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Method of Constructing the Fuzzy Regression Model of Bank Competitiveness

Abstract: The paper substantiates the need to consider economic efficiency indicators of bank activity as fuzzy quantities. Formulations of the problem of fuzzy regression analysis and modelling, available in literary sources, have been analyzed. Three main approaches to the fuzzy regression analysis are presented. The general mathematical and meaningful formulation of problem of a fuzzy multivariate regression analysis for commercial bank competitiveness has been proposed. Sequence of its solutions is described. The example of numerical computations for one of the large Ukrainian banks is given. Results of obtained solution were analyzed from the standpoint of reliability, accuracy and compared against the classical crisp regression analysis. Finishing steps for obtaining final accurate numerical results of solution process are described. In summary, convincing arguments concerning the expediency of application of this approach to the problem of determining the competitiveness of banks are formulated and presented.

Keywords: fuzzy regression model, bank competitiveness, fuzzy multiple regression, fuzzy modelling in banking

JEL classification: G20; G21; G32

1. Introduction

The study of conditions for the accounting and reporting of main economic indicators of banks showed that the value of some indicators should be seen as unclear values. This is especially true of performance indicators that determine bank effectiveness (Vojcheska, 2013; Andasarova, 2016). Therefore, description of bank competitiveness should be expanded with conditions of uncertainty using fuzzy sets theory. Benefits of using fuzzy regression models lie in the possibility of processing heterogeneous, fuzzy information (which is given in the form of sophisticated qualitative linguistic descriptions using non-metric scales and quantitative data) which definiteness conditions are doubtful. At the same time, strict preconditions for classic regression analysis does not allow its use under uncertainty, when information about the investigated dependence "inputs-output" has unclear linguistic evaluations such as "low", "medium", "high" and so on.

2. Analysis of existing approaches to constructing fuzzy regression models

Analysis of many scientific publications (Yager, 1986; Celmins, 1987; Diamond, 1988; Aliev, 1991; Sakawa, 1992; Redden, 1994; Wang, 2000; Yang, 2002; Kao, 2005; Krivonozhko, 2010; Yarushkina, 2010) showed that not all possible formulation of problems, which take into account the vagueness of input data and (or) model parameters were considered. Also many researches lack detailed description and recommendations for each stage of fuzzy regression modelling.

Based on the analysis of different approaches to constructing fuzzy regression models, we established that, in general, there are three main approaches in the development of these models: Fuzzy regression based on the criterion of minimizing vagueness, also known as Tanaka linear programming method (Tanaka, 1982), Fuzzy least squares method (approximation by distance, interval fuzzy least squares method), Multicriterial programming methods. So, the author recommends forming fuzzy linear regression model in the following sequence.

The first stage. Data should be presented by the following fuzzy linear model:

$$Y_i^* = A_1^* x_{i1} + \ldots + A_n^* x_{in} \tag{1}$$

where A_i is fuzzy type parameter.

When the values of x_i are the known, Y_i^* can be defined as (Tanaka, 1982):

$$\mu_{Y_i^*}(y) = 1 - \frac{|y_i - x_i^t \alpha|}{c^t |x_i|} \tag{2}$$

The second stage. The degree of approximation of calculated of fuzzy linear model (2) added values $Y_i = (y_i, e_i)$ is measured by level of confidence $\overline{h}_i$, which maximize h: $Y_i^h \subset Y_i^{*h}$, where

$$Y_i^h = \left\{ y \mid \mu_{Y_i}(y) \right\} \geq h,$$
$$Y_i^{*h} = \left\{ y \mid \mu_{Y_i^*}(y) \right\} \geq h \qquad (3)$$

The degree of approximation of fuzzy linear model to all data $Y_1,..., Y_N$ is defined as $\min_j \left[\overline{h}_j\right]$.

The third stage. Getting fuzzy coefficients A_i^*. The uncertainty of fuzzy linear model is determined as follows:

$$J = c_1 + ... + c_n . \qquad (4)$$

The problem is solved by obtaining fuzzy coefficients A_i^*, which minimize the variable J, $\overline{h}_i \geq H$ for all i. The level of confidence H is selected by the decision maker as the degree of approximation of the fuzzy linear model. Parameter $\overline{h}_i$ can be calculated from the formula (4):

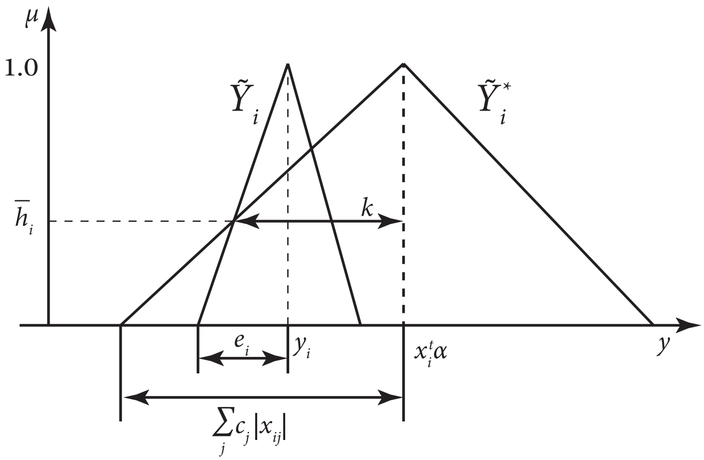
$$\overline{h}_i = 1 - \frac{\left| y_i - x_i^t \alpha \right|}{\sum_j c_j \left| x_{ij} \right| - e_i} . \qquad (5)$$

This is determined from the following relationship (Graph 1) (Tanaka, 1982):

$$\frac{1}{1 - \overline{h}_i} = \frac{\sum_j c_j \left| x_{ij} \right|}{k}, \qquad (6)$$

where $k = \left| y_i - x^t \alpha \right| + e_i (1 - \overline{h}_i).$ (7)

Graph 1. The degree of approximation of Y_i^* to fuzzy values $\tilde{Y}_i$



The task is to find the fuzzy parameters $A_i^* = (\alpha_i, c_i)$, which are the solution of the following linear programming problem:

$$J = c_1 + \ldots + c_n \to \min, \tag{8}$$

$$\begin{cases} \alpha^t x_i + (1-H)\sum_j c_j \left|x_{ij}\right| \geq y_i + (1-H)e_i \\ -\alpha^t x_i + (1-H)\sum_j c_j \left|x_{ij}\right| \geq -y_i + (1-H)e_i, \; i = \overline{1,N} \\ c_j > 0. \end{cases} \tag{9}$$

The first condition for the system (9) delivers what lower limit value of fuzzy variable $\tilde{Y}_i$ will be less than the actual values of the resulting characteristics, and the second condition means that the value of the upper limit of fuzzy variable $\tilde{Y}_i$ are greater than the factual value of the resulting characteristics.

In this way the interval of values of the simulated phenomena is formed. It should be said that this problem can be solved by using the «Search solution» add-on in MS Office Excel. Thus, the author (Tanaka, 1982) has shown that solving the ordinary linear programming problems can get the most accurate model for the above data. Moreover, the solution of the dual problem is easier than solving the direct problem because it reduces the number of calculations. If input data are negative, it is recommended to realize the same procedure for constructing fuzzy regression model. Please note that other types of fuzzy sets can also be used for such research.

However, a number of authors (Redden, 1994; Wang, 2000; Kao, 2005) have noted significant shortcomings of this approach. Firstly, the regression coefficients are highly sensitive to emissions of data (Redden, 1994). Secondly, the assessment has too extensive range (Wang, 2000). Thirdly, the target function is not interpreted as a similarity index of the desired and the actual behavior of model, as opposed to ordinary regression analysis (Ponomarenko and Malyarets, 2009). That is, increasing observations lead to more blurred evaluations, which are contrary to the general provisions of regression analysis. Because in classical regression analysis more observations provide more accurate results (Kao, 2005).

A similar assessment of this approach was given in the works (Redden, 1994; Wang, 2000). Practical verification of this model for indicators of competitiveness of bank really has led us to unreliable results. We solved the task in the MS Office Excel environment using add-in “Solver”. The obtained values were very large, some were negative, and this is not in line with the norms of the National Bank of Ukraine. The explanation of this phenomenon is that the model is highly sensitive to unexpected sudden fluctuations of data (Redden, 1994).

In (Aliev, 1991) at one time was proposed to apply criterion of minimization of fuzziness for assessing the fuzzy parameters of mathematical model presented by fuzzy multiple regression equation:

$$\widetilde{Y} = \widetilde{A}_0 + \widetilde{A}_1 x_1 + \ldots + \widetilde{A}_k x_k , \tag{10}$$

where $x_i, i = \overline{1, k}$ – the determined values, $\tilde{Y}$– fuzzy values. For estimation of fuzzy parameters of constructed model the criterion of minimization of fuzzy output values deviations $\tilde{Y}_i$ obtained by (10) was used from the corresponding sampling values Y_i according to the observations $i = \overline{1,n}$:

$$J = \underset{i=\overline{1,n}}{\Upsilon} (Y_i \,|-|\, \widetilde{Y}_i)^2 \rightarrow \min \tag{11}$$

where |–| – bounded difference of fuzzy numbers defined by the formula:

$$\mu_J(x) = \mu_{Y_i|-|\widetilde{Y}_i}(x) = \max(0, \mu_{Y_i}(x) - \mu_{\widetilde{Y}_i}(x)). \tag{12}$$

Bounded difference is the disjunction operations for fuzzy numbers [3].

The task of parameters estimation for equation (10) consists in determining the coefficients $\tilde{A}_i (i = \overline{1, n})$ which satisfy the condition (12). Equation (10) is a multi-dimensional function with fuzzy coefficients:

$$\widetilde{Y} = f(x_1,...,x_n, \widetilde{A}_1,...,\widetilde{A}_n), \tag{13}$$

considering this in (12), then

$$J = \mathop{\mathrm{Y}}_{i=\overline{1,n}} (Y_i \,|-|\, f(x_1,...,x_n, \widetilde{A}_1,...,\widetilde{A}_n))^2 \to \min. \tag{14}$$

In this way, the task of estimation for parameters of regression (10) has been reduced to minimizing multivariate functions with fuzzy variables.

Suppose that fuzzy coefficients $\tilde{A}_0, \tilde{A}_1, .., \tilde{A}_k$ are normal fuzzy sets on $\Re : \widetilde{A}_i = \mathop{\mathrm{Y}}_{a_i \in \Re} \mu_{\widetilde{A}_i}(a_i)/a_i$ and defines α - level sets of coefficients $\widetilde{A}_i :\ a_i^{\alpha} = \{a_i : a_i \in \Re,\ \mu_{\widetilde{A}_i}(a_i) \geq \alpha\},\ i = \overline{0, k},\ \alpha \in [0,1].$

For evaluation of fuzzy coefficients $\tilde{A}_0, \tilde{A}_1, .., \tilde{A}_k$ determine such ratios $a_0^{\alpha_j}, a_1^{\alpha_j}, ..., a_k^{\alpha_j},\ j = \overline{1, p}$ at each level α_j, $(\alpha : \{\alpha_0 = 0, \alpha_1, ..., \alpha_j, ..., \alpha_p = 1\})$, which satisfy condition $J_j = \sum_{i=1}^{N} (y_i^{\alpha_j} - \widetilde{y}_i^{\alpha_j})^2 \to \min,\ j = \overline{1, p},$ (15)
where $\widetilde{y}_i^{\alpha_j} = a_0^{\alpha_j} + a_1^{\alpha_j} x_1 + ... + a_k^{\alpha_j} x_k$.

So, for each level $\alpha : \{\alpha_0 = 0, \alpha_1,..., \alpha_j, ..., \alpha_p = 1\}$ a multiple regression equation can be written:

$$\begin{cases} y^{\alpha_0} = a_0^{\alpha_0} + a_1^{\alpha_0} x_1 + ... + a_n^{\alpha_0} x_n, \\ y^{\alpha_1} = a_0^{\alpha_1} + a_1^{\alpha_1} x_1 + ... + a_n^{\alpha_1} x_n, \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ y^{\alpha_p} = a_0^{\alpha_p} + a_1^{\alpha_p} x_1 + ... + a_n^{\alpha_p} x_n. \end{cases} \tag{16}$$

Equation (16) is an ordinary multiple regression equation. For the evaluation of fuzzy coefficients $\tilde{A}_0, \tilde{A}_1, .., \tilde{A}_k$ enough to define such $a_0^{\alpha_j}, a_1^{\alpha_j}, ..., a_n^{\alpha_j},\ \ j = \overline{1, p}$ in each level α_j, which satisfied the condition (15). Deterministic values $y_i^{\alpha_j}$, which are observed, obtained by approximation of fuzzy values of the output variable $\widetilde{y}_i^{\alpha_j}$ α - level sets in accordance with approximation of fuzzy coefficients $\tilde{A}_0, \tilde{A}_1, .., \tilde{A}_k$.

Thus, the output problem of evaluation for fuzzy coefficients of fuzzy regression equation (10) has been erected to the classic problems of parameter estimation of multiple regression (Diamond, 1988). Another way of constructing fuzzy linear regression is the approach based on the method of least squares FLSRA, which are proposed in (Celmins, 1987; Diamond, 1988). That approach involves the selection of fuzzy regression coefficients in such way so as to minimize the distance between fuzzy numbers (models output and data from input term-set). The methods of multicriterial programming and fuzzy least squares are applied for this purpose (Yang, 2002). The relevant approaches were formed and developed in the works (Sakawa, 1992; Yang, 2002).

3. Construction of fuzzy regression model for bank competitiveness

Let us assume that the resulting attribute y acquires fuzzy values. So, we will consider the construction of fuzzy multifactor linear regression model (of bank competitiveness) on the same data, which was calculated in the strict linear regression model. The model has fuzzy result and strict independent variables. We will search the fuzzy regression equation that describes the bank's competitiveness.

Let us consider dependence of the ratio of the loan portfolio to the bank obligations (y) (for the example of private Joint Stock Company «Commercial Bank Khreschatyk») from precise signs factors: of the bank attracted funds (x_1) and borrowed funds (x_2). According to the formula (10), we will seek equation in the form: $\widetilde{y} = \widetilde{A}_0 + \widetilde{A}_1 x_1 + \widetilde{A}_2 x_2$, (17)

where $\tilde{A}_0$, $\tilde{A}_1$, $\tilde{A}_2$ – fuzzy coefficients of model.

This model has a fuzzy resultative sign conditioned by strict independent variables. Such a type of fuzzy linear regression model allows taking into account random errors, the unpredictability of influence of external factors (e.g. outflow of foreign shareholders), which refers specifically to the resultative signs, because the accuracy of initial data were taken into consideration from the beginning of its construction. The availability of errors is obvious; therefore, to take account of the impact of knowingly inaccurate fuzzy data, we will construct a fuzzy linear regression model of competitiveness of the bank, which has a fuzzy result, however conditioned by strict input data.

The function of belonging (membership function) for the fuzzy characteristics should be determined next. For evaluation of fuzzy coefficients $\tilde{A}_i$ fuzzy sets used have been broken down into the following α- slices: α_1=0,5; α_2= 0,8; α_3=1.

As is well known, the concept of fuzzy set is based on the assumption that any element of it belongs to this set only to some extent. Therefore, the membership function is introduced and it describes the extent of belonging of the fuzzy number to interval [0,1]. In fact, the membership function reflects the confidence that the examined economic indicator of competitiveness of the bank will take a particular value within the limits of its carrier. The confidence given by α - slices, for example the value α=0,5 corresponds to 50% level of confidence. In general, a choice of confidence level depends on the researcher, depending on the prevailing position of the researcher of being either pessimistic or optimistic concerning the credibility of data. Wherein 50 percentile corresponding to a median value of fuzzy values *y*, which divides the distribution of function of the values of studied parameters in two equally probable parts.

The carrier of each fuzzy number, which corresponds to the values of resultant variable parameter $\tilde{y}$, is determined on the basis of the constructed membership function for *y* з using tools of descriptive statistics in the program Statgraphics Centurion. The carrier of fuzzy number is the interval *(c,d)* within which fuzzy number will be located. The boundaries of the interval are determined by membership function, which is the basis for acceptance of management solutions, fuzzy number with a corresponding α - slice. The fundamental point is the choice of the type of membership function. It is recommended to do this choice on the base of analysis of distribution for values of the trait *y* with calculation of numerical characteristics.

Equation (18) is a multi-dimensional function with fuzzy variables

$$\widetilde{y} = f(x_1,...,x_n,\widetilde{A}_1,...,\widetilde{A}_n). \tag{18}$$

To estimate fuzzy parameters of the constructed model the criterion that can be used is minimization for deviations of fuzzy output values $\tilde{y}_i$ obtained by (17), from the corresponding sampling values y_i according to observations $i=\overline{1,n}$ (11). In the notation proposed in (Zade, 2001), restricted difference is disjunction operations for fuzzy numbers, which are obtained by the formula (12). Thus, the problem of estimation for parameters of equation (18) consists in definition of coefficients $\tilde{A}_i$ which satisfies the condition (11).

To estimate fuzzy regression model coefficients, it is necessary to determine the boundaries (*c,d*) for the carrier of fuzzy value of fuzzy resultant variable $\tilde{y}$. So, it is recommended to introduce coefficients β_1, β_2 which calculated as the ratio of percentiles of indicator *y* and are determined by formulas:

$$\beta_1 = \frac{y_{\alpha=0,5}}{y_{\alpha=1}}; \quad (19) \qquad \beta_2 = \frac{y_{\alpha=0,8}}{y_{\alpha=1}}, \quad (20)$$

where $y_{\alpha=0,5}$ – statistical value of *y*, which corresponding to the 50-th percentile (median); $y_{\alpha=0,8}$ – statistical value of *y*, which corresponding to the 80-th percentile; $y_{\alpha=1}$ – statistical value of *y*, which corresponding to the 100-th percentile.

Usually these values are recommended to be calculated heuristically, for example, through experiments with the input data. Percentiles generalize information about the set of values of signs and characterizing value, which are achieved by preassigned percent of the total number of observations. That is, value *X*, which is observed in 50% observations, called the 50^{th} percentile.

Percentiles indicate what percentage of the values of the random variable, which is observed, is below the set level, and excess of set level is possible only with a given probability. For example, the value of X, to which 80% of observations is situated and above which 20% of observations is situated is called 80th percentile. This is to mathematically prove that such a form of description for variables of the investigated signs can be obtained on the basis of percentile values, which will not affect the emissions (abnormal values).

4. Numerical calculations according to the proposed model for bank "Khreschatyk"

For our task, percentiles for the corresponding value for the indicator y_{32} are: 1,0% – 0,4428; 5,0% – 0,4869; 10,0% – 0,5285; 25,0% – 0,60265; 50,0% – 0,6662; 75,0% – 0,7211; 80,0% – 0,7396; 85,0% – 0,7506; 90,0% – 0,7618; 95,0% – 0,8242; 99,0% – 0,8336.

We assume that the value of resultant signs *y* obtained at the value of *α* - slice $\alpha_3=1$ calculated correctly with 99.99 % probability, which corresponds to 100th percentile because the definition of fuzzy numbers implies that the value of its membership function cannot be greater than 1. This means that almost all values of competitiveness of the bank observed at $\alpha_3=1$ are lower than the level $x_m \in X, \mu_{\tilde{y}_i}(x_m)=1$, where in function increases on the left of x_m and on the right it decreases, where x_m is modal value of membership function $\mu_{\tilde{y}_i}(x)$.

Consequently, the calculation of confidence intervals for $\tilde{y}$ on 50th and 80th percentiles is recommended by the formulas (21), (22):

$$\alpha = 0{,}5 : c = \beta_1 y_i - s;\ d = \beta_1 y_i + s,\ i = \overline{1,n} \qquad (21)$$

$$\alpha = 0{,}8 : c = \beta_2 y_i - s;\ d = \beta_2 y_i + s,\ i = \overline{1,n}, \qquad (22)$$

where S – standard error, y_i – the value of resultant signs y defined in the i-th observation on the basis of bank reporting. The standard error is an indicator of reliability of the calculated parameters. The lower the standard deviation value, the more accurate the estimation.

The carrier (c,d) of fuzzy resultant sign $\tilde{y}$ at the appropriate level is determined by the formulas (4), (5). The required parameters β_1, β_2, s to calculate the boundaries of interval, which belongs fuzzy value of resultant variable $\tilde{y}$ at corresponding α - slice is recommended to calculate taking into account its numerical characteristics. Let calculate the parameters β_1, β_2 on the basis of percentile values for y:

$$\beta_1 = \frac{y_{\alpha = 0,5}}{y_{\alpha = 1}} = \frac{0{,}6662}{0{,}8336} = 0{,}7992 \quad (23); \quad \beta_2 = \frac{y_{\alpha = 0,8}}{y_{\alpha = 1}} = \frac{0{,}7396}{0{,}8336} = 0{,}8874. \quad (24).$$

Standard error is: $s = 0{,}018$. In accordance with the chosen level of confidence the value of input data x_i $(i = 1,2)$, which was observed, and output $\tilde{y}$ parameters at every level $\alpha_j (j = \overline{1,3})$ presented in Appendix (Tables 1-4). Each of these tables represents a deterministic relationship between the input and output parameters at every level $\alpha_j (j = \overline{1,3})$.

To evaluate the coefficients $a_i^{\alpha j}$ $(i = 1,2;\ j = \overline{1,3})$ on each level α_j fuzzy regression equation (18) according to (14) can be rewritten in the form:

$$y^{\alpha j} = a_0^{\alpha j} + a_1^{\alpha j} x_1 + a_2^{\alpha j} x_2, \qquad (25)$$

because it is proved that for each level of $\alpha : \{\alpha_0 = 0, \alpha_1, ..., \alpha_s, ..., \alpha_p = 1\}$, multiple regression equation (16) can be writen.

Thus, the equation (25) is a classical regression equation, and the program Statgraphics Centurion can be used to evaluate its coefficients. The results of calculations for coefficients (25) are as follows:

$a_0^{0,5} = (0{,}44; 0{,}47)$; $a_0^{0,8} = (0{,}50; 0{,}53)$; $a_0^1 = 0{,}57$; $a_1^{0,5} = (-1{,}68; -1{,}69)$; $a_1^{0,8} = (-1{,}89; -1{,}92)$;
$a_1^1 = -2{,}10$; $a_2^{0,5} = (-6{,}65; -6{,}74)$; $a_2^{0,8} = (-7{,}48; -7{,}32)$; $a_2^1 = -8{,}31$;

As a result, we receive equation for each α - slice at each interval of fuzziness (Table 5).

Table 5: The results of calculation of fuzzy linear regression model

Left limit of fuzziness interval (at appropriate level)	Right limit of fuzziness interval (at appropriate level)
The regression equation $\alpha_1 = 0,5$	
$y = 0,44 - 1,68x_1 - 6,65x_2$	$y = 0,47 - 1,69x_1 - 6,74x_2$
The regression equation $\alpha_2 = 0,8$	
$y = 0,50 - 1,89x_1 - 7,48x_2$	$y = 0,53 - 1,92x_1 - 7,32x_2$
The regression equation	
$y = 0,57 - 2,10x_1 - 8,31x_2$	

All calculated equations are statistically qualitative, that confirm the criteria of Student, Fisher and Durbin-Watson. To determine the fuzzy coefficients $\tilde{A}_i$ obtained values $a_i^{\alpha j}$ combined with the use of ratio (26) or (27), where $a_i^{\alpha} = \{a_i \mid \mu_{\tilde{A}_i}(a_i) \geq \alpha\}$.:

$$\tilde{A}_i = \bigcup_{a_i \in \Re} \mu_{\tilde{A}_i}(a_i)/a_i \quad (26); \qquad \mu_{\tilde{A}_i}(a_i) = \sup \min\{\alpha, \mu_{a_i^{\alpha}}\}, \quad (27).$$

Thus, the equation describing fuzzy dependence of bank competitiveness from signs x_1, x_2 takes the form:

$$\begin{aligned}\tilde{y} &= (0,5/0,44 + 0,8/0,5 + 1/0,57 + 0,8/0,53 + 0,5/0,47) + \\ &+ (0,5/-1,68 + 0,8/-1,89 + 1/-2,10 + 0,8/-1,92 + 0,5/-1,69)x_1 + \\ &+ (0,5/-6,65 + 0,8/-7,48 + 1/-8,31 + 0,8/-7,32 + 0,5/-6,74)x_2\end{aligned} \quad (28)$$

5. General sequence of constructing fuzzy regression model of bank competitiveness

Therefore, the logic of construction of fuzzy linear regression model for bank competitiveness consists of stages, as given below.

1. To form a system of factor signs of bank competitiveness $(x_{1i}, x_{2i}, ..., x_{ni})$ and resultant sign (y_i), $y_i = \tilde{f}(x_1, ..., x_n)$ (Table. 6).

Table 6: The indicative signs of bank competitiveness

N	x_1	x_2	…	x_n	y
1	x_{11}	x_{21}	…	x_{n1}	y_1
2	x_{12}	x_{22}	…	x_{n2}	y_2
…	…	…	…	…	…
N	x_{1N}	x_{2N}		x_{nN}	y_N

2. To choose function $\widetilde{y}=f(x_1,...,x_n,\widetilde{A}_1,...,\widetilde{A}_n)=\sum_{i=0}^{n}\widetilde{A}_i x_i$ (13), which approximates the function $\widetilde{f}(x_1,...,x_n)$, that a given by table 6, $\widetilde{A}_i$, $i=\overline{1,n}$ – fuzzy regression model coefficients.

3. To determine estimates for coefficients of function (13) the criterion of minimizing deviations of fuzzy output parameter values $\widetilde{\widetilde{y}}$ is used, obtained using (13) from its selective fuzzy values (Table 6) (14). The task of estimation of parameters for equation (13) consists in definition of coefficients $\widetilde{A}_i (i=\overline{0,n})$ which satisfies the condition (15).

4. To split fuzzy set of resultant values on α - slices.

5. To define at every level the input values of the independent variables x_i and resultant sign y_i.

6. To calculate value of effective sign (16) for each level.

7. To define fuzzy coefficients $\tilde{A}_i$ using relation (26) or (27).

8. To examine relative accuracy of fuzzy linear model by analyzing deviations of

$$e_{rel}=\frac{1}{n}\sum_{i=1}^{n}\left|\frac{\widetilde{y}_i-\widetilde{\widetilde{y}}_i}{\widetilde{y}_i}\right|\cdot 100\%, \tag{29}$$

If $e_{rel}\leq 10\%$ then accuracy of the model considered to be acceptable, and quantity of $e_{rel}\leq 5\%$ indicates a fairly high accuracy level.

9. To make the procedure of defuzzification for resultant variable to obtain accurate values.

10. To conduct the analysis the resulting model and on this basis to take rational managerial decision.

6. Steps for checking model adequacy

In general, the adequacy of fuzzy linear multivariable regression model can be checked based on any approach considered above. It is recommended to carry out the analysis of deviations or, as they say in econometrics, errors $\{\varepsilon_i=Y_i-\widetilde{Y}_i,\ i=\overline{1,n}\}$ for the purpose of checking the properties of random components (closeness to zero of mathematical expectation, the random nature of deviations, absence of autocorrelation). The verification should be conducted by methods that are used in the case of classical regression. Moving from fuzzy variables $\varepsilon_i = (m_{e_i}, \alpha_{e_i}, \beta_{e_i})$, where m_{e_i} – mode of fuzzy number, $\alpha_{e_i}, \beta_{e_i}$ – left and right border of fuzzy interval, respectively, to strict $e_i,\ i=\overline{1,n}$ should be implemented in a sequence, as shown below.

1. Defuzzification (adjustment to clearness according to the level of belonging of fuzzy set) on the basis of the resultant membership function $\mu_{e_i}(x)$ of fuzzy number $\varepsilon_i = (m_{e_i}, \alpha_{e_i}, \beta_{e_i})$ under given *LR* - functions, which calculate the numeric value of error:

$$\mu_{e_i}(x) = \begin{cases} L\left(\dfrac{m-x}{\alpha}\right), x \le m \\ R\left(\dfrac{x-m}{\beta}\right), x \ge m \end{cases} \quad (\alpha > 0,\ \beta > 0). \tag{30}$$

Defuzzification procedure determines the method of transition from fuzzy to crisp numbers. A number of methods are known for bringing to clearness (Shtovba, 2006), for example the method of center of gravity:

$$e_i = Defuz(\varepsilon_i) = D(\mu_{e_i}) = \frac{\sum_{i=1}^{k} x \cdot \mu_{e_i}(x)}{\sum_{i=1}^{k} \mu_{e_i}(x)},\ i=\overline{1,k} \tag{31}$$

2. Defuzzification on the basis of the resulting membership functions $\mu_{y_i}(x)$ and $\mu_{\tilde{y}_i}(x)$ fuzzy numbers $Y_i = (m_{y_i}, \alpha_{y_i}, \beta_{y_i})$ and $\tilde{Y}_i = (\tilde{m}_{y_i}, \tilde{\alpha}_{y_i}, \tilde{\beta}_{y_i})$ accordingly under given *LR* - functions. This operation allows to calculate the exact values of the dependent variable and its estimates by means of one of defuzzification methods (Shtovba, 2006). Thus an error $e_i,\ i=\overline{1,n}$ calculated as the difference:

$$e_i = Defuz(Y_i) - Defuz(\tilde{Y}_i) = D(\mu_{y_i}) - D(\mu_{\tilde{y}_i}). \tag{32}$$

3. The construction of γ - slices by the values of dependent variable $Y_i = (m_{y_i}, \alpha_{y_i}, \beta_{y_i})$ and its estimates $\tilde{Y}_i = (\tilde{m}_{y_i}, \tilde{\alpha}_{y_i}, \tilde{\beta}_{y_i})$:

$$\begin{aligned} \left[\underline{Y_i}(\gamma), \overline{Y_i}(\gamma)\right] &= [m_y - \alpha_y(1-\gamma), m_y + \beta_y(1-\gamma)],\ \gamma \in [0,1], \\ \left[\underline{\tilde{Y}_i}(\gamma), \overline{\tilde{Y}_i}(\gamma)\right] &= [\tilde{m}_y - \tilde{\alpha}_y(1-\gamma), \tilde{m}_y + \tilde{\beta}_y(1-\gamma)],\ \gamma \in [0,1] \end{aligned} \tag{33}$$

and drafting of residual sequence for each γ - slice by determining ε - point.

$$\varepsilon \in \left[\underline{Y_i}(\gamma), \overline{Y_i}(\gamma)\right] \Rightarrow \lambda_i^\gamma = \underline{Y_i}(\gamma)\cdot\varepsilon + \overline{Y_i}(\gamma)\cdot(1-\varepsilon),\ \gamma\in[0,1],$$
$$\varepsilon \in \left[\underline{\widetilde{Y}_i}(\gamma), \overline{\widetilde{Y}_i}(\gamma)\right] \Rightarrow \widetilde{\lambda}_i^\gamma = \underline{\widetilde{Y}_i}(\gamma)\cdot\varepsilon + \overline{\widetilde{Y}_i}(\gamma)\cdot(1-\varepsilon),\ \gamma\in[0,1]. \quad (34)$$

As a result, accurate error value $e_i, i=\overline{1,n}$ will be calculated using the formula: $e_i, \lambda_i^\gamma - \widetilde{\lambda}_i^\gamma,\ i=\overline{1,n}, \gamma\in[0,1]$.

It is possible to speak about the quality of the pair of fuzzy regression based on the values of the correlation coefficient. Let us deduce the formula calculating the correlation coefficients. Estimates of regression parameters according to the method of least squares can be founded by minimization of function (Sapkina, 2013):

$$F(\widetilde{A}_0, \widetilde{A}_1) = \sum_{i=1}^{n} D^2(Y_i, \widetilde{Y}_i) \rightarrow \min, \quad (35)$$

where $Y_i = (m_{yi}, \alpha_{yi}, \beta_{yi})$, and $\widetilde{Y}_i = (\widetilde{m}_{yi}, \widetilde{\alpha}_{yi}, \widetilde{\beta}_{yi})$ (fuzzy numbers of LR - type) are calculated by formulas (36, 37);

$$Y_i = A_0 + A_1 x_i + E_i, i=\overline{1,n}, \quad (36)$$

where $x_i \in \Re, A_0 = (m_{b0}, \alpha_{b0}, \beta_{b0}), A_1 = (m_{b1}, \alpha_{b1}, \beta_{b1})$ – theoretical coefficients (parameters of regression), $E_i = (m_{ei}, \alpha_{ei}, \beta_{ei})$ – random errors fuzzy numbers of LR - type, $i=\overline{1,n}$ – number of observation. At this Y_i, A_0, A_1 have the same membership function.

Empirical evaluation function of fuzzy linear pair regression looks as:

$$\widetilde{Y}_i = \widetilde{B}_0 + \widetilde{B}_1 x_i + E_i, i=\overline{1,n}, \quad (37)$$

where $x_i \in \Re$ – value of the independent variable, $\widetilde{Y}_i = (\widetilde{m}_{yi}, \widetilde{\alpha}_{yi}, \widetilde{\beta}_{yi})$ – estimates of the values of dependent (explanatory) variable, $\widetilde{B}_0 = (\widetilde{m}_{b0}, \widetilde{\alpha}_{b0}, \widetilde{\beta}_{b0})$, $\widetilde{B}_1 = (\widetilde{m}_{b1}, \widetilde{\alpha}_{b1}, \widetilde{\beta}_{b1})$ – estimations of the unknown parameters B_0, B_1, empirical (selective) regression coefficients.

In formula (38), $D^2(Y_i, \widetilde{Y}_i)$ – formula of distance between fuzzy variables Y_i and $\widetilde{Y}_i$ (Euclid's formula):

$$D = \sqrt{(m_{yi} - \widetilde{m}_{yi})^2 + (\alpha_{yi} - \widetilde{\alpha}_{yi})^2 + (\beta_{yi} - \widetilde{\beta}_{yi})^2}, \quad (38)$$

Considering rules of multiplication of fuzzy number on some constant, sums of fuzzy numbers and commutativity property, the target function (35) can be written as:

$$F(\widetilde{A}_0, \widetilde{A}_1) = \sum_{i=1}^{n} (\widetilde{A}_0 + \widetilde{A}_1 x_i - Y_i)^2 = \sum_{i=1}^{n} ((\widetilde{m}_{b0} + \widetilde{m}_{b1} x_i - m_{yi})^2 + (\widetilde{\alpha}_{b0} + \widetilde{\alpha}_{b1} x_i - \\ - \alpha_{yi})^2 + (\widetilde{\beta}_{b0} + \widetilde{\beta}_{b1} x_i - \beta_{yi})^2) \to \min. \tag{39}$$

Function $F(\widetilde{A}_0, \widetilde{A}_1)$ is a quadratic function of two parameters $\widetilde{A}_0 = (\widetilde{m}_{b0}, \widetilde{\alpha}_{b0}, \widetilde{\beta}_{b0})$ and $\widetilde{A}_1 = (\widetilde{m}_{b1}, \widetilde{\alpha}_{b1}, \widetilde{\beta}_{b1})$, $\{(x_i, Y_i)\}_{i=\overline{1,n}}$ – known observational data.

Function $F(\widetilde{A}_0, \widetilde{A}_1)$ is continuous, convex and bounded from below (*F>0*) i.e. has a minimum. The necessary condition for the existence of the minimum of function (39) is equality to zero of its partial derivatives for unknown variables $\widetilde{m}_{b0}, \widetilde{\alpha}_{b0}, \widetilde{\beta}_{b0}$ and $\widetilde{m}_{b1}, \widetilde{\alpha}_{b1}, \widetilde{\beta}_{b1}$.

$$\begin{cases} \dfrac{\partial F}{\partial \widetilde{m}_{b0}} = 2\sum_{i=1}^{n} (\widetilde{m}_{b0} + \widetilde{m}_{b1} x_i - m_{yi}) = 0, \\ \dfrac{\partial F}{\partial \widetilde{\alpha}_{b0}} = 2\sum_{i=1}^{n} (\widetilde{\alpha}_{b0} + \widetilde{\alpha}_{b1} x_i - \alpha_{yi}) = 0, \\ \dfrac{\partial F}{\partial \widetilde{\beta}_{b0}} = 2\sum_{i=1}^{n} (\widetilde{\beta}_{b0} + \widetilde{\beta}_{b1} x_i - \beta_{yi}) = 0. \end{cases} \tag{40}$$

$$\begin{cases} \dfrac{\partial F}{\partial \widetilde{m}_{b1}} = 2\sum_{i=1}^{n} (\widetilde{m}_{b0} + \widetilde{m}_{b1} x_i - m_{yi}) x_i = 0, \\ \dfrac{\partial F}{\partial \widetilde{\alpha}_{b1}} = 2\sum_{i=1}^{n} (\widetilde{\alpha}_{b0} + \widetilde{\alpha}_{b1} x_i - \alpha_{yi}) x_i = 0, \\ \dfrac{\partial F}{\partial \widetilde{\beta}_{b1}} = 2\sum_{i=1}^{n} (\widetilde{\beta}_{b0} + \widetilde{\beta}_{b1} x_i - \beta_{yi}) x_i = 0. \end{cases} \tag{41}$$

After transformations, we obtain the system of normal equations to determine the parameters of fuzzy linear pair regression (Sapkina, 2013):

$$\begin{cases} n\widetilde{m}_{b0} + \widetilde{m}_{b1} \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} m_{yi}, \\ n\widetilde{\alpha}_{b0} + \widetilde{\alpha}_{b1} \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} \alpha_{yi}, \\ n\widetilde{\beta}_{b0} + \widetilde{\beta}_{b1} \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} \beta_{yi}. \end{cases} \tag{42}$$

$$\begin{cases} \widetilde{m}_{b0} \sum_{i=1}^{n} x_i + \widetilde{m}_{b1} \sum_{i=1}^{n} x_i^2 = \sum_{i=1}^{n} x_i m_{yi}, \\ \widetilde{\alpha}_{b0} \sum_{i=1}^{n} x_i + \widetilde{\alpha}_{b1} \sum_{i=1}^{n} x_i^2 = \sum_{i=1}^{n} x_i \alpha_{yi}, \\ \widetilde{\beta}_{b0} \sum_{i=1}^{n} x_i + \widetilde{\beta}_{b1} \sum_{i=1}^{n} x_i^2 = \sum_{i=1}^{n} x_i \beta_{yi}. \end{cases} \tag{43}$$

Let us divide systems (42, 43) to n and obtain the following system of normal equations (Sapkina, 2013):

$$\begin{cases} \widetilde{m}_{b0} + \widetilde{m}_{b1}\overline{x} = \overline{m}_y, \\ \widetilde{\alpha}_{b0} + \widetilde{\alpha}_{b1}\overline{x} = \overline{\alpha}_y, \\ \widetilde{\beta}_{b0} + \widetilde{\beta}_{b1}\overline{x} = \overline{\beta}_y. \end{cases} \tag{44}$$

$$\begin{cases} \widetilde{m}_{b0}\overline{x} + \widetilde{m}_{b1}\overline{x}^2 = \overline{xm_y}, \\ \widetilde{\alpha}_{b0}\overline{x} + \widetilde{\alpha}_{b1}\overline{x}^2 = \overline{x\alpha_y}, \\ \widetilde{\beta}_{b0}\overline{x} + \widetilde{\beta}_{b1}\overline{x}^2 = \overline{x\beta_y}. \end{cases} \tag{45}$$

The relevant averages are determined by the formulas (46 - 49):

$$\overline{x} = \frac{\sum_{i=1}^{n} x_i}{n} \ (46); \quad \overline{x}^2 = \frac{\sum_{i=1}^{n} x_i^2}{n} \ (47); \quad \overline{m}_y = \frac{\sum_{i=1}^{n} m_{yi}}{n}, \ \overline{\alpha}_y = \frac{\sum_{i=1}^{n} \alpha_{yi}}{n}, \ \overline{\beta}_y = \frac{\sum_{i=1}^{n} \beta_{yi}}{n} \tag{48}$$

$$\overline{xm_y} = \frac{\sum_{i=1}^{n} x_i m_{yi}}{n}, \ \overline{x\alpha_y} = \frac{\sum_{i=1}^{n} x_i \alpha_{yi}}{n}, \ \overline{x\beta_y} = \frac{\sum_{i=1}^{n} x_i \beta_{yi}}{n} \tag{49}$$

Let us express the variables $\widetilde{m}_{b0}, \widetilde{\alpha}_{b0}, \widetilde{\beta}_{b0}$ in the system (44) via $\widetilde{m}_{b1}, \widetilde{\alpha}_{b1}, \widetilde{\beta}_{b1}$:

$$\begin{cases} \widetilde{m}_{b0} = \overline{m}_y - \widetilde{m}_{b1}\overline{x}, \\ \widetilde{\alpha}_{b0} = \overline{\alpha}_y - \widetilde{\alpha}_{b1}\overline{x}, \\ \widetilde{\beta}_{b0} = \overline{\beta}_y - \widetilde{\beta}_{b1}\overline{x}. \end{cases} \tag{50}$$

Substituting significances of the modal values, left and right coefficients of fuzziness for parameter $\widetilde{B}_0$ (all obtained from (50)) in the regression equation (37), we will have:

$$(\widetilde{m}_y, \widetilde{\alpha}_y, \widetilde{\beta}_y) = (\widetilde{m}_{b0}, \widetilde{\alpha}_{b0}, \widetilde{\beta}_{b0}) + (\widetilde{m}_{b1}, \widetilde{\alpha}_{b1}, \widetilde{\beta}_{b1})x \tag{51}$$

or $$(\widetilde{m}_y, \widetilde{\alpha}_y, \widetilde{\beta}_y) = (\overline{m}_y - \widetilde{m}_{b1}\overline{x}, \overline{\alpha}_y - \widetilde{\alpha}_{b1}\overline{x}, \overline{\beta}_y - \widetilde{\beta}_{b1}\overline{x}) + (\widetilde{m}_{b1}, \widetilde{\alpha}_{b1}, \widetilde{\beta}_{b1})x \tag{52}$$

Then we use the formula for adding fuzzy numbers and get [10]:

$$\begin{aligned} (\widetilde{m}_y, \widetilde{\alpha}_y, \widetilde{\beta}_y) &= (\overline{m}_y - \widetilde{m}_{b1}\overline{x} + \widetilde{m}_{b1}x, \overline{\alpha}_y - \widetilde{\alpha}_{b1}\overline{x} + \widetilde{\alpha}_{b1}x, \overline{\beta}_y - \widetilde{\beta}_{b1}\overline{x} + \widetilde{\beta}_{b1}x) = \\ &= (\overline{m}_y + \widetilde{m}_{b1}(x - \overline{x}), \overline{\alpha}_y + \widetilde{\alpha}_{b1}(x - \overline{x}), \overline{\beta}_y + \widetilde{\beta}_{b1}(x - \overline{x})). \end{aligned} \tag{53}$$

In this way, we get the following system of equations: $$\begin{cases} \widetilde{m}_y = \overline{m}_y + \widetilde{m}_{b1}(x - \overline{x}), \\ \widetilde{\alpha}_y = \overline{\alpha}_y + \widetilde{\alpha}_{b1}(x - \overline{x}), \\ \widetilde{\beta}_y = \overline{\beta}_y + \widetilde{\beta}_{b1}(x - \overline{x}). \end{cases} \quad (54)$$

The system (54) can be represented in an equivalent form:

$$\begin{cases} \dfrac{\widetilde{m}_y - \overline{m}_y}{\sigma_m^2} = \widetilde{m}_{b1} \dfrac{\sigma_x^2}{\sigma_m^2} \dfrac{(x - \overline{x})}{\sigma_x^2}, \\ \dfrac{\widetilde{\alpha}_y - \overline{\alpha}_y}{\sigma_\alpha^2} = \widetilde{\alpha}_{b1} \dfrac{\sigma_x^2}{\sigma_\alpha^2} \dfrac{(x - \overline{x})}{\sigma_x^2}, \\ \dfrac{\widetilde{\beta}_y - \overline{\beta}_y}{\sigma_\beta^2} = \widetilde{\beta}_{b1} \dfrac{\sigma_x^2}{\sigma_\beta^2} \dfrac{(x - \overline{x})}{\sigma_x^2}, \end{cases} \quad (55)$$

where σ_x^2 – sample variance of variable x, which determined by the formula:

$$\sigma_x^2 = \overline{x^2} - \overline{x}^2 = \left(\sum_{i=1}^{n} x_i^2\right) / n - \left(\left(\sum_{i=1}^{n} x_i^2\right) / n\right)^2, \quad (56)$$

and σ_m^2, σ_α^2, σ_β^2 – selective dispersions of values m_y, α_y, β_y:

$$\sigma_m^2 = \overline{m_y{}^2} - \overline{m}_y{}^2 = \left(\sum_{i=1}^{n} m_{yi}^2\right) / n - \left(\left(\sum_{i=1}^{n} m_{yi}^2\right) / n\right)^2, \quad (57)$$

$$\sigma_\alpha^2 = \overline{\alpha_y{}^2} - \overline{\alpha}_y{}^2 = \left(\sum_{i=1}^{n} \alpha_{yi}^2\right) / n - \left(\left(\sum_{i=1}^{n} \alpha_{yi}^2\right) / n\right)^2, \quad (58)$$

$$\sigma_\beta^2 = \overline{\beta_y{}^2} - \overline{\beta}_y{}^2 = \left(\sum_{i=1}^{n} \beta_{yi}^2\right) / n - \left(\left(\sum_{i=1}^{n} \beta_{yi}^2\right) / n\right)^2. \quad (59)$$

From the systems (55) follows that the value $$R = \left(\widetilde{m}_{b1} \frac{\sigma_x^2}{\sigma_m^2}, \widetilde{\alpha}_{b1} \frac{\sigma_x^2}{\sigma_\alpha^2}, \widetilde{\beta}_{b1} \frac{\sigma_x^2}{\sigma_\beta^2} \right) \quad (60)$$

shows how much values σ_m^2, σ_α^2, σ_β^2 in the average will change Y, if x will increase on value σ_x^2; fuzzy correlation coefficient R characterizing density of relationship between variables Y and x (Sapkina, 2013).

7. Defuzzification and analysis of the results

In order to find the accurate value of *r*, methods outlined below (Sapkina, 2013) can be applied.

1. Defuzzification using the resulting membership function $\mu_r(x)$ of fuzzy correlation coefficient $R = (m_r, \alpha_r, \beta_r)$ under given *LR* - functions; the method of center of gravity can be applied for this: $r = Defuz(R) = D(\mu_r) = \frac{\sum_{i=1}^{k} x_i \cdot \mu_r(x_i)}{\sum_{i=1}^{k} \mu_r(x_i)}$ where $i = \overline{1,k}$.

2. Formation of γ - slice by the values of of fuzzy correlation coefficient $R = (m_r, \alpha_r, \beta_r)$

$$\left[\underline{R}(\gamma), \overline{R}(\gamma)\right] = [m_r - \alpha_r(1-\gamma), m_r + \beta_r(1-\gamma)], \gamma \in [0,1].$$

Definition of strict correlation coefficient *r* for each γ - slice by determining ε - point:

$$\varepsilon \in \left[\underline{R}(\gamma), \overline{R}(\gamma)\right] \Rightarrow r = \underline{R}(\gamma) \cdot \varepsilon + \overline{R}(\gamma) \cdot (1-\varepsilon), \gamma \in [0,1].$$

The closer the absolute value of obtained precise correlation coefficient *r* is to one, the closer the relationship between the variables *Y* and *x*, thus it is possible to speak more confidently about the adequacy of the developed model.

There are various methods for bringing fuzzy numbers to precise numbers. The easiest way is to choose a precise number which corresponds to the maximum of membership function. This method can be used in the case of single-extremum membership functions. For multiextremal membership functions the following methods of defuzzification exist: center of gravity, medians, the biggest of maximums, the smallest of maximums, center of maximums. The final choice of method depends on the type of membership function of fuzzy numbers. In general, the choice of defuzzification algorithm depends on the features of the input data of studied process.

In his paper, (Shtovba, 2006) recommended to make defuzzification of the results by the center of gravity method because it gives best indicators of speed for settings (training) and precision of studied fuzzy models. Defuzzification of fuzzy set $\widetilde{A} = \sum_{i=1}^{k} \mu_{\widetilde{A}}(x_i) / x_i$ by the method of center of gravity is determined by the formula:

$$A = \frac{\sum_{i=1}^{k} x_i \cdot \mu_{\widetilde{A}}(x_i)}{\sum_{i=1}^{k} \mu_{\widetilde{A}}(x_i)}, \quad i = \overline{1,k}. \tag{61}$$

After realization of defuzzification for resultant fuzzy significance $\widetilde{y}$ using the center of gravity method, we will obtain such model: $\widetilde{y} = 0{,}51 - 1{,}89x_1 - 7{,}46x_2$.

7. Conclusions

The resulting values can be compared against the values of resultant indicators calculated by methods of conventional regression analysis: $y = 0{,}57 - 2{,}10x_1 - 8{,}31x_2$, $t_a = 11{,}11$; $t_{b_1} = -2{,}27$; $t_{b_2} = -7{,}87$; $F_{kr} = 54{,}04$; $R^2 = 90{,}38\%$; DW-statistic = 2,04 (P=0,2549). As a result, we can conclude that the fuzzy regression models give more reliable results. In doing so, they are qualitative and accurate and take into account the uncertainty of input data. They allow for determining the acceptable range of variations for resultant indicators, while retaining the impact of meaningful factors.

The features of developed models are to obtain resultant signs of bank competitiveness using multiple regression equations. They are built on each of the proposed α -slices, which describe the dependence of bank competitiveness performance characteristics from influential factors. Defuzzification value of resultant signs obtained by combining the regression models at $\alpha = 0{,}5$; $\alpha = 0{,}8$; $\alpha = 1$ by method of center of gravity.

In this approach, the accuracy of fuzzy regression model characterizes the proximity model and fact values for each observation. To characterize the degree of closeness the average relative error can be used:

$$e_{відн} = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{Y_i - \widetilde{Y}_i}{Y_i} \right| \cdot 100\%. \tag{62}$$

So, the closer founded the average relative error to zero, the more accurate the constructed fuzzy linear regression model (Sapkina, 2013). The verification of accuracy of constructed fuzzy linear regression multivariable model for the bank competitiveness showed that it is permissible.

The proposed approach provides new opportunities for bank competitiveness management. The developed fuzzy regression models allow us to calculate trustworthy intervals of change for values of activities results under any circumstances. Therefore, they allow for efficient managing of bank indicators and competitiveness and making the most rational decisions. Practical verification of the described methodical approach has confirmed that, in case of insufficient and uncertain information, analysis and assessment of bank competitiveness is appropriate to carry out by determining relevant dependencies using the fuzzy regression analysis tools.

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APPENDIX

Table 1: Value of competitiveness indicators of Commercial Bank Khreschatyk for $\alpha = 1$

№	Raised funds, hundreds of billion Hryvnia	Borrowed funds, hundreds of billion Hryvnia	Fuzzy number
	x_1	x_2	$\tilde{y}$
1	0,028499	0,00422	0,6227
2	0,030497	0,001391	0,6955
3	0,033558	0,001274	0,7047
4	0,034772	0,001411	0,7506
5	0,050065	0,001855	0,6624
6	0,050101	0,002109	0,6690
7	0,047949	0,001071	0,7147
8	0,045421	0,001196	0,7557
9	0,043954	0,000685	0,7618
10	0,040425	0,000666	0,8242
11	0,045852	0,000809	0,8336
12	0,051338	0,001438	0,7275
13	0,056119	0,001569	0,7072
14	0,052061	0,001589	0,7396
15	0,056197	0,001626	0,7126
16	0,058322	0,004539	0,6579
17	0,060957	0,004005	0,6634
18	0,06184	0,004445	0,6418
19	0,061032	0,004526	0,6819
20	0,061401	0,005963	0,6597
21	0,059139	0,00929	0,6596
22	0,057926	0,011107	0,5826
23	0,058872	0,029924	0,4428
24	0,067505	0,019956	0,4869
25	0,070108	0,011291	0,5700
26	0,07259	0,003629	0,5549
27	0,072957	0,003589	0,5343
28	0,075944	0,003541	0,5285

Table 2: Value of competitiveness indicators of Commercial Bank Khreschatyk for $\alpha = 0,5$

№	Raised funds, hundreds of billion Hryvnia	Borrowed funds, hundreds of billion Hryvnia	Fuzzy number $\widetilde{y}$	
	x_1	x_2	*c*	*d*
1	0,028499	0,00422	0,48016	0,51616
2	0,030497	0,001391	0,5384	0,5744
3	0,033558	0,001274	0,54576	0,58176
4	0,034772	0,001411	0,58248	0,61848
5	0,050065	0,001855	0,51192	0,54792
6	0,050101	0,002109	0,5172	0,5532
7	0,047949	0,001071	0,55376	0,58976
8	0,045421	0,001196	0,58656	0,62256
9	0,043954	0,000685	0,59144	0,62744
10	0,040425	0,000666	0,64136	0,67736
11	0,045852	0,000809	0,64888	0,68488
12	0,051338	0,001438	0,564	0,6
13	0,056119	0,001569	0,54776	0,58376
14	0,052061	0,001589	0,57368	0,60968
15	0,056197	0,001626	0,55208	0,58808
16	0,058322	0,004539	0,50832	0,54432
17	0,060957	0,004005	0,51272	0,54872
18	0,06184	0,004445	0,49544	0,53144
19	0,061032	0,004526	0,52752	0,56352
20	0,061401	0,005963	0,50976	0,54576
21	0,059139	0,00929	0,50968	0,54568
22	0,057926	0,011107	0,44808	0,48408
23	0,058872	0,029924	0,33624	0,37224
24	0,067505	0,019956	0,37152	0,40752
25	0,070108	0,011291	0,438	0,474
26	0,07259	0,003629	0,42592	0,46192
27	0,072957	0,003589	0,40944	0,44544
28	0,075944	0,003541	0,4048	0,4408

Table 3: Value of competitiveness indicators of Commercial Bank Khreschatyk for $\alpha = 0,8$

№	Raised funds, hundreds of billion Hryvnia	Borrowed funds, hundreds of billion Hryvnia	Fuzzy number	
			$\tilde{y}$	
	x_1	x_2	c	d
1	0,028499	0,00422	0,54243	0,57843
2	0,030497	0,001391	0,60795	0,64395
3	0,033558	0,001274	0,61623	0,65223
4	0,034772	0,001411	0,65754	0,69354
5	0,050065	0,001855	0,57816	0,61416
6	0,050101	0,002109	0,5841	0,6201
7	0,047949	0,001071	0,62523	0,66123
8	0,045421	0,001196	0,66213	0,69813
9	0,043954	0,000685	0,66762	0,70362
10	0,040425	0,000666	0,72378	0,75978
11	0,045852	0,000809	0,73224	0,76824
12	0,051338	0,001438	0,63675	0,67275
13	0,056119	0,001569	0,61848	0,65448
14	0,052061	0,001589	0,64764	0,68364
15	0,056197	0,001626	0,62334	0,65934
16	0,058322	0,004539	0,57411	0,61011
17	0,060957	0,004005	0,57906	0,61506
18	0,06184	0,004445	0,55962	0,59562
19	0,061032	0,004526	0,59571	0,63171
20	0,061401	0,005963	0,57573	0,61173
21	0,059139	0,00929	0,57564	0,61164
22	0,057926	0,011107	0,50634	0,54234
23	0,058872	0,029924	0,38052	0,41652
24	0,067505	0,019956	0,42021	0,45621
25	0,070108	0,011291	0,495	0,531
26	0,07259	0,003629	0,48141	0,51741
27	0,072957	0,003589	0,46287	0,49887
28	0,075944	0,003541	0,45765	0,49365

Table 4: Fuzzy values of resultant competitiveness indicator $\widetilde{y}$ for Commercial Bank Khreschatyk for $\alpha = 0{,}5$, $\alpha = 0{,}8$, $\alpha = 1$

№	$\alpha = 1$	$\alpha = 0{,}8$		$\alpha = 0{,}5$	
		c	*d*	*c*	*d*
1	0,6227	0,54243	0,57843	0,48016	0,51616
2	0,6955	0,60795	0,64395	0,5384	0,5744
3	0,7047	0,61623	0,65223	0,54576	0,58176
4	0,7506	0,65754	0,69354	0,58248	0,61848
5	0,6624	0,57816	0,61416	0,51192	0,54792
6	0,6690	0,5841	0,6201	0,5172	0,5532
7	0,7147	0,62523	0,66123	0,55376	0,58976
8	0,7557	0,66213	0,69813	0,58656	0,62256
9	0,7618	0,66762	0,70362	0,59144	0,62744
10	0,8242	0,72378	0,75978	0,64136	0,67736
11	0,8336	0,73224	0,76824	0,64888	0,68488
12	0,7275	0,63675	0,67275	0,564	0,6
13	0,7072	0,61848	0,65448	0,54776	0,58376
14	0,7396	0,64764	0,68364	0,57368	0,60968
15	0,7126	0,62334	0,65934	0,55208	0,58808
16	0,6579	0,57411	0,61011	0,50832	0,54432
17	0,6634	0,57906	0,61506	0,51272	0,54872
18	0,6418	0,55962	0,59562	0,49544	0,53144
19	0,6819	0,59571	0,63171	0,52752	0,56352
20	0,6597	0,57573	0,61173	0,50976	0,54576
21	0,6596	0,57564	0,61164	0,50968	0,54568
22	0,5826	0,50634	0,54234	0,44808	0,48408
23	0,4428	0,38052	0,41652	0,33624	0,37224
24	0,4869	0,42021	0,45621	0,37152	0,40752
25	0,5700	0,495	0,531	0,438	0,474
26	0,5549	0,48141	0,51741	0,42592	0,46192
27	0,5343	0,46287	0,49887	0,40944	0,44544
28	0,5285	0,45765	0,49365	0,4048	0,4408