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Guidelines to self-study work on the academic discipline "OPERATIONS RESEARCH"<br>for full-time students of training direction 6.030601 "Management"

Затверджено на засіданні кафедри економіки, організації та планування діяльності підприємства.

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Tasks for self-study on the academic discipline, guidelines to their implementation and questions to consolidate knowledge are given.

Recommended for full-time students of training direction 6.030601 "Management".

Надано завдання для самостійної роботи з навчальної дисципліни, методичні рекомендації до їх виконання та запитання для закріплення знань.

Рекомендовано для студентів напряму підготовки 6.030601 "Менеджмент" денної форми навчання.

## Introduction

Efficiency growth is closely linked with quantitative reasoning of decisionmaking. Due to this, a specialist in management should know and be able to put the economic and mathematical methods and models of operations research into practice.

A large number of factors and phenomena affect the results of commercial activities of enterprises. This effect is varied: some factors and phenomena have a major impact, others are weaker, and the others don't affect the activity at all. To develop an effective policy we must assess the relationship between the factors and phenomena themselves, and their impact on the results of business enterprises.

Mathematical methods are essential to the selection and formalization of the relationship between economic indicators, assessment of the form and parameters of their dependencies. Accordingly, the production manager should know and be able to use this method.

In addition, the future manager should know the features of economy, organization and production planning, take them into account when developing mathematical models of economic activity.

Future managers should also know the characteristics of economy, organization and planning, using methods of operations research to make decisions.

The academic discipline "Operations Research" refers to the cycle of regulatory subjects for students of training direction "Management".

The academic discipline "Operations Research" is based on the knowledge obtained by students after studying the subjects of economic and mathematical cycles such as "Economics", "Macroeconomics", "Higher and Applied Mathematics" "Statistics" and others. The general knowledge and skills, acquired by students mastering the academic discipline "Operations Research", can be expanded in the process of learning subjects in the speciality. During the study, students receive the necessary knowledge at lectures and practical studies, during the performance of individual tasks. Independent work of students is of great importance in the process of learning and consolidation of knowledge.

The purpose of the academic discipline is to master theoretical knowledge and practical skills in specialized mathematical economic methods to provide optimization of management, organization and planning tasks.

The task of the academic discipline is to apply operations research to testing economic theories by means of economic and mathematical methods and quantitative substantiation of optimal solutions.

The topic of the academic discipline is models and methods of system analysis, research methods and streamlining operations.

The academic discipline is taught after studying the academic discipline "Higher and Applied Mathematics". This links the disciplines of the mathematical cycle to economic sciences prior to the study of professional subjects, which are the basis for economic research.

The academic discipline is studied at lectures, laboratory studies, in the course of self-study work.

The academic discipline is taught within the training direction 6.030601 "Management" for full-time students.

## Plan of self-study work of students

For successful learning of the academic discipline, it is necessary to conduct self-study work of students in the form of lectures, preparation for practical seminars, and performing individual tasks.

Table 1 lists the questions for self-study work.
Table 1
The list of questions for self-study

| Theme | Question for independent work | Literature |
| :--- | :--- | :---: |
| 1 | 2 |  |
| Content module. Economic and mathematical methods |  |  |
| of operations research |  |  |
| 1. The subject and problems <br> of operations research, <br> methods of economic and <br> mathematical modelling |  |  |
| 1. Production of a complex system. | 2. Simulation as a method for | Main: |
| economic processes | $[2-4]$. |  |
| 4dditional: |  |  |
| 4. The tasks of organization | 1. Combinatorial problems on | Main: |
| and coordination | graphs. | $[1 ; 3 ; 4] ;$ |
|  | 2. The assignment problem. | Additional: |
|  | 3. The salesman problem | $[6 ; 7]$ |

Table 1 (the end)

| 1 | 2 | 3 |
| :--- | :--- | :---: |
| 7. Objectives and models <br> of optimum resource allo- <br> cation and replacement | 1. Methods for solving problems <br> of mathematical programming. <br> 2. Features of static and dynamic <br> models. | Main: $[2-4]$. <br> Additional: <br> $[5-7]$ |
|  | 2. Optimization of an enterprise. <br> 3. Distribution of investments <br> as a problem of linear program- <br> ming |  |
| 8. Multicriteria problems <br> in management | 1. A game model. <br> 2. Solving problems of uncertainty <br> and conflict | Main: $[3 ; 4]$. <br> Additional: <br> $[6 ; 7]$ |
| 9. Problems of uncertainty <br> and conflict | 1. Input-output tables. <br> 2. The method of the matrix model. | Main: $[3 ; 4]$. <br> Additional: <br> $[6 ; 7]$ |
| 3. Assessment of the technological <br> matrix |  |  |

Task 1. Theme "Problems and methods of queuing".
The content of the task. A branch of a radio repair company has $n$ masters. On average, $\square$ equipment comes for repair from the public during a working day. The flow of requests for repair is random and Poisson. The complexity of repair is also random and depends on the complexity of repair, skills of the repair masters and others factors. Statistics shows that the time for repair is affected by an exponential distribution law and on average during a day, each of the masters can repair about 2.5 devices $(\lambda=2.5)$.

The purpose of the task. Evaluate the work of the company branch which repairs radio devices, calculate the following basic characteristics of the queuing system (QS):

1. The possibility that all the masters are free from repairing the equipment, Ro.
2. The possibility of all the masters being busy working, Pn.
3. The average time for servicing (repairing) one unit.
4. The average queue length which determines the area for storage of equipment.
5. The average number of masters free from work.

The baseline data for doing the task are shown in Table 1.

## Baseline data for Task 1

| Variant number <br> (the last digit of the test book) | Number of <br> masters (n) | Number of repair <br> requests ( $\square)$ |
| :---: | :---: | :---: |
| $0-3$ | 4 | 5 |
| $4-6$ | 6 | 15 |
| $7-9$ | 8 | 20 |

## Guidelines to Task 1

1. The queuing theory (TMT), which is called the theory of queues abroad, is used for mathematical description of queuing systems. TMT studies the processes of queues, the relationship between their main characteristics and finding optimal regimes of queue flows.

The main objective of TMT is to find an optimal organization of service to achieve minimum losses from the service waiting time and downtime of the serving systems.

The basic concepts of the service system are the incoming flow of requests, service mechanism and the service discipline.

The request to perform work in TMT is called an order (an order or a client). Availability of an orders flow is also a common feature of queuing systems. The time of receiving an order in a service system is random and is often described by the Poisson law:

$$
\begin{equation*}
P_{x}=\frac{\lambda^{x}}{X!} e^{-\lambda}, \tag{1}
\end{equation*}
$$

where $\lambda$ is an average number of orders received per unit of time;
$x$ is the service parameter;
$X!$ is factorial $X$ or product of numbers $1,2,3 \ldots X$.

The service is provided by serving devices (humans, machines, etc.). The maintenance process is characterized by a capacity, i.e. the maximum number of orders served in a unit of time ( $\mu$ ), the duration of service (tobs = $=1 / \mu$ ), the number of service channels $(n)$, the service discipline (the rule that is followed to select the order for maintenance).

The law of distributing the service parameters acts as a mathematical model of service. The following distribution laws are used: normal or GaussLaplace law, Poisson's law, exponential, law etc.

By selecting the distribution law and calculating its parameters (for example, in Poisson's law case it is $\lambda$, let us suppose that $\lambda=\widetilde{\sigma}=3$, then $P_{x}=\frac{3^{x}}{X!} e^{-3}$ ), it is possible to tackle the problem.

1. Determine the quality of the service operation:
the probability of all the service channels being not busy, Po;
the probability of all the channels being busy, Pn;
the average processing time for one request;
the average waiting time for the request to start processing;
the average queue length, which determines the storage space for equipment;
the average number of channels free from work.
There are two methods for finding solutions: the analytical and statistical tests method or the Monte Carlo method.

The Monte Carlo method is an artificial reproduction of the service processes with different variants of its organization by using random numbers, which play the role of cumulative probabilities (integral values of the distribution law). The analytical method is used in cases when requests form a simple flow and are distributed according to Poisson's law.

Service systems can be open and closed. Closed MO systems are such systems in which the incoming stream of demands occurs in the system and its size is limited. An example of a closed system is a system of maintaining a group of machines by technicians which are the source of requests. Open systems are characterized by an infinite number of requirements. For example, the number of customers in a store, at the checkout, etc. If the request catches all the channels busy, then it leaves the system. If the channel is free, the request gets the service. In the analytical method of solving the service, quality indicators are determined using the probabilities of the states of the service system.
2. Determining the probabilities of the service system state with the analytical method of solution is as follows:

Determine the parameter $\alpha=\lambda / \mu$, i.e. the estimated number of service channels. If $\alpha=\lambda / \mu \mathrm{n}$, the queue cannot grow infinitely. Assume that $\mathrm{n}=5$, $\lambda=10$ units and $\mu=2.5$ device hour. Then $\alpha=\lambda / \mu=10 / 2.5=4<5$ (the queue cannot grow indefinitely).

Based on this data let's define the service quality indicators:

1. The probability that all the masters are free from repairing the equipment, is

$$
\begin{equation*}
P o=\left[\sum_{k=0}^{n-1} \frac{\alpha^{k}}{k!}+\frac{\alpha^{n}}{n!\left(1-\frac{\alpha}{n}\right)}\right]^{-1}, \tag{2}
\end{equation*}
$$

where $k$ is the current number of claims (demands);
$n$ is the number of service channels (technicians).
For our example $P_{o}=\frac{1}{\frac{4^{0}}{0!}+\frac{4^{1}}{1!}+\frac{4^{2}}{2!}+\frac{4^{3}}{3!}+\frac{4^{4}}{4!}+\frac{4^{5}}{5!\left(1-\frac{4}{5}\right)}}=0.013$.
Based on the calculation, only an average of $1.3 \%$ of master's time is free.
2. The probability that all the technicians are busy working:

$$
\begin{equation*}
\operatorname{Pn}=\frac{\alpha^{n}}{n!\left(1-\frac{\alpha}{n}\right)} P_{0} \text { with } \frac{\alpha}{n}<1 . \tag{3}
\end{equation*}
$$

For our example $P_{n}=\frac{4^{5}}{5!\left(1-\frac{4}{5}\right)} 0.013=0.554$. On average, the technicians are busy with repairs $55.4 \%$ of their time.
3. The average time of servicing (repairing) one unit per day:

$$
\begin{equation*}
\overline{\mathrm{t}}_{\mathrm{S}}=\frac{\mathrm{t}}{\mu} . \tag{4}
\end{equation*}
$$

In our case, $\overline{\mathrm{t}}_{\mathrm{S}}=\frac{\mathrm{t}}{\mu}=7 / 2.5=2.8$ hours / unit, where $t$ is the duration of the working day in hours. The average waiting time for the equipment (unit) to be started repairing per day:

$$
\begin{equation*}
\overline{\mathrm{t}}_{\mathrm{eq}}=\frac{\mathrm{P}_{\mathrm{n}}}{\mathrm{n}-\alpha} \overline{\mathrm{t}}_{\mathrm{s}} . \tag{5}
\end{equation*}
$$

In our case $t=\frac{0.554}{5-4} 2.8=1.55$ hour, meaning that each request is waiting for a repair for 1.55 hours on average.
5. The average queue length, which determines the area for storage of equipment:

$$
\begin{equation*}
\bar{M}=P_{n} \frac{\alpha}{n\left(1-\frac{\alpha}{n}\right)} . \tag{6}
\end{equation*}
$$

In our case $\bar{M}=0.554 \frac{4}{5\left(1-\frac{4}{5}\right)}=2.2$ items.
6. The average number of technicians free from operations:

$$
\begin{equation*}
\bar{n}_{0}=P_{0}\left[\sum_{k=0}^{n-1} \frac{n-k}{k!} \alpha^{k}\right] . \tag{7}
\end{equation*}
$$

In our case

$$
\bar{n}_{0}=0.013\left[\frac{(5-0) 4^{0}}{0!}+\frac{(5-1) 4^{1}}{1!}+\frac{(5-2) 4^{2}}{2!}+\frac{(5-3) 4^{3}}{3!}+\frac{(5-4) 4^{4}}{4!}\right]=0.95
$$

Thus, on average, during a working day, four of the five technicians are busy repairing. Conclusion: it is not necessary to increase the staff.

## Task 2. Theme "Network planning and management".

The content of the task. A company is interested in the release of a new device. This must be done in a short time. It is possible to reduce the technical preparation of the production time with network planning and management techniques. The initial data are the scope of work and its order of execution, which is shown in Table 2.

The purpose of the task. Justify a minimum period of project development.
Based on the initial data it is necessary to:

1) build a network diagram design;
2) justify the critical path and determine the minimum labor required for the project design;
3) calculate the temporal characteristics of events and activities;
4) construct a linear progress chart and diagram of labor needed by days of development;
5) justify actions to perform the conceptual design on time.

The variants for doing the task are listed in Table 3.
Table 2

## The content and order of work for Task 2

| The number and content of the operation | The execution order of the operation |
| :--- | :--- |
| 1. Concept development | Once the job is received |
| 2. Theoretical drawing development | After operation 1 |
| 3. Performing dimensional drawing | After operation 2, along with the <br> implementation of operation 4 |
| 4. Development of the general form <br> drawing | After work 2, along with the <br> execution of operation 3 |
| 5. Performing calculations | After operation 3 |
| 6. Performing calculations and paperwork | After operation 4 |
| 7. Designing the testing programs and <br> testing methods | After operations 5 and 4 |
| 8. Creating the explanatory note | After operations 6 and 7 |

Table 3
The background data to Task 2

| Variant | Duration of the operation, days / number of employees (people) |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | The number of the operation |  |  |  |  |  |  |  |  |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |  |
| 1 | $5 / 3$ | $4 / 4$ | $5 / 2$ | $3 / 3$ | $4 / 2$ | s | $4 / 3$ | $3 / 2$ |  |
| 2 | $5 / 3$ | $5 / 4$ | $6 / 2$ | $4 / 3$ | $4 / 2$ | s | $4 / 3$ | $3 / 2$ |  |
| 3 | $5 / 3$ | $6 / 4$ | $/ 2$ | $5 / 3$ | $4 / 2$ | s | $4 / 3$ | $3 / 2$ |  |
| 4 | $5 / 3$ | $7 / 4$ | $8 / 2$ | $6 / 3$ | $4 / 2$ | s | $4 / 3$ | $3 / 2$ |  |
| 5 | $5 / 3$ | $8 / 4$ | $9 / 2$ | $7 / 3$ | $4 / 2$ | s | $4 / 3$ | $3 / 2$ |  |
| 6 | $6 / 4$ | $5 / 5$ | $3 / 2$ | $3 / 3$ | $4 / 2$ | s | $4 / 3$ | $3 / 2$ |  |
| 7 | $6 / 4$ | $6 / 5$ | $4 / 2$ | $4 / 3$ | $4 / 2$ | s | $4 / 3$ | $3 / 2$ |  |
| 8 | $6 / 4$ | $7 / 5$ | $5 / 2$ | $5 / 3$ | $4 / 2$ | s | $4 / 3$ | $3 / 2$ |  |
| 9 | $6 / 4$ | $8 / 5$ | $6 / 2$ | $6 / 3$ | $4 / 2$ | s | $4 / 3$ | $3 / 2$ |  |
| 0 | $6 / 4$ | $9 / 5$ | $7 / 2$ | $7 / 3$ | $4 / 2$ | s | $4 / 3$ | $3 / 2$ |  |

The network diagram for individual tasks has the following form (Fig.1):


Fig. 1. The network diagram of individual tasks

Numbers above the arrows denote the number of jobs in the initial data. f. w. means fictitious work, the duration of which and the number of employees performing it equals to zero. It is required to put the duration of operations and the number of employees according to the variants of the task over the arrows.

## Guidelines to Task 2

1. Network planning methods and production management (SPM) relate to organizational management systems and are a set of graphic, calculation and control methods, organizational measures that ensure the modeling, analysis and restructuring of the plan of executing the work.

The essence of SPM is in modeling complex production processes using a network diagram. The network diagram is an arrow diagram that shows interaction between the individual stages of development, and a mathematical model of the SPM.

The network diagram makes it possible to determe:
a minimum term of development and its individual stages;
the need for resources (material, labor and financial) for each day of development;
conduct a fast correction of the development process in case the development conditions have changed.

The SPM procedure includes the following stages: preparation of the network diagram, calculating the network model, analysis and optimization of the network, planning and management of the implementation of works. The purpose
of SPM is to identify the time reserves and resources hidden in the irrational organization of production processes to prevent failures, better distribute work between the performers and improve management efficiency based on these changes.

The basic concepts of SPM are: an event, work, a path, a critical path, reserves. An event is the time of completion of one or more works. Events are numbered and affixed the work duration and the number of employees. An event has no duration, but it characterizes the time of its accomplishment. The numbering of events is performed by the method of deleting arcs. The essence of it is as follows: first, those events are numbered which include the work of the numbered events. The top of development often gives a zero number.

Work has duration $t i j$, where $i$ is the number of the events with which the work begins, and $j$ is the number of the events the work ends with. For example, the work has a duration of 41 days. In the STC language it will be written as follows: t2-5 = 41 days.

Work is the final process that connects the events and links one event to another. Jobs have the following values: the actual work takes time and resources; a sham operation (depending) shows the logical connections between the events and requires neither the time nor the resources (network graphs show fictitious work by dashed lines); wait is a job that requires time, but does not require resources.

In STC, a widespread notion is the path and the critical path. The path is a sequence of works from one event to another. The path length is the sum of the duration of work. From the start of the development to the end there are always several paths with different duration. A path having the maximum length is called critical.

When constructing the network diagram one should follow certain rules:
no work can be started before all the previous work is complete;
every work must be between two events;
events (i) $\underset{\mathrm{i}<\mathrm{j}}{\mathrm{t}_{\mathrm{ij}}}$ (i), with $i<j$, where $i$ is the number of the previous event, and $j$ is the number of the following event;
there should not be dead-ends (with no work resulting) and hovering events (with no work entering);
parallel execution of work requires the use of fictitious works, etc.

Solution of the network model assumes the calculation of the temporal characteristics of events and works and definition of the critical path.

Events have the following temporal characteristics:
an early term of accomplishment of an event is the maximum path from the initial event (or previous) to the given one:

$$
\begin{equation*}
T_{j}^{p}=\max \left\{T_{i}^{p}+t_{i-j} ;\right. \tag{8}
\end{equation*}
$$

a late term of accomplishment of an event:

$$
\begin{equation*}
T_{i}^{n}=\tau-T_{i}^{i c}, \tag{9}
\end{equation*}
$$

where $\tau$ is the length of the critical path;
$T_{i}^{\text {ic }}$ is an early term of accomplishment of events with inverse counting or the maximum path from this event to the final one.

$$
\begin{equation*}
T_{i}^{i c}=\max \left\{T_{j}^{i c}+t_{i-j}\right\} . \tag{10}
\end{equation*}
$$

The reserve time of the event $\mathrm{Rj}=\mathrm{Tjp}$ - Tjp shows how long the accomplishment of the events can linger without changing the time of the final event. The early and late periods of the events lying on the critical path coincide and their reserves are equal to zero.

Calculation of the temporal characteristics of events can be carried out by tabular or matrix methods.

A matrix is a table with the number of columns equal to the number of rows equal to the number of events plus 3.

For example, consider a network diagram of machine assembly shown in Fig. 2.


Fig. 2. A network diagram of the assembly of a machine

On the network diagram above, the arrows indicate the duration of work in days (in the numerator) and the number of employees (in the denominator). The fictitious work is designated by a dotted line; its length is equal to zero.

For this diagram we calculate the temporal characteristics of the events that are shown in Table 4.

Table 4

## Calculation of the temporal characteristics of events

| $\mathrm{T}^{\mathrm{p}}$ | i | 0 | 1 | 2 | 3 | 4 | $\mathrm{~T}^{\mathrm{ic}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $0 \leftarrow$ | 0 |  | $\leftarrow 4$ | 6 | 2 |  | 15 |
| $4 \rightarrow$ | 1 | $\rightarrow$ | $\rightarrow$ | 0 |  |  | 9 |
| 6 | 2 |  |  |  | 4 |  | 9 |
| 10 | 3 |  |  |  |  | $\downarrow 5 \leftarrow$ | $\leftarrow 5$ |
| 15 | 4 |  |  |  |  | $\rightarrow$ | $\rightarrow 0$ |
| $\mathrm{~T}^{\mathrm{n}}=\tau-\mathrm{T}^{\mathrm{ic}}$ | 0 | 6 | 6 | 10 | 15 |  |  |
| $\mathrm{R}=\mathrm{T}^{\mathrm{n}}-\mathrm{T}^{\mathrm{p}}$ | 0 | 2 | 0 | 0 | 0 |  |  |

Fill the table as follows: in the $i$ row and $j$ column duration of a related work is written. The number of the filled cells should correspond to the number of work in the network diagram, including the fictitious one. The filled cells will be only above the diagonal, since the number $\mathrm{i}<\mathrm{j}$.

The terms of accomplishment of the events are calculated by the above formulas. So, an early term of accomplishment of the events $i=0$ is assumed to be $\mathrm{Tp}=0$ and entered in the first column. The early term of accomplishments of event 1 is determined using Table 6: draw a mental line from $i=1$ to the right until it intersects with the diagonal and climb up to the first filled cell, with number 4 , which is duration of work $0-1$.

To this figure we add the number in the first column on the left (the term of accomplishment of the previous event $i=0$ ) and get 4 . If the event implies two or more works, then the term of its accomplishment is assumed to be the maximum number. For example, event 2 implies two works: $1-2(t 1-2=0)$ and $0-2(\mathrm{t} 0-2=6)$. In the left column, we obtain two values of the path $4+0$
and $0+6$. For an early term of accomplishment of event 4 as the final one equal to the critical path length $=15$ days.

The early term of accomplishment of an event with inverse counting is calculated similarly. Now a starting point of counting is the last event of model 4 for which T4ic $=0$. To determine T3ic, we must draw an imaginary line from $i=3$ of the far right column to the left until it intersects with the filled cell, with number 5 . This is the duration of the work, connecting event 4 with event 3 . Now you have to go down from it to the diagonal and turn right to column Tic, where the early term of the previous event 4 , equal to 0 is indicated. t sum $3-4+$ Tic $=5+0=5=$ T3ic is the early term of accomplishment of event 3 with inverse counting. If the line contains several filled cells, it is necessary to calculate the duration of several paths and select the maximum term of accomplishment as the event in question. This pattern is typical of event 0 . Here, with inverse counting, event 0 implies three works $0-1($ TOic $=13), 0-2($ TOic $=15), 0-3($ TOic $=7)$, and the term of accomplishment of event 0 with inverse counting is assumed to be 15. According to table formulas calculate the late term of accomplishment and the reserve of the event. The result is that TOic $=\mathrm{T} 4 \mathrm{r}=\mathrm{T} 4 \mathrm{p}=15$ days. Events lying on the critical path have no reserves (events $0,2,3,4$ ). The critical path passes through them and on the network diagram it is given in a bold line or red. The length of the critical path determines the minimum period of the entire development, or the maximum path from the initial to the final events. In our example, the minimum time for the assembly of a machine is 15 days.

On the basis of the temporal characteristics of events determine the temporal characteristics of the works:

The early start of work ( $\mathrm{i}-\mathrm{j}$ ) coincides with the early term of the of the previously accomplished events (i)

$$
\begin{equation*}
\mathrm{t} \mathrm{i}-\mathrm{jp.n}=\text { Tip. } \tag{11}
\end{equation*}
$$

Early termination of work $(\mathrm{i}-\mathrm{j})$ is the sum of the early onset and duration

$$
\begin{equation*}
\text { t i }- \text { jp.ok }=\text { Tip. }+\mathrm{ti} \mathrm{j} . \mathrm{j} . \tag{12}
\end{equation*}
$$

The late work termination limit coincides with the late term of accomplishment of the subsequent event

$$
\begin{equation*}
\mathrm{t} \mathrm{i}-\mathrm{jp.ok}=\mathrm{Tjp} . \tag{13}
\end{equation*}
$$

The late work beginning limit is equal to the difference between the late finish and duration of work

$$
\begin{equation*}
\mathrm{ti}-\mathrm{jp} . \mathrm{n} \text { Tjp }=-\mathrm{t} \mathrm{i}-\mathrm{j} ; \tag{14}
\end{equation*}
$$

Full-time reserve of work $(\mathrm{i}-\mathrm{j})$ equals to the difference between the late and early completion of work

$$
\begin{equation*}
\text { Ri }- \text { jpol }=\mathrm{ti}-\text { jp.ok }-\mathrm{ti}-\text { jp.ok Tjp }=-(\text { Tip. }+\mathrm{tij}) . \tag{15}
\end{equation*}
$$

Time reserve is the difference between the early start of the next work (Tjp) and early termination of the previous (considered) work

$$
\begin{equation*}
\text { Ri }-\mathrm{j} \text { St. Tjp }=-(\text { Tip. }+\mathrm{ti}-\mathrm{j}) . \tag{16}
\end{equation*}
$$

Works also have late independent reserves. Of all the reserves only the independent reserve can have a minus sign.

Works lying on the critical path do not have time reserves. Time characteristics of the work is shown in Table 5, which is filled based on the data from Table 4.

On the basis of the temporal characteristics of the work, a line graph (Fig. 3) and a chart of manpower requirements (Fig.4) is built. The line graph is a network diagram, built in the time scale. The abscissa represents the time of work, and the vertical axis is the network graph. Each work is postponed once, for this we must know the early onset, duration, and early termination of time reserve (col. 1, 2, 2, 4, 9 in Table 5).

The analysis of the network model includes an evaluation of the structure of the network diagram, the load of workers at all the stages of work, a possibility of shifting the start of work of noncritical areas. To do this work is classified according to the reserves value.

Optimization of the network diagram is the process of improvement of the organization of works based on the due date and available resources. It is carried out through redeployment of resources, intensification of work of the critical path (additional performers, equipment), the change in the nature of the range of works.

Table 5
Calculation of temporal characteristics of the works



Days
Fig. 3. Linear implementation schedule


Fig. 4. Diagram of manpower requirements
Optimization is carried out in three ways: by minimizing the time work, to minimizing the consumption of resources, the cost of work. The most widely used systems of STC are such in which the object of planning and control are terms of accomplishment of work. Optimization of such systems consists in reducing the duration of the critical path, which is performed by the reserve time identified in the analysis of the network, and will continue as long as it does not coincide with the duration of the specified period. The second area is the review of the provisional ratings of work. The duration of work may be changed by increasing the number of employees assigned to a particular work, better organization of labor, the technology of production, etc.

## Task 3.

The purpose of the task: develop a system of statistical warning of quality control of tractor parts.

The contents of the task. A part is processed on a six-spindle semiautomatic machine tool; control is carried out on the mean size of the part.

## The order of execution.

1. Analyze the accuracy of the technological process of the part machining:
build a number of parts by the diameter size distribution (the diameter of the parts is $16.9 \square 0.08 \mathrm{~mm}$ );
depict a histogram of distribution and on its basis, hypothesize about the theoretical distribution law;
construct a theoretical distribution curve and determine the nature of the deviations of the empirical distribution law of theory, using the graphical method, Pearson criterion.
2. Determine how accurate the process of machining is:
determine the coefficient of the machining accuracy and the value of marriage at the ideal setting of the machine;
determine the actual and allowable coefficient machine settings, as well as the value of marriage which is actually observed at the level of machine settings.
3. Calculate the parameters of the system of statistical quality control warning of tractor parts:
determine the control limits and the sample size for a system of preventive control, based on the arithmetic mean of the control. The producer and consumer risk is assumed to be 0.5 and 1.5 respectively;
construct a control diagram for eight samples.
The variant choice is given in Table 1, and the initial data is given in Table 2.
Table 1
The choice of the task variant

| Variant | Machine <br> number | Variant | Machine <br> number | Variant | Machine <br> number |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $1-3,9,10$ | 9 | $1-3,5,6$ | 17 | $2,3,5,8,10$ |
| 2 | $1-3,7,9$ | 10 | $1-3,6,10$ | 18 | $3,5,6,8,9$ |
| 3 | $1-3,5,10$ | 11 | $2,3,5-8$ | 19 | $3,5,6,9-10$ |
| 4 | $1-3,6,8$ | 12 | $3,5-7,9$ | 20 | $4-8$ |
| 5 | $1-3,7,10$ | 13 | $1-3,6,8$ | 21 | $4-7,9$ |
| 6 | $1-3,7,8$ | 14 | $1-3,6,9$ | 22 | $4-7,10$ |
| 7 | $1,3,5-7$ | 15 | $2,5-7,10$ | 23 | $1,4-6,8$ |
| 8 | $2,3,6,7,8$ | 16 | $1-3,8,10$ | 24 | $1,4,5,8,10$ |

Workpiece size, mm

|  | Machine number |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |  |
| 16.872 | 16.899 | 16.915 | 16.840 | 16.868 | 16.819 | 16.895 | 16.895 | 16.933 |  |
| 16.835 | 16.975 | 16.899 | 16.915 | 16.927 | 16.889 | 16.860 | 16.862 | 16.840 |  |
| 16.918 | 16.830 | 16.949 | 16.910 | 16.840 | 16.918 | 16.868 | 16.875 | 16.927 |  |
| 16.875 | 16.877 | 16.927 | 16.860 | 16.927 | 16.855 | 16.880 | 16.935 | 16.868 |  |
| 16.940 |  |  |  |  |  |  |  |  |  |
| 16.880 | 16.868 | 16.927 | 16.868 | 16.880 | 16.895 | 16.872 | 16.880 | 16.856 |  |
| 16.855 | 16.910 | 16.915 | 16.889 | 16.949 | 16.935 | 16.860 | 16.855 | 16.840 |  |
| 16.899 | 16.894 | 16.849 | 16.819 | 16.895 | 16.835 | 16.880 | 16.910 | 16.911 |  |
| 16.995 | 16.935 | 16.895 | 16.916 | 16.935 | 16.827 | 16.830 | 16.840 | 16.872 |  |
| 16.795 | 16.899 | 16.875 | 16.872 | 16.949 | 16.955 | 16.915 | 16.865 | 16.898 |  |
| 16.875 |  |  |  |  |  |  |  |  |  |
| 16.910 | 16.872 | 16.860 | 16.889 | 16.895 | 16.895 | 16.960 | 16.985 | 16.915 |  |
| 16.899 | 16.895 | 16.899 | 16.835 | 16.889 | 16.886 | 16.927 | 16.830 | 16.840 |  |
| 16.849 | 16.827 | 16.875 | 16.895 | 16.849 | 16.855 | 16.880 | 16.960 | 16.895 |  |
| 16.927 | 16.899 | 16.918 | 16.860 | 16.940 | 16.915 | 16.872 | 16.855 | 16.918 |  |
| 16.880 | 16.889 | 16.855 | 16.889 | 16.915 | 16.895 | 16.895 | 16.818 | 16.830 |  |
| 16.868 | 16.860 | 16.899 | 16.855 | 16.895 | 16.879 | 16.855 | 16.875 | 16.935 |  |
| 16.910 | 16.895 | 16.875 | 16.895 | 16.855 | 16.899 | 16.885 | 16.827 | 16.875 |  |
| 16.895 | 16.855 | 16.895 | 16.849 | 16.899 | 16.849 | 16.868 | 16.849 | 16.918 |  |
| 16.915 | 16.975 | 16.889 | 16.840 | 16.872 | 16.835 | 16.868 | 16.855 | 16.975 |  |
| 16.895 | 16.855 | 16.849 | 16.899 | 16.868 | 16.848 | 16.940 | 16.849 | 16.918 |  |
| 16.915 | 16.975 | 16.889 | 16.872 | 16.835 | 16.868 | 16.855 | 16.975 | 16.840 |  |
| 16.940 |  |  |  |  |  |  |  |  |  |

Cumulative distribution function

| T | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0 | 0.5000 | 5040 | 5080 | 5120 | 5160 | 5199 | 5239 | 5275 | 5319 | 5359 |
| 0.1 | 5398 | 5438 | 5478 | 5517 | 5557 | 5596 | 5636 | 5675 | 5714 | 5754 |
| 0.2 | 5793 | 5832 | 5871 | 5940 | 5948 | 5987 | 6026 | 6064 | 6103 | 6141 |
| 0.3 | 6179 | 6217 | 6255 | 6293 | 6331 | 6338 | 6406 | 6443 | 6480 | 6517 |
| 0.4 | 6554 | 6591 | 6628 | 6664 | 6700 | 6736 | 6772 | 6808 | 6844 | 6879 |
| 0.5 | 6915 | 6950 | 6985 | 7019 | 7054 | 7088 | 7123 | 7157 | 7190 | 7224 |
| 0.6 | 7258 | 7921 | 7324 | 7357 | 7389 | 7422 | 7454 | 7486 | 7516 | 7549 |
| 0.7 | 7580 | 7612 | 7624 | 7673 | 7707 | 7734 | 7764 | 7794 | 7823 | 7852 |
| 0.8 | 7881 | 7910 | 7939 | 7967 | 8000 | 8023 | 8051 | 8079 | 8106 | 8133 |
| 0.9 | 8159 | 8186 | 8212 | 8228 | 8234 | 8239 | 8315 | 8348 | 8365 | 8389 |
| 1.0 | 8413 | 8438 | 8461 | 8485 | 8508 | 8631 | 8554 | 8577 | 8600 | 8621 |
| 1.1 | 8643 | 8665 | 8686 | 8708 | 8729 | 8749 | 8770 | 8790 | 8810 | 8830 |
| 1.2 | 8849 | 8867 | 8888 | 8907 | 8925 | 8944 | 8962 | 8980 | 8997 | 9015 |
| 1.3 | 9032 | 9049 | 9066 | 9082 | 9099 | 9105 | 9131 | 9147 | 9162 | 9177 |
| 1.4 | 9192 | 9207 | 9222 | 9236 | 9251 | 9265 | 9265 | 9279 | 9293 | 9319 |
| 1.5 | 93319 | 93448 | 93574 | 93699 | 93822 | 93943 | 94062 | 94171 | 94295 | 94408 |
| 1.6 | 94520 | 94630 | 94738 | 94950 | 96053 | 95154 | 95254 | 95254 | 95352 | 95449 |
| 1.7 | 95543 | 95637 | 95728 | 95848 | 95907 | 95994 | 96008 | 96164 | 96246 | 96327 |
| 1.8 | 96407 | 96485 | 96562 | 96638 | 96712 | 96784 | 96856 | 96926 | 96993 | 97062 |
| 1.9 | 97128 | 97195 | 97257 | 97320 | 97381 | 97441 | 97500 | 97558 | 97615 | 97670 |

## Task 4.

The purpose of the task is to build a mathematical model of building a plan of cutting rods, providing a minimum amount of waste.

## The order of execution.

In accordance with the production task the procurement department must cut from bills steel bars of three kinds.

To do this:

1. Make a plan of cutting rods providing a minimum amount of waste.
2. Determine the absolute value of the waste and utilization of materials.

The initial data (by variants) are presented in Table 3.

## Task 5.

## The contents of the task.

A company may additionally issue 3 types of products $\mathrm{A}, \mathrm{B}$ and C . The company has a production capacity of process equipment on which these products are manufactured. There are some complexities related to the manufacturing of this product by type of equipment and profits from its distribution. These data are presented in Table 5.

On the basis of these data, it is necessary to justify the program, which gives the maximum profit.

The purpose of the task is to build a mathematical model of the optimal production program, which gives the maximum profit.

Table 4

## Labour intensity of product manufacturing, $\mathrm{n}-\mathrm{h}$

| Equipment group | Description of <br> an article |  |  | Equipment runtime fund, <br> hours |
| :--- | :---: | :---: | :---: | :---: |
|  | A | Б | B | 18200 |
| Lathes | 2 | 3 | 5 |  |
| Milling machines | 3 | 6 | 1 | 12000 |
| Drilling machines | 1,5 | 1,1 | 0,5 |  |
| Profits from sales of products, <br> UAH | 45 | 33 | 27 |  |

Input

| Variant | Length, <br> $m$ | Blank 1 <br> Number, <br> pcs |  | Length, <br> m | Quantity, <br> pcs | Length, <br> m | Quantity, <br> pcs |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 220 | 1.5 | 380 | Length, <br> m |  |  |
| 2 | 6.0 | 220 | 1.6 | 380 | 1.7 | 80 | 3.5 |
| 3 | 6.0 | 220 | 1.7 | 380 | 1.8 | 80 | 3.5 |
| 4 | 6.0 | 220 | 1.8 | 380 | 1.9 | 80 | 3.5 |
| 5 | 6.0 | 220 | 1.9 | 380 | 2.0 | 80 | 3.5 |
| 6 | 6.0 | 240 | 1.5 | 380 | 1.6 | 80 | 3.5 |
| 7 | 6.0 | 240 | 1.6 | 380 | 1.7 | 90 | 3.5 |
| 8 | 6.0 | 240 | 1.7 | 380 | 1.8 | 80 | 3.5 |
| 9 | 6.0 | 240 | 1.8 | 380 | 1.9 | 80 | 3.5 |
| 10 | 6.0 | 240 | 1.9 | 380 | 2.0 | 80 | 3.5 |
| 11 | 6.0 | 260 | 1.5 | 380 | 1.6 | 80 | 3.5 |
| 12 | 6.0 | 260 | 1.6 | 380 | 1.7 | 80 | 3.5 |
| 13 | 6.0 | 260 | 1.7 | 380 | 1.8 | 80 | 3.5 |
| 14 | 6.0 | 260 | 1.8 | 380 | 1.9 | 80 | 3.5 |
| 15 | 6.0 | 260 | 1.9 | 380 | 2.0 | 80 | 3.5 |
| 16 | 6.0 | 280 | 1.5 | 380 | 1.6 | 80 | 3.5 |
| 17 | 6.0 | 280 | 1.6 | 380 | 1.7 | 80 | 3.5 |
| 18 | 6.0 | 280 | 1.7 | 380 | 1.8 | 80 | 3.5 |
| 19 | 6.0 | 280 | 1.8 | 380 | 1.9 | 80 | 3.5 |
| 20 | 6.0 | 280 | 1.9 | 380 | 2.0 | 80 | 3.5 |
| 21 | 6.0 | 300 | 1.5 | 380 | 1.6 | 80 | 3.5 |
| 22 | 6.0 | 300 | 1.6 | 380 | 1.7 | 80 | 3.5 |
| 23 | 6.0 | 300 | 1.7 | 380 | 1.8 | 80 | 3.5 |
| 24 | 6.0 | 300 | 1.8 | 380 | 1.9 | 80 | 3.5 |
| 25 | 6.0 | 300 | 1.9 | 380 | 2.0 | 80 | 3.5 |

## Test questions for self-study

1. Explain the aims and objectives of the discipline "Operations Research".
2. Define the subject and object of the discipline "Operations Research".
3. What are the operations and methods of research?
4. Define the concept of a system.
5. Describe the basic properties of a system.
6. What are the main stages of the economic problems solution using economic and mathematical methods?
7. Advantages and disadvantages of the modelling method.
8. What is a mathematical model of operations?
9. Description of operations research models.
10. Stages of mathematical models construction of operations research.
11. The main problem of using operations research methods.
12. Basic concepts of the queuing theory.
13. Basic laws of economic indicators distribution.
14. The integral and differential distribution function.
15. Histogram and polygon distribution.
16. Performance of evaluation of the distribution choice.
17. Cumulates and their use in queuing.
18. Methods for solving queuing.
19. The Monte Carlo method, its nature and use.
20. Random numbers and their use in queuing.
21. Numerical and graphical models. Basic principles of their construction.
22. Calculation criteria for choosing an optimal variant of queuing.
23. Application of queuing.
24. The essence of SPM.
25. The concepts of SPM.
26. The principles of grid models.
27. Characterization of events.
28. Description of work.
29. Optimizing grid models.
30. Construction of the line graphics performance charts and manpower requirements.
31. Basic principles of optimizing a grid model.
32. The practice of grid planning methods.
33. The essence of statistical quality control.
34. What is precision?
35. What is the width of the control limits and the sample size?
36. What are the risks of a producer and a consumer?
37. How to measure risks of a producer and a consumer?
38. What is a control chart?
39. Name the basic principles of the control chart.
40. The practice of statistical quality control.
41. Basic concepts of the inventory management system.
42. The main objective of inventory management.
43. What is the criterion for optimality in problems of inventory management?
44. Characterize the model of Wilson.
45. List the major costs associated with the formation of reserves.
46. Describe the model of industrial supplies.
47. Name the main features of the stochastic model in inventory management.
48. The method of optimal planning.
49. Describe the mathematical model of optimal scheduling.
50. Characterization of methods for solving models of optimal planning.
51. The main features of the simplex method.
52. What is optimal for each iteration of the simplex table?
53. Methods of solving the problems of optimal planning.
54. Is it possible to have an optimal plan of artificial variables?
55. Is it possible to have an optimal plan with extra variables?
56. Describe the mathematical model of the production program optimization.
57. Describe the mathematical model of load equipment optimization.
58. Describe the mathematical model of material optimization.
59. Describe the mathematical model of mixture.
60. Features of mathematical model for equipment replacement.
61. Define the term "game".
62. Selection of strategy players.
63. What is a saddle point?
64. Describe the nature of the game.
65. An optimal criterion in multicriteria problems.
66. Techniques of multicriteria decision problems.
67. How can single figure bonuses be selected in solving production problems?
68. How are interconnection rate bonuses and bonuses provided?
69. How are mathematical functions used to ensure the linkage results and prizes?
70. What is the scale of bonuses?
71. How is a scale of standards built?
72. What is the premium of using the scale of bonuses?

## Recommended literature

## 1. Main

1. Балашевич В. А. Математические методы в управлении производством / В. А. Балашевич. - Мн. : "Вышэйшая школа", 1976. - 336 с.
2. Клебанова Т. С. Методы исследования операций : учеб. пособ. / Т. С. Клебанова, В. А. Забродский и др. - Х. : Изд. ХГЭУ, 1999. - 160 с.
3. Исследование операций в экономике : учеб. пособ. для вузов / Н. Ш. Кремер, В. В. Федосеев, А. Н. Гармаш и др. - М. : ЮНИТИ, 1999. 392 c.
4. Омелаенко Н.Н. Конспект лекций по курсу "Исследование операций" / Н. Н. Омелаенко. - Х. : Изд. ХНЭУ, 2006. - 98 с.
5. Экономико-математические методы и прикладные модели : учеб. пособ. для вузов / В. В. Федосеев, А. Н. Гармаш, Д. М. Дайитбегов и др. ; под ред. В. В. Федосеева. - М. : ЮНИТИ, 1999. - 392 с.

## 2. Additional

1. Горчаков А. А. Компьютерные экономико-математические модели / А. А. Горчаков, И. В. Орлова. - М. : Компьютер: ЮНИТИ, 1995. - 136 с.
2. Дегтярев Ю. И. Исследование операций / Ю. И. Дегтярев. - М. : Высшая школа, 1986. - 320 с.
3. Зайченко Ю. П. Исследование операций / Ю. П. Зайченко. - К. : Вища школа, 1988. - 552 с.
4. Омелаенко Н. Н. Практические задания и методические рекомендации к их выполнению по курсу "Исследование операций" для студентов специальностей 8.050201, 8.050108 дневной формы обучения / Н. Н. Омелаенко. - Х. : Изд. ХНЕУ, 2005. - 16 с.
5. Сетевые графики в планировании / под ред. И. М. Разумова, Л. Д. Белова. - М. : Высшая школа, 1981. - 168 с.

## 3. Information resources

1. State Agency of Ukraine for Investments and Innovations. - Access mode : http://www.in.gov.ua.
2. State Statistics Committee of Ukraine. - Access mode : http://www.ukrstat.gov.ua.
3. National Bank of Ukraine. - Access mode : http://www.bank.gov.ua.
4. Press Service of the National Bank of Ukraine. - Access mode : http://pr.bank.gov.ua/ukr.

## Guidelines

 to self-study work on the academic discipline "OPERATIONS RESEARCH"for full-time students of training direction 6.030601 "Management"

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