

MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE
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Guidelines
to laboratory works
in the academic discipline
"OPERATIONS RESEARCH"
for full-time students
of training direction 6.030601 "Management"

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Затверджено на засіданні кафедри економіки, організації та планування діяльності підприємства.

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Guidelines to laboratory works in the academic discipline and questions for knowledge consolidation are given.

Recommended for full-time students of training direction 6.030601 "Management".

Наведено методичні рекомендації до виконання лабораторних робіт з навчальної дисципліни та питання для закріплення знань.

Рекомендовано для студентів напряму підготовки 6.030601 "Менеджмент" денної форми навчання.

Introduction

Efficiency growth is closely linked with quantitative reasoning of decisions making. Due to this, a specialist in management should know and be able to put into practice the economic and mathematical methods and models of operations research.

The results of the commercial activities of enterprises are affected by a large number of factors and phenomena. This effect is varied: some factors and phenomena have a major impact, others are weaker, and the others are not affected. To develop an effective policy we must be able to assess the relationship between the factors and phenomena themselves, and their impact on the results of business enterprises.

Mathematical methods are essential to select and formalize the relationship between economic indicators, to assess the form and parameters of their dependencies. Accordingly, the production manager should know and be able to use this method.

Also, the future manager should know the features of economy, organization and production planning, taken into account when developing mathematical models of economic activity.

Future managers must also know the characteristics of the economy, organization and planning, using methods of operations research for better decision making.

Academic discipline "Operations Research" refers to the cycle of regulatory subjects for students of specialization area "Management".

The subject "Operations Research" is based on knowledge obtained by students after subjects of economic and mathematical cycles, "Economics", "Macroeconomics", "Higher and Applied Mathematics" "Statistics" and others. General knowledge and skills, acquired by students of the subject "Operations Research", can be expanded in the process of learning subjects in the speciality. During the study students receive the necessary knowledge at lectures and practicals, in the performance of individual tasks. Of great importance in the process of learning and consolidation of knowledge is independent work of students.

The purpose of the subject is to master theoretical knowledge and practical skills in specialized mathematical economic methods to provide optimization of management, organization and planning tasks.

The task of the subject is to apply operations research to test economic theories by means of economic and mathematical methods and quantitative substantiation of optimal solutions.

The topic of the subject are models and methods of system analysis, research methods and streamlining operations.

The discipline is taught after the subjects "Higher and Applied Mathematics". This links the disciplines of mathematical cycle of economic sciences, prior to the study of professional subjects which are the basis for economic research.

The subject consists of lectures, laboratory studies, self-study work.

Academic discipline is taught within the specialization area 6.030601 "Management" for full-time students.

Plans of Laboratory Works

Themes of laboratory works. Laboratory works are a form of academic classes, where students are organized for a detailed analysis of some theoretical discipline. For this purpose objectives and guidelines for their implementation are given. Successful problem solving requires the use of batch programs.

During laboratory works the students form skills of operation research methods application for solving specific economic problems. At laboratory works each student performs individual tasks. For their successful implementation the previous lecture material on a particular theme must be revised, and used in the accomplishment of software tasks. Program Excel is used.

Three practical problems are presented:

1. Theme 2 "Objectives and Methods of Queuing".
2. Theme 5 "Methods of Planning and Management".
3. Theme 6 "Optimization Tasks of Inventory Management".

The list of laboratory works is presented in Table 1.

Table 1

List of themes for modules of laboratory studies

Task	Themes	Hours	References
Semantic module. Economic and mathematical methods of operations research			
Task 1. Justification of size adjusters using queuing theory	Workshop 1. Build a model of equipment maintenance	2	References: main: [2 – 4]; ancillary: [6; 7]
	Workshop 2. Build a model of the equipment	2	References: main: [1; 3; 4]; ancillary: [6; 7]
	Workshop 3. Construct a numerical model of equipment maintenance	2	References: main: [2 – 4]; ancillary: [5 – 7]
	Workshop 4. Construct a graphical model of service	2	References: main: [3; 4]; ancillary: [6; 7]
	Workshop 5. Calculating the cost of work time for repairing and service. Substantiate the number of adjusters	2	References: main: [3; 4]; ancillary: [6; 7]
Task 2. Justification of complexity. Conceptual design	Workshop 6. Build a network model for project development	2	References: main: [1; 3; 4]; ancillary: [6]
	Workshop 7. Solve the network model	2	References: main: [2 – 4]; ancillary: [5 – 7]
	Workshop 8. Construct a line graph chart of project development needs of workers	2	References: main: [2 – 4]; ancillary: [5 – 7]
Task 3. Substantiation of the order quantity	Workshop 9. Substantiate the amount of parts, and beverages	2	References: main: [3; 4]; ancillary: [6; 7]
Total		18	

Semantic Module. Economic and Mathematical Methods of Operations Research

Task 1. Justification of size adjusters using queuing theory

The purpose of the task. To justify the norm of machine service for the adjuster.

The content of the task. Valid standards for determining the norm of service do not fully take certain production conditions into account. Application of the queuing theory can eliminate this disadvantage and perform a simulation of the production process, i.e. to play various variants of service organization over on paper. Using the chosen optimality criterion – the minimum value of the cost of operation and maintenance – it's necessary to justify the most economical variant of machine service organization performed by the adjuster.

Guidelines to the task:

1. Develop an economic and mathematical model of the machines run time:

1.1. Construct a histogram of the distribution of the machines run time.

1.2. Select the type of curve that describes the distribution and find its formula.

1.3. Check the proximity of empirical data to theoretical data using Pearson criterion χ^2 .

1.4. Build integrated cumulative curve for the time of the machines.

2. Develop a mathematical model of service time machines:

2.1. Build ground distribution of service time.

2.2. Select the type of curve that describes the distribution and find its formula.

2.3. Check the proximity of empirical data to theoretical criterion using Pearson χ^2 .

2.4. Build the mass diagram (integral cumulative curve) for the machines run time.

3. Solving the model:

3.1. Build a numerical model of adjuster service of two, three and four machines using the table of random numbers and cumulative curves.

3.2. Build a graphical model of service and determine the average value of machines' downtime due to absence (being busy) of adjuster in different variants of service organization.

4. Prove the most economical variant of service organization. To do this, determine the cost price of work and service in each variant and build a graph of dependence between the cost price and the number of machines at service. Choose the most economical variant of service organization.

Output from this task:

machine run time automatically (option task in Table 2, the outgoing data is grouped and presented in Table 3);

the service tools (data grouped and presented in Table 5 and 6, then they must be entered in Table 4).

Table 2

Selecting data of machine run time from Table 3

Variant	Machines	Variant	Machines
1, 2, 3	1 – 10	16, 17, 18	1 – 5, 8 – 12
4, 5, 6	1 – 9, 11	19, 20, 21	1 – 4, 7 – 12
7, 8, 9	1 – 8, 11, 12	22, 23, 24	1 – 3, 6 – 12
10, 11, 12	1 – 7, 10 – 12	25, 26, 27	1 – 2, 5 – 12
13, 14, 15	1 – 6, 9 - 12	28, 29, 30	1 – 2, 6 – 12

Table 3

Hours machines, min.

Machines	Number of Measurement									
	1	2	3	4	5	6	7	8	9	10
1	10	12	35	12	24	32	25	34	23	21
2	22	23	25	26	25	25	24	21	22	23
3	20	15	24	25	32	21	29	26	21	29
4	21	23	19	24	26	18	17	23	27	27
5	22	27	26	16	19	22	27	28	18	29
6	31	25	20	23	32	23	18	31	29	25
7	24	17	23	22	25	30	19	25	19	28
8	21	23	30	23	15	24	22	30	24	26
9	16	27	18	21	23	28	20	24	25	27
10	21	24	22	25	22	26	21	27	20	23
11	22	21	26	29	26	28	27	24	24	25
12	24	21	23	24	25	26	28	25	22	26

Table 4

Time of service and repetition frequency

Interval (n)	1	2	3	4	5	6	7	8
Time Service (X), min.								
Repetition frequency (m)								

To fill Table 4 time service take from Table 5, and the frequency of repetition from Table 6.

Table 5

Time Service (X)

Variant	Interval (n)							
	1	2	3	4	5	6	7	8
1, 2, 3	5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5
4, 5, 6	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0
7, 8, 9	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5
10, 11, 12	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
13, 14, 15	7.0	7.5	8.0	8.5	9.0	9.5	10.0	10.5
16, 17, 18	7.5	8.0	8.5	9.0	9.5	10.0	10.5	11.0
19, 20, 21	8.0	8.5	9.0	9.5	10.0	10.5	11.0	11.5
22, 23, 24	8.5	9.0	9.5	10.0	10.5	11.0	11.5	12.0
25, 26, 27	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5
28, 29, 30	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0

Table 6

Repetition frequency (m) service time

Variant										Repetition frequency (m)							
										Interval (n)							
										1	2	3	4	5	6	7	8
1	4	7	10	13	16	19	22	25	28	12	27	30	23	13	6	3	1
2	5	8	11	14	17	20	23	26	29	13	26	31	22	13	6	3	1
3	6	9	12	15	18	21	24	27	30	14	26	30	22	13	6	3	1

Methodical Recommendations for the Task 1

1. Construction of mathematical model of the machines run time.

1.1. Grouping of the machines run time:

a) the number of intervals n:

$$n = 1 + 3,32 \log N, \tag{1}$$

where N – number of observations (in the task N = 100);

b) the interval width lx:

$$lx = (X_{\max} - X_{\min}) / n; \tag{2}$$

c) interval series (Table 7).

Table 7

Grouping of the machines run time

Most interval	Range of X	Empirical frequency m
1	From (Xmin) to (Xmin + lx)	m_1
2	From (Xmin + lx) to (Xmin + 2 · lx)	m_2
...
8	From (Xmin + n · lx) to (Xmax)	m_8
		$\sum m_i = N = 100$

1.2. Construct a distribution histogram of the machine run time and choose the distribution law (recommended to take normal law).

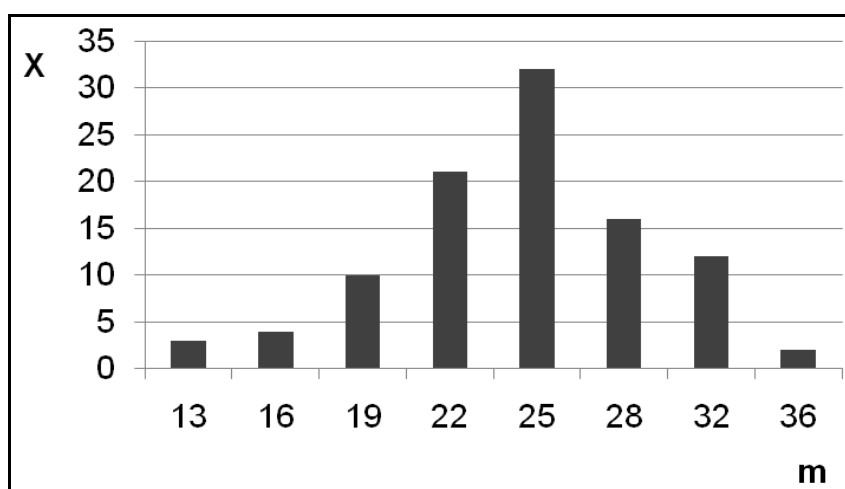


Fig. 1. A distribution histogram of the machine run time (conventional data)

1.3. Justification of correctness of the chosen law with Pearson criterion χ^2 .

Calculation of Pearson χ^2 and cumulative probabilities for the normal distribution law is convenient to carry out in Table 8.

Table 8

Calculation of Pearson χ^2 and cumulative probabilities for the normal distribution law

n	X	m	X'	X'm	(X') ² m	t	f(t)	m'	χ^2	F(t)
1	2	3	4	5	6	7	8	9	10	11
1	4	3	-4							
...
5	16	28	0							
...
8	25	4	3							
		$\sum m$		\sum	\sum			$\sum m'$	$\sum = \chi^2_{calc}$	

In Table 8:

n – number of intervals;

N – number of observations (100);

X – intervals' endings from Table 7;

X' – conditional variable, which is calculated by the formula

$$X' = (X - a_x) / I_x,$$

where a_x – number of 2nd column of 7, which has a maximum frequency in the 3rd column;

I_x – interval width;

m – empirical frequency from Table 7.

Average value of conditional variable:

$$\bar{X}' = \frac{\sum X' m}{\sum m}; \quad (3)$$

conditional variable variance:

$$\sigma_{X'}^2 = \frac{\sum (X')^2 m}{\sum m} - (\bar{X}')^2; \quad (4)$$

normalized value:

$$t = \frac{X' - \bar{X}'}{\sigma_{X'}}. \quad (5)$$

$f(t)$ – differential distribution function (Table 9);

theoretical frequency:

$$m' = f(t) \cdot (\sum m) / \sigma_{X'}, \quad (6)$$

where $F(t)$ – cumulative distribution function (Table 10);

$\sum m$ must be equal to $\sum m'$.

Criterion Pearson χ^2 is calculated by the formula:

$$\chi^2_p = \chi^2_p = \sum_1^n \frac{(m_i - m'_i)^2}{m'_i}, \quad (7)$$

where n – number of intervals;

m_i – empirical frequency of the i -th interval;

m'_i – theoretical frequency of the i -th interval.

The closer the value of χ^2_p to zero, the better theoretical and empirical distributions interact. Significance evaluation of criteria is conducted as follows: having the significance level $\alpha = 5\%$ (probability of mismatch) and determine the number of degrees of freedom $\tau = n - k$, where n – number of intervals, k – number imposed relations. For normal law $k = 3$, so you need to know three parameters (\bar{X} , σ , N) to calculate differential and integral distribution functions.

If $\chi^{2\text{calc}} < \chi^{2\text{theor}}$ for a given level of significance α and the number of degrees of freedom τ the theoretical distribution will relate to the empirical one correctly. In this case, the parameters of empirical law \bar{X} and σ can be used to express the theoretical distribution law and describe the outgoing data. Tabular values of the criterion are listed in table 11.

If the selected function is suitable, then record the mathematical model of equipment run time. To do this, use the integral function:

$$F(N, \bar{X}, \sigma) = \frac{1}{\sqrt{2\pi}} \int_0^X e^{-\frac{(X - \bar{X})^2}{2}} dx, \quad (8)$$

It's necessary to make the transition of the empirical average and standard deviation from the conditional variables to outgoing data by formulas (9 and 10):

$$X = a_x + X' \cdot i_x; \quad (9)$$

$$\sigma_X = i_x \sigma_{X'} \quad (10)$$

Construct a cumulative distribution function (cumulation) of the machines run time (Fig. 2).

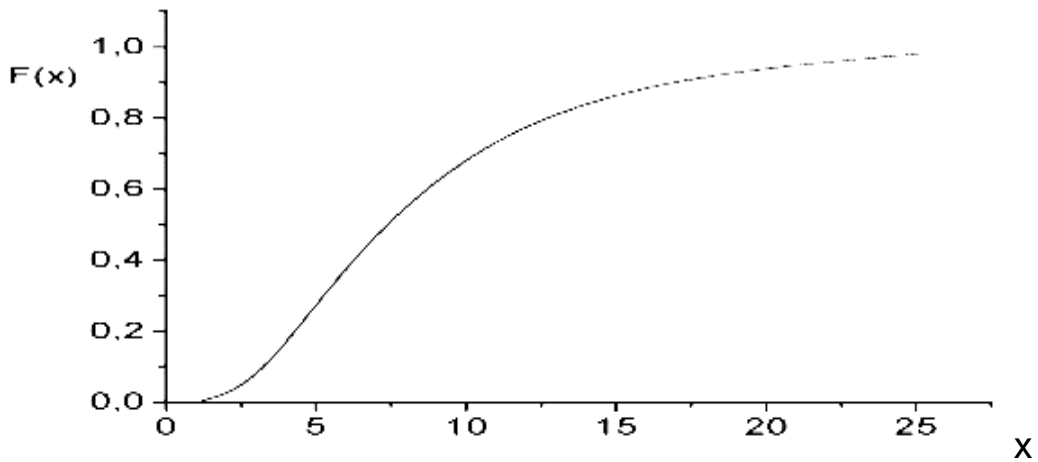


Fig. 2 Cumulative distribution function of the machines run time (conventional data)

2. Construct a mathematical model of the service time.

2.1. Construct a distribution polygon of the service time and choose the law of the Poisson distribution (Fig. 3).

To construct a polygon are middle intervals.

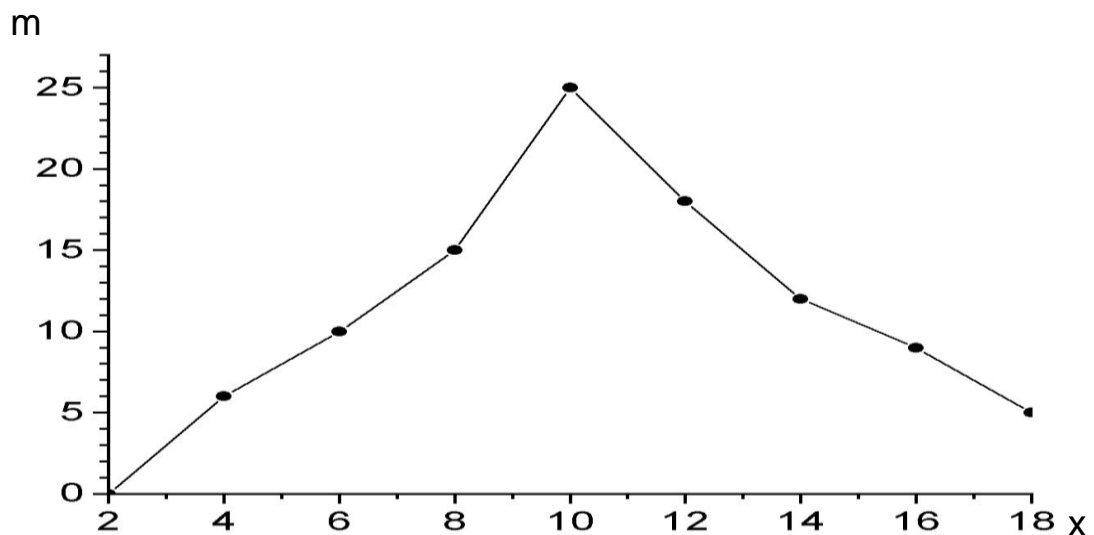


Fig. 3 Polygon distribution of the service time (conventional data)

2.2. Calculation of Pearson χ^2 and cumulative probabilities for a Poisson distribution are given in Table 9.

Table 9

Calculation of Pearson χ^2 and cumulative probabilities for a Poisson

n	X	M	X'	X'm	(X') ² m	P _x	m'	χ^2	$\sum P_x = F(x)$
1	2	3	4	5	6	7	8	9	10
1	4.5	3	0	0	0	0.050	5,8 \cong 6	1.50	0.050
2	5.5	12	1	12	12	0.149	17,1 \cong 17	1.47	0.199
...
8	7
		$\sum m$		\sum	\sum		$\sum m'$	$\sum = \chi^2_p$	

In Table 9:

n – number of intervals;

X – the service;

m and m' – empirical and theoretical frequency;

X' – conditional variable, calculated by the formula:

$$X' = (X - a_x) / l_x, \quad (11)$$

where a_x – first value of the 2nd column;

l_x – width interval.

Value of the average conditional variable:

$$\bar{X}' = \frac{\sum X'm}{\sum m}. \quad (12)$$

Variance of the conditional variable:

$$\sigma_{X'}^2 = \frac{\sum (X')^2 m}{\sum m} - (\bar{X}')^2, \quad (13)$$

where P_x – differential Poisson function (see Table 9).

$$m' = P_x \cdot \sum m,$$

where $\sum m$ must be equal to $\sum m'$;

$F(x) = \sum P_x$ – cumulative probability or Poisson integral function.

Verification of the chosen law by the criterion of Pearson χ^2 . If the selected function is suitable, shift from conventional variables to the outgoing data, and record the model using the empirical average.

Differential Poisson function has the form:

$$P_x = \frac{\lambda^x}{x!} e^{-\lambda}, \quad (14)$$

where $e = 2,71828$; $\lg e = 0,434245$; $N = \bar{X} = \sigma_x^2$.

2.3. Construct an integral distribution function (cumulation) of the machines service time (Fig. 4).

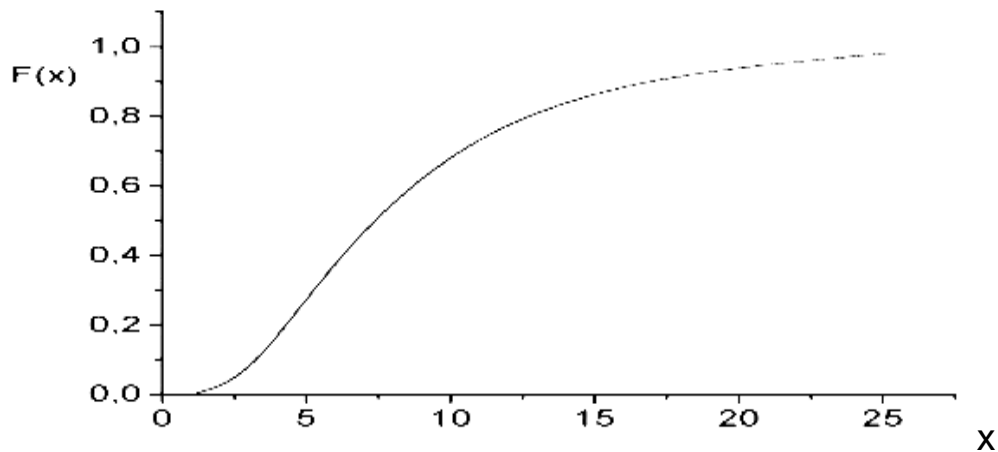


Fig. 4. **Cumulative distribution function of service time (conventional data)**

2.4. Construct a numerical model of service for 2, 3- and 4-machines. Numerical model for 2-machine is shown in Table 10.

Table 10

Numerical model service 2 looms

1 machine				2 machine			
Work		Service		Work		Service	
RN	OT	RN	ST	RN	OT	RN	ST
0.80	18	0.20	18	0.70	14	0.12	8
0.60	12	0.12	8	0.72	16	0.15	10
0.14		0.15		0.58	10	0.28	
0.08		0.68		0.26		0.59	
0.99		0.35		0.41		0.34	

Random numbers (RN) act as the accumulation of probabilities. For each option of service 5 values should be taken for each column of Table 12. Operation time (OT) is taken from cumulative curve of the machines run time (Fig. 2), and time of service (TS) is taken from the cumulative curve of service time (Fig. 4).

2.5. Basing on the numerical models of service, to build a graphical model (Fig. 5) for three variants of service ($i = 3$), when the adjuster serves 2, 3 or 4 machines.

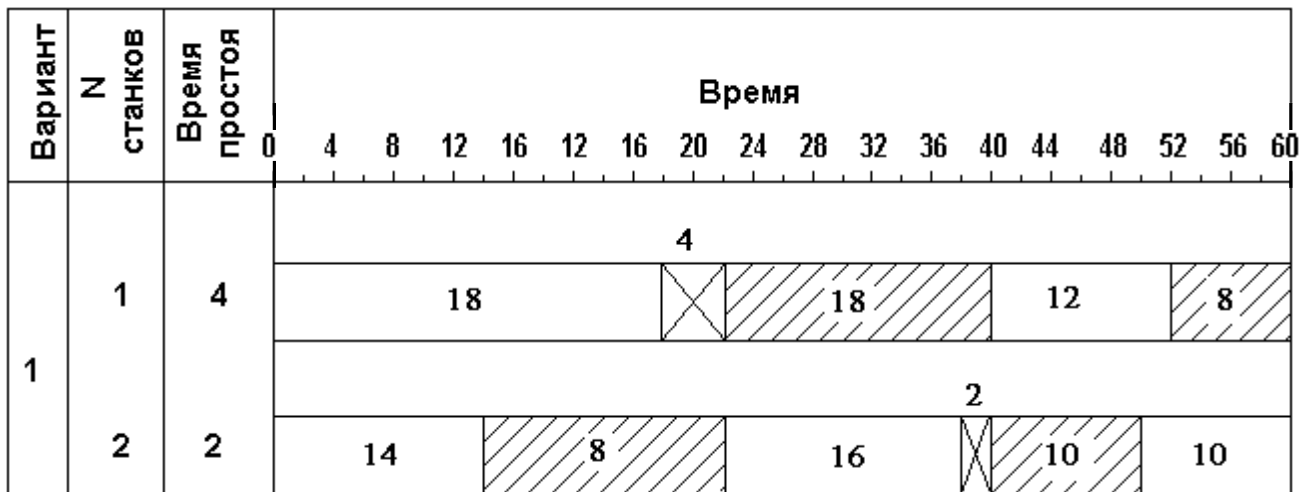


Fig. 5. A graphical model of service of two machines

Below there are other variants of service (3 and 4 machines).

8. Calculate the cost (C_i) of one minute of work and service in the i -th variant of the service:

$$C_i = (3 + K \cdot N_i) / (60 \cdot N_i - t_{opr\ i} - t_{np\ i}), \quad (15)$$

where C – salary per hour (10 UAH);

N_i – number of machines that are serviced in the i -th variant of the service organization;

K – content of equipment cost per hour of its work (8 UAH);

$t_{org\ i}$ – organizational service time (taken to be 0);

$t_{dt\ i}$ – equipment downtime in the i -th service options take the graphics service model (Fig. 5).

Calculations are to be made for three options for service when adjuster serves 2, 3 or 4 machines.

Construct a graph of the cost of service depending on the variant of the service organization. In order to do this, put a variant of organization on the X axis (number of machines – 2, 3, 4), the axis – costs of every variant.

One variant of (2, 3 or 4 machines), cost of which is minimal, will be accepted as a norm of service.

Table 11

The value of the differential function $f(t)$

T	0	1	2	3	4	5	6	7	8	9
0,0	0,399	3989	3989	3988	3986	3984	3982	3980	3977	3973
0,1	3970	3965	3961	3956	3951	3945	3939	3932	3925	3918
0,2	3910	3902	3894	3885	3876	3867	3857	3847	3836	3825
0,3	3814	3802	3790	3778	3765	3752	3739	3726	3712	3697
0,4	3683	3668	3653	3637	3621	3605	3589	3572	3555	3538
0,5	3521	3503	3485	3467	3448	3429	3410	3391	3372	3352
0,6	3332	3312	3292	3271	3251	3230	3209	3187	3166	3144
0,7	3223	3101	3079	3056	3034	3011	2989	2966	2943	2920
0,8	2897	2874	2850	2827	2803	2780	2756	2732	2709	2685
0,9	2661	2637	2613	2589	2565	2541	2516	2492	2468	2444
1,0	2420	2396	2371	2347	2323	2299	2275	2251	2227	2208
1,1	2179	2155	2131	2107	2083	2059	2036	2012	1989	1965
1,2	1942	1919	1895	1872	2849	1826	1804	1781	1758	1736
1,3	1714	1691	1669	1647	1626	1604	1582	1561	1539	1518
1,4	1497	1476	1456	1435	1415	1394	1374	1354	1334	1315
1,5	1295	1276	1257	1238	1219	1200	1182	1163	1145	1127
1,6	1109	1092	1074	1057	1040	1023	1006	0989	0973	0957
1,7	0940	0925	0909	0893	0863	0848	0833	0818	0804	0804
1,8	0790	0775	0761	0748	0734	0721	0707	0694	0681	0669
1,9	0656	0664	0632	0620	0608	0596	0584	0753	0562	0551
2,0	0540	0529	0519	0508	0498	0498	0478	0468	0459	9449
2,1	0440	0431	0422	0413	0404	0396	0388	0379	0371	0363
2,2	0355	0347	0339	0332	0325	0317	0310	0303	0297	0290
2,3	0287	0277	0370	0264	0258	0252	0246	0241	0235	0229
2,4	0224	0219	0213	0208	0203	0198	0194	0189	0184	0180
2,5	0175	0171	0167	0163	0158	0154	0151	0147	0143	0139
2,6	0136	0132	0129	0126	0122	0119	0116	0113	0110	0107
2,7	0104	0101	0099	0096	0098	0091	0068	0086	0084	0081
2,8	0079	0077	0075	0073	0071	0089	0087	0065	0063	0061
2,9	0060	0058	0056	0055	0053	0051	0050	0048	0047	0046
3,0	0044	0043	0042	0040	0039	0038	0037	0036	0035	0034
3,1	0033	0032	0031	0030	0029	0028	0027	0027	0025	0025
3,2	0024	0023	0022	0022	0021	0020	0020	0019	0018	0018
3,3	0017	0017	0016	0016	0015	0015	0014	0014	0013	0013
3,4	0012	0012	0012	0011	0011	0010	0010	0010	0009	0009
3,5	0009	0008	0008	0008	0008	0007	0007	0007	0007	0005
3,6	0006	0006	0006	0005	0005	0005	0005	0005	0005	0004
3,7	0004	0004	0004	0004	0004	0004	0003	0003	0003	0003
3,8	0003	0003	0003	0003	0003	0002	0002	0002	0002	0002
3,9	0002	0002	0002	0002	0002	0002	0002	0002	0001	0001

Table 12

The value of the cumulative distribution function $F(t)$

t	0	1	2	3	4	5	6	7	8	9
0,0	0,50	5040	5080	5120	5160	5199	5239	5275	5319	5359
0,1	5398	5438	5478	5517	5557	5596	5636	5675	5714	5754
0,2	5793	5832	5871	5940	5948	5987	6026	6064	6103	6141
0,3	6179	6217	6255	6293	6331	6338	6406	6443	6480	6517
0,4	6554	6591	6628	6664	6700	6736	6772	6808	6844	6879
0,5	6915	6950	6985	7019	7054	7088	7123	7157	7190	7224
0,6	7258	7921	7324	7357	7389	7422	7454	7486	7516	7549
0,7	7580	7612	7624	7673	7707	7734	7764	7794	7823	7852
0,8	7881	7910	7939	7967	8000	8023	8051	8079	8106	8133
0,9	8159	8186	8212	8228	8234	8239	8315	8348	8365	8389
1,0	8413	8438	8461	8485	8508	8631	8554	8577	8600	8621
1,1	8643	8665	8686	8708	8729	8749	8770	8790	8810	8830
1,2	8849	8867	8888	8907	8925	8944	8962	8980	8997	9015
1,3	9032	9049	9066	9082	9099	9105	9131	9147	9162	9177
1,4	9192	9207	9222	9236	9251	9265	9265	9279	9293	9319
1,5	9332	9345	9357	9369	9382	9394	9406	9417	9429	9441
1,6	9452	9463	9474	9495	9605	9515	9525	9525	9535	9545
1,7	9554	9564	9573	9585	9591	9599	9601	9616	9625	9633
1,8	9641	9649	9656	9664	9671	9678	9686	9693	9699	9706
1,9	9713	9719	9726	9732	9738	9744	9750	9756	9762	9767
2,0	9770	9773	9783	9788	9793	9798	9803	9808	9812	9817
2,1	9821	9826	9830	9834	9838	9842	9846	9850	9854	9857
2,2	9861	9865	9868	9871	9875	9878	9881	9884	9887	9890
2,3	9893	9896	9898	9901	9904	9906	9909	9911	9917	9916
2,4	9918	9920	9922	9925	9927	9929	9931	9939	9934	9936
2,5	9938	9940	9941	9943	9945	9946	9948	9949	9951	9952
2,6	9953	9955	9956	9957	9959	9960	9961	9962	9963	9964
2,7	9965	9965	9967	9968	9969	9970	9971	9972	9973	9974
2,8	9974	9975	9976	9977	9977	9978	9979	9980	9980	9981
2,9	9981	9982	9983	9983	9983	9984	9984	9985	9986	9986
3,0	9987	9939	9987	9988	9988	9989	9989	9989	9989	9990
3,1	9990	9991	9992	9991	9992	9992	9992	9992	9993	9993
3,2	9993	9993	9994	9994	9994	9994	9994	9995	9995	9995
3,3	9995	9995	9996	9996	9996	9996	9996	9996	9996	9997
3,4	9997	9997	9997	9997	9997	9997	9997	9997	9998	9998

Table 13

Poisson distribution
where $\lambda = \bar{X}$ [3]

n \ \bar{X}	1,8	1,9	2,0	2,1	2,2	2,3	2,4	2,5	2,6	2,7	2,8	2,9
0	0,16	150	135	122	111	100	091	082	074	067	065	055
1	298	284	271	257	244	231	218	205	193	181	170	160
2	268	270	271	270	268	265	261	257	251	245	238	231
3	161	171	180	189	197	203	209	214	218	220	222	224
4	072	081	090	099	108	117	125	134	141	149	156	162
5	026	031	036	042	048	054	060	067	074	080	087	094
6	008	010	012	015	017	021	024	028	032	036	041	045
7	002	003	003	004	005	007	008	010	012	014	016	019
8	001	001	001	001	001	002	002	003	004	005	006	007
9									001	001	002	002
10												001

Table 14

The value of Pearson's χ^2 depending on n and α

n ↓ K	$\alpha \rightarrow 0,9$	0,80	0,70	0,50	0,30	0,20	0,10	0,05	0,02	0,01
1	0,016	0,064	0,148	0,45	1,07	1,64	2,71	3,84	5,41	6,64
2	0,211	0,466	0,713	1,39	2,41	3,22	4,60	5,99	8,82	9,21
3	0,584	1,005	1,424	2,37	3,66	4,64	6,25	7,85	9,84	11,34
4	1,064	1,649	2,20	3,36	4,88	5,99	7,78	9,46	11,6	13,28
5	1,610	2,34	3,00	4,35	6,06	7,29	9,24	11,06	13,3	15,09
6	2,20	3,07	3,83	5,35	7,23	8,56	10,64	12,59	15,03	16,31
7	2,83	3,82	4,67	6,35	8,38	9,80	12,02	14,07	16,6	18,48
8	3,49	4,59	5,53	7,34	9,52	11,03	13,36	15,61	18,1	20,1
9	4,17	5,38	6,39	8,34	10,66	12,24	14,68	16,92	19,6	21,7
10	4,86	6,18	7,27	9,34	11,78	13,44	15,99	18,31	21,2	23,2

Table of random numbers

1	2	3	4	5	6	7	8	9	10
1,534	7,106	2,836	7,833	6,574	7,545	7,590	5,574	1,202	7,712
6,128	8,993	4,102	2,551	0,330	2,358	6,427	7,067	9,325	2,454
6,047	8,566	8,644	9,341	9,297	6,751	3,500	8,754	2,913	1,258
0,806	5,201	5,705	7,355	1,448	9,562	7,514	9,205	0,402	2,427
9,915	8,274	4,525	5,695	5,725	9,630	7,172	6,988	0,227	4,264
2,882	7,158	4,341	3,463	1,178	5,786	1,173	0,670	0,820	5,067
9,213	1,223	4,388	9,760	6,691	6,861	8,214	8,813	0,611	3,131
8,410	9,836	3,899	3,683	1,253	1,683	6,988	9,978	8,026	6,751
9,974	2,362	2,103	4,326	385	9,079	6,187	2,721	1,489	4,216
3,402	8,162	8,226	0,782	3,364	7,871	4,500	5,598	9,491	3,816
8,188	6,596	1,492	2,139	8,823	6,878	0,613	7,161	0,241	3,834
3,825	7,020	1,124	7,483	9,155	4,919	3,209	5,959	2,364	2,555
9,801	8,788	6,338	6,899	3,303	0,807	0,968	0,539	4,205	8,257
5,603	1,251	6,352	6,467	0,231	3,556	2,569	9,446	4,174	9,219
0,714	3,757	0,378	8,266	8,854	1,374	6,687	1,221	0,678	3,714
5,617	5,652	7,627	0,372	8,151	3,668	1,994	4,402	2,124	0,016
6,789	6,279	7,306	1,856	7,023	9,043	7,161	7,526	6,913	6,396
6,705	4,978	8,621	1,790	4,433	6,628	0,854	9,127	3,445	1,111
3,840	1,086	0,774	9,241	9,297	4,239	1,739	7,734	0,119	2,436
7,662	3,939	2,165	3,273	0,551	1,645	8,477	1,877	5,327	8,629
7,639	2,868	4,391	2,950	7,122	7,325	9,727	0,080	7,464	7,947
3,237	7,203	4,246	7,329	7,936	0,065	4,146	0,866	4,916	8,648
3,917	6,271	1,721	5,469	1,914	8,653	0,387	2,756	6,073	8,984
9,138	9,395	6,006	6,423	7,977	1,873	7,103	4,267	9,316	7,206
8,358	5,896	6,286	9,242	5,040	8,509	2,941	3,913	3,028	1,563
1,030	5,094	1,745	2,975	2,018	7,340	6,547	0,207	5,587	0,300
6,606	6,305	1,564	6,628	7,822	7,142	6,564	1,659	5,369	1,659
4,533	8,841	4,922	9,365	1,361	6,691	1,633	6,764	0,747	3,881
4,258	2,012	0,992	0,106	1,542	4,760	0,392	4,057	0,092	5,203
5,224	5,128	8,949	7,928	7,267	0,116	1,476	2,009	1,772	3,860
6,872	7,492	7,962	1,867	7,437	1,526	3,516	9,129	4,159	8,064
8,638	8,407	7,198	0,956	0,950	7,753	5,144	3,914	5,596	6,104
9,958	7,172	5,822	4,224	6,701	7,559	4,985	4,856	4,461	6,147
0,265	3,086	2,996	0,699	3,584	9,702	1,665	0,446	9,107	6,437
8,997	5,441	7,878	9,404	0,487	2,939	3,805	9,127	7,887	5,197

Table 15 (the end)

1	2	3	4	5	6	7	8	9	10
5,552	3,529	9,627	9,362	6,298	6,021	0,024	9,520	9,154	0,643
9,383	6,640	7,394	9,592	9,903	7,699	8,939	9,972	1,257	0,994
9,903	4,959	0,332	9,109	0,182	6,721	9,163	9,008	2,542	4,461
6,530	5,070	7,589	6,928	6,014	1,832	9,307	5,107	1,354	9,255
9,679	8,953	8,310	2,060	6,267	1,773	7,979	6,741	6,033	3,588
5,765	4,987	1,639	3,512	9,843	5,286	3,786	2,384	4,919	5,611
7,198	2,447	6,716	0,391	5,585	1,106	5,330	0,504	6,346	3,679
2,385	0,605	2,678	1,399	2,371	7,968	1,212	9,569	8,650	5,841
0,732	8,732	8,660	5,836	9,065	4,603	0,029	8,042	0,151	0,345
1,642	6,094	3,795	3,600	4,532	9,740	0,376	4,384	9,203	5,387
4,514	1,956	7,212	0,687	7,632	2,126	0,846	7,055	4,106	9,157
8,744	5,580	8,038	9,087	7,222	0,424	0,028	4,511	3,191	9,846
3,729	6,225	5,397	6,790	2,157	3,414	6,509	5,204	4,779	5,641
8,858	3,147	8,410	2,873	1,290	9,796	8,873	7,585	7,185	4,726
3,522	5,601	6,197	6,051	3,470	8,283	5,702	0,103	8,726	5,282

Task 2. Justification of Complexity. Conceptual Design

The purpose of the task. To substantiate the duration of the conceptual design development.

The content of the task. A firm is interested in production of a new device. It is necessary to reduce the time of technical preparation of production that can be achieved by using methods of network planning and management.

Outgoing data:

The content of works and how to implement them are listed in Table 16.

On basis of this data it's necessary to:

- 1) build a network diagram and project development network graph;
- 2) calculate a critical path, temporal characteristics of events and activities;
- 3) construct a line graph of work performance and chart of needs for workforce by days of development;
- 4) justify measures aimed at implementation in terms of conceptual design.

Variants of the problem (Table 17) recorded in Table 16.

Table 16

The content and order of the work

Number and content of	The procedure of works	Duration	Quantity
1. Action chart development	After receiving the task		
2. Lines drawing development	After completing work 1		
3. Constructing outline drawing	After completing work 2 in parallel with the implementation of work 4		
4. General drawing development	After completing work 2 in parallel with the implementation of work 3		
5. Performing technical calculations	After execution of works 3 and 4		
6. Conducting technical and economical calculations and design of other documents	After completing the work 4		
7. Program and testing method development	After completing work 5		
8. Creating an explanatory note	After completing works 6 and 7		

Table 17

Outgoing data

Variant	Duration of work, days/number of employees (office)							
	Number of							
	1	2	3	4	5	6	7	8
1	2	3	4	5	6	7	8	9
1	6/3	15/3	7/3	10/4	6/4	10/4	8/4	5/2
2	6/3	12/2	8/5	10/3	6/4	10/4	8/4	5/2
3	6/3	13/3	8/5	10/4	6/4	10/4	8/4	5/3
4	6/3	14/4	9/5	10/4	6/4	10/4	8/4	5/3
5	6/3	15/4	8/5	10/3	6/4	10/4	8/4	5/3
6	6/3	16/4	7/5	10/4	6/4	10/4	8/4	5/3
7	6/3	17/4	9/5	10/3	6/4	10/4	8/4	5/3
8	6/3	10/4	8/5	11/4	6/4	10/3	8/4	5/3
9	6/3	9/4	10/5	11/3	6/4	10/3	8/4	5/3
10	6/3	8/4	9/5	11/3	6/3	10/3	8/4	5/3
11	6/3	7/4	9/5	11/4	6/3	10/4	8/4	5/3

Table 17 (the end)

1	2	3	4	5	6	7	8	9
12	5/3	12/4	8/5	11/4	6/3	10/3	8/4	5/3
13	5/3	14/4	8/5	11/3	6/4	10/4	8/4	5/3
14	5/3	16/4	8/5	13/4	6/4	10/4	8/4	5/3
15	5/3	15/4	7/5	14/4	6/4	10/4	8/4	5/3
16	7/3	16/4	7/5	11/5	6/4	10/4	8/4	5/3
17	7/3	17/4	7/5	11/5	6/4	10/4	8/4	5/3
18	7/3	18/4	7/5	11/5	6/4	10/3	8/4	5/3
19	7/3	19/4	6/5	12/5	6/3	10/3	8/4	5/3
20	7/3	20/4	7/5	11/5	6/3	10/4	8/4	5/3
21	8/3	15/4	8/5	10/5	6/4	10/4	8/4	5/3
22	8/3	14/4	10/5	12/4	6/3	10/3	8/4	5/3
23	9/3	15/4	11/5	14/3	6/3	10/3	8/4	5/3
24	9/3	13/4	8/5	10/3	6/3	10/3	8/4	5/3
25	9/3	12/4	8/5	10/3	6/3	10/3	8/4	5/3
26	9/3	11/4	7/5	9/4	6/3	10/3	8/4	5/3
27	9/3	6/3	6/4	10/4	6/4	10/4	8/4	5/3
28	10/4	8/3	8/4	10/5	6/2	9/5	8/3	6/3

Methodical Recommendations to Task 2

1. Events have the following temporal characteristics:

early term of event execution – the maximum path from the initial event (or prior) to the given one:

$$T_j^{\text{early}} = \max \{T_i^e + t_{i-j}\}, \quad (16)$$

later term of event execution:

$$T_i^{\text{late}} = \tau - T_i^{\text{rev}}, \quad (17)$$

where τ – length of the critical path;

T_i^{rev} – early term of event execution in case of the reverse account or maximum path from the given event to the end event:

$$T_i^{\text{rev}} = \max \{T_j^{\text{rev}} + t_{i-j}\}, \quad (18)$$

event slack time:

$$R_j = T_j^{\text{late}} - T_j^{\text{early}}, \quad (19)$$

shows the term for which event execution can be delayed without changing the timing of the final event. The events that lie on the critical path have matching early and late terms and reserves are equal to zero.

Calculation of the temporal characteristics of events can be done by tabular or matrix methods. Matrix is a table in which the number of columns equals to the number of rows and equals to the number of events plus 3 (Table 18).

Table 18

Calculation of the temporal characteristics of events

T^{early}	$i \backslash j$	0	1	2	3	4	...	T^{rev}
0←	0		←4	6	2			
4→	1	→	→↑	0				
	2				4			
	3					↓5←		←5
	4					→		→0
...							...	
$T^{\text{late}} = \tau - T^{\text{rev}}$		0	6	6				
$R = T^{\text{late}} - T^{\text{early}}$		0	2	0				

2. On the basis of the temporal characteristics of events determine temporal characteristics of work:

early beginning of the work i-j coincides with the early timing of the previous event i:

$$t_{i-j}^{\text{eb}} = T_i^{\text{early}}. \quad (20)$$

Early completion of the i-j is the sum of early beginning and duration:

$$t_{i-j}^{\text{ec}} = T_i^{\text{early}} + t_{i-j}. \quad (21)$$

Late boundary completion of the work coincides with the late timing of the following event:

$$t_{i-j}^{lbc} = T_j^{late}. \quad (22)$$

Late boundary beginning of the work is the difference between late completion and duration of work:

$$t_{i-j}^{lbb} = T_j^{late} - t_{i-j}. \quad (23)$$

Full-time reserve of work i-j is the difference between late and early completion of work:

$$R_{i-j}^{full} = T_j^{late} - (T_i^{early} + t_{i-j}). \quad (24)$$

Free float time is the difference between the early beginning of the next work (T_j^p) and the early completion of the previous (considered) work:

$$R_{i-j}^{fft} = T_j^{early} - (T_i^{early} + t_{i-j}). \quad (25)$$

Late reserve:

$$R_{i-j}^{lr} = T_j^{late} - (T_i^{late} + t_{i-j}). \quad (26)$$

Independent reserve:

$$R_{i-j}^{ind} = T_j^{early} - (T_i^{late} + t_{i-j}). \quad (27)$$

Among all the reserves only independent reserve can have a negative sign. Works that are lying on the critical path, do not have reserve time. Calculation of the temporal characteristics of the works is presented in Table 19, filling which data from pervious Table 18 is used.

Table 19

Calculation of the temporal characteristics of work

Number of work i-j	Operation time t_{i-j}	Early beginning of work, T_i^{early}	Early completion of work, $T_j^{early} + t_{ij}$	Later completion of work T_j^{late}	Later beginning of work, $T_j^{late} - t_{ij}$	Early beginning of the next work, T_j^{early}	Full time reserve, $R_{i-j}^{full} = T_j^{late} - (T_i^{early} + t_{i-j})$	Free float time, $R_{i-j}^{fft} = T_j^{early} - (T_i^{early} + t_{i-j})$
0-1	4	0	4	6	2	4	0	0

Task 3. Justification of the Order Size

Task 1

The content of the task. The company acquires equipment used in the manufacture of mixtures. It is possible to order the required number of spare parts. It is known that in the work process of this equipment it's necessary to replace one of the main details. Therefore, to ensure continuous equipment operation there should be a reserve of such details. The detail price is \$100. The company suffers losses due to equipment stop because of lack of details in amount of 200 USD per day. The known probability distribution detail release of the system is shown in Table 20.

The purpose of the task. To determine the required number of details when purchasing the equipment, where the total cost of their purchase and losses due to downtime would be minimal.

Table 20

The probability of detail breakdown

The number of parts	0 0	1 1	2 2	3	4 4	5 5	> 5
Probability	0,9	0,05	0,02	0,01	0,01	0,01	0 0

Task 2

The content of the task. The company manufactures equipment and spare parts, which come to the warehouse and are shipped by kits at the request of customers. The demand for parts is random, but its probability distribution is known (Table 21). The cost of keeping one set for the month equals to 10 USD, and the penalty for failure to supply kits in recent months – 200 USD.

The purpose of the task. Find the optimal number of kits, where the total costs associated with their storage and unsatisfied demand would be minimal.

Table 21

Probability of demand

The number of details	0 0	1 1	2 2	3	4 4	5 5	> 5
Probability	0,1	0,2	0,2	0,3	0,3	0,1	0,1

Task 4. To substantiate the Order Quantity

The content of the task. The company sells fresheners. At the end of each week manager decides on increase (decrease) in the delivery from the warehouse of the fresheners. There are three basic solutions:

- 1) increase the volume of order by 10 boxes (strategy P_1);
- 2) increase the volume of order by 5 boxes (strategy P_2);
- 3) leave the order size unchanged (strategy P_3).

At the time of the decision it is not possible to estimate the expected weather conditions. If there is heat (denote this state forecast through O_1), then the number of buyers will increase and there can be a shortage of drinks; if the weather is cold and rainy (O_2) and a lot of drinks were delivered, they will not be realized. In both cases, the firm suffers losses. Known volumes of losses and profits depending on weather conditions are listed in Table 22.

Table 22

The size of the profit and loss

Wholesale purchases strategy	Weather strategy	
	O_1	O_2
P_1	400	- 500
P_2	200	- 200
P_3	0 0	600

The objective of the task. To determine the optimal size of the order for fresheners.

Self-study Work of Students

For successful learning of the subject it is necessary to conduct self-study work of students in the form of lectures, preparation for practical seminars, performance of individual tasks.

Table 23 lists the questions for self-study work.

List of questions for self-study

Theme title	Question for self-study work	References
Semantic module. Economic and mathematical methods of operations research		
Subject and problems of operations research, methods of economic and mathematical modelling	1. Production of a complex system. 2. Simulation a method for economic processes	References: main: [2 – 4]; ancillary: [6; 7]
The tasks of organizing and coordinating	1. Combinatorial problems on graphs. 2. Assignment problem. 3. Salesman problem	References: main: [1; 3; 4]; ancillary: [6; 7]
Objectives and models of optimum resource allocation and replacement	1. Methods for solving problems of mathematical programming. 2. Features of static and dynamic models. 3. Optimization of the enterprise. 4. Distribution of investments – as a problem of linear programming	References: main: [2 – 4]; ancillary: [5 – 7]
Multicriteria problems in management	1. Game model. 2. Solving problems of uncertainty and conflict	References: main: [3; 4]; ancillary: [6; 7]
Problems of uncertainty and conflict	1. Input-output tables. 2. Method of matrix model. 3. Assessment of technological matrix	References: main: [3; 4]; ancillary: [6; 7]

Test Questions for Self-study

1. Explain the aims and objectives of the discipline "Operations Research".
2. Define the subject and object of the discipline "Operations Research".
3. What is the operations and methods of research?
4. Define the concept of a system.

5. Describe the basic properties of the system.
6. What are the main stages of the economic problems solution using economic and mathematical methods?
7. Define advantages and disadvantages of modelling method.
8. What is a mathematical model of the operations?
9. Explain description of operations research models.
10. Name stages of mathematical models construction of operations research.
11. Name the main problem of using operations research methods.
12. Name basic concepts of queuing theory.
13. Define basic laws of economic indicators distribution.
14. Define integral and differential distribution function.
15. Define histogram and polygon distribution.
16. Explain performance evaluation of the distribution choice.
17. Explain cumulates and its use in queuing.
18. Name methods for solving queuing.
19. Define Monte Carlo method, its nature and use.
20. Define Random numbers and their use in queuing.
21. Define numerical and graphical models and basic principles of their construction.
22. Name calculation criteria for choosing optimal variant of queuing.
23. Define application of queuing.
24. Explain the essence of the of SPU.
25. Name concepts of SPU.
26. Name principles of grid models.
27. Define characterization of events.
28. What is description of work?
29. What is optimizing grid models.
30. Explain construction of the line graphics performance charts and manpower requirements.
31. Name basic principles of optimizing grid model.
32. What is the practice of grid planning methods.
33. What is the essence of statistical quality control?
34. What is the precision?
35. What is the width of the control limits and the sample size?
36. What are the risks of a producer and a consumer?

37. How to measure risks of a producer and a consumer?
38. What is a control chart?
39. Name the basic principles of the control chart.
40. Define the practice of statistical quality control.
41. Name basic concepts of inventory management system.
42. Define the main objective of inventory management.
43. What is the criterion for optimality in problems of inventory management?
44. Characterize the model of Wilson.
45. List the major costs associated with the formation of reserves.
46. Describe the model of industrial supplies.
47. Name the main features of the stochastic model in inventory management.
48. Define the method of optimal planning.
49. Describe the mathematical model of optimal scheduling.
50. Name characterization methods for solving models of optimal planning.
51. Name the main features of the simplex method.
52. What is optimal for each iteration of the simplex table?
53. Explain methods of solving the problems of optimal planning.
54. Is it possible to have an optimal plan of artificial variables?
55. Is it possible to have an optimal plan with extra variables?
56. Describe the mathematical model of the production program optimization.
57. Describe the mathematical model of load equipment optimization.
58. Describe the mathematical model of material optimization.
59. Describe the mathematical model of the mixture.
60. Name features of mathematical model for equipment replacement.
61. Define the term "game".
62. Explain selection of strategy players.
63. What is a saddle point?
64. Describe the nature of the game.
65. Name optimal criterion in multicriteria problems.
66. Name techniques of multicriteria decision problems.
67. How to select the single figure bonuses in solving production problems?

68. How to provide interconnection rate bonuses and bonuses?

69. How to use mathematical functions to ensure the linkage results and prizes?

70. What is the scale of bonuses?

71. How to build standards scale?

72. What is the premium using the scale bonuses?

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