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Dynamics of Formation of Transitional Prices on the Chain of Sequential Markets: Analytical Model

Abstract. Although the problem of formation of market prices, determination of equilibrium prices within the model "Demand – Supply" is quite known and a great number of both theoretical works and works that summarize the results of observations are devoted to its research, this problem remains relevant, especially as to the dynamics of pricing processes and the stability of equilibrium prices in relation to changes in parameters that characterize the state of the system. Most studies addressing these issues focus on either a particular local market or the global market for some products in general. The purpose of this work is to build a mathematical model that would allow us to analyze general issues related to the formation of transitional prices in the finite N-dimensional chain of sequential markets in accordance with the scheme of market equilibrium. An analytical model is proposed that makes it possible to study the dynamics of prices in adjacent markets. Within this model, which is based on the determination of processes using a system of integral equations, it was assumed that the impact on the chain of sequential markets and the response to this impact are continuous over time. The dynamic aspect of the proposed pricing model in the vertical sequence of markets is the existence of an "after-effect", which is described in an integral form by the delay distributed over time. The issues of adequacy of the model were examined, its internal coherence was studied, the correctness of the transition from the mathematical model of dynamics as a system of integral equations to the model in the form of a system of linear algebraic equations was substantiated. The conditions for the existence of the solution for this system of equations and the area of its stability are formulated. The mathematical model proposed in this paper allows for a qualitative analysis of the system states (by phase trajectories). Examples of numerical implementation of our analytical model for two and three sequential markets are given, equilibrium prices for each link of the chain of sequential markets are determined. Applying simulation modelling, the stability of the solution in relation to changes in such parameters of the model as the elasticity of demand and supply in the market under study and cross-elasticities in adjacent markets as well as the impact of these parameters on such dynamic indicators of the market system as the rate of attainment of equilibrium was examined

Keywords: market vertical, pricing, phase trajectories, Volterra integral equations, model adequacy, simulation modelling, elasticities of supply and demand

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INTRODUCTION

At all stages of production of goods and services, as well as their sale, the structures of interaction of economic agents are quite diverse. However, generally there are two types of relationships distinguished, according to which the classification of relations between economic agents is carried out. Firstly, it is a hierarchical type of relations, i.e. vertical interaction or even integration; secondly, these are relations of competition or cooperation at the same level, i.e. horizontal interaction. In the international model of differentiated goods, the "Rule of Three" market structure (RoT-market) can be implemented, which allows for competition both within the country and between firms of different countries [1]. In this case, the long-term risk of a firm facing domestic competition may be higher or lower

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than the short-term risk, whereas a foreign monopolist has less long-term risk.

While the features of horizontal interaction are explored in many works of both practical and theoretical direction, the issues of vertical interaction are more complex, at least in theoretical terms, but they are becoming increasingly relevant due to the rapid development of globalization processes [2]. The problem of vertical interaction in the product market has become especially acute in recent years due to the COVID-19 epidemic. This has even led to a failure in economic activity and, as a result, affected the efficiency of participants throughout the market vertical [3]. Thus, there is a need to build mathematical models describing the dynamics of processes in a market sequence. Thus, in his review "Mathematical Problems of the Next Century" [4], the leading American mathematician Steve Smale, an expert in the field of mathematical economics, identified the problem of introducing dynamics into economic theory as one of such problems (the so-called Problem 8: Introduction of Dynamics into Economic Theory, according to his definition). And this theory of dynamic equilibrium should be compatible with the theory (static) of equilibrium, which was laid down in the works of Leon Walras [5] and acquired its modern appearance thanks to the works of such scientists as Paul Samuelson [6] and John Hicks [7].

Sales of many goods including, for example, agricultural products, coal, oil, gas, electricity, are carried out through sequential markets. Let's consider the features of the vertical structure of a market. For the commodity market, the links of such chain are extraction of raw materials, its processing, production of semi-finished products (or components), production of finished goods, trade (sale), i.e. delivery to the end consumer. The vertical sequence of commodity markets can be represented by the scheme shown in Figure 1.

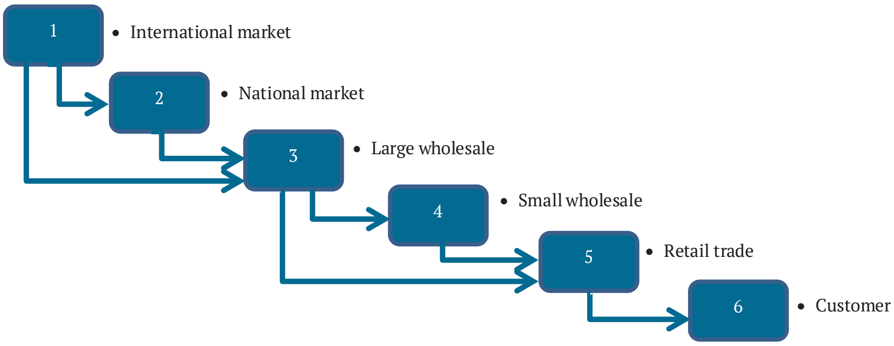


Figure 1. The sequence of links of the vertical chain of markets (hierarchical structure)

In its most extensive form, such sequence is most often observed in commodity markets. However, some types of services, for example, IT technologies business, also have a similar vertical structure of the market [5; 8]. Depending on the type of goods, some chains in this sequence may be missing. If the hierarchy of service markets is considered, such chain includes a minimum number of links. The shortest vertical chains can consist of such levels: 1 – 3 – 5 – 6 or 2 – 4 – 5 – 6. In fact, the vertical sequence of markets reflects a chain of successive stages of value creation. The total sum of value added generated at each level of this chain determines the final value of goods.

Numerous studies are devoted to assessing the quantitative changes [9] that occur in markets as a result of integration processes, and determining links between factors [10] that define the direction and speed of these processes. Exponential sliding averages may be characteristic of the system states, and the possible states themselves are described with Markov chains [11]. But this analysis reveals mainly the consequences, not the causes of the processes. In order to understand the possible direction of evolution of individual links in the sequence of markets and their consequences, it is necessary to take into account the very nature of qualitative connections between these links as well as to determine the quantitative indicators characterizing these connections with a variety of mathematical tools. Then, based on the analysis of the obtained results, it is necessary to develop a dynamic mathematical model of the system to determine the trajectories of the system development and identify the boundaries of stability of equilibrium states.

Modern theoretical studies of economic and social problems are based on mathematical models of sufficiently large dimensionality, with which the dynamics of the system states by development trajectories is described, i.e., a qualitative analysis of the system is carried out. This analysis is the basis for predicting the development of processes that can occur in the system depending on the implementation of a particular scenario, therefore, on the value of the parameters that are provided in the model. Thus, one of the first reports considered at the meeting of the Club of Rome, namely "Boundaries of Growth" by Dennis L. Meadows [12], included forecasts about human population and the depletion of natural resources. The basis for these forecasts were (this should be emphasized) dynamic mathematical models, in which the state of the system was described through a system of differential equations. The value of this approach is that a comparative analysis of the 1972 mathematical model and the 1993 World3 model made it possible to create a global development strategy aimed at stabilizing the system that can be called "Humanity". The argument in favor of the effectiveness of the application of mathematical modelling in economic and social research also lies in the fact that the analysis of the dynamics of processes that have taken place worldwide over the past 30 years confirmed the consistency of forecasts by the Meadows model [13]. This example most convincingly demonstrates the feasibility of creating mathematical models of dynamics to determine possible directions of development of economic and social systems, the availability of powerful computing equipment allows to consider scenarios that are determined by a large number of variables, and the possibility of their implementation depends on a large number of parameters. One of the advantages of mathematical modelling is that this approach allows to obtain a fundamental solution of the problem, and by substituting certain values of parameters, you have the ability to determine the prevailing impact, thus the development of the system can be set in the desired direction.

The need to develop mathematical models of the system that would take into account as many significant factors as possible is also explained by the presence of psychological factors affecting pricing, for example, in the financial market [14]. Thus, it has been proved that the need to make decisions in conditions of uncertainty can lead to an increase in the probability of systematic errors in the pricing process.

If you consider the relations between participants that determine the pricing process at different levels of the market vertical [15; 16], then at the lowest level, the relationship between the buyer and the seller has the form of a one-time agreement, and all information about the product is reflected by its price. The next step is repeated transactions, leading to the formation of weak, but certain relations between the buyer and the seller. However, industrial marketing, especially within the global industry [17], is generally based on closer long-term relations that rely heavily on the interaction between market participants. Such relations are inherent in a higher level of the market vertical. Strong partnership relations are also possible; their formation is associated with the transition to quality management (which makes products more attractive to the buyer). A strategic alliance, an example of which is a joint venture, corresponds to the highest level. At the present stage of development, this type of interaction becomes an integral part of a firm's global strategy, since it accelerates the development of its technological capabilities, reduces risks and promotes access to new resources and markets. Thus, there is an evolution of marketing management manifested in the transition from viewing marketing as competition, when the interaction of market participants takes the form of individual transactions, to viewing it as cooperation and formation of vertical relations. Although transactions are considered the most common marketing relations between actors, vertical integration is considered the most complete when raw material suppliers and product sales companies are part of the main product company. Apparently, the vertical market structure involves different types of relations between the buyer and the seller.

Although many works are devoted to the issues of interaction between sequential links of the market vertical, they all have a predominantly practical direction, and only a limited range of works relates to the study of theoretical principles of the dynamics of pricing processes for the market vertical as a single system.

The purpose of the work is to build a mathematical model to determine the formation of transition prices and equilibrium prices for each link in the chain of successive markets and, based on the obtained analytical model, to study the influence of external factors on the dynamics of these processes. As external factors, the elasticities of the supply and demand functions within the market of that link of the vertical sequence for which the study is being carried out, and cross-elasticities for adjacent markets (previous relative to the investigated market and the next after it) will be considered.

MATERIALS AND METHODS

In modern science and technology, models based on differential and integral analysis are successfully applied to model dynamic systems. This approach is also becoming common in the research of economic and social systems. Now we can provide examples of a considerable number of economic problems, for the formalization of which differential and integral equations are used [18-20]. It should be noted that models based on the application of differential equations can be used when the impact on the system and the system's response to this impact are measured both at some fixed points in time and in continuous measurement, while the application of integral equations implies that this impact is distributed over time. However, both integral and differential equations describe the case when the processes that determine the state of the system are continuous in time. When time is considered discrete, difference equations are used to build dynamic models of economic systems [21; 22]. There are also attempts to apply non-local fractional operators, which go beyond the traditional use of elements of differential and integral calculus in building economic models of dynamics [23]. The processes occurring in the system can be considered Markov ones, i.e. when at random moment of time the evolution of the system is determined only by its current state and does not depend on how the system has reached this state. Markov chains are applied to determine possible transitions [11]. Functional autoregressive models are also used for modelling the price market [24]. Recently, another area of study of dynamic systems has emerged, which is related to the application of elements of fuzzy logic in creation mathematical models. In this case, for example, demand and supply are described by fuzzy numbers, and the learning function changes within fuzzy limits [25]. Thus, there is a natural interest in the extended application of mathematical apparatus to the modelling of economic dynamics in the research of pricing processes in the system of sequential markets.

This paper investigates the structure of interaction in sequentially connected markets, the number of which in general can be equal to *N*, based on a simplified scheme of sequential markets, which is presented in Figure 2.

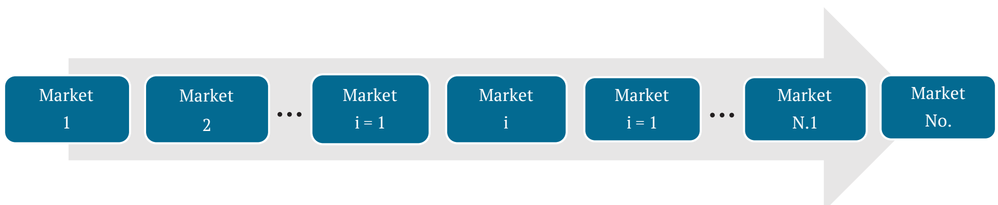


Figure 2. Simplified scheme of the system of sequential markets

To build a deterministic analytical model of sequential markets, principles of the theory of integral equations, the Laplace transform, the theory of matrices and determinants were used, and to test the theoretical principles on the dynamics of the formation of equilibrium prices, simulation modelling using MS Excel was made within the framework of this mathematical model.

RESULTS AND DISCUSSION

The organization of each market is based on mechanisms of the mutual consistency of supply and demand, which are, respectively, functions of price. Let's assume that the demand function D_i in each market depends only on the price p_i in that market, i.e. $D_i=D_i(p_i)$, where $i=\overline{1,N}$. Whereas the supply function S_i will depend not only on the price p_i in the given market, but also on the prices p_{i+1} and p_{i-1} in adjacent markets, i.e. $S_i=S_i(p_{i-1}, p_i, p_{i+1})$, where $i=\overline{2,N-1}$. Let's consider boundary cases. Thus, at i=1 we will have $S_1=S_1(p_1, p_2)$, and at $i=N$, respectively, we will have $S_N=S_N(p_{N-1}, p_N)$. To formalize the functions of supply and demand, let's assume that they have a linear dependence on price arguments. This assumption, although not expressed, is taken for granted in the interpretation of market equilibrium by both Walras [5] and Marshall [26]. So, let's write the functions of supply and demand as follows:

$$D_i = d_i^o - d_i p_i \quad \text{for} \quad i = \overline{1,N}\,, \tag{1}$$

$$S_i = c_i p_{i-1} + a_i p_i + b_i p_{i+1} - s_i^o \text{ for } \quad i = \overline{2,\ N-1}\,, \tag{2}$$

where constant values d_i^o, s_i^o have autonomous, i.e. independent of prices, values of demand and, consequently, supply functions; parameters d_i and a_i are elasticities of demand and supply at a price p_i; parameters c_i and b_i are the values of cross-elasticities at prices p_{i-1} and p_{i+1} (prices in adjacent markets).

In some cases the marginal supply function takes the form of:

$$S_1 = a_1 p_1 + b_1 p_2 - s_1^o\,, \tag{2'}$$

or

$$S_N = c_N p_{N-1} + a_N p_N - s_N^o\,. \tag{2''}$$

Using the relations (1), (2), (2') and (2"), it is not difficult to create a system of algebraic equations for finding equilibrium (static) values of the price of goods in each of the markets, i.e., the value $p_1^*\ p_2^*,\dots,\ p_3^*$ provided that $D_i=S_i$, where $i=\overline{1,N}$.

When creating mathematical models of problems of a non-stationary (dynamic) nature, it is quite common to apply integral equations of the Volterra type, which have a variable integration area [27; 28]. The application of integral equations is a method of mathematical modelling, as a result of which a relationship is established between the known source data and the determined characteristics of the phenomenon under study. Most often, integral equations are used to model physical processes, but recently there has been an expansion of their application not only to other branches of natural science, but also to economic processes and phenomena. It is this mathematical apparatus that the authors chose in order to create a dynamic model of a sequential chain of markets.

The dynamic version of the mathematical model of the system of sequential markets has a slightly different basis compared to the static model, since it is necessary to someway take into account the inertia of price evolution. This can be done using a latency factor, which in time dimension can be either a fixed lag or distributed over all previous time periods. In this study, when creating the model, the second option was chosen, i.e. it is assumed that there is a distributed lateness in a continuous time segment. So, let's assume that the demand at a fixed point in time is equal to the supply over the entire previous period of time:

$$D_i(p_i(t)) = \int_0^t K_i(t,\tau) S_i(p_{i-1}(\tau), p_i(\tau), p_{i+1}(\tau))d\tau \text{ for } \quad i = \overline{1,N}\,, \tag{3}$$

where $K_i(t,\tau)$ is the function of the two arguments, which is the kernel of the integral equation, characterizes the way the "dynamic memory" about the previous supply values is organised in relation to the fixed point in time for which demand is determined. In general, it is a function describing the system's response to external influence at a certain point in time. Obviously, in order to ensure causality, the time variable for all of these functions must be descending.

The system of integral ratios (3) is the basis for determining the price dynamics in each of the studied markets. For the sake of simplifying this system of functional equations, we will consider that all functions $K_i(t,\tau)$ depend on the difference of their arguments in the same way, i.e.

$$K_i(t,\tau) = K(t-\tau) \text{ for } \ \mathrm{i} = \overline{1,N}. \tag{4}$$

With the assumption (4) considered, the system of equations (3) takes the form:

$$\begin{cases} d_1^o - d_1 p_1(t) = \int_0^t K(t-\tau)\cdot(a_1 p_1(\tau) + b_1 p_2(\tau) - s_1^o)d\tau, \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ d_i^o - d_i p_i(t) = \int_0^t K(t-\tau)\cdot(c_i p_{i-1}(\tau) + a_i p_i(\tau) + b_i p_{i+1}(\tau) - s_i^o)d\tau, \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ d_N^o - d_N p_N(t) = \int_0^t K(t-\tau)\cdot(c_N p_{N-1}(\tau) + a_N p_N(\tau) - s_N^o)d\tau. \end{cases} \tag{5}$$

The system (5) is a system of linear integral equations of the Volterra equation type. It can be solved by traditional methods of the theory of integral equations [20; 29]. However, this system contains a very large number of parameters, so we will look for other solutions. Let's each equation of the system is divided by its corresponding d_i ($i=\overline{1,N}$), and we get a system of integral equations with fewer parameters:

$$\begin{cases} \mathrm{p}_d - p_1(t) = \int_0^t K(t-\tau)\cdot(a\cdot p_1(\tau) + b\cdot p_2(\tau) - a\cdot \mathrm{p}_s)d\tau, \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \mathrm{p}_d - p_i(t) = \int_0^t K(t-\tau)\cdot(c\cdot p_{i-1}(\tau) + a\cdot p_i(\tau) + b\cdot p_{i+1}(\tau) - a\cdot \mathrm{p}_s)d\tau, \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \mathrm{p}_d - p_N(t) = \int_0^t K(t-\tau)\cdot(c\cdot p_{N-1}(\tau) + a\cdot p_N(\tau) - a\cdot \mathrm{p}_s)d\tau. \end{cases} \tag{6}$$

The dynamic aspect of the proposed model is the existence of an "after-effect", which is described in an integral form by a distributed time lag. Thus, if the kernel $K(t\text{-}\tau)$ is not taken into account, the system (6) contains five parameters, namely:

$a = \frac{a_i}{d_i}$ – is relative elasticity of supply as to demand for the i market;

$b = \frac{b_i}{d_i}, c = \frac{c_i}{d_i}$ – are relative cross (reciprocal) elasticities of supply as to demand in markets that are adjacent to the market under study;

$p_d = \frac{d_i^o}{d_i}$ – is price at "zero" demand;

$p_s = \frac{s_i^o}{a_i}$ – is price at "zero" supply.

According to the scheme of formation of "Demand and Supply" market equilibrium by Walras (Fig. 3) prices at zero demand and zero supply are defined as the points of intersection of the graphs of the corresponding functions with the axis of ordinates.

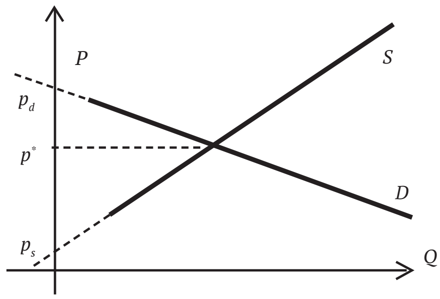


Figure 3. Market equilibrium model by Walras

It should be noted that this well-known scheme is generally given to illustrate the concept of an equilibrium point itself as a point of intersection of graphs of functions describing supply and demand. In this article, the scheme is necessary for a clear explanation of the concepts of zero demand and zero supply which are considered when creating a mathematical model of market dynamics. Given that the right-hand part of each of the equations in the system (6) is an integral convolution, it is advisable to apply the integral Laplace transform to this system:

$$p_i(\lambda) = \int_0^\infty e^{-\lambda t} p_i(t)dt \text{ for } \quad i = \overline{1, N}. \tag{7}$$

As a result, the system of linear algebraic equations relative to $p_i(\lambda)$ acquires such a view:

$$\begin{pmatrix} 1+aK(\lambda) & bK(\lambda) & \dots & 0 & 0 & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & cK(\lambda) & 1+aK(\lambda) & bK(\lambda) & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 & 0 & 0 & \dots & cK(\lambda) & 1+aK(\lambda) \end{pmatrix} \begin{pmatrix} p_1(\lambda) \\ \dots \\ p_i(\lambda) \\ \dots \\ p_N(\lambda) \end{pmatrix} = \begin{pmatrix} \frac{\mathrm{p}_d + a\cdot \mathrm{p}_s K(\lambda)}{\lambda} \\ \dots \\ \frac{\mathrm{p}_d + a\cdot \mathrm{p}_s K(\lambda)}{\lambda} \\ \dots \\ \frac{\mathrm{p}_d + a\cdot \mathrm{p}_s K(\lambda)}{\lambda} \end{pmatrix}, \tag{8}$$

where $K(\lambda)$ is a fractional rational function of the parameter λ. For example, if the kernel is recorded as $K(t-\tau) = exp\left(-\frac{t-\tau}{T}\right)$, then $K(\lambda) = \frac{1}{T\lambda+1}$, where T is a certain constant characteristic of time, namely the parameter that determines lateness. This parameter is distributed by the exponential law from 0 to t. Dividing each equation of the system (8) by the value $K(\lambda)$, we obtain a new system of linear algebraic equations which can be presented in matrix form:

$$\Phi(\lambda) \cdot \mathrm{P}(\lambda) = F(\lambda) \tag{9}$$

where

$$\Phi(\lambda) = \begin{pmatrix} a_0(\lambda) & b & 0 & \dots & 0 & 0 \\ c & a_0(\lambda) & b & \dots & 0 & 0 \\ \dots & \dots. & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & c & a_0(\lambda) \end{pmatrix}; \mathrm{P}(\lambda) = \begin{pmatrix} \mathrm{p}_1(\lambda) \\ \mathrm{p}_2(\lambda) \\ \dots \\ \mathrm{p}_N(\lambda) \end{pmatrix}; F(\lambda) = \frac{\mathrm{p}_d \cdot K^{-1}(\lambda) + a\cdot \mathrm{p}_s}{\lambda} \begin{pmatrix} 1 \\ 1 \\ \dots \\ 1 \end{pmatrix},$$

accordingly,

$$a_0(\lambda) = K^{-1}(\lambda) + a.$$

The matrix equation (9) will have the following solution:

$$\mathrm{P}(\lambda) = \Phi^{-1}(\lambda) \cdot F(\lambda), \tag{10}$$

if there is an inverted matrix $\Phi^{-1}(\lambda)$ for the matrix $\Phi(\lambda)$. Thus, it is necessary to check whether the determinant of the matrix $\Phi(\lambda)$ is different from 0. It should be noted that the matrix $\Phi(\lambda)$ is a tridiagonal matrix or Jacobi matrix [30, p.34]. There is a recurrent ratio with the corresponding initial conditions for calculating the determinant of such matrix:

$$\Delta_N(\lambda) = a_0(\lambda) \cdot \Delta_{N-1}(\lambda) - bc \cdot \Delta_{N-2}(\lambda). \tag{11}$$

Upon converting it, we get the formula:

$$\Delta_N = \frac{\left(a_0 + \sqrt{a_0^2 - 4bc}\right)^{N+1} - \left(a_0 - \sqrt{a_0^2 - 4bc}\right)^{N-1}}{2^{N+1} \cdot \sqrt{a_0^2 - 4bc}}. \tag{12}$$

Let's determine under what conditions the matrix $\Phi(\lambda)$ becomes degenerate, since in this case the matrix equation (9) will not have a solution. To do this, we need to solve the following equation $\Delta_N(\lambda)$=0. Therefore, from the formula (12) we obtain

$$\left(\frac{a_0 + \sqrt{a_0^2 - 4bc}}{2\sqrt{bc}}\right)^{2(N+1)} = 1. \tag{13}$$

Given that $a_0(\lambda)=K^{-1}(\lambda)+a$, this means that:

$$\frac{K^{-1}(\lambda)+a}{2\cdot\sqrt{bc}} = \sqrt[2(N+1)]{1}, \text{ or } K^{-1}(\lambda)+a = 2\sqrt{bc}\cdot cos\frac{\pi m}{N+1} \text{ for } \quad m=\overline{0,N}. \tag{14}$$

In the case where $K(\lambda)=\frac{1}{T\lambda+1}$ there is a relationship:

$$T\lambda_m + 1 + a = 2\sqrt{bc}\cdot cos\frac{\pi m}{N+1} \text{ for } m=\overline{0,N}. \tag{14'}$$

Hence you can find that:

$$\lambda_m = \frac{1}{\mathrm{T}}\cdot\left(-1-\mathrm{a}+2\sqrt{bc}\cdot cos\frac{\pi m}{N+1}\right) \text{ for } m=\overline{0,N}. \tag{15}$$

Thus, to ensure the fulfillment of the stability conditions for the system (9), i.e. the existence of its solution (10), it is necessary and sufficient that the following condition is met: $\lambda_m<0$. Therefore, as follows from the ratio (15), it is necessary to solve the inequality:

$$1+\mathrm{a} > 2\sqrt{bc}. \tag{16}$$

Taking into account the meaning of the parameters, the condition (16) acquires this economic interpretation. In order for the system of algebraic linear equations (9) relative to the price of a particular product in each of the sequential markets in the market vertical to have a stable solution, the following condition must be met: the arithmetic mean of the elasticities of demand and supply for each of the markets must be greater than the geometric mean of the cross-elasticities of supply in their adjacent markets.

Let's consider how this mathematical model is implemented on the example of a small number of sequential markets. Further we will examine the sequential chain of markets, for which the markets adjacent to the studied one are symmetrical, i.e. $b=c$ as well as $K(\lambda)=\frac{1}{T\lambda+1}$ and, respectively, $a_0(\lambda)=K^{-1}(\lambda)+a$.

Example 1 (analytical model). Let there be only two sequential markets, i.e. N=2. In this case the system of linear algebraic equations in matrix form (9) is recorded as follows:

$$\begin{pmatrix} a_0(\lambda) & b \\ b & a_0(\lambda) \end{pmatrix}\cdot\begin{pmatrix} \mathrm{p}_1(\lambda) \\ p_2(\lambda) \end{pmatrix} = \frac{p_d\cdot(T\lambda+1)+a\cdot p_s}{\lambda}\cdot\begin{pmatrix} 1 \\ 1 \end{pmatrix}. \tag{17}$$

Therefore, you can write down the solution of the system of equations (17) in matrix form:

$$\begin{pmatrix} p_1(\lambda) \\ p_2(\lambda) \end{pmatrix} = \frac{1}{a_0^2(\lambda)-b^2}\begin{pmatrix} a_0(\lambda) & -b \\ -b & a_0(\lambda) \end{pmatrix}\cdot\frac{p_d\cdot(T\lambda+1)+a\cdot p_s}{\lambda}\cdot\begin{pmatrix} 1 \\ 1 \end{pmatrix}. \tag{18}$$

It follows from the ratio (18) that:

$$p_1(\lambda)=p_2(\lambda)=\frac{p_d\cdot(T\lambda+1)+a\cdot p_s}{\lambda\cdot(a_0(\lambda)+b)}=\frac{p_d\cdot(T\lambda+1)+a\cdot p_s}{\lambda\cdot(T\lambda+1+a+b)}, \tag{19}$$

or

$$p_1(\lambda)=p_2(\lambda)=\frac{p_d+a\cdot p_s}{\lambda\cdot(1+a+b)}+\frac{(a+b)\cdot p_d-a\cdot p_s}{(1+a+b)\cdot(\lambda+(1+a+b)/T)}. \tag{20}$$

Using the inverted Laplace transform, we obtain a formula that determines the change in the transitional market price over time:

$$P_1(t)=P_2(t)=p_{1,2}^*+\frac{(a+b)\cdot p_d-a\cdot p_s}{(1+a+b)}\cdot exp\left(-\frac{1+a+b}{T}\cdot t\right), \tag{21}$$

where $p_{1,2}^*$ is the equilibrium price for each of the markets.

This price is determined by the ratio between the price at "zero" demand, the price at "zero" supply for this market and elasticities in this market and the ones adjacent to it:

$$p_{1,2}^*=\frac{p_d+a\cdot p_s}{1+a+b}. \tag{22}$$

Example 2 (analytical model). Let there be three sequential markets, i.e. N=3. In this case, the system of linear algebraic equations in matrix (9) is recorded as follows:

$$\begin{pmatrix} p_1(\lambda) \\ p_2(\lambda) \\ p_3(\lambda) \end{pmatrix} = \frac{p_d\cdot(T\lambda+1)+a\cdot p_s}{\lambda\cdot a_0(\lambda)\cdot(a_0^2(\lambda)-2b^2)}\cdot\begin{pmatrix} a_0^2(\lambda)-b^2 & -a_0(\lambda)\cdot b & b^2 \\ -a_0(\lambda)\cdot b & a_0^2(\lambda) & -a_0(\lambda)\cdot b \\ b^2 & -a_0(\lambda)\cdot b & a_0^2(\lambda)-b^2 \end{pmatrix}\cdot\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}. \tag{23}$$

Therefore, the solution to the system of equations (23) can be written as follows:

$$\begin{pmatrix} p_1(\lambda) \\ p_2(\lambda) \\ p_3(\lambda) \end{pmatrix} = \frac{p_d\cdot(T\lambda+1)+a\cdot p_s}{\lambda\cdot a_0(\lambda)\cdot(a_0^2(\lambda)-2b^2)}\cdot\begin{pmatrix} a_0^2(\lambda)-b^2 & -a_0(\lambda)\cdot b & b^2 \\ -a_0(\lambda)\cdot b & a_0^2(\lambda) & -a_0(\lambda)\cdot b \\ b^2 & -a_0(\lambda)\cdot b & a_0^2(\lambda)-b^2 \end{pmatrix}\cdot\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}. \tag{24}$$

After the necessary transformations we get the ratio to determine the price in each of the markets:

$$p_{1,3}(\lambda)=\frac{\mathrm{a}_0(\lambda)-b}{\mathrm{a}_0^2(\lambda)-2b^2}\cdot\frac{p_d\cdot(T\lambda+1)+a\cdot p_s}{\lambda} \tag{25}$$

$$p_2(\lambda)=\frac{a_0(\lambda)-2b}{a_0^2(\lambda)-2b^2}\cdot\frac{p_d\cdot(T\lambda+1)+a\cdot p_s}{\lambda}. \tag{26}$$

From (25) and (26) we find equilibrium prices for each of the markets:

$$p_{1,3}^*=\frac{1+a-b}{(1+a)^2-2b^2}\cdot(p_d+a\cdot p_s); \tag{27}$$

$$p_2^*=\frac{1+a-2b}{(1+a)^2-2b^2}\cdot(p_d+a\cdot p_s), \tag{28}$$

and the ratios defining transition regime prices for the first and third markets:

$$P_{1,3}(t)=p_{1,3}^*+\mathrm{A}_1\cdot exp\left(\frac{-1-a+b\sqrt{2}}{T}\cdot t\right)+\mathrm{A}_2\cdot exp\left(\frac{-1-a-b\sqrt{2}}{T}\cdot t\right),$$

where:

$$A_1 = \frac{(1+(\sqrt{2}-1)\cdot b)\cdot p_d + a\cdot p_s - (1+a+b\sqrt{2})\cdot p_{1,3}^*}{2\sqrt{2}\cdot b};$$
$$A_2 = \frac{((\sqrt{2}+1)\cdot b-1)\cdot p_d - a\cdot p_s + (1+a-b\sqrt{2})\cdot p_{1,3}^*}{2\sqrt{2}\cdot b}, \qquad (29)$$

as well as for the transition price in the second market:

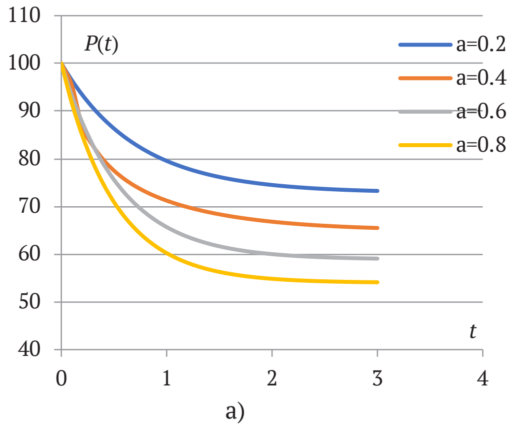
$$P_2(t) = p_2^* + D_1 \cdot exp\left(\frac{-1-a+b\sqrt{2}}{T}\cdot t\right) + D_2 \cdot exp\left(\frac{-1-a-b\sqrt{2}}{T}\cdot t\right)$$

where

$$D_1 = \frac{(1+(\sqrt{2}-2)\cdot b)\cdot p_d + a\cdot p_s - (1+a+b\sqrt{2})\cdot p_2^*}{2\sqrt{2}\cdot b};$$
$$D_2 = \frac{((\sqrt{2}+2)\cdot b-1)\cdot p_d - a\cdot p_s + (1+a-b\sqrt{2})\cdot p_2^*}{2\sqrt{2}\cdot b}. \qquad (30)$$

Note that in the general case for the systems (9) and (10) there are analytical solutions that are based on the explicit form of the inverted matrix $\Phi^{-1}(\lambda)$. Its elements take the form of Chebyshov polynomials of the second kind [31; 32], but this issue lies beyond the scope of this work. Based on the analytical models given in Examples 1 and 2, we will analyze the impact of the elasticities of supply and demand functions on the magnitude of equilibrium prices and the dynamics of changes in transition market price for the case of two and three sequential markets using simulation modelling.

Example 1 (results of simulation modelling). When calculating the analytical model for two sequential markets, the following values of the system parameters were adopted, namely: for both markets, the price at zero demand was taken as 100 percent and amounted to p_d=100 conventional units, and the price at zero supply was 10 percent of the price of zero demand, i.e. p_s=0.1·p_d=10 conventional units. Let us remind that the change in transitional market prices over time for both the first and second markets is described by the same ratio (21). Figure 4 shows the results of the study of the impact of "demand-supply" elasticities that occur in the market under study (Fig. 4a) and in the adjacent market (Fig. 4b) on the dynamics of changes in transitional prices.



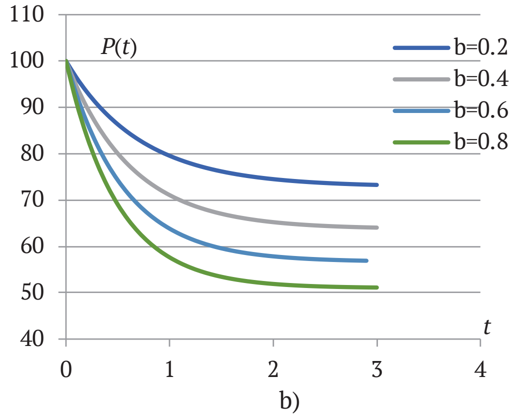


Figure 4. Dynamics of transition prices in two adjacent markets depending "demand-supply" elasticities in the market under study (a) and in the adjacent market (b)

As can be seen in Figure 4, the transitional prices in both the first and second markets decrease monotonically over time. This result is not unexpected since the ratio (21) contains an exponent. It should be noted that although the growth of elasticity "demand-supply" both in the market itself, for which the study is conducted, and cross-elasticity accelerate the price reduction, they do not noticeably affect the speed of reaching the equilibrium price.

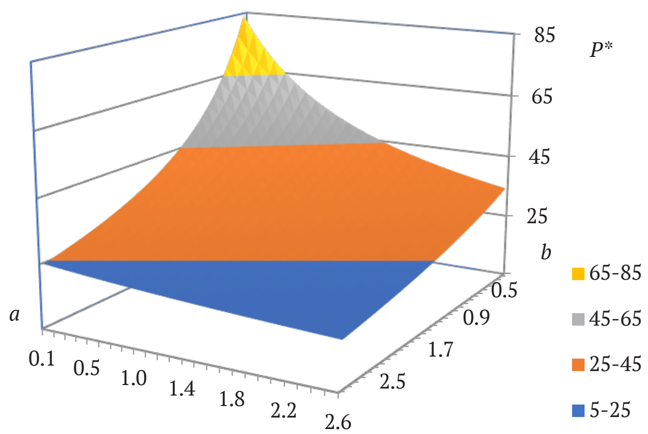


Figure 5. Equilibrium price as a function of elasticities in two adjacent markets

This can be explained by the fact that with increasing elasticity the value of the equilibrium price in each of the markets significantly decreases. Thus, Figure 5 presents a diagram showing the changes in equilibrium prices in the chain of adjacent markets with a simultaneous change in elasticities both in the market under study and in cross-elasticities.

The features described above are inherent in both inelastic and elastic markets. It is also interesting to note that for the case of two sequential markets it was found that the violation of condition (16) does not affect the results of the calculations in any way.

Example 2 (results of simulation modelling). In order to compare the results of calculations, when calculating the analytical model for three sequential markets the same values of the system parameters were adopted, namely: the price at zero demand was p_d=100 conventional units, and the price at zero supply was equal to $p_s=0.1\cdot p_d=10$ conventional units. Let us remind that the change in transitional market prices over time for the first and third markets is described by the same ratio (29), while for the second market the change in transitional market prices over time is described by the ratio (30).

Let's consider the dynamics of transition prices in each of the three markets, depending on the elasticities in the market under study and cross-elasticities. Unlike with the sequence of two markets, the change in transitional prices is not monotonous for a chain of three markets. With certain elasticities, the price reaches the minimum value in a short time, and then it increases slightly and reaches the equilibrium value. As an example, Table 1 shows the results of calculations of such characteristics as the equilibrium price in each of the three markets, the lowest value of the price (*min* $p_{1,3}$ and *min* p_2) that it achieves throughout the duration of the study (50 conventional units), the time period corresponding to the attainment of this minimum value $t(p{=}min\ p_{1,3})$ and $t(p{=}min\ p_2)$ depending on the elasticity in the market under study, provided that the cross elasticities are constant and equal b=0.3.

Table 1. Characteristics of price dynamics in three sequential markets

a	0.2	0.4	0.6	0.8	1.0	1.2	1.4
$p^*_{1,3}$	72.86	64.27	57.90	52.94	48.95	45.67	42.90
p^*_2	48-57.	46.74	44,54	42.35	40.31	38.45	36.77
min $p_{1,3}$	72.65	64.27	57.90	52.94	48.95	45.67	42.90
min p_2	48-57.	46.74	44.53	42.34	40-30	38.44	36.77
$t(p{=}min\ p_{1,3})$	3.2	5.0	18.8	15.8	11.6	12.0	10.4
$t(p{=}min\ p_2)$	27.8	21.6	4.6	3.6	3.2	3.0	3.0

As shown in Table 1, for the given conditions the equilibrium prices in all three markets decrease monotonically with an increase in the relative elasticity of supply as to the demand in the market under study. For the first and third markets, this trend was observed at almost any ratio of elasticities in adjacent markets (Fig. 6a), since at low values of the parameter b, the growth of the parameter a ensured the fulfillment of the condition (16). Violation of this condition occurred only when combining small values of the parameter a with large values of the parameter b. For the second market, a monotonic decrease in the equilibrium price was found only at small values of cross-elasticities when the condition (16) was not violated. At large values of the parameter b there was initially a slight increase in the equilibrium price, which was then followed by its monotonic decrease (Fig. 6b);

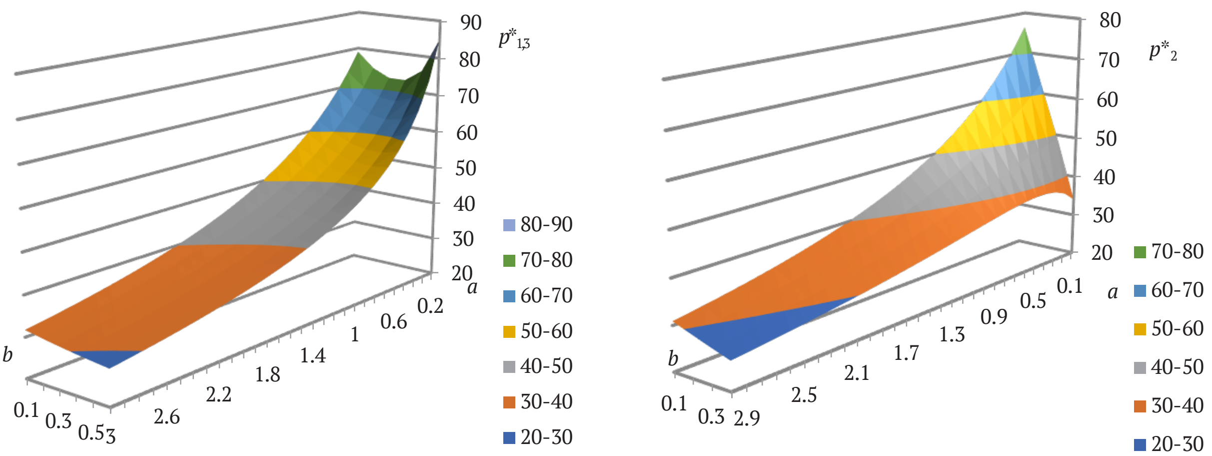


Figure 6. Equilibrium price depending on the elasticities for the first and third (a) and for the second market (b) in three adjacent markets

Let's note another feature of pricing, which is illustrated by the data given in the four bottom lines of Table 1, i.e. the process of establishing an equilibrium price is not monotonous in time. It was found that with certain a and *b* parameter ratios and the "transition price-time" function reaches a minimum value in a sufficiently short period of time and then it reaches an equilibrium value. Thus, when the a=0.2 difference between the lowest price and the equilibrium price for the first and third markets is 0.29% of the equilibrium price, whereas for the second market the transition price changes monotonically over time and its minimum value is equal to the equilibrium price. Conversely, at a=0.6 (and more) transition prices in the first and third markets change monotonically. In the second market, the minimum price is observed at t=4.6, then the price rises to an equilibrium value. Under these conditions, the difference between the lowest price and the equilibrium price for the second market is only 0.01% of the equilibrium price. But this difference grows with the increase in elasticity in the market under study and at a=0.9 it already reaches 0.03% of the equilibrium price. We should also note that the time period, during which the transition price becomes equal to the equilibrium price, decreases with increasing elasticity. This is observed for all links of the chain of three markets, although the equilibrium in the second market is reached in a shorter period of time than in the first or third ones.

It should be noted that the results in Table 1 are only one example of the impact of elasticities of "demand-supply" curves on price dynamics. Similar calculations within the scope of this work were carried out for other elasticity ratios both in the market under study and for cross-elasticities as well as for other values of marginal prices.

Let's compare the obtained results with the findings of other researchers. As noted in the literature review, the problem of equilibrium in the system of "demand-supply" is multifaceted and has a very large number of research areas. As a rule, they consider the interaction of two actors in the horizontal market [33; 34]. This is due to the fact that the interaction of actors in the vertical market is considered more complex [35]. Game theory [36] and differential calculus [37] are most often used as a mathematical apparatus for modeling. In this study, we propose a general model of the sequence of N-dimensional vertical markets, which takes into account the effect of time-distributed lateness, i.e. when present demand is affected by supply in the past. Unlike other models, this model of economic dynamics is a system of integral equations. The same distributed lateness was investigated in the paper [38], but on the example of a specific market – the labor market. Just like in our study, the results of simulation modelling also confirmed the existence of two points of equilibrium for the labor market. Similar results regarding the existence of attractors were also obtained using simulation modelling in the research [38]. Thus, although research in this direction is quite active, however, compared to the model proposed in this paper, it is devoted to partial issues with a limited number of participants. Yet, upon considering these issues, the obtained results coincide with those obtained for the general model of economic dynamics proposed in this paper.

CONCLUSIONS

The results of analytical research and simulation modelling have shown that the mathematical model proposed in this paper adequately describes the dynamics of pricing for a system of sequential markets and gives the opportunity to determine the dependence of this process on a set of parameters that characterize demand and supply in each of the markets. The dynamic aspect of the proposed model is the existence of an "after-effect", which is described in an integral form by a distributed time lag. At the first stage, we recorded the initial mathematical model of sequentially connected markets as a system of Volterra integral equations. This model than was subsequently transformed into a system of linear algebraic equations using the Laplace integral transform in order to determine the price in each of the markets at each time point. The existence of a tridiagonal Jacobi matrix is specific to the resulting system of linear algebraic equations, which makes it possible to find analytical solutions. Moreover, the conditions for the stability of analytical solutions in the vicinity of the equilibrium state of the system of sequentially connected markets were formulated on the basis of these equations. The application of simulation modelling for systems consisting of two and three sequential markets allowed us to identify which processes occur when the stability condition is violated.

The results of calculations by the proposed analytical model give scope for additional research. The authors envisage further development of this model in the study of the impact of parameters characterizing demand and supply in each of the markets on the dynamics of pricing for four or more sequentially connected markets and in a more detailed analysis of the processes that are observed when the conditions of solution stability are violated.

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Динаміка формування перехідних цін на ланцюгу послідовних ринків: аналітична модель

Анотація. Хоча проблема формування ринкових цін, визначення рівноважних цін в межах моделі «Попит - пропозиція» є достатньо відомою, і її дослідженню присвячено чимало як теоретичних робіт, так і робіт, в яких узагальнюються результати спостережень, ця проблема залишається актуальною, особливо в тій її частині, що стосується динаміки процесів ціноутворення і стійкості рівноважних цін відносно зміни параметрів, що характеризують стан системи. У більшості досліджень, що присвячені цим питанням, основна увага приділяється або певному локальному ринку, або глобальному ринку деякої продукції в цілому. Метою даної роботи є побудова математичної моделі, яка б дозволяла у загальному вигляді здійснювати аналіз питань, що пов'язані з формуванням перехідних цін на скінченому N-мірному ланцюгу послідовних ринків згідно зі схемою ринкової рівноваги. Запропоновано аналітичну модель, що дозволяє досліджувати динаміку цін на суміжних ринках. У межах цієї моделі, що базується на визначенні процесів за допомогою системи інтегральних рівнянь, передбачалось, що вплив на ланцюг послідовних ринків і реакція на цей вплив є неперервними у часі. Динамічний аспект запропонованої моделі ціноутворення на вертикальній послідовності ринків полягає в існуванні «ефекту післядії», який в інтегральній формі описується запізненням, розподіленим у часі. Розглянуті питання адекватності моделі, проведено дослідження її внутрішньої узгодженості, обґрунтована коректність переходу від математичної моделі динаміки як системи інтегральних рівнянь до моделі у формі системи лінійних алгебраїчних рівнянь. Сформульовані умови існування розв'язку цієї системи рівнянь і визначена область його стійкості. Математична модель, що запропонована у даній роботі, дозволяє здійснювати якісний аналіз станів системи (за фазовими траєкторіями). Наведено приклади чисельної реалізації отриманої аналітичної моделі для випадку двох і трьох послідовних ринків, визначені рівноважні ціни для кожної ланки ланцюга послідовних ринків. За допомогою імітаційного моделювання досліджувалась стійкість розв'язку відносно зміни таких параметрів моделі, як еластичності попиту і пропозиції на ринку, що досліджується, і перехресних еластичностей на суміжних з ним ринках, а також вплив цих параметрів на такі динамічні показники ринкової системи, як швидкість досягнення рівноваги

Ключові слова: ринкова вертикаль, ціноутворення, фазові траєкторії, інтегральні рівняння Вольтерра, адекватність моделі, імітаційне моделювання, еластичності попиту і пропозиції