# MATHEMATICAL MODEL OF HIGH-PERFORMANCE DIAMOND GRINDING

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**Abstract:** Analytical dependences to determine the steady cutting relief of the diamond wheel are given. It is theoretically and experimentally established the presence of a minimum of the relative consumption of diamond when grinding with diamond wheels on metal bonds, the value of which is determined by the ratio of the volume of intergranular space required for free placement of chips, and the magnitude of linear wear of grains before their volume destruction. It is shown that the minimum of the relative consumption of diamond and the maximum of the conditional cutting stress correspond to approximately the same grinding mode. From these positions, the influence of the strength of the metal being processed on the parameters of the cutting relief of the diamond wheel and the technological parameters of grinding has been established, as well as the practical recommendations have been given.

**Keywords:** mathematical model, diamond grinding, energy balance, machining productivity, relative diamond consumption, conditional cutting stress, surface roughness.

## Introduction

Grinding by diamond wheels on durable metal bonds provides high quality and productivity. It is the main method of machining metallic and non-metallic materials of increased hardness (hard alloys, wear-resistant coatings, diamonds, ceramics and ferrites, technical glass, etc.) [1]. Grinding efficiency is greatly enhanced when using electrophysical and chemical methods of dressing independently or in conjunction with the process of cutting conductive materials [2-4]. Under these conditions, it can be maximize the potential of the diamond wheel, for which it is necessary to correctly determine the optimal grinding conditions, including the parameters of the cutting mode, the characteristics of the diamond wheel, etc. Therefore, the paper proposed a mathematical model of high-performance diamond grinding, which allows a scientifically sound approach to the choice of optimal processing conditions for the implementation of diamond grinding.

## Analytical research

When solving problems of analytical optimization of diamond grinding, researchers, as a rule, proceed from geometric and kinematic ideas about the process of mass removal of metal by cutting grains. The physical side of grinding, which takes into account the wear of the diamond wheel and the continuous change in the parameters of its cutting relief, is not taken into account in the calculations. Optimization is thus considered for a particular cutting relief of the diamond wheel, which, regardless of changing grinding conditions, remains constant.

Studies have shown [4, 5] that for each combination of the parameters of the grinding mode, the characteristics of the diamond wheel and the material being processed, a very specific stable cutting relief is formed on the working surface of the diamond wheel. In this case it is realized a minimum of the relative consumption of diamond, which is due to the transition of one mechanism of wear of the diamond wheel to another - the dominant volume destruction of grains to their premature loss from the bond from the action of static loads (Fig. 1).

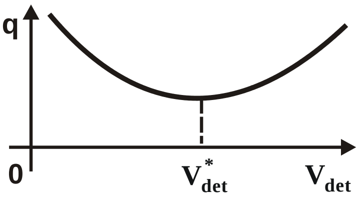


Fig. 1. Character of change in relative consumption of diamond with increasing the speed of workpiece during circular longitudinal grinding

The decrease in the relative consumption of diamond q with a change in the speed of the workpiece in the range (0, $V^*_{det}$) is associated with an increase in the linear wear of the grains before their bulk destruction, since a smoother relief is formed on the working surface of the wheel. When $V_{det} = V^*_{det}$, the cutting force acting on the maximum protruding grain, at the same time is to the force destroying the grain, and the force holding it together with the bond. Under the condition $V_{det} < V^*_{det}$ the force of grain retention in a bond is greater than the destructive force, and under the condition $V_{det} > V^*_{det}$ – vice versa. The critical depth of embedding of grains in a bond at minimum points of the relative consumption of diamond, regardless of the combination of grinding mode parameters, always remains approximately the same, while the amount of linear wear of grains before their bulk destruction, the maximum cut thickness and the intergranular space of the diamond wheel for free placement of chips are changing. This condition is true for various characteristics of diamond wheels, grinding modes and therefore can be taken as a condition for optimizing grinding, because it reflects not only the geometric and kinematic, but also the physical side of the actual grinding. Thus, using the principle of energy equilibrium of the "wheel – part" system, expressed in equality of internal and external forces acting on the diamond grain, and minimizing the relative consumption of diamond, one can compare various grinding options that are comparable in physical essence and choose the best option.

The basic optimization equation has the following form: the sum of linear wear of grains before their bulk destruction, the maximum slice thickness and the height of the intergranular space for free placement of chips is a constant value equal to the maximum height of protrusion above the bond of unworn grain. The expression for the maximum slice thickness is obtained on the basis of probabilistic calculations for the optimal cutting relief of the wheel and takes into account the linear wear of the grains before their bulk destruction or falling out of the bond.

The condition for the stabilization of the cutting relief of the diamond wheel was taken to be equality for the maximum protruding grain cutting force and force destroying the grain. Calculations performed for the circular outer longitudinal diamond grinding of carbide and high-speed multi-blade tools showed that the most preferred option is deep grinding with a small peripheral speed of the workpiece and a longitudinal feed close to the height of the diamond wheel. Moreover, the greater the depth of grinding, the less must be the peripheral speed of the workpiece. With its decrease and, accordingly, an increase in the depth of grinding, linear wear of grains until their bulk destruction is maximum, i.e. the cutting properties of diamonds are most used. In this case, the maximum slice thickness is minimal.

The relative consumption of diamond takes the lowest values despite the fact that the largest wear areas are formed on the cutting grains, the relief of the diamond wheel is the least developed, and the conventional cutting stresses are maximum. The work of grains goes into the mode of their volumetric destruction, which is more economically efficient compared to the work of the diamond wheel in the mode of intensive precipitation of low-worn grains.

To study the regularities of changing the minimum values of the relative consumption of diamond depending on the grinding conditions, it was experimentally investigated the parameters of diamond grinding process of 160 mm carbide disc mills on a 3B12 circular grinding machine with a modernized workpiece rotation drive, allowing the workpiece speed to be varied within 1 ... 10 m/min (Fig. 2) [5; 6].

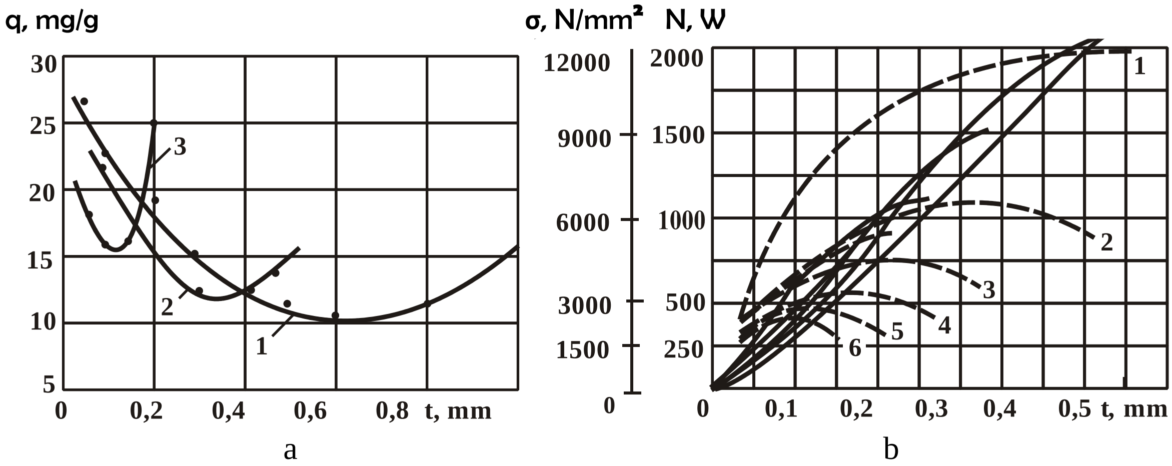


Fig. 2. Dependences of the relative consumption of diamond q (a), conditional cutting stress σ (– –) and grinding power N (—) (b) due to grinding depth t: material to be processed – hard alloy WC85%-Co15%; diamond wheel – 1A1 300 25 5 AC6 250/200 M1-01 4; 35 m/s; 22.5 mm/rev.; coolant – 3% solution $NaNO_3$; $V_{det}$ : a) 1 – 1 m/min, 2 – 2 m/min, 3 – 6 m/min, b) 1 – 1 m/min; 2 – 2 m/min; 3 – 3 m/min; 4 – 4 m/min; 5 – 5 m/min; 6 – 6 m/min

As follows from Fig. 2, with a round outer longitudinal deep-seated diamond grinding with a low workpiece speed and a longitudinal feed close to the height of the wheel, the dependence of the relative consumption of diamond is always extreme, regardless of the combination of grinding mode parameters. An increase in the speed of the workpiece $V_{det}$ causes an increase in the minimum of the relative consumption of diamond $q_{min}$ and shifts it to a zone of lesser grinding depths t. The maximum conditional cutting stress $\sigma_{max}$ and the minimum relative consumption of diamond $q_{min}$ correspond to virtually the same grinding mode. Moreover, a higher value of $q_{min}$ corresponds to a smaller value of $\sigma_{max}$. At a speed $V_{det}$ of 1 m/min, the cutting capabilities of the diamond wheels are used to the maximum. In this case, the linear wear of the grain to its bulk destruction, and, consequently, the maximum depth of penetration of the cutting grain into the processed material, counting it from the top of an unworn, maximally protruding grain, H, Fig. 3 [5], increases, and the minimum relative diamond consumption takes the smallest value. To verify the correctness of this conclusion, it is necessary to compare the value of H for various values $q_{min}$, taking into account the dependency:

$$H = \frac{9b \cdot V_{det} \cdot \sqrt{2 \cdot t \cdot \rho}}{k \cdot V_c \cdot S_{lim}}, \qquad (1)$$

where k – surface concentration of grains, pcs/m$^2$; b – maximal height of protrusion above a bond of unworn grains, m; $V_c$, $V_{det}$ – respectively, speed of the wheel ("circle") and workpiece ("detail"), m/s; t – depth of grinding, m; $\rho = 1/R_c + 1/R_{det}$; $R_c$, $R_{det}$ – respectively, radius of the wheel and workpiece, m; $S_{lim}$ – maximal cross-sectional area of the slice, m$^2$.

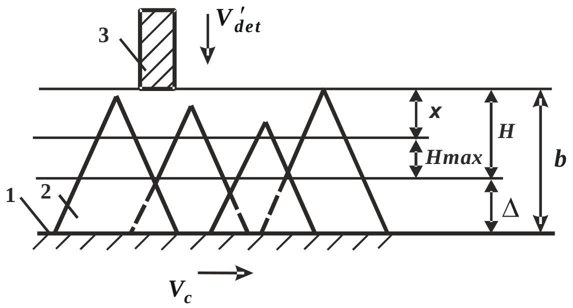


Fig. 3. Design scheme of grinding process parameters (x – value of linear grain wear; $H_{max}$ – maximal slice thickness): 1 – level of bond; 2 – cutting grain; 3 – workpiece

It is necessary to imagine $S_{lim} = P/\sigma_{max}$, where P is the force destroying the grain. Conventional cutting stress, according to Fig. 2, b, is directly proportional to the grinding depth t, and it can be approximated by dependence $\sigma_{max} = A_0 \cdot t$, where $A_0$ is the coefficient determined from the graph. Then

$$H = \frac{9 \cdot b \cdot A_0 \cdot V_{det} \cdot t \cdot \sqrt{2t \cdot \rho}}{k \cdot V_c \cdot P}. \qquad (2)$$

Based on Fig. 2, a, the product of parameters $V_{det} \cdot t$ that determines the machining productivity Q in points $q_{min}$, slightly decreases with increasing grinding depth t, and therefore, depending on (2), it can be assumed to be constant. With this in mind, the parameter H increases with increasing t.

The change in the parameter H depending on the grinding mode is associated with different height of the intergranular space Δ required for free placement of chips, since the layer of chips formed along the entire length of the arc of the wheel contact with the workpiece prevents further wear of the grains to their bulk destruction and thus reduces their active (cutting) part. To confirm the extended position by analytical way, it is necessary to determine the nature of the change in the parameter H depending on the height of the intergranular space Δ required for free placement of chips, as well as the nature of the change in the maximum thickness of the cut Hmax (Fig. 3). In order to simplify calculations, the volume of chips $\vartheta$ is represented by the volume of a homogeneous liquid filling the entire intergranular space of a wheel.

The thickness of the chip layer Δ was determined from the condition that during the time Δτ the workpiece moved by an amount $V_{det} \cdot \Delta\tau$, the grains of the wheel went the way $Vc \cdot \Delta\tau$, and the removed material uniformly filled the layer thickness Δ of the intergranular space of the contacting working surface of the wheel (Fig. 3):

$$\Delta = \frac{\vartheta}{B_1 \cdot (V_c \cdot \Delta\tau)} = \frac{Q \cdot \Delta\tau}{B_1 \cdot (V_c \cdot \Delta\tau)} = \frac{V_{det} \cdot t}{V_c}, \qquad (3)$$

where $B_1$ – is the longitudinal feed,min/rev.

It can be concluded from the analysis of the condition of optimal self-sharpening of the wheel, determined by the equality of the force of holding the grain in the bond and the force of destroying the grain, that the depth of the critical embedding of the grain in the bond for all values qmin remains constant. Consequently, the sum of the values of H and Δ at all points also remains the same. As a result, using the mathematical expression (2) and (3) for parameters H and Δ, we obtained:

$$\frac{9 \cdot b \cdot V_{det} \cdot \sqrt{2 \cdot t \cdot \rho}}{k \cdot V_c \cdot S_{lim}} + \frac{V_{det} \cdot t}{V_c} = c, \qquad (4)$$

where c is a constant.

Solving the dependence (4) relatively $V_{det}$, we have:

$$V_{det} = \frac{c \cdot V_c \cdot k \cdot S_{lim}}{\left(9 \cdot b \cdot \sqrt{2 \cdot t \cdot \rho} + t \cdot k \cdot S_{lim}\right)}. \qquad (5)$$

The following is obtained after substituting dependencies (5) into dependencies (1), (3) and into the well-known expression to determine the mode machining productivity $Q = B_1 \cdot V_{det} \cdot t$:

$$H = \frac{9 \cdot b \cdot c \cdot \sqrt{2 \cdot t \cdot \rho}}{9 \cdot b \cdot \sqrt{2 \cdot t \cdot \rho} + t \cdot k \cdot S_{lim}}; \quad (6)$$

$$\Delta = \frac{c \cdot t \cdot k \cdot S_{lim}}{\left(9 \cdot b \cdot \sqrt{2 \cdot t \cdot \rho} + t \cdot k \cdot S_{lim}\right)}; \quad (7)$$

$$Q = \frac{c \cdot V_{кр} \cdot k \cdot S_{lim} \cdot B_1 \cdot t}{\left(9 \cdot b \cdot \sqrt{2 \cdot t \cdot \rho} + t \cdot k \cdot S_{lim}\right)}. \quad (8)$$

As it follows from dependencies (6) - (8), as the grinding depth t increases, the parameter H increases, but Δ and Q decrease, that confirms the advanced position about the influence of the height of the wheel intergrain space in points $q_{min}$ on the amount of linear wear of the grain before its volume destruction.

It is important to establish the nature of the change in the maximal slice thickness $H_{max}$ depending on the speed of the workpiece $V_{det}$ in points $q_{min}$. For this, we use the dependency that follows from the representations [5, 6]:

$$H_{max} = \frac{1}{1 + \sqrt{1 - \dfrac{k^2 \cdot V_c^2 \cdot S_{lim}^2}{81 \cdot tg^2\gamma \cdot b^2 \cdot V_{det}^2 \cdot 2 \cdot t \cdot \rho}}} \cdot$$

$$\cdot \frac{k \cdot V_c \cdot S_{lim}^2}{9 \cdot tg^2\gamma \cdot b \cdot V_{det} \cdot \sqrt{2 \cdot t \cdot \rho}}. \quad (9)$$

As follows from relationship (9), the maximal slice thickness $H_{max}$ at points $q_{min}$ decreases and causes a decrease in values of $q_{min}$ with an increase in the depth of grinding t, that is consistent with the main provisions of the mathematical model of grinding. In contrast to the known dependencies for determining the parameter $H_{max}$, the obtained relationship (9), besides the geometrical and kinematic parameters of grinding, contains a number of physical characteristics, which indicates a more complex formation of the cutting relief of the diamond wheel during grinding and its wear than it is usually assumed in the calculations. Experimental studies of the roughness of the machined surface at points $q_{min}$ show a decrease $R_a$ with increasing grinding depth t (Fig. 4), which is also associated with a greater degree of smoothness of the cutting relief of the diamond wheel.

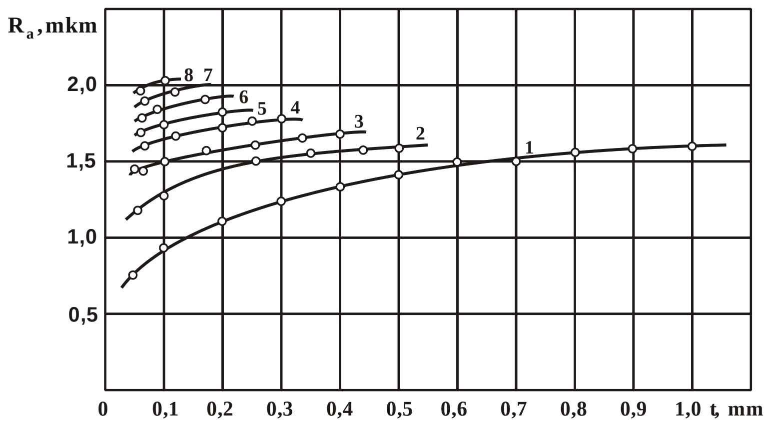


Fig. 4. Dependencies $R_a$ on t for round grinding:
diamond wheel – 1A1 300 25 AC6 200/160 A1 4 M1-01 ($V_c$ = 35 m/s);
processed material – hard alloy WC92%-Co8%; 1 ... 6 – speed details $V_{det}$,
respectively: 1 ... 6 – 1 ... 6 m/min, 7 – 8 m/min, 8 – 10 m/min

To assess the effect of the strength of the processed material on the condition of the optimum self-sharpening of the diamond wheel, one should use the obtained analytical dependencies for comparative calculations on various processed materials with different strength, determined by the parameter $S_{lim}$.

From analysis the dependences (6), (8) and (9), it can be seen that, under the condition t = const, an increase in the parameter $S_{lim}$ for $q_{min}$ causes an increase in H and a decrease in the processing capacity Q and the maximal slice thickness $H_{max}$. Consequently, when grinding difficult-to-work materials, it is possible to increase the linear wear of the grains before their bulk destruction and reduce $q_{min}$. However, it comes at a lower machining productivity Q, since with an increase in the parameter $S_{lim}$ under the condition t = const, according to (5), the speed of the workpieces $V_{det}$ decreases. Such a seemingly paradoxical result is well confirmed experimentally [5]. In Table 1 shows the experimental values of the machining productivity Q, the relative consumption of diamond q, grinding power N and the conditional cutting stress σ, obtained by round diamond grinding of hard alloy WC85%-Co15% and Steel 45.

Table 1. Experimental values of machining productivity Q, relative diamond consumption q, grinding power N, and conditional cutting stress σ for workpiece speed $V_{det}$ = 2 m/min (numerator) and $V_{det}$ = 4 m/min (denumerator), under longitudinal feed $B_1 = 22.5$ mm/rev. and wheel speed $V_c$ = 35 m/s

| t, mm | Q, $mm^3$/min | q, mg/g | N, W | $\sigma \cdot 10$, $N/mm^2$ |
|---|---|---|---|---|
| Hard alloy WC85%-Co15% | | | | |
| 0,1 | 4500 / 9000 | 0,4 / 0,3 | 180 / 320 | 240 / 210 |
| 0,2 | 9000 / 18000 | 0,26 / 0,35 | 400 / 500 | 264 / 168 |
| Steel 45 | | | | |
| 0,1 | 4500 / 9000 | 0,13 / 0,12 | 450 / 800 | 600 / 540 |
| 0,2 | 9000 / 18000 | 0,12 / 0,145 | 200 / 1500 | 540 / 498 |

As can be seen from the Table 1 for all modes of grinding, power and conditional cutting stress is higher for Steel 45, and the relative consumption of diamond is higher for hard alloy WC85%-Co15%. Thus, experimental studies have confirmed the correctness of the analytical optimization of the parameters of diamond grinding. At points of minimum, the relative consumption of diamond decreases with an increase in the depth of grinding, the machining productivity remains almost unchanged, and the roughness parameter $R_a$ decreases.

## Conclusion

The mathematical model of grinding developed on the principle of energy equilibrium allows solving various engineering problems as applied to the grinding process, in particular, revealing the essence of influence of the diamond wheel characteristics on tool working capacity, analytical design of high-performance diamond grinding processes, etc.

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