitation of the … .
3. Positive … values.
4. To … a general value.
5. The … of a sequence.
6. The terms of the … .
7. The … of a sequence.
8. Bounded for all … of its argument.
9. A finite set of … .
10. … values of the argument.
11. All real numbers … than q.
12. … provided.

Task 35. Complete the table.

Verb	Noun	Adjective
limit		
define		
restrict		

represent		
extend		
adopt		

Focus on Speaking

Task 36. Make up sentences of your own according to each model; the forms given in bold type in each model are to be used as constants.

1. We **propose** that this number **should be** positive.

We insist
We suggest that this number should be positive.
We demand

2. The function can take values as large as we please **provided we take *x* large enough.**

3. $(1 + x)^n > nx$, **since** every term in the sum is positive.

4. It is convenient **to extend** the definition of a function.

Focus on Reading

Task 37. These are the key terms to the text under study. Read them carefully and find the best explanations. Use a dictionary for help [6].

1) landmark	a) a narrow flat bar of iron or steel
2) curve	b) the process, instance, or state of being divided again following upon an earlier division
3) embrace	c) the graph of a function with one independent variable
4) strip	d) an important or unique decision, event, fact, discovery, etc.
5) approximate	e) to comprise or include as an integral part
6) subdivision	f) to find an expression for (some quantity) accurate to a specified degree

Task 38. Read the text and answer the questions.

1. What did the study of integration first arise from?
2. What was it originally developed quite independently of?
3. Why was the discovery that these two processes were closely related an important landmark in the history of analysis?
4. Why would it be wrong to assume that a region bounded by curved lines has a number associated with it, which we may call its area?
5. Are there functions with whose graphs we should not associate an area?

Integration

The study of integration first arose from the need to calculate areas and volumes. It was originally developed quite independently of differentiation, and the discovery that these two processes were closely related was an important landmark in the history of analysis.

We must not allow geometrical intuition to cloud the analytical problem.

In particular, it would be wrong to assume that a region bounded by curved lines has a number associated with it which we may call its "area". On the other hand, once the definition of the integral has been developed, it will be found, to embrace functions – for example, certain discontinuous functions – with whose graphs we should not expect to associate an area.

The geometrical illustration which we shall use throughout this chapter is the "area under a curve"; that is, the area bounded by two vertical lines and those parts of the x-axis and of the curve which lie between these lines.

We shall adopt the definition associated with the name of Riemann. The basic idea is that the region is divided into a number of vertical strips, each of which is supposed to have: an area approximately equal to that of a certain rectangle. The integral is then defined as a bound of the set of approximate values so obtained (an integral can also be defined as a limit; the definition as a bound is more convenient, as it is easier to prove that a function is bounded than that it has a limit).

In recent years a number of alternative definitions of an integral have been given which embrace an even wider class of functions than does Riemann's definition.

Definition. If there is a closed interval, any finite set of numbers such as

$$a < x_1 < x_2 < ... < x_{n-1} < b$$

is called a subdivision of $[a, b]$.

This subdivision will be denoted by the symbol

$$\{a, x_1, x_2, ..., x_{n-1}, b\}.$$

Subdivisions will frequently be referred to by single Greek letters α, β, γ, etc.

Associated with the subdivision $\alpha = \{a, x_1, x_2, ..., x_{n-1}, b\}$ are the n intervals $[a, x_1]$, $[x_1, x_2]$, ..., $[x_{n-1}, b]$. Each of these is called a subinterval of α.

It is sometimes convenient to write $a = x_0$ and $b = x_n$.

(Adapted from [4])

Task 39. Read the text again and answer the questions.
1. What definition shall we adopt in this lesson?
2. What is the basic idea of this definition?
3. What is the integral then defined as?
4. What can the integral also be defined as?
5. Why is the definition as a bound more convenient?
6. What other definition of a function does your friend know?
7. What is a subdivision called?
8. What is a subinterval of *a* called?
9. Do you often need in practice the definition of an integral?
10. Why would it sometimes be convenient to write $a = x_0$ and $b = x_n$?

Task 40. Define the following notions: integral, subdivision, subinterval.

Task 41. Translate the sentences into your own language.
1. We divide the region into a number of vertical strips.
2. We show that the two processes are related.
3. We assume this fact.
4. We develop the definition of an integral.
5. They use geometrical illustration throughout this section.
6. We deal only with bounded functions and finite intervals.
7. We consider now a set of *n* rectangles.
8. We simplify the situation in the diagram.
9. This number corresponds to the sup of the areas of the rectangles.
10. The notation is not complicated.

Focus on Vocabulary

Task 42. Choose words from the list to fill in the gaps.

Subdivision, geometrical, differentiation, area, embrace, vertical, approximately, strips, set, curve, study.

1. The … of integration.
2. Independently of … .
3. … intuition.
4. … functions.
5. To associate an … .

6. Area under a … .
7. Two … lines.
8. A number of vertical … .
9. An area … equal.
10. The … of approximate values.
11. Associated with the … .

Task 43. Complete the table.

Verb	Noun	Adjective
calculate		
develop		
associate		
divide		
prove		
consider		

Focus on Speaking

Task 44. Use the following word combinations in sentences of your own.

Geometrical intuition, to embrace functions, approximately, equal to, a convenient definition.

Task 45. Prove the lemma.

If β is a subdivision of $[a, b]$ formed by adding one more number in $[a, b]$ to the subdivision α, then $S(\beta) \geq S(\alpha)$.

Useful phrases

First of all you ...
My view is …, because …
The fact is …
On the one hand …, on the other hand …
I think that / In my opinion …
The first thing you have to do is …
The next thing you do is ...

Focus on Reading

Task 46. These are the key terms to the text under study. Read them carefully and find the best explanations. Use a dictionary for help [6].

1) differential	a) the change of a function, $f(x)$, with respect to an infinitesimally small change in the independent variable, x; the limit of $[f(a + \Delta x) - f(a)/\Delta x$, at $x = a$, as the increment, Δx, tends to 0. Symbols: $df(x)/dx$, $f'(x)$, $Df(x)$
2) equation	b) to be a multiple of, leaving no remainder
3) derivative	c) containing, or involving one or more derivatives or differentials
4) contain	d) a mathematical statement that two expressions are equal: it is either an identity in which the variables can assume any value, or a conditional equation in which the variables have only certain values (roots)
5) contribution	e) occurring, existing, or operating at the same time; concurrent
6) linear	f) something that you give or do in order to help something be successful
7) combination	g) the unique set of values that yield a true statement when substituted for the variables in an equation solution
8) simultaneous	h) not representing any specific value
9) solution	i) a group formed in this way. The number of combinations of n objects taken r at a time is $n!/[(n - r)!r!$. Symbol C_r^n
10) arbitrary	j) of or relating to the first degree

Task 47. Read the text and answer the questions.

1. What do we call an ordinary differential equation?
2. What is the order of the equation?
3. What is the degree of the equation?
4. What must the equation be squared for?
5. Where is the definition of order also adopted?
6. Which equations shall we refer to as linear?
7. Which equations will be called nonlinear?
8. What is the reason for this?
9. What shall we replace derivatives by in many methods of solution?
10. What nonlinear terms will the resulting algebraic equation contain?

Ordinary Differential Equations

An ordinary differential equation is an equation involving one or more derivatives of the dependent variable y with respect to a single independent variable x.

The order of the equation is that of the highest derivative contained in it, so that the general differential equation of order n can be written in the form

$$F(y^{(n)}, y^{(n-1)} \dots y', y, x) = 0 \tag{1}$$

the symbol y^r denoting $d^r y/dx^r$. This is often replaced by y^{I}, y^{II}, y^{III}, y^{IV} etc., with unbracketed roman superscripts, when a specific order is intended. The degree of the equation is defined mathematically to be that of its highest order derivative, when the equation has been made rational as far as the derivatives are concerned. The equation

$$y'' = [xy^{5/2} + y']^{1/2} \tag{2}$$

for example, is of order 2 and degree 2, since the equation must be squared to rationalize the contributions from the derivatives.

The definition of order is adopted in numerical work also, but the definition of degree is not very relevant. We shall refer to an equation of the form

$$y^{(n)} + f_{n-1}(x)\, y^{(n-1)} + f_{n-2}(x) y^{(n-2)} + \dots + f_1(x)y' + f_0(x)y = g(x) \tag{3}$$

in which the $f_i(x)$ are functions of x only, as linear, and any other type of equation will be called nonlinear. The reason for this is fairly clear: in many methods of solution we shall replace derivatives by linear combinations of several discrete values $y(x_i)$ and, if f_i in (3) also contains y or any derivative, the resulting algebraic equation will contain nonlinear terms y^s, $s \neq 1$.

Simultaneous ordinary differential equations involve several dependent variables y, z, … and their derivatives. The equations do not necessarily have the same order or degree, but there are usually the same number of equations as the number of unknown functions. General simultaneous equations in two variables will have the form

$$\left.\begin{aligned} F_1(y^{(p)}, y^{(p-1)} \dots y', y; z^{(q)}, z^{(q-1)}, \dots, z', z) = 0 \\ F_2(y^{(r)}, y^{(r-1)} \dots y', y; z^{(s)}, z^{(s-1)}, \dots, z', z) = 0 \end{aligned}\right\} \tag{4}$$

It can be proved that the most general solution of an ordinary differential equation of order n contains k arbitrary constants. This general solution is called the Complete Primitive, and a Particular Integral is obtained by giving specific values to these arbitrary constants.

Nonlinear equations may also have singular solutions, not obtainable from the complete primitive.

For the linear differential equation (3) the particular integral, as before, is any solution $y_0(x)$ which contains no arbitrary constants. The Complementary Function is of the form

$$y(x) = \sum_{1}^{n} A_r y_r(x), \tag{5}$$

in which the functions $y_r(x)$ are independent solutions of the homogeneous equation obtained from (3) by replacing $g(x)$ by zero, and the arbitrary constants A_r appear linearly. The general solution of (3) is then given by

$$y(x) = y_0(x) + \sum_{1}^{n} A_r y_r(x). \tag{6}$$

(Adapted from [4])

Task 48. Read the text again and answer the questions.

1. How many dependent variables y, z, ... and their derivatives do simultaneous ordinary differential equations involve?
2. Do these equations necessarily have the same order or degree?
3. How many of equations are usually there?
4. What form do general simultaneous equations have?
5. What is the general solution called?
6. In what way is a particular integral obtained by?
7. May nonlinear equations also have singular solutions?
8. What is the particular integral for the linear differential equation (3)?
9. What form does the complementary function take?
10. What expression is the general solution of (3) given by?

Task 49. Define the following notions: ordinary differential equations; order of the equation; degree of the equation; linear equation; nonlinear equation; simultaneous ordinary differential equations; complete primitive.

Task 50. Translate the sentences into your own language.

1. We need tables of e^x and $\tan^{-1}x$ for effective tabulation of y.
2. A numerical solution contains no arbitrary constants by definition.
3. The prescribed conditions are involved from the start in a numerical method in direct contrast to analytical methods.

4. The particular integral for the linear differential equation (3) is any solution $y_0(x)$ which contains no arbitrary constants.

5. Much computation may be needed to produce numerical results from an analytical solution.

6. We concern the solution to such problems in this section.

7. +2 and –3 are two numbers; the former is positive, the latter is negative.

8. $3 \times 2 = 6$ and $a \times b = ab$ are two expressions; the former is an arithmetical one and the latter is an algebraical one.

9. These two texts deal with differential equations. The former is concerned with ordinary differential equations, the latter with analytical and numerical solutions.

Focus on Vocabulary

Task 51. Choose words from the list to fill in the gaps.

Single, involving, differential, contributions, obtainable, form, unknown, values, highest, solutions, algebraic, simultaneous.

1. An ordinary … equation.
2. An equation … one or more derivatives.
3. A … independent variable *x*.
4. The … derivative contained in it.
5. Rationalize the … from the derivatives.
6. An equation of the … .
7. The resulting … equation.
8. The number of … functions.
9. General … equations.
10. Giving specific … .
11. Have singular … .
12. Not … from the complete primitive.

Task 52. Complete the text with the following words and phrases.

Elementary, analytical, mathematical, numerical, calculated, computation, restriction, solution, integral, derivatives, prescribed, discrete, differential, requirement.

Analytical and Numerical Solutions

With **1** … methods we try to obtain a solution in explicit form (1), or implicit form (2), containing the required number of arbitrary constants. The functions f_1 and f_2 may include only the **2** … functions sin *x*, e^x, etc., or involve higher **3** … functions such as those of Airy and Bessel. In practical work, however, the most important usual **4** … is that *y* should be expressible quantitatively in terms of *x*, by means of a graph if no great precision is needed, or otherwise by a table of values of *y* **5** … at various points in the range of *x.* In a useful analytical solution, therefore, the functions contained in it must be well-known and adequately tabulated.

Even in this case, however, much **6** … may be needed to produce numerical results from an analytical solution. For example, the simple **7** … equation

$$y' - \frac{2y}{1-x^4} = 0 \tag{1}$$

has the solution

$$y = A\left(\frac{1+x}{1-x}\right)^{1/2} e^{tan^{-1}x}. \tag{2}$$

For effective tabulation of *y* we need tables of $x^{1/2}$, e^x, and tan^{-1}x, and we shall generally have to interpolate, at least in the exponential table.

A further **8** … on analytical methods is that they can only rarely produce closed solutions, of the required type, to differential equations which arise in practical problems.

For example, the equation

$$y' - \frac{2y}{1-x^4} = x. \tag{3}$$

A seemingly trivial amendment of (3) has the solution

$$y = \left(\frac{1+x}{1-x}\right)^{1/2} e^{tan^{-1}x}\left\{ jx\left(\frac{1+x}{1-x}\right)^{1/2} e^{tan^{-1}x} dx + A\right\} \tag{4}$$

and it is unlikely that the indefinite **9** … can be expressed in closed form in terms of known functions.

When a convenient analytical **10** … can be obtained it has of course advantages over **11** … methods. In particular, we have available the complete primitive containing arbitrary constants, and any particular integral is obtained easily by giving specific values to these constants. The latter are usually determined from a prescribed knowledge of some values of *y* or its **12** … , or both, at one or more points in the range of *x*. If several problems are presented with the same differential equation but different **13** … conditions, the complete primitive is the same in all cases, only the arbitrary constants needing recalculation. This advantage is not in general shared by numerical methods.

A numerical solution is a graph or table of *y* against *x*. A numerical method, from our point of view, is one which, produces **14** … values of *y* without the assistance of an analytical solution. By definition, a numerical solution contains no arbitrary constants, so that we always obtain particular integrals rather than complete primitives. Indeed in the direct contrast to analytical methods, a change in these conditions usually presents a completely new problem, even for the same differential equation. Singular solutions, also, are seldom required.

(Adapted from [4])

Focus on Speaking

Task 53. Work in pairs and discuss the following questions.

1. What solutions do we try to obtain with analytical methods?
2. What is the most important usual requirement in practical work?
3. What are the restrictions on analytical methods?
4. What advantages does an analytical solution have over numerical methods?
5. What is a numerical solution?
6. What does a numerical method produce?

Task 54. Use the following word combinations in sentences of your own.

A homogeneous equation; to adopt a definition; as far as this is concerned; an arbitrary constant; an (arbitrary) singular solution; a discrete value.

Useful phrases

First of all you ...

The fact is …

On the one hand …, on the other hand …

I think that / In my opinion …

Unit 5. Algebra

Task 1. Answer the questions.

1. What is algebra for you? Give five associations with this word.
2. What do you know about the concept of modern algebra?

Task 2. Read the text and find information to answer the questions (Task 1).

Our algebra of real numbers developed through the centuries from considerations of problems in arithmetic. The study of the algebra of real numbers and the recent recognition of the fundamental importance of the basic principles have led to the development of what is now called modern algebra or abstract algebra.

One of the earliest pioneers in this direction was the French genius Evariste Galois (1811 – 1832). Although he lived a tragic life and died in a foolish duel at the age of 20, his work led to the development of the modern theory of groups and fields.

The concepts of modern algebra have been found to be extremely useful in other branches of mathematics, as well as in the physical and social sciences. A chemist may use modern algebra in a study of the structure of crystals; a physicist may use modern algebra in designing an electronic computer; a mathematician may use modern algebra in a study of logic.

In the algebra of real numbers we study the properties of addition and multiplication of real numbers, which follow as a consequence of certain basic principles. In modern algebra we may work with any set of objects. We need not work just with real numbers. We consider certain operations on these objects. These operations need not be addition and multiplication. We agree that certain basic principles are satisfied by these operations. These basic principles need not be the same as our basic principles for addition and multiplication of real numbers. Then we derive various properties which follow as consequences of the assumed basic principles.

([Adapted from [3])

Focus on Reading

Task 3. These are the key terms to the text under study. Read them carefully and find the best explanations. Use a dictionary for help [6].

1) undergo	a) to calculate the difference between (two numbers or quantities) by subtraction
2) quintic	b) a generalized complex number consisting of four components
3) quaternion	c) spread throughout
4) dimension	d) to experience
5) subtract	e) showing or feeling great enthusiasm
6) ecstatic	f) relating to the fifth degree
7) obsession	g) a state of mind occupied by some ideas in an inordinate degree
8) pervade	h) the number of coordinates required to locate a point in space

Task 4. Read the text below (Task 5) and choose the best heading to each part of it.

A. Origins.

B. The Abstract.

C. The Modern World.

D. Algebra.

Task 5. Read the text again. Are these statements true or false?

1. Algebra gives us a distinctive way of solving problems, a deductive method with a twist.
2. We couldn't find this by a trial and error method but algebra is more economical.
3. Mathematics underwent a big change when it passed from the science of arithmetic to the science of symbols or algebra.
4. Algebra wasn't a significant element in the work of Islamic scholars in the ninth century.
5. A significant event in modern algebra occurred in 1826 when the Irishman William Rowan Hamilton discovered the quaternions.
6. In the 20th century the dominant paradigm of algebra was the axiomatic method.

1. ...

Algebra gives us a distinctive way of solving problems, a deductive method with a twist. That twist is "backwards thinking". For a moment consider the problem of taking the number 25, adding 17 to it, and getting 42. This is forwards thinking. We are given the numbers and we just add them together. But instead suppose we were given the answer 42, and asked a different question? We now want the number which when added to 25 gives us 42. This is where backwards thinking comes in. We want the value of x which solves the equation $25 + x = 42$ and we subtract 25 from 42 to give it to us.

Word problems which are meant to be solved by algebra have been given to schoolchildren for centuries:

My niece Michelle is 6 years of age, and I am 40.

When will I be three times as old as her?

We could find this by a trial and error method but algebra is more economical. In x years from now Michelle will be $6 + x$ years and I will be $40 + x$. I will be three times older than her when $3 \times (6 + x) = 40 + x$.

Multiply out the left-hand side of the equation and you'll get $18 + 3x =$ $= 40 + x$, and by moving all the xs over to one side of the equation and the numbers to the other, we find that $2x = 22$ which means that $x = 11$. When I am 51 Michelle will be 17 years old.

What if we wanted to know when I will be *twice* as old as her? We can use the same approach, this time solving $2 \times (6 + x) = 40 + x$ to get $x = 28$. She will be 34 when I am 68. All the equations above are of the simplest type – they are called "linear" equations. They have no terms like x^2 or x^3, which make equations more difficult to solve. Equations with terms like x^2 are called "quadratic" and those with terms like x^3 are called "cubic" equations. In past times, x^2 was represented as a square and because a square has four sides the term quadratic was used; x^3 was represented by a cube.

Mathematics underwent a big change when it passed from the science of arithmetic to the science of symbols or algebra. To progress from numbers to letters is a mental jump but the effort is worthwhile.

2. ...

Algebra was a significant element in the work of Islamic scholars in the ninth century. Al-Khwarizmi wrote a mathematical textbook which contained the Arabic word *al-jabr*. Dealing with practical problems in terms of linear and quadratic equations, al-Khwarizmi's "science of equations" gave us the word "algebra". Still later Omar Khayyam is famed for writing the *Rubaiyat* and the immortal lines (in translation): *A Jug of Wine, a Loaf of Bread – and Thou Beside me singing in the Wilderness.* But in 1070, aged 22, he wrote a book on algebra in which he investigated the solution of cubic equations.

Girolamo Cardano's great work on mathematics, published in 1545, was a watershed in the theory of equations for it contained a wealth of results on the cubic equation and the quartic equation – those involving a term of the kind x^4. This flurry of research showed that the quadratic, cubic and quartic equations could all be solved by formulae involving only the operations +, –, ×, ÷ (the last operation means the q-th root). For example, the quadratic equation $ax^2 + bx + c = 0$ can be solved using the formula: $\frac{-b \pm \sqrt{a^2} - 4ac}{2a}$.

If you want to solve the equation $x^2 - 3x + 2 = 0$ all you do is feed the values $a = 1$, $b = -3$ and $c = 2$ into the formula.

The formulae for solving the cubic and quartic equations are long and unwieldy but they certainly exist. What puzzled mathematicians was that they could not produce a formula which was generally applicable to equations involving x^5, the "quintic" equations. What was so special about the power of five?

In 1826, the short-lived Niels Abel came up with a remarkable answer to this quintic equation conundrum. He actually proved a negative concept, nearly always a more difficult task than proving that something can be done. Abel proved there could not be a formula for solving all quintic equations, and concluded that any further search for this particular holy grail would be futile. Abel convinced the top rung of mathematicians, but news took a long time to filter through to the wider mathematical world. Some mathematicians refused to accept the result, and well into the 19th century people were still publishing work which claimed to have found the non-existent formula.

3. ...

For 500 years algebra meant "the theory of equations" but developments took a new turn in the 19th century. People realized that symbols in algebra could represent more than just numbers – they could represent "propositions" and so algebra could be related to the study of logic. They could even represent higher-dimensional objects such as those found in matrix algebra.

And, as many non-mathematicians have long suspected, they could even represent nothing at all and just be symbols moved about according to certain (formal) rules.

A significant event in modern algebra occurred in 1843 when the Irishman William Rowan Hamilton discovered the quaternions. Hamilton was seeking a system of symbols that would extend two-dimensional complex numbers to higher dimensions. For many years he tried three-dimensional symbols, but no satisfactory system resulted. When he came down for breakfast each morning his sons would ask him, "Well, Papa, can you *multiply* triplets?" and he was bound to answer that he could only add and subtract them.

Success came rather unexpectedly. The three-dimensional quest was a dead end – he should have gone for four-dimensional symbols. This flash of inspiration came to him as he walked with his wife along the Royal Canal to Dublin. He was ecstatic about the sensation of discovery. Without hesitation, the 38-year-old vandal, Astronomer Royal of Ireland and Knight of the Realm, carved the defining relations into the stone on Brougham Bridge – a spot that is acknowledged today by a plaque. With the date scored into his mind, the subject became Hamilton's obsession. He lectured on it year after year and published two heavy weight books on his "westward floating, mystic dream of four".

One peculiarity of quarterions is that when they are multiplied together, the order in which this is done is vitally important, contrary to the rules of ordinary arithmetic. In 1844 the German linguist and mathematician Hermann Grassmann published another algebraic system with rather less drama. Ignored at the time, it has turned out to be far reaching. Today both quaternions and Grassmann's algebra have applications in geometry, physics and computer graphics.

4. ...

In the 20th century the dominant paradigm of algebra was the axiomatic method. This had been used as a basis for geometry by Euclid but it wasn't applied to algebra until comparatively recently.

Emmy Noether was the champion of the abstract method. In this modern algebra, the pervading idea is the study of structure where individual examples are subservient to the general abstract notion. If individual examples have the same structure but perhaps different notation they are called isomorphic.

The most fundamental algebraic structure is a group and this is defined by a list of axioms. There are structures with fewer axioms (such as group-oids, semi-groups and quasi-groups) and structures with more axioms (like rings, skew-fields, integral domains and fields). All these new words were imported into mathematics in the early 20th century as algebra transformed itself into an abstract science known as "modern algebra".

(Adapted from [5])

Task 6. Translate the paragraph "The Modern World" into your own language. Let your friend check your work.

Focus on Vocabulary

Task 7. Choose words from the list to fill in the gaps.

A trial and error, backwards, dimensions, system, equations, applications, method, thinking, two-dimensional.

1. … thinking.
2. Forwards … .
3. … method.
4. Linear … .
5. … complex numbers.
6. Higher … .
7. Algebraic … .
8. … in geometry, physics and computer graphics.
9. Abstract … .

Task 8. Answer the questions.

1. What signs are used in algebra?
2. What do signs (+) and (–) indicate?
3. How is the sign (±) read?
4. What is the equality sign?
5. What is the meaning of the multiplication sign?
6. What is the meaning of the division sign?
7. What does the expression (a > 6) mean?

Task 9. Complete the text with the following words and phrases.

Brackets, minus, double sign, greater than, braces, equals, *a* is greater than 6, dot, division, parentheses, operation, plus, less than, 3 is less than 5, multiplication.

Signs Used in Algebra

In algebra, the signs **1** … (+) and **2** … (–) have their ordinary meaning, indicating addition and subtraction and also serve to distinguish between opposite kinds of numbers, positive (+) and negative (–). In such an **3** … as +10 – 10 = 0, the minus sign means that the minus 10 is combined with the plus 10 to give a zero result or that 10 is subtracted from 10 to give a zero remainder.

The so-called "**4** … " (±), which is read "plus-or-minus", is sometimes used. It means that the number or symbol which it precedes may be "either plus or minus" or "both plus and minus".

As in arithmetic, the equality sign (=) means "**5** … " or "is equal to".

The **6** … sign (×) has the same meaning as in arithmetic. In many cases, however, it is omitted. A **7** … (·) placed between any two numbers a little above the line (to distinguish it from a decimal point) is sometimes used as a sign of multiplication.

The **8** … sign (÷) has the same meaning as in arithmetic. It is frequently replaced by the fraction line; thus means the same as 6 : 3 and in both cases the result or quotient is 2. The two dots above and below the line in the division sign (÷) indicate the position of the numerator and denominator in a fraction, or the dividend and divisor in division.

9 … ( ), **10** … [ ], **11** … { } , and other inclosing signs are used to indicate that everything between the two signs is to be treated as a single quantity and any sign placed before it refers to everything inside as a whole and to every part of the complete expression inside.

Another sign which is sometimes useful is the sign which means "greater than" or "less than". The sign (>) means "**12** … " and the sign (<) means "**13** … ". Thus, *a* > 6 means that "**14** … ", and 3 < 5 means "**15** … ".

The sign "three dots" (…) at the corners of a triangle, means "hence" or "therefore".

(Adapted from [1])

Task 10. Translate into your own language.

ab means the same as $a \times b$ and $2 \times c$ means the same as 2*c*, twice *c*. We cannot write 23, however, for 2×3 as 23 has another meaning, namely, the number twenty three. Therefore, in general, the multiplication sign (×) may be omitted between algebraic symbols or between an algebraic symbol and an ordinary arithmetical number, but not between two arithmetical numbers.

Another sign which is sometimes used is the inclined fraction line (/); thus 6/3 means the same as 6 : 3 or $\frac{6}{3}$. This form has the advantage of being compact and also allowing both dividend and divisor (or numerator and denominator) to be written or printed on the same line.

Task 11. Complete the text with the following words and phrases.

Quotient, formula, unknown, average, value, equation.

The Formula

The selling price of an article is equal to the sum of the cost of the article and the gain on the cost. If we let *S.P.* stand for the selling price, *C* for the cost and *G* for the gain, then a rule for finding the selling price can be written as the **1** … $S.P. = C + G$.

Often we have to write the rule which is being expressed briefly as a formula. State the rule for the **2** … $r = \frac{d}{t}$. If *r* is the average rate, *d* is the distance, and *t* is the time, then the average rate is equal to the **3** … of the distance divided by the time. Or, the **4** … rate is equal to the distance divided by the time.

If the **5** … of every letter but one in a formula is known, the value of the unknown letter can be found. This is known as calculating a formula for the **6** … . To evaluate a formula we substitute numerical values for literal numbers, and solve the problem.

(Adapted from [1])

Task 12. Translate into your own language.

A formula is a general expression for solving certain problems or cases. It is a relation established amongst quantities any one of which may be taken as the unknown if the other quantities are known or can be ascertained. In finding formulas, we are usually given the values of the related numbers to determine how each one relates to or depends upon the other. The related numbers are called variables, for their values vary. When we understand how the numbers vary, we can express in a formula the relationship between the variables.

Task 13. Read the text below (Task 14) and choose the best heading to each part of it.

A. Adding and Subtracting Like Terms.
B. Coefficients.
C. Combining Terms.
D. Factors.

Task 14. Answer the questions.

1. What is the result of multiplication called?
2. What numbers are called factors?
3. What coefficient is called a numerical coefficient?
4. When is the coefficient considered to be 1?
5. By what are the terms separated?
6. What is the purpose of adding or subtracting numbers?
7. What do like terms have?
8. How do we simplify an algebraic expression?

Factors, Coefficients and Combining Terms

1. ...

If two or more numbers (arithmetic or literal) are multiplied, the result of the multiplication is called a product. Each number that has been multiplied to arrive at that product is called a factor of the product. For example, since $2 \times 7 = 14$, 2 and 7 are factors of their product, 14. Similarly, the number 210 can be written as $2 \times 3 \times 5 \times 7$. Two, three, five and seven are called the prime factors of 210. A prime factor is a factor that is not divisible by anything other than itself or unity. Factor 6 can be written in the form $2 \cdot 3$. 30 has 2, 3 and 5 as factors. Consider the number $6ab$, which can be written as $2 \cdot 3 \cdot a \cdot b$. Then $6ab$ has the following factors: 2, $3ab$, a, b, 6 and so on.

When a product is broken down into its factors, it is broken down into numbers which, multiplied together, will equal the product. 2 and 2 are factors of 4. 2 and x are factors of $2x$.

2. ...

Any factor of a product may be called the coefficient of the product of the remaining factors. For example, in the expression 7*xyz*, 7 is the coefficient of the remaining factors *xyz*, or 7*x* is the coefficient of the remaining factors *yz*, etc.

A coefficient which is an arithmetic number is called a numerical coefficient. Thus, 8 is the numerical coefficient in the expression 8*xy*. If a letter is written without a number before it, the coefficient is understood to be 1. For example, *x* means 1*x*, and *ab* means 1*ab*.

3. ...

An algebraic expression consists of one or more terms. If an algebraic expression consists of more than one term, as for example, 3*a*2*bc,* the terms are separated by plus (+) or minus (–) signs.

A term or a monomial consists of numbers connected only by signs of multiplication or division. For example, 2*xy* and *ab* are terms or monomials. Thus, the algebraic expression 3*x* – 2*ab* + 4 has 3 terms: 3*x,* 2*ab* and 4.

4. ...

The purpose of adding or subtracting numbers or objects is to find out how many of the same kind we have.

The sum of 3*ab* and 7*ab* is 10*ab,* because 3*ab*'s and 7*ab*'s more like them would yield 10*ab*'s. However, 2*a* and 3*b* cannot be added because these are unlike terms.

Like terms have the same literal factors. Thus, 3*a* and 5*a* are like terms, and *xy* and 4*xy* are like terms. Unlike terms do not have the same literal factors. 3*d*, 7*x,* 2*y* and 5*xy* are all unlike terms.

An algebraic expression containing two or more terms can be simplified by combining like terms. Since unlike terms cannot be added or subtracted we merely indicate their addition or subtraction by signs. For example, 3*x* + + 6*a* – 2*b*.

(Adapted from [1])

Task 15. Read the text and answer the questions.

1. Into how many groups are algebraic expressions divided?
2. What is a monomial algebraic expression?
3. By what is a monomial represented?

4. What algebraic expression is called polynomial?
5. What are terms of a polynomial?

Monomial and Polynomial

Algebraic expressions are divided into two groups according to the last algebraic operation indicated.

A monomial is an algebraic expression whose last operation in point of order is neither addition nor subtraction.

Consequently, a monomial is either a separate number represented by a letter or by a figure, e.g. $-a$, +10, or an *ab* product, e.g. *ab,* $(a + b)c$, or a quotient, e.g. $\frac{a}{b}$, or a power, e.g. b^2, but it must never be either a sum or a difference.

If a monomial is a quotient, it is called a fractional monomial; all the other monomials are called integral monomials. Thus, $\frac{a-b}{c}$ is a fractional monomial, while $(x - y)ab$; $a(x + y)^2$ are integral monomials.

An algebraic expression which consists of several monomials connected by the + and – signs, is known as a polynomial. Such is, for instance, the expression $ab - a + b - 10 + \frac{a-b}{c}$.

Terms of a polynomial are separate expressions which form the polynomial by the aid of the + and – signs. Usually, the terms of a polynomial are taken with the signs prefixed to them; for instance, we say: term $-a$, term $+b^2$, and so on. When there is no sign before the first term it is *ab* or +*ab*.

A binomial is an algebraic expression of two terms; a trinomial is an expression of three terms and so on.

(Adapted from [1])

Task 16. Complete the table.

Verb	Noun	Adjective
operate	operation	operative
serve		
express		
indicate		
divide		
represent		
connect		

Task 17. Complete the table.

Algebraic, integral, addition, last, while, point, several, separate, sign, easily, fractional, difference, term.

Noun	**Adjective**	**Adverb**

Task 18. Translate into your own language.

An algebraic expression of one term is called a monomial or simple expression. An algebraic expression of more than one term is called a polynomial; a polynomial of two terms is called a binomial.

$3a + 2b$ and $x^2 - y^2$ are binomials, $a + b + c$ is a trinomial.

Task 19. Read the text. Are these statements true or false?

1. The algebraic formula for the area of a square is $A = s^2$.
2. The square of a number is called the third power of the number.
3. The cube of a number is the second power of the number.
4. The power of a number is indicated by a small figure written a little below and to the right.

Powers and Roots

The subject of *powers* is closely related to multiplication. The subject of *roots* is closely related to division.

The algebraic formula for the area of a square is $A = s^2$. A represents *area* and s represents one *side* of the square. We read s^2 as "s (side) squared"; it means $s \times s$. In like manner, $3^2 = 3 \times 3 = 9$, and $10^2 = 10 \times 10 = 100$. To square a number, we simply multiply it by itself. The square of a number is called the second power of the number.

The formula for the volume of a cube is $V = e^3$. In this formula e represents the length of one edge of the cube. We read e^3 as "e cubed"; it means $e \times e \times e$. In like manner $2^3 = 2 \times 2 \times 2 = 8$, and $5^3 = 5 \times 5 \times 5 = 125$. The cube of a number is the *third power* of the number.

In the same way we can find the fourth power, the fifth power, or any other power of a number. For example, 3^4 (3 to the fourth power) = 3 × 3 × × 3 × 3 = 81; 10^5 (100 to the fifth power) = 10 × 10 × 10 × 10 × 10 = 100,000. It will be seen that, except for very small numbers, the higher powers of a number are very large.

The first power of a number is simply the number itself. For example, $7^1 = 7$. We do not ordinarily talk about the first power of a number.

The power of a number is indicated by a small figure written a little above and to the right. This small figure is called the exponent. In 8^3, for example, the exponent is 3; it tells us that 8 is to be raised 36 to the third power. The exponent 1 is seldom written; if no exponent appears on a number, it is understood that the exponent is 1.

In algebra it is shown that any number with an exponent of 0 is equal to 1. Thus, $3^0 = 1$; $10^0 = 1$. It is also shown in algebra that exponents can be negative numbers. Any number with a negative exponent is equal to 1 divided by that number with a positive exponent. For example, $x^{-3} = 1/x^3$.

(Adapted from [3])

Task 20. Translate into your own language.

The sign $\sqrt[2]{\ }$ means "a square root of". We know that $6^2 = 36$; the square root of 36 is 6, or $\sqrt[2]{36 = 6}$.

The sign $\sqrt[3]{\ }$ means "a cube root of". Similarly there are 4th roots, 5th roots, 10th roots, and so on. Then the cube of 4 is 64; that is $4^3 = 64$. The cube root of 64 is 4; that is, $\sqrt[3]{64 = 4}$.

(Adapted from [3])

Task 21. Read the text and answer the questions.

1. What operation should be performed to square a number?
2. What is a perfect square?
3. What do we do to get the square root of a number?
4. What is the process of finding a root called?
5. How do we check the accuracy of a root?

Squares and Square Roots

To square a number, you have learned, you must multiply that number by itself. The square root of a number is just the opposite. When you find the square root of a number, you find what number multiplied by itself gives you the number you began with. The sign for the square root is $\sqrt[2]{\ }$. Thus, the square root of 25 is represented by $\sqrt[2]{25=5}$. 25 is a perfect square. That is, a whole number 5 multiplied by itself will give you 25. Most numbers are not perfect squares. In that case, to get the square root of a number we may either find it by taking an arithmetic square root or by using a table.

The process of finding a root is known as evolution; it is the inverse of involution, because by the aid of this process we try to find that which is given only when raising a number to a power (viz. the base of the power), while the data given is just what is sought for raising a number to a power (viz. the power itself). Therefore the accuracy of the root taken may always be checked by raising the number to the power. For instance, in order to check the equality $\sqrt[3]{125=5}$, it is sufficient to cube 5; obtaining the quantity under the radical sign, we conclude that the cube root of 125 has been found correctly.

(Adapted from [1])

Task 22. Complete the table.

Verb	Noun	Adjective
inverse		
obtain		
		square
multiply		
	evolution	

Task 23. Translate into your own language.

To square a number; the number you began with; what is sought for; by raising the number to the power.

Task 24. Read the text and answer the questions.

1. What is an equation?
2. What are the expressions on either side of the sign of equality called?
3. What should be done to keep the balance of the equation?
4. How do we check an equation?
5. What operations must one do when solving an equation by the combination of rules?

Equations

An equation is a statement of equality. The statement may be true for[1] all values of the letters.

The value of the letters for which the equation is true is the root or solution of the equation.

When a statement of equality of this kind is given, our interest is in finding[2] the value of the letter for which it is true. The following rules aid in finding the root.

1. The roots of an equation remain the same if the same expression is added to or subtracted from both sides of the equation.

2. The roots of an equation remain the same if both sides of the equation are multiplied or divided by the same expression other than zero and not involving the letter whose value is in question[3].

The equation $2x = 4$ where x is the unknown, is true for $x = 2$. To illustrate the first of the above two rules, add $5x$ to both sides of the equation $2x = 4$. We get $2x + 5x = 4 + 5x$ which, like equation $2x = 4$ is true for only $x = 2$. To illustrate the importance of the restriction in the second of the above two laws, multiply both sides of the equation by x and get $(2x)x = (4)x$ which is true not only for $x = 2$ but also for $x = 0$.

It is always a good plan to check the accuracy[4] of one's work by substituting the result in the original equation to see whether the equation is true for this value.

Rule 1 is applied very frequently. It is, therefore, desirable to state it in a way which mechanizes its application.

If the equation $4x = 28 – 3x$ is given, in applying Rule 1, $3x$ may be added to both sides of the equation, yielding $4x + 3x = 28 – 3x + 3x = 28$.

The result of the operation consists in omitting[5] the term $+3x$ to the left side. We call this operation transposition of the term $3x$. This operation is an application of Rule 1 and may be explained in the following way:

Any term of one side of an equation may be transposed to the other side if its sign is changed.

Example. Find the value of x which satisfies $3x + 7(4 – x) + 6x = 15$. Clearing of parentheses and combining terms:

$3x + 28 – 7x + 6x = 15$; $2x + 28 = 15$.

Transposing +28 from the left side:

$2x = 15 – 28$; $2x = –13$.

Dividing each side by 2, according to Rule 2:

$x = -13/2$; $x = -6.5$.

An equation which can be reduced to the form $ax + b = 0$ ($a \neq 0$), is called a linear equation in x.

To solve an equation containing fractions, first reduce each fraction to its lowest terms. Then multiply each side of the equation by the least common denominator of all the denominators. This process is called clearing of fractions.

A quadratic equation is one which can be reduced to the form $ax^2 + bx + c = 0$ ($a \neq 0$) where a, b and c are known and x is unknown.

(Adapted from [1])

Notes:

1. May be true for – может быть действительным для, пригодным для, верным, справедливым.
2. Our interest is in finding – нам интересно найти.
3. In question – искомое (которое неизвестно).
4. To check the accuracy – чтобы проверить точность.
5. Consists in omitting – состоит в устранении.

Task 25. Translate into your own language.

If the same number is added to each side of an equation, the equality of the two sides is not altered. If the same number is subtracted from each side of an equation, the equality of the two sides is not altered. If both sides of an equation are multiplied by the same number, the equality of the two sides is not altered. If both sides of an equation are divided by the same number, the equality of the two sides is not altered.

Task 26. Read the text. Are these statements true or false?

1. Cayley was the first inventor of matrices.
2. Cayley's idea of a matrix comes from Hamilton's theory of quaternion.
3. Properties of transformations on linear equations serve as the basis of Cayley's theory of matrices.
4. The law of multiplication of matrices does not relate to the theory of linear transformations.

Matrices

Although the idea of a matrix was implicit in the quaternion (4-tuples) of N. Hamilton and also in the "extended magnitude" (*n*-tuples) of H. Grassmann, the credit for inventing matrices is usually given to Cayley with a date of 1857, even though Hamilton obtained one of two isolated results in 1852. Cayley said that he got the idea of a matrix either directly from that of a determinant, or as a convenient mode of expression of the equations $x = ax + by$, $y = cx + dy$. He represented this transformation and developed an algebra of matrices by observing properties of transformations on linear equations:

$$\begin{cases} x = ax + by \\ y = cx + dy \end{cases} \rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Cayley also showed that a quaternion could be represented in matrix form as shown above where a, b, c, d are suitable complex numbers. For example, if we let the quaternion units 1, i, j, k be represented by

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \text{ and } \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$$

the quaternion $4 + 5i + 6j + 7k$ can be written as shown below:

$$\begin{bmatrix} 4+5i & 6+7i \\ -6+7i & 4-5i \end{bmatrix}$$

This led P. G. Tait, a disciple of Hamilton, to conclude erroneously that Cayley had used quaternion as his motivation for matrices. It was shown by Hamilton in his theory of quaternion that one could have a logical system in which the multiplication is not commutative. This result was undoubtedly of great help to Cayley in working out his matrix calculus because matrix multiplication is also noncommutative. In 1925 Heisenberg discovered that the algebra of matrices is just right for the noncommutative maths describing phenomena in quantum mechanics.

Cayley's theory of matrices grew out of his interest in linear transformations and algebraic invariants, an interest he shared with J. J. Silvester. In collaboration with J. J. Silvester, Cayley began the work on the theory of algebraic invariants which had been in the air for some time and which, like matrices, received some of its motivation from determinants. They investigated algebraic expressions that remained invariants (unchanged except, possibly, for a constant factor) when the variables were transformed by substitutions representing translations, rotations, dilatations ("stretching" from the origin), reflections about an axis, and so forth.

There are three fundamental operations in matrix algebra: addition, multiplication and transposition, the last not occuring in ordinary algebra. The law of multiplication of matrices which Cayley invented and his successors have approved, takes its rise in the theory of linear transformations. Linear combinations of matrices with scalar coefficients obey the rules of ordinary algebra. A transposition is a permutation which interchanges two numbers and leaves the other fixed, or in other words: the formal operation leading from x to x' and also that leading from x' to x is called transposition. A matrix of m rows and n columns has rank r, when not all its minor determinants of order r vanish, while of order $r + 1$ do. A matrix and its transposition have the same rank. The rank of a square matrix is the greatest number of its rows or columns which are linearly independent.

Today, matrix theory is usually considered as the main subject of linear algebra, and it is a mathematical tool of the social scientist, geneticist, statistician, engineer and physical scientist.

(Adapted from [7])

Task 27. Read the text and answer the questions.

1. Who was the first to create a matrix? When was this?
2. Did anyone get the idea of a matrix before that?
3. What did Hamilton show in his theory of quaternion?
4. Why did Hamilton's theory of quaternion help Cayley to work out his matrix theory?
5. What did Heisenberg discover in 1925?
6. Who did Cayley collaborate with on the theory of algebraic invariants? What did they investigate?
7. What are three fundamental operations in matrix algebra?

Task 28. Read the text and answer the questions.

1. What equations are termed linear?
2. What is the first operation in solving a system of two linear equations in two unknowns?
3. What do you obtain by adding or subtracting two equations?
4. What operation do you perform to find the second unknown quantity?

Systems of Two Linear Equations in Two Unknowns

Consider the equation

$$x - 2y = 5 \qquad (1)$$

In this equation $x = 7$ and $y = 1$, but also $x = 5$ and $y = 0$. There are many such pairs of values which satisfy the equation (1). To find pairs other than those given, choose a value of one letter, say y arbitrary, and then from (1) find the corresponding value of x. For example, let $y = 3$. Then from (1)

$$x = 5 + 2y, \tag{2}$$

where $x = 5 + 2 \cdot 3$, $x = 5 + 6 = 11$ and the pair of values $x = 11$, $y = 3$ satisfies the equation (1). The method for finding the pair of values satisfying both equations indicated above usually applies to pairs of equations of the form:

$$\left.\begin{array}{l} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{array}\right\}, \tag{3}$$

where a_1, a_2, b_1, b_2, c_1, c_2 are known, and x and y are unknown quantities.

The equations (3) are termed linear because the unknown x and y enter to the first power only.

To solve a system of two linear equations in two unknowns[1], solve for one unknown in one equation and substitute this result in the other equation, thus obtaining one equation in one unknown.

An alternative way[2] of solving a system of two linear equations, which is usually more convenient, is given by the following rule: multiply the two equations with numerical factors which are chosen so that the coefficient of one of the two unknowns have the same numerical values in both equations.

By adding or subtracting the two equations, a new equation with only one unknown quantity is obtained. Solve this equation. In order to find the second unknown quantity, substitute the value which has been found and solve for the remaining unknown quantity. An alternative method for finding the second unknown is to repeat the above process of finding the equal coefficient for the other unknown.

(Adapted from [1])

Notes:

1. Equations in two unknowns – уравнения с двумя неизвестными.
2. An alternative way – другой способ.

Task 29. Translate into your own language.

In order to solve two equations in two unknowns, it is necessary to eliminate one of the unknowns by combining the two equations into one equation, which only contains one of the unknowns. This simple equation is then readily solved for that unknown in the usual way. With one of the original unknowns now known, its value can be substituted for the symbol in one of the equations, and from the resulting simple equation, the other unknown can be found. There are several methods of eliminating one of the unknowns and combining the two original equations into one.

Focus on Speaking

Task 30. Read these phrases and translate into your own language.

Subtraction: $4 - 3 = 1$ is read: three from four is one; four minus three is one.

Addition: $a + b + c$ is read: *a* plus *b* is *c*; *a* and *b* is equal to *c*.

Multiplication: $2 \times 3 = 6$; $2 \cdot 3 = 6$ is read: two multiplied by three is six.

Division: $35 \div 5 = 7$ is read: thirty five divided by five is 7; five into thirty five goes seven times.

Involution or raising to power: 3^2 is read: three to the second power; 3 squared.

5^3 is read: five cubed; 5 to the third power; 5 to power three.

x^2 is read: *x* is called the base of the power; 2 is called an exponent or index of the power.

Evolution: $\sqrt{9 = 3}$ is read: the square root of nine is three.

$\sqrt[3]{27 = 3}$ is read: the cube root of twenty seven is three.

$\sqrt{\ }$ is called "the radical sign" or "the sign of the root".

Fractions: 1/9 is read: a ninth, one ninth; 2/123 is read: two one hundred and twenty-thirds; 3 2/5 is read: three and two fifths.

Task 31. Make up sentences of your own using the words and expressions given below.

In one unknown, in two unknowns, in three unknowns, to satisfy the equation, the method for finding, to obtain an equation, to establish.

Useful phrases

My view is …, because …

Surely the main point is …

The fact is …

On the one hand …, on the other hand …

Another problem is that …
I think that / In my opinion …
As far as I'm concerned / In my view …

Task 32. Read the text. Are the statements true or false?
1. A point is an idea about any dot on a surface.
2. A point does not have exact dimension and location.
3. We can easily measure the length, the thickness and the width of a line.
4. A line is limited by two endpoints.
5. A line segment is also a subset of a line.
6. Although a ray has an endpoint, we cannot define its length.

Points and Lines

Geometry is a very old subject. It probably began in Babylonia and Egypt. As the knowledge of the Egyptians spread to Greece, the Greeks found the ideas about geometry very intriguing and mysterious. The Greeks began to ask "Why? Why is that true?". In 300 B.C all the known facts about Greek geometry were put into a logical sequence by Euclid. His book, called "The Elements", is one of the most famous books of mathematics. In recent years, men have improved on Euclid's work. Today geometry includes not only the shape and size of the earth and all things on it, but also the study of relations between geometric objects. The most fundamental idea in the study of geometry is the idea of a point and a line.

The world around us contains many physical objects from which mathematics has developed geometric ideas. These objects can serve as models of the geometric figures. The edge of a ruler, or an edge of this page is a model of a line. We have agreed to use the word "line" to mean a straight line. A geometric line is the property these models of lines have in common; it has length but no thickness and no width; it is an idea. A particle of dust in the air or a dot on a piece of paper is a model of a point. A point is an idea about an exact location; it has no dimensions. We usually use letters of the alphabet to name geometric ideas. For example, we speak of the following models of a point as point *A*, point *B* and point *C*.

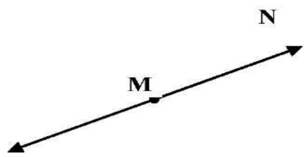
.B

A. **C.**

We speak of the following as line *AB* or line *BA*.

The arrows on the model above indicate that a line extends indefinitely in both directions. Let us agree to use the symbol ↔ to name a line. $\overline{AB}$ means line *AB*. Can you locate a point *C* between *A* and *B* on the drawing of $\overline{AB}$ above? Could you locate another point between *B* and *C*? Could you continue this process indefinitely? Why? Because between two points on a line there is another point. A line consists of a set of points. Therefore a piece of the line is a subset of a line. There are many kinds of subsets of a line. The subset of $\overline{AB}$ shown above is called a line segment. The symbol for the line segment *AB* is $\overline{AB}$. Points A and B are the endpoints, as you may remember. A line segment is a set of points on the line between them. How do line segments differ from a line? Could you measure the length of a line? Of a line segment? A line segment has definite length but a line extends indefinitely in each of its directions.

Another important subset of a line is called a ray. The part of $\overline{MN}$ shown below is called ray *MN*. The symbol for ray *MN* is $\overline{MN}$.



A ray has indefinite length and only one endpoint. The endpoint of a ray is called its vertex. The vertex of $\overline{MN}$ is *M*. In the drawings above you see pictures of a line, a line segment and a ray – not the geometric ideas they represent.

(Adapted from [7])

Task 33. Read the text once more and answer the questions.

1. Where did the history of geometry begin?
2. Who was considered the first starting geometry?
3. What was the name of the mathematician who first assembled Greek geometry in a logical sequence?
4. How have mathematicians developed geometric ideas?

5. Why can you locate a point *C* between *A* and *B* on the line *AB*?
6. How does a line segment differ from a line, a ray?

Task 34. Complete the following statements.
1. A point is … .
2. A line is … .
3. A line segment is … .
4. A ray is … .

Focus on Reading

Task 35. Read the text. Are the statements true or false?

1. The *set of positive integers* is called a *subset* of the set of real numbers and consists of all the positive numbers.

2. The *set of negative integers* is also a *subset* of the set of real numbers and consists of all the negative numbers: –1, –2, –3, and so on.

3. The *set of integers* doesn't consist of all the positive integers, all the negative integers, and zero.

4. Rational numbers can't be represented as fractions.

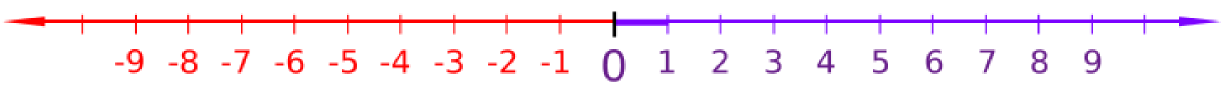
5. Every rational number corresponds to a point on the number line.

6. The real numbers which are rational are called *irrational* numbers.

The Number Line and the Rational Numbers

The set of real numbers may be pictured as the set of points on a line, and we speak of the set of real numbers as the *number line.* A point on the line is chosen to represent the number 0 and other points are chosen for the numbers +1, +2, +3, +4, and so on.

The *set of positive integers* is called a *subset* of the set of real numbers and consists of all the positive numbers. The *set of negative integers* is also a *subset* of the set of real numbers and consists of all the negative numbers: –1, –2, –3, and so on.



The drawing of the number line may be extended to the right and to the left as necessary.

The *set of integers* consists of all the positive integers, all the negative integers, and zero.

When one integer is divided by another nonzero integer, the quotient is called a rational number. Rational numbers may be represented as fractions. For example, the fraction represents the quotient of +1 divided by +2. The symbol above the line is called the numerator of the fraction, and the symbol below the line is called the denominator of the fraction.

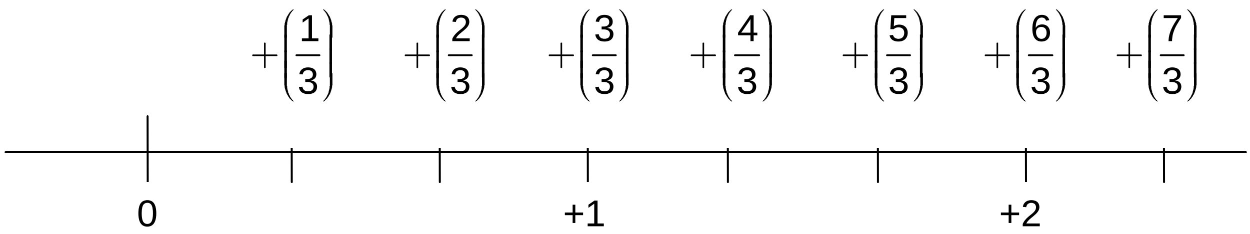
Rational numbers may also be represented as decimals. A fraction can be converted by division to the decimal representation as follows:

$$\frac{+1}{+2} = +0.5; \ \frac{-3}{+5} = -0.6.$$

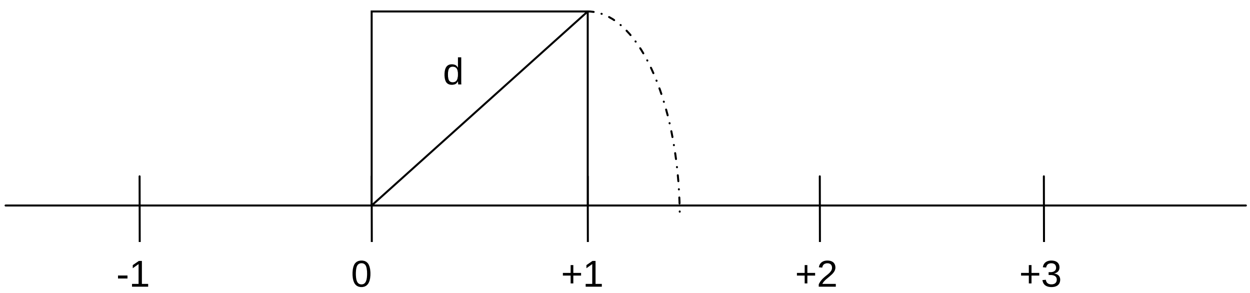
You may have noticed that the rational numbers include both positive and negative integers. For example, –4 is a rational number because $\frac{-8}{+2} = -4$.

The Irrational Numbers

In the real number system, every rational number corresponds to a point on the number line. For example, the point corresponding to $+\left(\frac{7}{3}\right)$ may be found by dividing the segment between 0 and +1 into thirds and then constructing the segment 7 times as long as the segment from 0 to $+\left(\frac{1}{3}\right)$.



However, not every point that can be shown on a real-number line designates a rational number. The ancient Greek geometers were the first to discover that there are some real numbers which are not rational. They showed, that if we construct a square measuring 1 unit by 1 unit, the length of the diagonal (denoted by *d* in the drawing) is not a rational number.



The Pythagorean theorem, named after the Greek geometer Pythagoras, states that the square of the hypothenuse of a right triangle is equal to the sum of the squares of the legs. By this theorem,

$$d^2 = 1^2 + 1^2 = 1 + 1 = 2. \text{ So, } d = \sqrt{2}.$$

The real numbers which are not rational are called *irrational* numbers. Other irrational numbers are $\sqrt{3}$, $\sqrt{5}$, and π. (Say "square root of three", "square root of five", and "pi". Pi is the circumference of a circle whose diameter is 1 unit in length).

(Adapted from [3])

Task 36. Read the text. Are the statements true or false?

1. Each ordered pair (*x, y)* tells you how to locate every point in the plane.
2. We know each of the numbers of a pair to be either positive or negative.
3. The operation of forming the Cartesian product is commutative.
4. To every ordered pair of real numbers there correspond several points on the plane.
5. There is no one-to-one correspondence between real numbers and the points on a line.

The Coordinate Plane

Now we want you to consider two sets: *A* and *B*, such that *A* = {*a*, *b*, *c*} and *B* = {*d*, *e*}. We will form a new set from sets *A* and *B*, which we will call the Cartesian product, or simply the product set, by forming all possible ordered pairs (*x*, *y*) such that *x* is from set *A* and *y* is from set *B*. This new set is denoted by *A* × *B* (read *A* cross *B*):

$$A \times B = \left\{ \begin{matrix} a, d & , & a, e \\ b, d & , & b, e \\ c, d & , & c, e \end{matrix} \right\}.$$

Let us use the notation *n*(*A*) to mean the number of elements in the set *A* and *n*(*A* × *B*) to mean the number of elements (ordered pairs) in *A* × *B*. Observe that *n*(*A* × *B*) = 6 and that *n*(*A*) = 3 and *n*(*B*) = 2. Since 3 × 2 = 6, we see there is a relationship of some importance between the set operation of forming the Cartesian product and multiplication of numbers *n*(*A*) × *n*(*B*) = = *n*(*A* × *B*). Now let us form *B* × *A*:

$$B \times A = \left\{ \begin{matrix} d, a & , & d, b & , & d, c \\ e, a & , & e, b & , & e, c \end{matrix} \right\}.$$

You may have noticed that no elements (ordered pairs) of $B \times A$ are the same as those of $A \times B$, though their numbers are still the same. This means that $A \times B \neq B \times A$, while $n(A) \times n(B) = n(B) \times n(A)$. Forming the product set is a noncommutative operation. In this case it is a noncommutative multiplication.

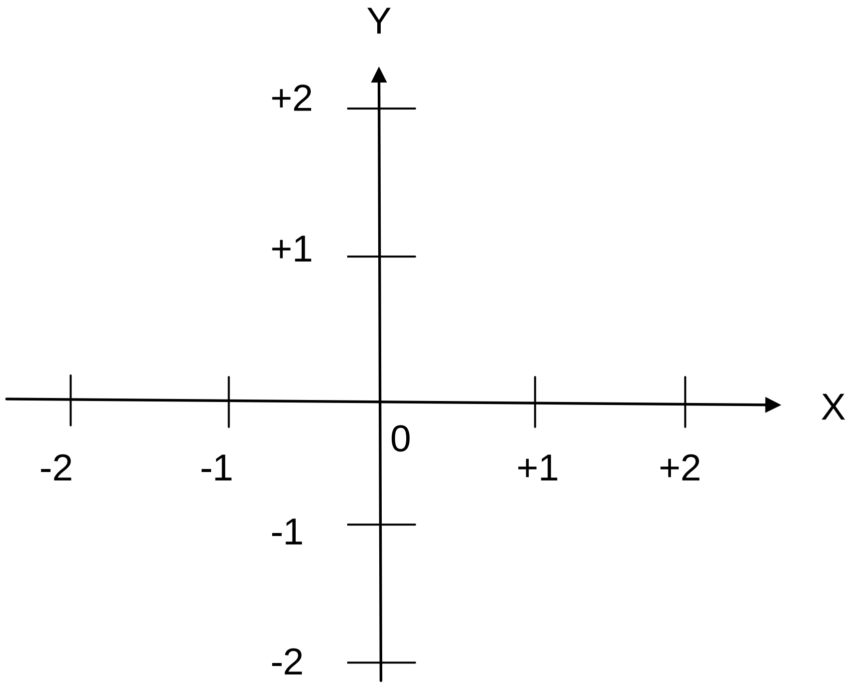
In our next step we do something that at first will seem purposeless. Given that set $A = \{a, b, c\}$, we will form the new set $A \times A$:

$$A \times A = \begin{Bmatrix} a, a \ , & a, b \ , & a, c \\ b, a \ , & b, b \ , & b, c \\ c, a \ , & c, b \ , & c, c \end{Bmatrix}.$$

This is, of course, the Cartesian product of the set A with itself, and you will wonder what you can do with it. Its use will become clear if we let $X = \{0, 1, 2\}$ and let $Y = \{0, 1, 2\}$. Then find $X = Y$ the Cartesian product of a set with itself since $X = Y$:

$$X \times Y = \begin{Bmatrix} (0,0), & (0,1), & (0,2) \\ (1,0), & (1,1), & (1,2) \\ (2,0), & (2,1), & (2,2) \end{Bmatrix}.$$

We then interpret this set of ordered pairs of numbers as a set of points in a plane such that to each point there corresponds one ordered pair of numbers and vice versa. Now it is necessary for us to set up a model for geometric interpretation. To do this we intersect two number lines at the zero point, or origin of the graph, so that the lines are perpendicular to each other. Label the number lines as shown in the following figure by choosing X to denote the set of points on the horizontal line and Y to denote the set of points on the vertical line. Now we assign positive numbers to the right half line of X and negative numbers to the left half line of X. Similarly we assign positive numbers to the upper half line of Y and negative numbers to its lower half line. The two number lines are called axes. We speak of the x axis when we refer to the horizontal number line and of the y axis when we refer to the vertical number line. We now have an interpretation such that every ordered pair of numbers labels a point in the plane determined by the X and Y axes.



Since we find each of the axes to represent an ordered set of points and both axes to cooperate in determining the plane, such a system is said to be a coordinate system and the plane determined by it is said to be a coordinate plane. Each ordered pair (*x*, *y*) tells you how to locate a point in the coordinate plane, by starting from the origin. (*x*, *y*) means: first move *x* units from (0, 0) along the *x* axis to the right or left (indicated by + or – preceding the first numeral of the pair); then move *y* units from that point parallel to the *y* axis (up or down as indicated by + or – preceding the second numeral of the pair).

(Adapted from [7])

Task 37. Read the text and answer the questions.

1. What are the two number lines as we have used them for the coordinate systems called?
2. What is the horizontal number line often referred to?
3. What is the vertical number line often referred to?
4. Into how many parts do the two axes of the coordinate system divide the plane?
5. If both coordinates of a point are 0, where is the point located?
6. What does a coordinate of a point tell you?
7. What does each of the axes represent?

Unit 6. Probability and Statistics

Task 1. Answer the questions.

1. What does the term "probability" mean? Give five associations with this word.

2. What do you know about probability theory? How has the theory developed?

3. What is the main question in probability theory?

4. Is the concept of probability widely applied?

Task 2. Match the words and phrases with the definitions and then find them in the text.

1) controversial	a) reliable; certain not to fail
2) underpinnings	b) produce, generate
3) secure	c) any supporting structure or system
4) occurrence	d) causing a lot of disagreement, because many people have strong opinions about the subject being discussed
5) spawn	e) large in size
6) enormous	f) something that occurs; a happening; event

Task 3. Read the text and find information to answer the questions (Task 1).

When probability theory is applied, the results can be controversial, but at least the mathematical underpinnings are reasonably secure. In 1933, Andrey Nikolaevich Kolmogorov was instrumental in defining probability on an axiomatic basis – much like the way the principles of geometry were defined two millennia before.

Probability is defined by the following axioms:

1) the probability of all occurrences is 1;

2) probability has a value which is greater than or equal to zero;

3) when occurrences cannot coincide, their probabilities can be added. From these axioms, dressed in technical language, the mathematical properties of probability can be deduced. The concept of probability can be widely applied. Much of modern life cannot do without it. Risk analysis, sport, sociology, psychology, engineering design, finance, and so on – the list is

endless. Who'd have thought the gambling problems that kick-started these ideas in the 17th century would spawn such an enormous discipline? What were the chances of that?

Focus on Reading

Task 4. These are the key terms to the text under study. Read them carefully and find the best explanations. Use a dictionary for help [6].

1) assumption	a) readily perceived; considerable
2) assignment	b) to behave or manage (oneself)
3) approach	c) to make, show, or recognize a difference or differences (between or among)
4) conduct	d) a function that associates specific values with each variable in a formal expression
5) sensible	e) a statement that is used as the premise of a particular argument but may not be otherwise accepted
6) distinguish	f) a method of doing sth or dealing with a problem

Task 5. Read the text below (Task 6) and choose the best heading to each of its parts.

A. Origins of Probability.
B. Playing Craps.
C. Probability.
D. The Monkey on a Typewriter.

Task 6. Read the text again. Are these statements true or false?

1. Probability theory has a bearing on uncertainty and is an essential ingredient in evaluating risk.
2. It is not a problem to quantify probability.
3. It is possible to pinpoint an exact number for the probability.
4. The probability of an impossible event is 0 and a certainty is 1.
5. The mathematical theory of probability came to the fore in the 18th century with discussions on gambling problems.
6. There are 7 combinations that each add up to 6 in the modern game of craps.
7. Alfred will have typed "to be or" in about 6.1 years.

1. ...

What is the chance of it snowing tomorrow? What is the likelihood that I will catch the early train? What is the probability of you winning the lottery? Probability, likelihood, chance are all words we use every day when we want to know the answers. They are also the words of the mathematical theory of probability.

Probability theory is important. It has a bearing on uncertainty and is an essential ingredient in evaluating risk. But how can a theory involving uncertainty be quantified? After all, isn't mathematics an exact science?

The real problem is to quantify probability.

Suppose we take the simplest example on the planet, the tossing of a coin. What is the probability of getting a head? We might rush in and say the answer is 1/2 (sometimes expressed as 0.5 or 50 %). Looking at the coin we make the assumption it is a fair coin, which means that the chance of getting a head equals the chance of getting a tail, and therefore the probability of a head is 1/2.

Situations involving coins, balls in boxes, and "mechanical" examples are relatively straightforward. There are two main theories in the assignment of probabilities. Looking at the two-sided symmetry of the coin provides one approach. Another is the relative frequency approach, where we conduct the experiment a large number of times and count the number of heads. But how large is large? It is easy to believe that the number of heads relative to the number of tails is roughly 50 : 50 but it might be that this proportion would change if we continued the experiment.

But what about coming to a sensible measure of the probability of it snowing tomorrow? There will again be two outcomes: either it snows or it does not snow, but it is not at all clear that they are equally likely as it was for the coin. An evaluation of the probability of it snowing tomorrow will have to take into account the weather conditions at the time and a host of other factors. But even then it is not possible to pinpoint an exact number for this probability. Though we may not come to an actual number, we can usefully describe a "degree of belief" that the probability will be low, medium or high. In mathematics, probability is measured on a scale from 0 to 1. The probability of an impossible event is 0 and a certainty is 1. A probability of 0.1 would mean a low probability while 0.9 would signify a high probability.

2. ...

The mathematical theory of probability came to the fore in the 17th century with discussions on gambling problems between Blaise Pascal, Pierre de Fermat and Antoine Gombaud (also known as the Chevalier de Méré). They found a simple game puzzling. The Chevalier de Méré's question is this: which is more likely, rolling a "six" on four throws of a dice, or rolling a "double six" on 24 throws with two dice? Which option would you put your shirt on?

The prevailing wisdom of the time thought the better option was to bet on the double six because of the many more throws allowed. This view was shattered when the probabilities were analyzed. Here is how the calculations go: Throw one dice: the probability of *not* getting a six on a single throw is ⅚, and in four throws the probability of this would be 5/6 × 5/6 × 5/6 × 5/6 which is $(5/6)^4$. Because the results of the throws do not affect each other, they are "independent' and we can multiply the probabilities. The probability of at least one six is therefore $1 – (5/6)^4 = 0.517746$ ...

Throw two dice: the probability of *not* getting a double six in one throw is 35/36 and in 24 throws this has the probability $(35/36)^{24}$.

The probability of at least one double six is therefore

$1 – (35/36)^{24} = 0.491404$ ...

3. ...

The two dice example is the basis of the modern game of craps played in casinos and online betting. When two distinguishable dice (red and blue) are thrown there are 36 possible outcomes and these may be recorded as pairs (*x*, *y*) and displayed as 36 dots against a set of *x/y* axes – this is called the "sample space" (Fig 6.1).

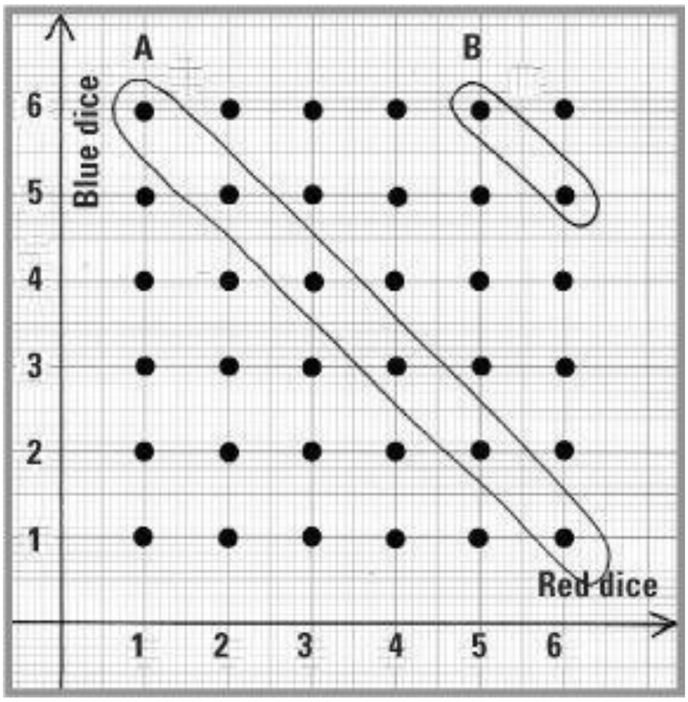


Fig. 6.1. **Sample space (for 2 dice)**

Let's consider the "event" A of getting the sum of the dice to add up to 7. There are 6 combinations that each add up to 7, so we can describe the event by $A = \{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\}$ and ring it on the diagram. The probability of A is 6 chances in 36, which can be written $Pr(A) = 6/36 =$ $= 1/6$. If we let B be the event of getting the sum on the dice equal to 11, we have the event $B = \{(5, 6), (6, 5)\}$ and $Pr(B) = 2/36 = 1/18$.

In the dice game craps, in which two dice are thrown on a table, you can win or lose at the first stage, but for some scores all is not lost and you can go onto a second stage. You win at the first throw if either the event A or B occurs – this is called a "natural". The probability of a natural is obtained by adding the individual probabilities, 6/36 + 2/36 = 8/36. You lose at the first stage if you throw a 2, 3 or a 12 (this is called "craps"). A calculation like that above gives the probability of losing at the first stage as 4/36. If a sum of either 4, 5, 6, 8, 9 or 10 is thrown, you go onto a second stage and the probability of doing this is 24/36 = 2/3.

In the gaming world of casinos the probabilities are written as odds. In craps, for every 36 games you play, on average you will win at the first throw 8 times and not win 28 times so the odds against winning on the first throw are 28 to 8, which is the same as 3.5 to 1.

4. ...

Alfred is a monkey who lives in the local zoo. He has a battered old typewriter with 26 keys for the letters of the alphabet, a key for a full stop, one for a comma, one for a question mark and one for a space – 30 keys in all. He sits in a corner filled with literary ambition, but his method of writing is curious – he hits the keys at random.

Any sequence of letters typed will have a nonzero chance of occurring, so there is a chance he will type out the plays of Shakespeare word perfect. More than this, there is a chance (even though smaller) he will follow this with a translation into French, and then Spanish, and then German. For good measure we could allow for the possibility of him continuing on with the poems of William Wordsworth. The chance of all this is minute, but it is certainly not zero. This is the key point. Let's see how long he will take to type the soliloquy in *Hamlet*, starting off with the opening "To be or". We imagine 8 boxes which will hold the 8 letters including the spaces.

T	o		b	e		o	r

The number of possibilities for the first position is 30, for the second it is 30, and so on. So the number of ways of filling out the 8 boxes is 30 × 30 × × 30 × 30 × 30 × 30 × 30 × 30. The chance of Alfred getting as far as "To be or" is 1 chance in 6.561×10^{11}. If Alfred hits the typewriter once every second there is an expectation he will have typed "To be or" in about 20,000 years, and proved himself a particularly long-lived primate. So don't hold your breath waiting for the whole of Shakespeare. Alfred will produce nonsense like "xo,h?yt?" for a great deal of the time.

(Adapted from [5])

Task 7. Translate the paragraph "Origins of Probability" into your own language. Let your friend check your work.

Focus on Vocabulary

Task 8. Read this phrase. What does it mean?

Two's company, but three's a crowd.

Task 9. Choose words from the list to fill in the gaps.

Rolling, tail, betting, probability, coin, impossible, key, certainty, outcomes, heads, throw.

1. Tossing of a … .
2. The chance of getting a … .
3. Count the number of … .
4. A sensible measure of the … .
5. The probability of an … event is 0 and a … is 1.
6. … a "double six".
7. … one dice.
8. Online … .
9. 36 possible … .
10. The … point.

Task 10. Complete the text with the following words and phrases.

Outcome, elements, assigning, hopeless, truth, assertion, prediction, measure, statement, unique, assumption.

Probability Theory

We often hear statements of the following kind: "It is likely to rain today", "I have a fair chance of passing this course", "There is an even chance that a coin will come up heads", etc. In each case our **1** … refers to a situation in which we are not certain of the **2** … , but we express some degree of confidence that our **3** … will be verified. The theory of probability provides a mathematical framework for such assertions.

Consider an experiment whose outcome is not known. Suppose that someone makes an **4** … *p* about the outcome of the experiment, and we want to assign a probability to *p.* When statement *p* is considered in isolation, we usually find no natural assignment of probabilities. Rather, we look for a method of **5** … probabilities to all conceivable statements concerning the outcome of the experiment. At first this might seem to be a **6** … task, since there is no end to the statements we can make about the experiment. However we are aided by a basic principle:

Fundamental **7** … : Any two equivalent statements will be assigned the same probability.

As long as there are a finite number of logical probabilities, there are only a finite number of **8** … sets, and hence the process of assigning probabilities is a finite one. We proceed in three steps: (1) we first determine *U*, the possibility set, that is, the set of all logical possibilities; (2) to each subset *X* of *V* we assign a number called the **9** … $m(X)$; (3) to each statement *p* we assign $m(P)$, the measure of its truth sets, as a probability. The probability of statement *p* is denoted by $Pr\ [p]$.

It is important to remember that there is no **10** … method for analyzing logical possibilities. In a given problem we may arrive at a very fine or a very rough analysis of possibilities, causing *U* to have many or few **11** … .

Having chosen *U*, the next step is to assign a number to each subset *X* of *U,* which will in turn be taken to be the probability of any statement having truth set *X*.

(Adapted from [9])

Focus on Reading

Task 11. Match the words and phrases with the definitions and then find them in the text (Task 12).

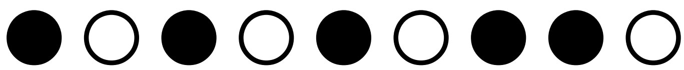
1) trial	a) an approving attitude
2) favour (favors US)	b) the probability, expressed as a ratio, that a certain event will take place
3) failure	c) the act or an instance of trying or proving
4) odds	d) lack of success

Task 12. Read the text. Are these statements true or false? Correct the false statements.

1. The trial can succeed in nine ways when you suppose that you have nine circles.
2. The sum of the probability of success and failure is equal to 1.
3. The probability that an event will occur can be more than 1.
4. In tossing two coins the fact that one fell heads would not affect the way the other fell.

Probability of Occurence

Look at these circles and let's perform an experiment.



Here are nine circles. Five are black, four are white. If you were told to cover one circle with your finger, you might choose any one of the nine. But you are more likely to choose a black circle than a white, because there are more black circles than white ones. Indeed, the probability that you will cover a black circle is 5/9 of the number of black circles to the total number of circles.

In mathematical language the choice, the probability of success is the ratio of the number of ways in which the trial can succeed to the total number of ways in which the trial can result. Here nothing favors the choice of any particular circle; they are all on the same page, and you are just as likely to cover one as another. The trial can result in five ways; there are five black circles. The trial can result in nine ways; there are nine circles in all. If p represents the probability of success, then $p = 5/9$.

Similarly, the probability of failure is the ratio of the number of ways in which the trial can fail to the total number of ways in which it can result. If q represents the probability of failure, in this case $q = 4/9$. Notice that the sum of probability of success and failure is 1. If you put your finger on a circle, it is certain to be either a black circle or a white one, for no other kind of circle is present. Thus $p + q = 5/9 + 4/9 = 1$. The probability that an event will occur cannot be more than 1. When $p = 1$, success is a certainty. When $q = 1$, failure is sure.

Let S represent the number of ways in which a trial can succeed. And let f represent the number of ways in which a trial can fail. Then

$$p = \frac{S}{S+f}; \; q = \frac{f}{S+f}; \; p+q = \frac{S}{S+f} + \frac{f}{S+f} = 1.$$

When S is greater than f, the odds are S to f in favor of success, thus the odds in favor of covering a black circle are 5 to 4. Similarly, when f is greater than S, the odds are f to S against success. And when S and f are equal, the chances are even; success and failure are equally likely. Tossing a coin illustrates a case in which S and f are equal. There are two sides to a coin, and there is no reason why a normal coin should fall one side up rather than the other. So if you toss a coin and call heads, the probability that it will fall heads is 1/2. Suppose you toss a coin a hundred times, for each of the hundred trials, the probability that the coin will come down heads is 1/2.

You might expect fifty of the tosses to be heads. Of course, you may not get fifty heads. But the more times you toss a coin, the closer you come to the realization of what you expect.

If p is the probability of success on one trial, and K is the number of trials, then the expected number is Kp. Mathematical expectation in this case is defined as Kp.

Task 13. Read the text once more and think how to answer the following questions.

1. What does the article deal with?
2. If you were shown 9 red circles and 6 black circles and were asked to choose one of them, which of these circles would you be likely to choose? Why?
3. Can you give the definition of the probability of failure? What is it?
4. What are the odds in case $f > S$?
5. What are the odds in case $f < S$?
6. Suppose $S = f$, what would the chances be?
7. Could you give some examples to illustrate a case when S and f are equal?

Task 14. Fill in each gap using a word from the text.

1. There are differences of opinion among mathematicians and philosophers about … theory.

2. Suppose two dice are thrown. What are the chances that the … of the faces is five?

3. Two coins are … simultaneously. Since a coin will come down … ( ) or tail (T), each possible outcome is a member of $A \times A$ where $A = \{ ... , T\}$.

4. To describe this sample space … each situation in terms of events and discuss the chances of each event … .

5. When we try to do something several times we say that we have had several … .

Focus on Speaking

Task 15. Match the words and phrases with the definitions and then find them in the text.

1) scatter	a) persuade (a person) to believe or realize
2) astounding	b) to produce a result, answer
3) resume	c) a hole or recess in a dovecote for pigeons to nest in
4) pigeonhole	d) the possession and use of property by or without agreement and without any claim to ownership
5) minuscule	e) following chronological sequence
6) convince	f) to throw about in various directions; strew
7) unravel	g) begin again or continue after an interruption
8) compute	h) way that is shorter than the usual one; short cut
9) cutoff	i) a small compartment
10) sophisticated	j) to calculate or reckon
11) yield	k) causing amazement and wonder
12) triplet	l) to calculate
13) consecutive	m) written in small letters (colloq. extremely small or unimportant)
14) cell	n) a group or set of three similar things
15) occupancy	o) to explain or solve
16) figure out	p) (of machines, methods, etc.) complex and refined

Task 16. Read the text and discuss the birthday problems. Work in pairs and make a list of them then compare it with other students. Do you agree with the conclusion at the end of the text? Draw your own conclusion.

The Birthday Problem

Imagine you are on the top deck of the Clapham omnibus with nothing in particular to do but count your fellow passengers going off to work in the early morning. As it is likely that all the passengers are independent of each other, we may safely assume that their birthdays are randomly scattered throughout the year. Including you there are only 23 passengers on board. It is not many, but enough to claim there is a better than even chance that two passengers share a birthday. Do you believe it? Millions do not but it is absolutely true. Even a seasoned expert in probability, William Feller, thought it astounding.

The Clapham omnibus is now too small for our needs so we resume the argument in a large room. How many people must gather in the room so that it is certain that two people share the same birthday? There are 365 days in a standard year (and we'll ignore leap years just to make things simpler) so if there were 366 people in the room, at least one pair would definitely have the same birthday. It cannot be the case that they all have different ones.

This is the pigeonhole principle: if there are $n + 1$ pigeons who occupy n pigeonholes, one hole must contain more than one pigeon. If there were 365 people we could not be certain there would be a common birthday because the birthdays could each be on different days of the year. However, if you take 365 people at random this would be extremely unlikely and the probability of two people not sharing a birthday would be minuscule. Even if there are only 50 people in the room there is a 96.5 % chance that two people share a birthday. If the number of people is reduced still further the probability of two sharing a birthday reduces. We find that 23 people is the number for which the probability is just greater than 1/2 and for 22 people the probability that a birthday is shared is just less than 1/2. The number 23 is the critical value. While the answer to the classic birthday problem is surprising it is not a paradox.

Can We Prove It?

How can we be convinced? Let's select a person at random. The probability that another person has the same birthday as this person is 1/365 and so the probability these two do not share a birthday is one minus this (or 364/365). The probability that yet another person selected at random shares a birthday with the first two is 2/365 so the probability this person does not share a birthday with either of the first two is one minus this (or 363/365).

The probability of none of these three sharing a birthday is the multiplication of these two probabilities, or (364/365) × (363/365) which is 0.9918.

Continuing this line of thought for 4, 5, 6, … people unravels the birthday problem paradox. When we get as far as 23 people with our pocket calculator we get the answer 0.4927 as the probability that none of them shares a birthday. The negation of "none of them sharing a birthday" is "at least two people share a birthday" and the probability of this is 1 – 0.4927 = 0.5073, just greater than the crucial 1/2.

If $n = 22$, the probability of two people sharing a birthday is 0.4757, which is less than 1/2. The apparent paradoxical nature of the birthday problem is bound up with language. The birthday result makes a statement about two people sharing a birthday, but it does not tell us which two people they are. We do not know where the matches will fall. If Mr. Trevor Thomson whose birthday is on 8 March is in the room, a different question might be asked.

How many birthdays coincide with Mr. Thomson's?

For this question, the calculation is different. The probability of Mr. Thomson not sharing his birthday with another person is 364/365 so that the probability that he does not share his birthday with any of the other $n – 1$ people in the room is $(364/365)^n – 1$. Therefore the probability that Mr. Thomson does share his birthday with someone will be one minus this value.

If we compute this for $n = 23$ this probability is only 0.061151 so there is only a 6 % chance that someone else will have their birthday on 8 March, the same date as Mr. Thomson's birthday. If we increase the value of n, this probability will increase. But we have to go as far $n = 254$ (which includes Mr. Thomson in the count) for the probability to be greater than 1/2. For $n = 254$, its value is 0.5005. This is the cutoff point because $n = 253$ will give the value 0.4991 which is less than 1/2. There will have to be a gathering of 254 people in the room for a chance greater than 1/2 that Mr. Thomson shares his birthday with someone else. This is perhaps more in tune with our intuition than with the startling solution of the classic birthday problem.

Other Birthday Problems

The birthday problem has been generalized in many ways. One approach is to consider three people sharing a birthday. In this case 88 people would be required before there is a better than even chance that three people will share the same birthday. There are correspondingly larger groups if four people, five people, … are required to share a birthday. In a gathering of

1000 people, for example, there is a better than even chance that nine of them share a birthday.

Other forays into the birthday problem have inquired into near birthdays. In this problem, a match is considered to have occurred if one birthday is within a certain number of days of another birthday. It turns out that a mere 14 people in a room will give a greater than even chance of two people having a birthday in common or having a birthday within a day of each other.

A variant of the birthday problem which requires more sophisticated mathematical tools is the birthday problem for boys and girls: if a class consists of an equal number of boys and girls, what would be the minimum group that would give a better than even chance that a boy and a girl shared a birthday?

The result is that a class of 32 (16 girls and 16 boys) would yield the minimum group. This can be compared with 23 in the classic birthday problem.

By changing the question slightly we can get other novelties (but they are not easy to answer). Suppose we have a long queue forming outside a Bob Dylan concert and people join it randomly. As we are interested in birthdays we may discount the possibility of twins or triplets arriving together. As the fans enter they are asked their birthdays. The mathematical question is this: how many people would you expect to be admitted before two consecutive people have the same birthday? Another question: How many people go into the concert hall before a person turns up with the same birthday as Mr. Trevor Thomson (8 March)?

The birthday calculation makes the assumption that birthdays are uniformly distributed and that each birthday has an equal chance of occurring for a person selected at random. Experimental results show this is not exactly true (more are born during the summer months) but it is close enough for the solution to be applicable.

Birthday problems are examples of occupancy problems, in which mathematicians think about placing balls in cells. In the birthday problem, the number of cells is 365 (these are identified with possible birthdays) and the balls to be placed at random in the cells are the people. The problem can be simplified to investigate the probability of two balls falling in the same cell. For the boys-and-girls problem, the balls are of two colours.

It is not only mathematicians who are interested in the birthday problem. Satyendra Nath Bose was attracted to Albert Einstein's theory of light based on photons. He stepped out of the traditional lines of research and considered the physical setup in terms of an occupancy problem. For him, the cells were not days of the year as in the birthday problem but energy levels of the photons. Instead of people being put into cells as in the birthday problem he distributed numbers of photons. There are many applications of occupancy problems in other sciences. In biology, for instance, the spread of epidemics can be modelled as an occupancy problem – the cells in this case are geographical areas, the balls are diseases and the problem is to figure out how the diseases are clustered.

The world is full of amazing coincidences but only mathematics gives us the way of calculating their probability. The classical birthday problem is just the tip of the iceberg in this respect and it is a great entry into serious mathematics with important applications.

(Adapted from [9])

Useful phrases

My view is …, because …
Surely the main point is …
The fact is …
On the one hand …, on the other hand …
Another problem is that …
I think that / In my opinion …
As far as I'm concerned / In my view …

Task 17. Answer the questions.

1. What does the term "statistics" mean? Give five associations with this word. Do you know the origin of the word "statistics"?

2. What do you know about statistics? Give real-world examples that require the use of statistics.

3. Is statistics largely grounded in mathematics?

Task 18. Match the words and phrases with the definitions and then find them in the text (Task 19).

1) poll	a) any gadget or device
2) bias	b) a forecast
3) widget	c) the result or quantity of such a voting
4) prediction	d) selectivity in a sample which influences its distribution and so renders it unable to reflect the desired population parameters

Task 19. Read the text and choose the best answer to each question according to the information given in the text.

What is Statistics

Your company has created a new drug that may cure arthritis. How would you conduct a test to confirm the drug's effectiveness?

The latest sales data have just come in, and your boss wants you to prepare a report for management on places where the company could improve its business. What should you look for? What should you not look for?

You and a friend are at a baseball game, and out of the blue he offers you a bet that neither team will hit a home run in that game. Should you take the bet?

You want to conduct a poll on whether your school should use its funding to build a new athletic complex or a new library. How many people do you have to poll? How do you ensure that your poll is free of bias? How do you interpret your results?

A widget maker in your factory that normally breaks 4 widgets for every 100 it produces has recently started breaking 5 widgets for every 100. When is it time to buy a new widget maker? (And just what is a widget, anyway?)

These are some of the many real-world examples that require the use of statistics.

Statistics, in short, is the study of data. It includes descriptive statistics (the study of methods and tools for collecting data, and mathematical models to describe and interpret data) and inferential statistics (the systems and techniques for making probability-based decisions and accurate predictions based on incomplete (sample) data).

As its name implies, statistics has its roots in the idea of "the state of things". The word itself comes from the ancient Latin term *statisticum collegium*, meaning "a lecture on the state of affairs". Eventually, this evolved into the Italian word *statista*, meaning "statesman", and the German word *Statistik*, meaning "collection of data involving the State". Gradually, the term came to be used to describe the collection of any sort of data.

As one would expect, statistics is largely grounded in mathematics, and the study of statistics has lent itself to many major concepts in mathematics: probability, distributions, samples and populations, the bell curve, estimation, and data analysis.

(Adapted from [11])

Focus on Reading

Task 20. These are the key terms to the text under study (Task 22). Read them carefully and find the best explanations. Use a dictionary for help [6].

1) dizzy	a) cause to be or become; make
2) procedure	b) to grow strongly and vigorously
3) equation	c) unusually large in size
4) glaze	d) conclude
5) vast	e) to collect or be collected gradually; muster
6) render	f) causing or tending to cause vertigo or bewilderment
7) meaningless	g) measure or standard used for comparison
8) preserve	h) relating to a nerve or the nervous system
9) thrive	i) a mode of performing a task
10) neural	j) to maintain protection and favourable conditions for something
11) gather	k) a mathematical statement that two expressions are equal
12) mislead	l) innumerable
13) infer	m) to fit or cover with glass
14) yardstick	n) to give false or misleading information to
15) myriad	o) futile or empty of meaning

Task 21. Read the text below (Task 22) and choose the best heading to each of its parts.

A. Categorical Analysis.

B. Classification.

C. Design of Experiments.

D. Modern Regression.

E. Probability Theory and Mathematical Statistics.

F. Sampling.

G. Survival Analysis.

H. Time Series.

Task 22. Read the text. Are the statements true or false?

1. We're always more adept at using a tool if we can't understand why we're using that tool.

2. One can take this argument a step further to claim that a vast number of students will never actually use a t-test.

3. He had not a large number of subjects.

4. The first puzzle always took longer because the subjects were first learning how to work the puzzle.

5. If we wanted to measure the population of some harmful beetle and its effect on trees, we would not be forced to travel into some forest land and make observations.

6. Regression models relate variables to each other in a linear fashion.

7. Statistical classification is a large field containing methods such as linear discriminant analysis, classification trees, neural networks and other methods.

8. Many types of research don't look at data that are gathered over time.

9. Categorical Analysis is used in a myriad of places, from political polls to analysis of census data to genetics and medicine.

Subjects in Modern Statistics

A remarkable amount of today's modern statistics comes from the original work of R. A. Fisher in the early 20th century. Although there are a dizzying number of minor disciplines in the field, there are some basic, fundamental studies.

The beginning student of statistics will be more interested in one topic or another depending on his or her outside interest. The following is a list of some of the primary branches of statistics.

1. ...

Those of us who are purists and philosophers may be interested in the intersection between pure mathematics and the messy realities of the world. A rigorous study of probability – especially the probability distributions and the distribution of errors – can provide an understanding of where all these statistical procedures and equations come from. Although this sort of rigor is likely to get in the way of a psychologist (for example) learning and using statistics effectively, it is important if one wants to do serious (i.e. graduate-level) work in the field.

That being said, there is good reason for all students to have a fundamental understanding of where all these "statistical techniques and equations" are coming from! We're always more adept at using a tool if we can understand why we're using that tool. The challenge is getting these important ideas to the non-mathematician without the student's eyes glazing over.

One can take this argument a step further to claim that a vast number of students will never actually use a t-test – he or she will never plug those numbers into a calculator and churn through some esoteric equations – but by having a fundamental understanding of such a test, he or she will be able to understand (and question) the results of someone else's findings.

2. ...

One of the most neglected aspects of statistics – and maybe the single greatest reason that statisticians drink – is experimental design. So often a scientist will bring the results of an important experiment to a statistician and ask for help analyzing results only to find that a flaw in the experimental design rendered the results useless. So often we statisticians have researchers come to us hoping that we will somehow magically "rescue" their experiments.

A friend provided me with a classic example of this. In his psychology class he was required to conduct an experiment and summarize its results. He decided to study whether music had an impact on problem solving. He had a large number of subjects (myself included) solve a puzzle first in silence, then while listening to classical music and finally listening to rock and roll, and finally in silence. He measured how long it would take to complete each of the tasks and then summarized the results.

What my friend failed to consider was that the results were highly impacted by a *learning effect* he hadn't considered. The first puzzle always took longer because the subjects were first learning how to work the puzzle. By the third try (when subjected to rock and roll) the subjects were much more adept at solving the puzzle, thus the results of the experiment would seem to suggest that people were much better at solving problems while listening to rock and roll!

The simple act of randomizing the order of the tests would have isolated the "learning effect" and in fact, a well-designed experiment would have allowed him to measure both the effects of each type of music *and* the effect of learning. Instead, his results were meaningless. A careful experimental design can help preserve the results of an experiment, and in fact some designs can save huge amounts of time and money, maximize the results of an experiment, and sometimes yield additional information the researcher had never even considered!

3. ...

Similar to the Design of Experiments, the study of sampling allows us to find a most effective statistical design that will optimize the amount of information we can collect while minimizing the level of effort. Sampling is very different from experimental design however. In a laboratory we can design an experiment and control it from start to finish. But often we want to study something outside of the laboratory, over which we have much less control.

If we wanted to measure the population of some harmful beetle and its effect on trees, we would be forced to travel into some forest land and make observations, for example: measuring the population of the beetles in different locations, noting which trees they were infesting, measuring the health and size of these trees, etc. Sampling design gets involved in questions like "How many measurements do I have to take?" or "How do I select the locations from which I take my measurements?" Without planning for these issues, researchers might spend a lot of time and money only to discover that they really have to sample ten times as many points to get meaningful results or that some of their sample points were in some landscape (like a marsh) where the beetles thrived more or the trees grew better.

4. ...

Regression models relate variables to each other in a linear fashion. For example, if you recorded the heights and weights of several people and plotted them against each other, you would find that as height increases, weight tends to increase too. You would probably also see that a straight line through the data is about as good a way of approximating the relationship as you will be able to find, though there will be some variability about the line. Such linear models are possibly the most important tool available to statisticians. They have a long history and many of the more detailed theoretical aspects were discovered in the 1970s. The usual method for fitting such models is by "least squares" estimation, though other methods are available and are often more appropriate, especially when the data are not normally distributed.

What happens, though, if the relationship is not a straight line? How can a curve be fit to the data? There are many answers to this question. One simple solution is to fit a quadratic relationship, but in practice such a curve is often not flexible enough. Also, what if you have many variables and relationships between them are dissimilar and complicated?

Modern regression methods aim at addressing these problems. Methods such as generalized additive models, projection pursuit regression, neural networks and boosting allow for very general relationships between explanatory variables and response variables, and modern computing power makes these methods a practical option for many applications.

5. ...

Some things are different from others. How? That is, how are objects classified into their respective groups? Consider a bank that is hoping to lend money to customers. Some customers who borrow money will be unable or unwilling to pay it back, though most will pay it back as regular repayments. How is the bank to classify customers into these two groups when deciding which ones to lend money to?

The answer to this question no doubt is influenced by many things, including a customer's income, credit history, assets, already existing debt, age and profession. There may be other influential, measurable characteristics that can be used to predict what kind of customer a particular individual is. How should the bank decide which characteristics are important, and how should it combine this information into a rule that tells it whether or not to lend the money?

This is an example of a classification problem, and statistical classification is a large field containing methods such as linear discriminant analysis, classification trees, neural networks and other methods.

6. ...

Many types of research look at data that are gathered over time, where an observation taken today may have some correlation with the observation taken tomorrow. Two prominent examples of this are the fields of finance (the stock market) and atmospheric science. We've all seen those line graphs of stock prices as they meander up and down over time. Investors are interested in predicting which stocks are likely to keep climbing (i.e. when to buy) and when a stock in their portfolio is falling. It is easy to be misled by a sudden jolt of good news or a simple "market correction" into inferring – incorrectly – that one or the other is taking place!

In meteorology scientists are concerned with the venerable science of predicting the weather. Whether trying to predict if tomorrow will be sunny or determining whether we are experiencing true climate changes (i.e. global warming) it is important to analyze weather data over time.

7. ...

Suppose that a pharmaceutical company is studying a new drug which it is hoped will cause people to live longer (whether by curing them of cancer, reducing their blood pressure or cholesterol and thereby their risk of heart disease, or by some other mechanism). The company will recruit patients into a clinical trial, give some patients the drug and others a placebo, and follow them until they have amassed enough data to answer the question of whether, and by how long, the new drug extends life expectancy.

Such data present problems for analysis. Some patients will have died earlier than others, and often some patients will not have died before the clinical trial completes. Clearly, patients who live longer contribute informative data about the ability (or not) of the drug to extend life expectancy. So how should such data be analyzed?

Survival analysis provides answers to this question and gives statisticians the tools necessary to make full use of the available data to correctly interpret the treatment effect.

8. ...

In laboratories we can measure the weight of fruit that a plant bears, or the temperature of a chemical reaction. These data points are easily measured with a yardstick or a thermometer, but what about the colour of a person's eyes or her attitudes regarding the taste of broccoli? Psychologists can't measure someone's anger with a measuring stick, but they can ask their patients if they feel "very angry" or "a little angry" or "indifferent". Entirely different methodologies must be used in statistical analysis from these sorts of experiments.

Categorical Analysis is used in a myriad of places, from political polls to analysis of census data to genetics and medicine.

(Adapted from [11])

Focus on Vocabulary

Task 23. Choose words from the list to fill in the gaps.

Fundamental, straight line, branches, take, survival, effectively, understanding, analyzing, interest, errors.

1. … studies.

2. Outside … .
3. The primary … of statistics.
4. The distribution of … .
5. Using statistics … .
6. To have a fundamental … .
7. … results.
8. How long it would … .
9. A … through the data.
10. … analysis.

Focus on Speaking

Task 24. Discuss the question "Why Should I Learn Statistics?" Work in pairs and make a list of reasons.

Task 25. Read the text and find ideas similar to yours.

Why Should I Learn Statistics?

Imagine reading a book for the first few chapters and then becoming able to get a sense of what the ending will be like – this is one of the great reasons to learn statistics. With the appropriate tools and solid grounding in statistics, one can use a limited sample (e.g. read the first five chapters of *Pride & Prejudice*) to make intelligent and accurate statements about the population (e.g. predict the ending of *Pride & Prejudice*). This is what knowing statistics and statistical tools can do for you.

In today's information – overloaded age, statistics is one of the most useful subjects anyone can learn. Newspapers are filled with statistical data, and anyone who is ignorant of statistics is at risk of being seriously misled about important real-life decisions such as what to eat, who is leading the polls, how dangerous smoking is, etc. Knowing a little about statistics will help one to make more informed decisions about these and other important questions. Furthermore, statistics are often used by politicians, advertisers, and others to twist the truth for their own gain. For example, a company selling the cat food brand "Cato" (a fictitious name here), may claim quite truthfully in their advertisements that eight out of ten cat owners said that their cats preferred Cato brand cat food to "the other leading brand" cat food.

What they may not mention is that the cat owners questioned were those they found in a supermarket buying Cato.

"The best thing about being a statistician is that you get to play in everyone else's backyard". (John Tukey, Princeton University)

More seriously, those proceeding to higher education will learn that statistics is the most powerful tool available for assessing the significance of experimental data, and for drawing the right conclusions from the vast amounts of data faced by engineers, scientists, sociologists, and other professionals in most spheres of learning. There is no study with scientific, clinical, social, health, environmental or political goals that does not rely on statistical methodologies. The basic reason for that is that variation is ubiquitous in nature and probability and statistics are the fields that allow us to study, understand, model, embrace and interpret variation.

(Adapted from [12])

Task 26. Do you agree with the statement "The best thing about being a statistician is that you get to play in everyone else's backyard"? Give your reasons.

Useful phrases

Agreement

Yes, I agree entirely here.
I fully agree.
I couldn't agree more.
I am of the same opinion.
You know, that's exactly what I think.
It really looks like that.
Yes, that's true.
That's my way of looking at it too.
What you say is perfectly true.
That's a good point.
That's just what I was thinking.

Disagreement

There may be something in what you say but ...
I see your point but ...
I can't possibly ...

I shouldn't say so.
I've got an argument to oppose.
It's not at all the same thing.
On the one hand ..., on the other hand ...
Well, I'm not so sure.
I wouldn't say that exactly.
It might be right but ...

Focus on Reading

Task 27. Match the words and phrases with the definitions and then find them in the text.

1) pivotal	a) the difference between an observed value in a series of such values and their arithmetic mean
2) pedigree	b) confine, bound, limit
3) flee (fled)	c) the scalar product of the operator
4) divergence	d) to run move quickly; rush; speed
5) hung over	e) a considered opinion; judgment
6) deviation	f) of crucial importance
7) estimation	g) suffering from the effects of a hangover
8) restrict	h) a genealogical tree

Task 28. Read the text. Are these statements true or false? Correct the false statements.

1. The "normal" curve has been called the equivalent of the straight line in mathematics.

2. The normal curve is not prescribed by a specific mathematical formula which creates a bell-shaped curve; a curve with one hump and which tails away on either side.

3. If the standard deviation is small the data is close together and has little variability, but if it is large, the data is spread out.

4. The chance of throwing a head each time is $p = 2/3$.

5. The larger the value of n the better the approximation and tossing the coin 100 times qualifies as large.

The Normal Curve

The "normal" curve plays a pivotal role in statistics. It has been called the equivalent of the straight line in mathematics. It certainly has important mathematical properties but if we set to work analyzing a block of raw data we would rarely find that it followed a normal curve exactly.

The normal curve is prescribed by a specific mathematical formula which creates a bell-shaped curve; a curve with one hump and which tails away on either side. The significance of the normal curve lies less in nature and more in theory, and in this it has a long pedigree. In 1733 Abraham de Moivre, a French Huguenot who fled to England to escape religious persecution, introduced it in connection with his analysis of chance. Pierre Simon Laplace published results about it and Carl Friedrich Gauss used it in astronomy, where it is sometimes referred to as the Gaussian law of error.

Adolphe Quetelet used the normal curve in his sociological studies published in 1835, in which he measured the divergence from the "average man" by the normal curve. In other experiments he measured the heights of French conscripts and the chest measurements of Scottish soldiers and assumed these followed the normal curve. In those days there was a strong belief that most phenomena were "normal" in this sense.

The Cocktail Party

Let's suppose that Georgina went to a cocktail party and the host, Sebastian, asked her if she had come far? She realized afterwards it was a very useful question for cocktail parties – it applies to everyone and invites a response. It is not taxing and it starts the ball rolling if conversation is difficult.

The next day, slightly hung over, Georgina travelled to the office wondering if her colleagues had come far to work. In the staff canteen she learned that some lived around the corner and some lived 50 miles away – there was a great deal of variability. She took advantage of the fact that she was the Human Resources Manager of a very large company to tack a question on the end of her annual employee questionnaire: "How far have you travelled to work today?" She wanted to work out the average distance of travel of the company's staff. When Georgina drew a histogram of results the distribution showed no particular form, but at least she could calculate the average distance travelled (Fig. 6.1).

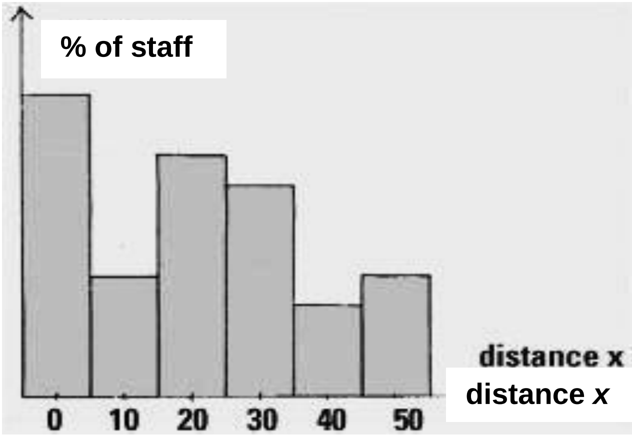


Fig. 6.1. **Georgina's histogram of distance travelled by her colleagues to work**

This average turned out to be 20 miles. Mathematicians denote this by the Greek letter "mu", written μ, and so here $\mu = 20$. The variability in the population is denoted by the Greek letter sigma, written σ, which is sometimes called the standard deviation. If the standard deviation is small, the data is close together and has little variability, but if it is large, the data is spread out. The company's marketing analyst, who had trained as a statistician, showed Georgina that she might have got around the same value of 20 by sampling. There was no need to ask all the employees. This estimation technique depends on the Central Limit Theorem.

Take a random sample of staff from all of the company's workforce. The larger the sample, the better, but 30 employees will do nicely. In selecting this sample at random it is likely there will be people who live around the corner and some long-distance travellers as well. When we calculate the average distance for our sample, the effect of the longer distances will average out the shorter distances. Mathematicians write the average of the sample as $\bar{x}$, which is read as "*x* bar". In Georgina's case, it is most likely that the value of $\bar{x}$ will be near 20, the average of the population. Though it is certainly possible, it is unlikely that the average of the sample will be very small or very large.

The Central Limit Theorem is one reason why the normal curve is important to statisticians. It states that the actual distribution of the sample averages $\bar{x}$ approximates to a normal curve whatever the distribution of x. What does this mean? In Georgina's case, x represents the distance from the workplace and is the average of a sample. The distribution of x in Georgina's histogram is nothing like a bell-shaped curve, but the distribution of $\bar{x}$ is, and it is centred on $\mu = 20$ (Fig. 6.2).

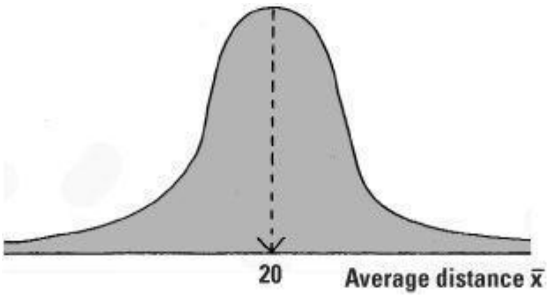


Fig. 6.2. **How the sample average is distributed**

This is why we can use the average of a sample as an estimate of the population average *μ*. The variability of the sample averages is an added bonus. If the variability of the *x* values is the standard deviation *σ*, the variability of $\bar{x}$ is $\sigma/\sqrt{n}$ where *n* is the size of the sample we select. The larger the sample size, the narrower the normal curve, and the better the estimate of *μ*.

Other Normal Curves

Let's do a simple experiment. We'll toss a coin four times. The chance of throwing a head each time is $p = 1/2$. The result for the four throws can be recorded using *H* for heads and *T* for tails, arranged in the order in which they occur. Altogether there are 16 possible outcomes. For example, we might obtain three heads in the outcome *THHH*. There are in fact four possible outcomes giving three heads (the others are *HTHH*, *HHTH*, *HHHT*) so the probability of three heads is 4/16 = 0.25.

With a small number of throws, the probabilities are easily calculated and placed in a table, and we can also calculate how the probabilities are distributed. The number of combinations row can be found from Pascal's triangle:

Number of heads	0	1	2	3	4
Number of combinations	1	4	6	4	1
Probability	0.0625	0.25	0.375	0.25	0.0625
	(= 1/16)	(= 4/16)	(= 6/16)	(= 4/16)	(= 1/16)

This is called a binomial distribution of probabilities, which occurs where there are two possible outcomes (here a head or a tail). These probabilities may be represented by a diagram in which both the heights and areas describe them (Fig. 6.3).

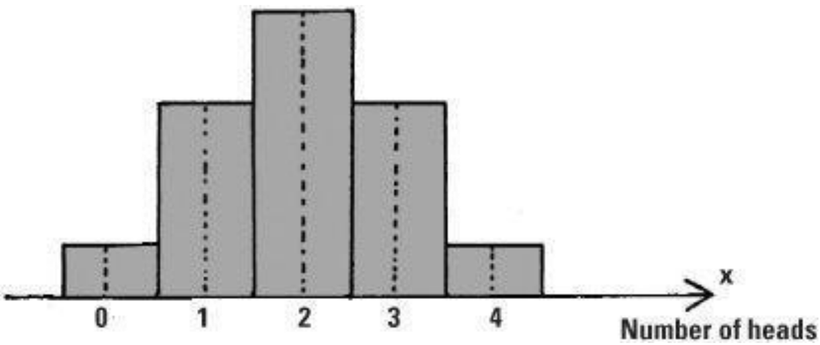


Fig. 6.3. **The number of heads in four tosses of a coin, according to the binomial distribution**

Tossing the coin four times is a bit restrictive. What happens if we throw it a large number, say 100, times? The binomial distribution of probabilities can be applied where n = 100, but it can usefully be approximated by the normal bell-shaped curve with mean μ = 50 (as we would expect 50 heads when tossing a coin 100 times) and variability (standard deviation) of σ = 5 (Fig. 6.4). This is what de Moivre discovered in the 16th century.

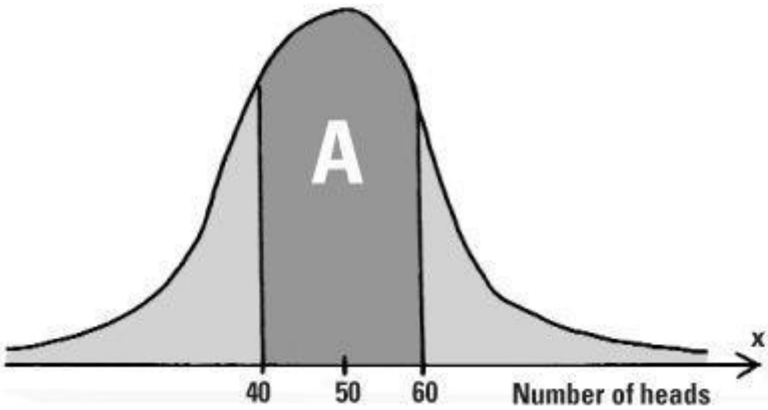


Fig. 6.4. **Distribution of the probability for the number of heads in 100 throws of a coin**

For large values of n, the variable x which measures the number of successes fits the normal curve increasingly well. The larger the value of n, the better the approximation and tossing the coin 100 times qualifies as large. Now let's say we want to know the probability of throwing between 40 and 60 heads. The area A shows the region we're interested in and gives us the probability of tossing between 40 and 60 heads which we write as $prob(40 \le x \le 60)$. To find the actual numerical value we need to use precalculated mathematical tables, and once this has been done, we find $prob(40 \le x \le 60) = 0.9545$. This shows that getting between 40 and 60 heads in 100 tosses of a coin is 95.45 %, which means that this is very likely.

The area left over is 1 – 0.9545 which is a mere 0.0455. As the normal curve is symmetric about its middle, half of this will give the probability of getting more than 60 heads in a 100 tosses of the coin. This is just 2.275 % and represents a very slim chance indeed. If you visit Las Vegas this would be a bet to leave well alone.

(Adapted from [5])

Focus on Vocabulary

Task 29. Choose words from the list to fill in the gaps.

Average, bell-shaped, straight line, at random, binomial distribution, estimation, the coin, actual, numerical, strong, deviation, the population.

1. The … in mathematics.
2. A … curve.
3. A … belief.
4. The … distance of travel.
5. The standard … .
6. … technique.
7. In selecting this sample … .
8. The average of … .
9. A … of probabilities.
10. Tossing … four times.
11. The … value.

Focus on Grammar

Task 30. Write sentences using the verbs.

↗	↘	→
increase	decrease	maintain
raise	drop	keep
step up	cut	stay at
expand	go down	remain stable
improve	decline	
grow	fall	
rise		

Focus on Speaking

Task 31. Choose a topic, draw a graph or a diagram and make your own presentation.

Useful phrases for presentations

Signalling a start

Right, ladies and gentlemen. Shall we begin?

OK, I'd like to begin by ...

Right then, everybody ...

Greetings

Good morning, ladies and gentlemen.

I'd like to welcome you all on behalf of NKS.

I'd like to say how happy I am to be here today.

Introducing yourself

My name is John Brown.

I'm the Marketing Manager here.

I'd like to introduce myself. My name is ...

Let me introduce myself. My name is ...

Introducing people

I'd like to introduce Mr. Loop from Intercom.

May I introduce ...

Let me introduce ...

The objective of your presentation

My objective this afternoon is to inform you ...

I'm here today to give you ...

My purpose today is to introduce you to ...

My aim this morning is to ...

The length of time your presentation will take

My presentation will last twenty-five minutes.

I will talk for fifteen minutes.

Structuring

My presentation is split into three key areas.

Firstly ... Secondly ... Thirdly ...

To begin with ... After this ... Finally ...

Saying when your audience may ask questions

There will be time for questions at the end ...

Please feel free to interrupt me as I go along.

Introducing topics

I want to look at ...

I'd like to review ...

I want to discuss ...

I'm going to analyse ...

I want to cover ...

I'd like to talk about ...

Moving on

Let's now move on to ...

I'd like to go on to ...

This brings me to ...

I now want to ...

Looking back

Let's look back for a moment ...

To go back to ...

As I said before ...

Adding

In addition ...

I might add that ...

Furthermore ...

Moreover ...

Dealing with interruptions

Perhaps I could return to that point later on?

If I might just finish?

If you'd allow me to continue?

Summarising and concluding

To sum up then, ...

To summarise my main points ...

I'd like to conclude by reminding you that ...

Let me end by ...

I'd like to finish ...

In conclusion, may I remind you ...

Finally, ...

Asking for questions

You no doubt have many questions ...
If there are any questions ...
I'm sure you have many questions ...

Playing for time

That's an interesting question.
I'm glad you asked that question.
You've raised an important point there.
That's a difficult question.

Closing question time

Perhaps on that note we could end?
I'm afraid that's all we have time for.

Thanking

I'd like to thank you all on behalf of Company X.
I'd like to thank you all for your participation.

Key

Unit 1

Task 1. 1 – c; 2 – d; 3 – a; 4 – e; 5 – b.

Task 3. 1 – g; 2 – d; 3 – e; 4 – b; 5 – c; 6 – k; 7 – a; 8 – j; 9 – l; 10 – h; 11 – f; 12 – i.

Task 6. 1 – false; 2 – false; 3 – true; 4 – false; 5 – false; 6 – true; 7 – false.

Task 10. 1 – h; 2 – d; 3 – e; 4 – f; 5 – g; 6 – b; 7 – a; 8 – i; 9 – j; 10 – c.

Task 11. 1 – sciences; 2 – calculate; 3 – methods; 4 – language; 5 – branches.

Task 12. 1 – a; 2 – b; 3 – c; 4 – d; 5 – a.

Task 13. 1 – d; 2 – f; 3 – e; 4 – a; 5 – c; 6 – b; 7 – l; 8 – j; 9 – s; 10 – h; 11 – n; 12 – q; 13 – p; 14 – k; 15 – r; 16 – m; 17 – g; 18 – o; 19 – i.

Task 14. 1 – Actuary studies; 2 – Auditing; 3 – Engineering; 4 – Statistics.

Task 16. 1 – Mathematics; 2 – Arithmetic and Algebra; 3 – Geometry and Trigonometry; 4 – Graphs and Analytic Geometry.

Task 17. were known, have been developed, is based, have been devised, are represented, is called.

Unit 2

Task 2. 1 – b; 2 – c; 3 – d; 4 – a.

Task 5. 1 – decimal; 2 – translated; 3 – system; 4 – resistance; 5 – replaced; 6 – limited; 7 – computation.

Task 6. 1 – characters; 2 – contain; 3 – invention; 4 – suffice; 5 – proper.

Task 7. 1 – f; 2 – e; 3 – g; 4 – a; 5 – b; 6 – d; 7 – c.

Task 9. 1 – c; 2 – a; 3 – b.

Task 11. 1 – j; 2 – f; 3 – k; 4 – a; 5 – h; 6 – c; 7 – b; 8 – e; 9 – d; 10 – g; 11 – i.

Task 12. 1 – Addition; 2 – Subtraction; 3 – Multiplication; 4 – Division.

Task 13. 1 – i; 2 – g; 3 – e; 4 – a; 5 – h; 6 – c; 7 – d; 8 – f; 9 – b.

Task 14. 1 – b; 2 – c; 3 – d 4 – a.

Task 19. 1 – c; 2 – e; 3 – a; 4 – b; 5 – d.

Task 25. 1 – b; 2 – a.

Task 28. 1 – f; 2 – d; 3 – h; 4 – c; 5 – g; 6 – a; 7 – e; 8 – b.

Task 29. 1 – d; 2 – f; 3 – e; 4 – a; 5 – c; 6 – b.

Task 32. 1 – d; 2 – f; 3 – e; 4 – a; 5 – c; 6 – b.

Task 43. 1 – c; 2 – f; 3 – e; 4 – a; 5 – d; 6 – b.

Task 47. picto, bar, line circle.

Task 52. 1 – has gone down; 2 – increased; 3 – hasn't changed; 4 – have remained; 5 – have dropped; 6 – didn't rise.

Unit 3

Task 2. 1 – g; 2 – n; 3 – h; 4 – j; 5 – m; 6 – c; 7 – a; 8 – i; 9 – e; 10 – d; 11 – l; 12 – o; 13 – k; 14 – b; 15 – f.

Task 4. 1 – Two premises and a conclusion; 2 – Propositional logic; 3 – Other logics.

Task 5. 1 – true; 2 – false; 3 – true; 4 – false; 5 – true; 6 – false.

Task 7. 1 – summer; 2 – inescapable; 3 – makes; 4 – validity; 5 – question; 6 – conclusion; 7 – connotations; 8 – argument; 9 – syllogism; 10 – assume; 11 – active.

Task 9. In order; However; Similarly; Thus; Such.

Task 12. 1 – d; 2 – g; 3 – b; 4– j; 5 – e; 6 – a; 7 – k; 8 – i; 9 – l; 10 – c; 11 – f; 12 – h.

Task 15.

Direction	**Verb**	**Noun**
↗	Rose (to) Increased (to) Went up (to) Climbed (to) Boomed	A rise An increase Growth An upward trend A boom (a dramatic rise)
↘	Fell (to) Declined (to) Decreased (to) Dipped (to) Dropped (to) Went down (to) Slumped (to) Reduced (to)	A decrease A decline A fall A drop A slump (a dramatic fall) A reduction

→	Levelled out (at) Did not change Remained stable (at) Remained steady (at) Stayed constant (at) Maintained the same level	A levelling out No change
→	Fluctuated (around) Peaked (at) Plateaued (at) Stood at (we use this phrase to focus on a particular point before we mention the movement, for example: In the first year, unemployment stood at …)	A fluctuation A peak (of) Reached a plateau

Unit 4

Task 3. 1 – d; 2 – e; 3 – g; 4 – a; 5 – f; 6 – b; 7 – c.

Task 4. 1 – false; 2 – true; 3 – true; 4 – false; 5 – true; 6 – true.

Task 6. 1 – new; 2 – rules; 3 – incline; 4 – curving; 5 – crate; 6 – unchanging; 7 – energy; 8 – constantly; 9 – problem; 10 – chunks; 11 – infinitely.

Task 8. 1 – concepts; 2 – proceeding; 3 – mathematical; 4 – associating; 5 – object; 6 – pairing; 7 – resulting; 8 – mapping; 9 – intuitive; 10 – characterizes.

Task 11. 1 – c; 2 – e; 3 – a; 4 – i; 5 – g; 6 – b; 7 – f; 8 – d; 9 – h.

Task 17. 1 – domain; 2 – argument; 3 – values; 4 – function; 5 – between; 6 – mathematical; 7 – connection; 8 – large; 9 – unbounded; 10 – tending.

Task 25. 1 – c; 2 – f; 3 – d; 4 – a; 5 – g; 6 – h; 7 – e; 8 – b.

Task 28. 1 – labour; 2 – logarithms; 3 – operation; 4 – multiplication; 5 – system; 6 – base; 7 – exponent; 8 – decimal.

Task 29. has; counted; is said; would; did; gets; add; would; added; is.

Task 30. 1 – f; 2 – i; 3 – b; 4 – c; 5 – h; 6 – a; 7 – e; 8 – d; 9 – g.

Task 34. 1 – positive; 2 – domain; 3 – integral; 4 – represent; 5 – argument; 6 – sequence; 7 – bounds; 8 – values; 9 – numbers; 10 – nonintegral; 11 – greater; 12 – applicable.

Task 37. 1 – d; 2 – c; 3 – e; 4 – a; 5 – f; 6 – b.

Task 42. 1 – study; 2 – differentiation; 3 – geometrical; 4 – embrace; 5 – area; 6 – curve; 7 – vertical; 8 – strips; 9 – approximately; 10 – set; 11 – subdivision.

Task 46. 1 – c; 2 – d; 3 – a; 4 – b; 5 – f; 6 – j; 7 – i; 8 – e; 9 – g; 10 – h.

Task 51. 1 – differential; 2 – involving; 3 – single; 4 – highest; 5 – contributions; 6 – form; 7 – algebraic; 8 – unknown; 9 – simultaneous; 10 – values; 11 – solutions; 12 – obtainable.

Task 52. 1 – analytical; 2 – elementary; 3 – mathematical; 4 – requirement; 5 – calculated; 6 – computation; 7 – differential; 8 – restriction; 9 – integral; 10 – solution; 11 – numerical; 12 – derivatives; 13 – prescribed; 14 – discrete.

Unit 5

Task 4. 1 – Algebra; 2 – Origins; 3 – The modern world; 4 – The abstract.

Task 5. 1 – true; 2 – false; 3 – true; 4 – false; 5 – false; 6 – true.

Task 7. 1 – backwards; 2 – thinking; 3 – a trial and error; 4 – equations; 5 – two-dimensional; 6 – dimensions; 7 – system; 8 – applications; 9 – method.

Task 9. 1 – plus; 2 – minus; 3 – operation; 4 – double sign; 5 – equals; 6 – multiplication; 7 – dot; 8 – division; 9 – parentheses; 10 – brackets; 11 – braces; 12 – greater than; 13 – less than; 14 – *a* is greater than 6; 15 – 3 is less than 5.

Task 11. 1 – equation; 2 – formula; 3 – quotient; 4 – average; 5 – value; 6 – unknown.

Task 13. 1 – Factors; 2 – Coefficients; 3 – Combining Terms; 4 – Adding and Subtracting Like Terms.

Task 19. 1 – true; 2 – false; 3 – false; 4 – false.

Task 34. 1 – true; 2 – true; 3 – true; 4 – false; 5 – true; 6 – false.

Task 38. 1 – coordinates; 2 – negative; 3 – right; 4 – units.

Unit 6

Task 2. 1 – d; 2 – c; 3 – a; 4 – f; 5 – b; 6 – e.

Task 4. 1 – e; 2 – d; 3 – f; 4 – b; 5 – a; 6 – c.

Task 5. 1 – Probability; 2 – Origins of Probability; 3 – Playing Craps; 4 – The Monkey on a Typewriter.

Task 6. 1 – true; 2 – false; 3 – false; 4 – true; 5 – false; 6 – true; 7 – false.

Task 9. 1 – coin; 2 – tail; 3 – heads; 4 – probability; 5 – impossible / certainty; 6 – rolling; 7 – throw; 8 – betting; 9 – outcomes; 10 – key.

Task 10. 1 – statement; 2 – outcome; 3 – prediction; 4 – assertion; 5 – assigning; 6 – hopeless; 7 – assumption; 8 – truth; 9 – measure; 10 – unique; 11 – elements.

Task 11. 1 – c; 2 – a; 3 – d; 4 – b.

Task 15. 1 – f; 2 – k; 3 – g; 4 – c; 5 – m; 6 – a; 7 – o; 8 – l; 9 – h; 10 – p; 11 – b; 12 – n; 13 – e; 14 – i; 15 – d; 16 – j.

Task 18. 1 – c; 2 – d; 3 – a; 4 – b.

Task 20. 1 – f; 2 – i; 3 – k; 4 – m; 5 – c; 6 – a; 7 – o; 8 – j; 9 – b; 10 – h; 11 – e; 12 – n; 13 – d; 14 – g; 15 – l.

Task 21. 1 – Probability Theory and Mathematical Statistics; 2 – Design of Experiments; 3 – Sampling; 4 – Modern Regression; 5 – Classification; 6 – Time Series; 7 – Survival Analysis; 8 – Categorical Analysis.

Task 22. 1 – false; 2 – true; 3 – false; 4 – true; 5 – false; 6 – true; 7 – true; 8 – false; 9 – true.

Task 23. 1 – fundamental; 2 – interest; 3 – branches; 4 – errors; 5 – effectively; 6 – understanding; 7 – analyzing; 8 – take; 9 – straight line; 10 – survival.

Task 27. 1 – f; 2 – h; 3 – d; 4 – c; 5 – g; 6 – a; 7 – e; 8 – b.

Task 28. 1 – true; 2 – false; 3 – true; 4 – false; 5 – true.

Task 29. 1 – straight line; 2 – bell-shaped; 3 – strong; 4 – average; 5 – deviation; 6 – estimation; 7 – at random; 8 – the population; 9 – binomial distribution; 10 – the coin; 11 – actual numerical.

Glossary

Addition – a mathematical operation in which the sum of two numbers or quantities is calculated; usually indicated by the symbol "+".

Algebra – dealing with letters instead of numbers so as to extend arithmetic. Algebra is now a general method applicable to all mathematics and its applications. The word "algebra" derives from *al-jabr* used in an Arabic text of the ninth century AD.

Algorithm – a mathematical recipe; a set routine for solving a problem.

Axiom – a statement, for which no justification is sought, that is used to define a system. The term "postulate" served the same purpose for the Greeks but for them it was a self-evident truth.

Binary number system – a number system based on two symbols, 0 and 1, fundamental for computer calculation.

Cardinality – the number of objects in a set. The cardinality of the set {*a*, *b, c, d,* e} is 5, but cardinality can also be given meaning in the case of infinite sets.

Commutative – multiplication in algebra is commutative if $a \times b = b \times a$, as in ordinary arithmetic (e.g. 2 × 3 = 3 × 2). In many branches of modern algebra this is not the case (e.g. matrix algebra).

Conic section – the collective name for the classical family of curves which includes circles, straight lines, ellipses, parabolas and hyperbolas. Each of these curves is found as cross-sections of a cone.

Corollary – a minor consequence of a theorem.

Counter example – a single example that disproves a statement. The statement "All swans are white" is shown to be false by producing a black swan as a counterexample.

Calculate – to solve (one or more problems) by a mathematical procedure; compute.

Denominator – the bottom part of a fraction. In the fraction 3/7, the number 7 is the denominator.

Differentiation – a basic operation in calculus which produces the derivative or rate of change. For an expression describing how distance depends on time, for example, the derivative represents the velocity. The derivative of the expression for velocity represents acceleration.

Discrete – a term used in opposition to "continuous". There are gaps between discrete values, such as the gaps between the whole numbers 1, 2, 3, 4, …

Distribution – the range of probabilities of events that occur in an experiment or situation. For example, the Poisson distribution gives the probabilities of *x* occurrences of a rare event happening for each value of *x*.

Divident – a number to be divided by a divisor.

Divisor – a whole number that divides into another whole number exactly. The number 2 is a divisor of 6 because 6 ÷ 2 = 3. So 3 is another because 6 ÷ 3 = 2.

Empty set – a set with no objects in it. Traditionally denoted by Ø, it is a useful concept in set theory.

Exponent – a notation used in arithmetic. Multiplying a number by itself, 5 × 5 is written 5^2 with the exponent 2. The expression 5 × 5 × 5 is written 5^3, and so on. The notation may be extended: for example, the number $5^{1/2}$ means the square root of 5. Equivalent terms are power and index.

Fraction – a whole number divided by another, for example 3/7.

Geometry – dealing with the properties of lines, shapes, and spaces, the subject was formalized in Euclid's "The Elements" in the third century BC. Geometry pervades all of mathematics and has now lost its restricted historical meaning.

Greatest common divisor, gcd – the gcd of two numbers is the largest number which divides into both exactly. For example, 6 is the gcd of the two numbers 18 and 84.

Hexadecimal system – a number system of base 16 based on 16 symbols, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, and F. It is widely used in computing.

Hypothesis – a tentative statement awaiting either proof or disproof. It has the same mathematical status as a conjecture.

Imaginary numbers – numbers involving the "imaginary" $i = \sqrt{-1}$. They help form the complex numbers when combined with ordinary (or "real') numbers.

Integration – a basic operation in calculus that measures area. It can be shown to be the inverse operation of differentiation.

Irrational numbers – numbers which cannot be expressed as a fraction (e.g. the square root of 2).

Iteration – starting off with a value *a* and repeating an operation is called iteration. For example, starting with 3 and repeatedly adding 5 we have the iterated sequence 3, 8, 13, 18, 23, …

Matrix – an array of numbers or symbols arranged in a square or rectangle. The arrays can be added together and multiplied and they form an algebraic system.

Multiplication – an arithmetical operation, defined initially in terms of repeated addition, usually written $a \times b$, $a \cdot b$, or ab. Multiplication by fractions can then be defined in the light of the associative and commutative properties; multiplication by $1/n$ is equivalent to multiplication by 1 followed by division by n for example $0.3 \times 0.7 = 0.3 \cdot 7/10 = (0.3 \times 7)/10 = 2.1/10 = 0.21$.

Numerator – the top part of a fraction. In the fraction 3/7, the number 3 is the numerator.

One-to-one correspondence – the nature of the relationship when each object in one set corresponds to exactly one object in another set, and vice versa.

Optimum solution – many problems require the best or optimum solution. This may be a solution that minimizes cost or maximizes profit, as occurs in linear programming.

Place-value system – the magnitude of a number depends on the position of its digits. In 73, the place value of 7 means "7 tens" and of 3 means "3 units".

Prime number – a whole number that has only itself and 1 as divisors. For example, 7 is a prime number but 6 is not (because 6 : 2 = 3). It is customary to begin the prime number sequence with 2.

Rational numbers – numbers that are either whole numbers or fractions.

Remainder – if one whole number is divided by another whole number, the number left over is the remainder. The number 17 divided by 3 gives 5 with remainder 2.

Sequence – a row (possibly infinite) of numbers or symbols.

Series – a row (possibly infinite) of numbers or symbols added together.

Set – a collection of objects: for example, the set of some items of furniture could be F = {chair, table, sofa, stool, cupboard}.

Square number – the result of multiplying a whole number by itself. The number 9 is a square number because 9 = 3 × 3. The square numbers are 1, 4, 9, 16, 25, 36, 49, 64, … .

Square root – the number which, when multiplied by itself, equals a given number. For example, 3 is the square root of 9 because 3 × 3 = 9.

Squaring the circle – the problem of constructing a square with the same area as that of a given circle – using only a ruler for drawing straight lines and a pair of compasses for drawing circles. It cannot be done.

Symmetry – the regularity of a shape. If a shape can be rotated so that it fills its original imprint it is said to have rotational symmetry. A figure has mirror symmetry if its reflection fits its original imprint.

Theorem – a term reserved for an established fact of some consequence.

Transcendental number – a number that cannot be the solution of an algebraic equation, like $ax^2 + bx + c = 0$ or one where x has an even higher power. The number π is a transcendental number.

Twin primes – two prime numbers separated by at most one number. For example, the twins 11 and 13. It is not known whether there is an infinity of these twins.

Unit fraction – fractions with the top (numerator) equal to 1. The ancient Egyptians partly based their number system on unit fractions.

***x* – *y* axes** – the idea due to René Descartes of plotting points having an x-coordinate (horizontal axis) and y-coordinate (vertical axis).

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Unit 1

* Figures indicate the number of the task.

Unit 2

Unit 3

Unit 4

Unit 5

Unit 6

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Contents

НАВЧАЛЬНЕ ВИДАННЯ

Борова Тетяна Анатоліївна
Мілов Олександр Володимирович

АНГЛІЙСЬКА МОВА ДЛЯ БІЗНЕС-АНАЛІТИКІВ

Навчальний посібник

У трьох частинах

ЧАСТИНА 1. МАТЕМАТИЧНІ ОСНОВИ БІЗНЕС-АНАЛІЗУ

(англ. мовою)

Відповідальний за видання *Т. А. Борова*

Відповідальний редактор *М. М. Оленич*

Редактор *З. В. Зобова*

Коректор *З. В. Зобова*

Запропоновано матеріал щодо оволодіння англійською мовою для бізнес-аналітиків. Завдання сприяють формуванню професійної комунікативної компетентності математичного спрямування. Структура посібника відповідає сучасній системі організації навчального процесу з вивчення англійської мови професійного спілкування. Посібник може бути використаний як для навчання у групах, так і для самостійного вивчення.

Рекомендовано для студентів спеціальності 051 "Економіка", викладачів, а також усіх, хто вивчає та використовує англійську мову у своїй професійній діяльності, що пов'язана з використанням математичних методів в економіці.

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