

Extinction of solutions for quasi-linear parabolic equations

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Investigations are devoted to the study of the extinction of solutions in finite time to initial-boundary value problems for a wide classes of nonlinear parabolic equations of the second and higher orders with a degenerate absorption potential, whose presence plays a significant role for the mentioned nonlinear phenomena.

So, behavior of solutions to the parabolic equation of non-stationary diffusion with double nonlinearity and a degenerate absorption term:

$$(|u|^{q-1}u)_t - \sum_{i=1}^n \frac{\partial}{\partial x_i} \left(|\nabla_x u|^{q-1} \frac{\partial u}{\partial x_i} \right) + a_0(x)|u|^{\lambda-1}u = 0 \quad \text{in } \Omega \times (0, T),$$

where Ω is bounded domain in \mathbb{R}^N , $N \geq 1$, $0 \in \Omega$, $a_0(x) \geq d_0 \exp\left(-\frac{\omega(|x|)}{|x|^{q+1}}\right)$, $x \in \Omega \setminus \{0\}$, $d_0 = \text{const} > 0$, $0 \leq \lambda < q$, $\omega(\cdot) \in C([0, +\infty))$, $\omega(0) = 0$, $\omega(\tau) > 0$ when $\tau > 0$ was studied. As well known the extinction property means that any solution of the mentioned equation vanishes in Ω in a finite time. Modifying the local energy approach of [1], we obtain a condition of Dini type on the function $\omega(\cdot)$ that ensures the extinction.

Also we investigate the property of extinction in the finite time of solutions to the initial-boundary problem for $2m$ order nonlinear parabolic equation with absorption of the following type:

$$(|u|^{q-1}u)_t + (-1)^m \sum_{|\eta|=m} D_x^\eta \left(|D_x^m u|^{q-1} D_x^\eta u \right) + a(x)|u|^{\lambda-1}u = 0 \quad \text{in } \Omega \times (0, +\infty),$$

where Ω is bounded domain in \mathbb{R}^N , $N \geq 1$, $0 \in \Omega$, $m \geq 1$, $0 \leq \lambda < q$, an absorption potential $a(x)$ is nonnegative, measurable, bounded in Ω function. Using the semi-classical technic of [2], we find sufficient conditions, which guarantee the extinction for the mentioned equation above. These conditions are depending on N , m , and on the parameter of homogeneous nonlinearity of the main part in the equation q .

This work was financial supported in part by Akhiezer Fund and the trilateral German-Russian-Ukrainian summer school "Spectral Theory, Differential Equations and Probability".

References

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