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Numerical calculation of multidimensional integrals depended on input information about the function in mathematical modelling of technical and economic processes

Numerical calculation of multidimensional integrals depended on input information about the function in mathematical modelling of technical and economic processes

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Abstract. The object of study is a numerical integration of the functions of several variables using new information operators. The cubature formulas of the approximate calculation of double and triple integrals for different information operators are presented. Information about function is given not only by the values of a function in nodes, but also as a set of traces of a function on planes or as a set of traces of a function on lines. Attention is paid to the application of such approximate calculation of integrals in various fields of science.

1. Introduction

The development of information technology has contributed to the emergence of new approaches to obtaining, processing and analyzing information in scientific research. Scientists build new mathematical models or improve well-known ones using different types of data. Today the problem should be solved in cases when the input information about function is a set of traces of function on stripes, planes, lines or as a set of values of the function in the points. Other types of information provisions, as well as the so-called information operators, which are currently widely used in various fields of science, can be found in [1]. The use of this type of operators has proven to be effective in solving problems of the computed tomography, aerospace research, digital signal, and image processing [2].

Mathematical modeling is an important part of modern scientific research. Modeling in scientific research has been used since ancient times and gradually captured all new areas of scientific knowledge. One of the most important and most frequently seen problems in mathematical modeling is the problem of numerical integration. That is why it is essential to highlight modern algorithms in numerical integration, to analyze known results and obtain new ones. This paper demonstrates methods which help to understand how we can use new information operators for numerical integration of functions of many variables.

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2. Analysis of recent studies and publications

Scientists were interested in the problems of multidimensional numerical integration since the 50s of last century. Among such scientists we can note N.S. Bakhvalov [3], P. C. Hammer and A. H. Stroud [4]. However, even now this area of research remains relevant. This is due to the fact that many technical tasks require multidimensional integration skills. Obviously, a number of problems can be solved based on classical methods. These usually include methods in terms of quadrature rule, such as Newton Kotes formulas and Gaussian quadrature [5-7]. However, more modern methods are studied and can be used in solving technical or economic problems. In [8], hybrid functions and Haar wavelets have been applied for numerical solution of double and triple integrals with variable limits of integration. This approach is the generalization and improvement of the methods where the numerical methods are only applicable to the integrals with constant limits [9]. Such approach has advantages over the classical methods based on quadrature rule. The new methods are more efficient. General solution based on linear Legendre multi-wavelets for single, double and triple integrals is proposed in [10]. The main benefits of this method are its being simply applicable and efficient. The error analysis for single, double and triple integrals shows the efficiency of the method. In this work, numerical examples for the integrals are conducted by using linear Legendre multi-wavelets in order to validate the error estimation.

It should be noted that all the above classical and more modern methods of multidimensional numerical integration uses nodes as input information. The aim of the article is to highlight modern approaches in mathematical modeling, to construct the cubature formulas for the approximate calculation of the multidimensional integrals in the case when the sets of traces of the function on the planes and on the lines will be used as input information about the functions.

3. The approximate calculation of double integrals for different information operators

Definition 1. Under the traces of a function $g(x,y)$ on the lines $x_k = k\Delta - \Delta/2$, $y_j = j\Delta - \Delta/2$, $k, j = \overline{1,l}$, $\Delta = 1/l$, we understand, respectively, the function of one variable $g(x_k, y)$, $0 \le y \le 1$, $g(x, y_j)$, $0 \le x \le 1$.

For numerical evaluation of double integral

$$I = \int_0^1 \int_0^1 g(x,y)dxdy$$

in the case when the input information about function $g(x,y)$ is given as a set of traces of function on the lines in [11] the formula

$$\Phi = \sum_{k=1}^{l} \int_{x_{k-\frac{1}{2}}}^{x_{k+\frac{1}{2}}} dx \int_0^1 g(x_k, y)dy + \sum_{j=1}^{l} \int_0^1 g(x, y_j)dx \int_{y_{j-\frac{1}{2}}}^{y_{j+\frac{1}{2}}} dy - \sum_{k=1}^{l} \sum_{j=1}^{l} g(x_k, y_j) \int_{x_{k-\frac{1}{2}}}^{x_{k+\frac{1}{2}}} dx \int_{y_{j-\frac{1}{2}}}^{y_{j+\frac{1}{2}}} dy,$$

$$x_k = k\Delta - \Delta/2, \quad y_j = j\Delta - \Delta/2, \quad k, j = \overline{1,l}, \ \Delta = 1/l$$

is offered.

Theorem 1. [11] Suppose that $\left|g^{(1,1)}(x,y)\right| \le \tilde{M}$ and the function is given by $N = 2l$ traces on a system of mutually perpendicular lines $g(x_k, y)$, $k = \overline{1,l}$, $g(x, y_j)$, $j = \overline{1,l}$ in the domain $G = [0,1]^2$. The following estimate is valid for the cubature formula:

$$|I - \Phi| \le \frac{\tilde{M}}{16\ell^2} = \frac{\tilde{M}}{4N^2}.$$

Furthermore, in [11] calculation of double integral was studied in the case when the information about the function $g(x,y)$ is given in points.

Theorem 2. [11] Let $g(x,y) \in H_1^{2,r}(M,\widetilde{M}) = \{g(x,y): |g^{(1,0)}(x,y)| \le M, |g^{(0,1)}(x,y)| \le M, |g^{(1,1)}(x,y)| \le \widetilde{M}\}$ and values $g_{kj} = g(x_k, y_j)$, $k = \overline{1,m_1}$, $j = \overline{1,m_2}$ given no more than in $N = m_1 m_2$, $m_1 = m_2 = l^2$, $N = l^4$ fixed nodes $(x_k, y_j) \in G$. For the cubature formula

$$\widetilde{\Phi} = \sum_{k=1}^{\ell}\sum_{\tilde{j}=1}^{\ell^2} g(x_k, \tilde{y}_{\tilde{j}}) \int_{x_{k-\frac{1}{2}}}^{x_{k+\frac{1}{2}}} dx \int_{\tilde{y}_{\tilde{j}-\frac{1}{2}}}^{\tilde{y}_{\tilde{j}+\frac{1}{2}}} dy + + \sum_{j=1}^{\ell}\sum_{\tilde{k}=1}^{\ell^2} g(\tilde{x}_{\tilde{k}}, y_j) \int_{\tilde{x}_{\tilde{k}-\frac{1}{2}}}^{\tilde{x}_{\tilde{k}+\frac{1}{2}}} dx \int_{y_{j-\frac{1}{2}}}^{y_{j+\frac{1}{2}}} dy -$$

$$\sum_{k=1}^{l}\sum_{j=1}^{l} g(x_k, y_j) \int_{x_{k-\frac{1}{2}}}^{x_{k+\frac{1}{2}}} dx \int_{y_{j-\frac{1}{2}}}^{y_{j+\frac{1}{2}}} dy,$$

$$x_k = k\Delta - \frac{\Delta}{2}, \quad y_j = j\Delta - \frac{\Delta}{2}, \quad k,j = \overline{1,\ell}, \quad \Delta = \frac{1}{\ell},$$

$$\tilde{x}_{\tilde{k}} = \tilde{k}\Delta_1 - \frac{\Delta_1}{2}, \ \tilde{y}_{\tilde{j}} = \tilde{j}\Delta_1 - \frac{\Delta_1}{2}, \quad \tilde{k},\tilde{j} = \overline{1,\ell^2}, \Delta_1 = \frac{1}{\ell^2}$$

the following estimate of approximation is valid: $|I - \widetilde{\Phi}| \le \frac{\widetilde{M}+8M}{16}\frac{1}{\ell^2} = \frac{\widetilde{M}+8M}{16}\frac{1}{\sqrt{N}}$.

To achieve the error $O\left(\frac{1}{\sqrt{N}}\right)$ the cubature formula $\widetilde{\Phi}$ uses $O(l^3) = O\left(N^{\frac{3}{4}}\right)$ values of the function $g(x,y)$. To obtain the same order of accuracy, the classical formula will use $N = l^4$ values of the function $g(x,y)$.

4. The cubature formula for approximate calculation of triple integral using the information about the function on the traces on planes

Definition 2. Under the traces of function $f(x,y,z)$ on the planes $x_k = k\Delta - \Delta/2$, $y_j = j\Delta - \Delta/2$, $z_s = s\Delta - \Delta/2$, $k, j, s = \overline{1,l}$, $\Delta = 1/l$ we understand a function of two variables $f(x_k, y, z)$, $0 \le y \le 1$, $0 \le z \le 1$, $f(x, y_j, z)$, $0 \le x \le 1$, $0 \le z \le 1$, $f(x,y,z_s)$, $0 \le x \le 1$, $0 \le y \le 1$.

Here we suggest an algorithm of numerical calculation of triple integral

$$I^* = \int_0^1\int_0^1\int_0^1 f(x,y,z)dxdydz$$

on $[0,1]^3$ by formula

$$\Phi^* = \int_0^1\int_0^1\int_0^1 Jf(x,y,z)dxdydz,$$

$$Jf(x,y,z) = J_1 f(x,y,z) + J_2 f(x,y,z) + J_3 f(x,y,z) -$$
$$- J_1 J_2 f(x,y,z) - J_2 J_3 f(x,y,z) - J_1 J_3 f(x,y,z) + J_1 J_2 J_3 f(x,y,z),$$

$$J_1 f(x,y,z) = \sum_{k=1}^{l} f(x_k, y, z) h_{1k}(x), \ J_2 f(x,y,z) = \sum_{j=1}^{l} f(x, y_j, z) h_{2j}(y), \ J_3 f(x,y,z) = \sum_{s=1}^{l} f(x,y,z_s) h_{3s}(z),$$

$$X_k = [x_{k-1/2}, x_{k+1/2}], \ Y_j = [y_{j-1/2}, y_{j+1/2}], \ Z_s = [z_{s-1/2}, z_{s+1/2}],$$

$$h_{1k}(x) = \begin{cases} 1, x \in X_k, \\ 0, x \notin X_k, \end{cases} \quad h_{2j}(y) = \begin{cases} 1, y \in Y_j, \\ 0, y \notin Y_j, \end{cases} \quad h_{3s}(z) = \begin{cases} 1, z \in Z_s, \\ 0, z \notin Z_s. \end{cases}$$

Theorem 3. Suppose that $\left|f^{(1,1,1)}(x,y,z)\right| \le K$, then $\left|I^* - \Phi^*\right| \le \frac{K}{64 l^3}$.

Proof. We find the error of approximation of the integral I^* by the cubature formula Φ^*:

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$$\left|I^*-\Phi^*\right|=\left|\int_0^1\int_0^1\int_0^1\left(f(x,y,z)-Jf(x,y,z)\right)dxdydz\right|\le\sum_{k=1}^{1}\sum_{j=1}^{1}\sum_{s=1}^{1}\int_{x_{k-\frac{1}{2}}}^{x_{k+\frac{1}{2}}}\int_{y_{j-\frac{1}{2}}}^{y_{j+\frac{1}{2}}}\int_{z_{s-\frac{1}{2}}}^{z_{s+\frac{1}{2}}}\left|f(x,y,z)-Jf(x,y,z)\right|dxdydz=$$

$$=\sum_{k=1}^{1}\sum_{j=1}^{1}\sum_{s=1}^{1}\int_{x_{k-\frac{1}{2}}}^{x_{k+\frac{1}{2}}}\int_{y_{j-\frac{1}{2}}}^{y_{j+\frac{1}{2}}}\int_{z_{s-\frac{1}{2}}}^{z_{s+\frac{1}{2}}}\left|\int_{x_k}^{x}\int_{y_j}^{y}\int_{z_s}^{z}f^{(1,1,1)}(\xi,\eta,\zeta)\,d\xi d\eta d\varsigma\right|dxdydz\le$$

$$\le K\sum_{k=1}^{1}\sum_{j=1}^{1}\sum_{s=1}^{1}\int_{x_{k-\frac{1}{2}}}^{x_{k+\frac{1}{2}}}\left|x-x_k\right|dx\int_{y_{j-\frac{1}{2}}}^{y_{j+\frac{1}{2}}}\left|y-y_j\right|dy\int_{z_{s-\frac{1}{2}}}^{z_{s+\frac{1}{2}}}\left|z-z_s\right|dz=$$

$$=K\sum_{k=1}^{1}\sum_{s=1}^{1}\sum_{s=1}^{1}\left(-\frac{\left(x-x_k\right)^2}{2}\Bigg|_{x_{k-\frac{1}{2}}}^{x_k}+\frac{\left(x-x_k\right)^2}{2}\Bigg|_{x_k}^{x_{k+\frac{1}{2}}}\right)\times$$

$$\times\left(-\frac{\left(y-y_j\right)^2}{2}\Bigg|_{y_{j-\frac{1}{2}}}^{y_j}+\frac{\left(y-y_j\right)^2}{2}\Bigg|_{y_j}^{y_{j+\frac{1}{2}}}\right)\left(-\frac{\left(z-z_s\right)^2}{2}\Bigg|_{z_{s-\frac{1}{2}}}^{z_s}+\frac{\left(z-z_s\right)^2}{2}\Bigg|_{z_s}^{z_{s+\frac{1}{2}}}\right)=$$

$$=K\mathrm{l}^3\frac{\Delta^2}{4}\frac{\Delta^2}{4}\frac{\Delta^2}{4}=\frac{K}{64}\frac{1}{\mathrm{l}^3}.$$

In Table 1 we demonstrate the results of numerical experiment; the calculations were made in MathCad 15.

Table 1. Numerical calculation I^* for $f(x,y,z)=\sin(x+y+z)$ by Φ^*.

l	Exact value of integral I^*	Approximate value of integral Φ^*	Approximation error $\left\lvert I^*-\Phi^*\right\rvert$	Estimation of error $\frac{M}{64\mathrm{l}^3}$
4	0,879354930645401	0,879354946260593	$1,56\cdot10^{-8}$	$2,44\cdot10^{-4}$
8	0,879354930645401	0,879354930888389	$2,42\cdot10^{-10}$	$3,05\cdot10^{-5}$
10	0,879354930645401	0,879354930645662	$2,61\cdot10^{-13}$	$1,56\cdot10^{-5}$

5. Application of multidimensional numerical integration in various fields of science

The approximate calculation of integrals from highly oscillating functions occupies a special place in the numerical calculation of functions of several variables. Such integrals are widely used in digital signal and image processing, astronomy, physics and in optic research. Cubature formulas for approximate calculating of integrals of highly oscillating functions of many variables are built using various information about the function. The developed algorithms belong to Filon type methods. An overview of Filon type methods in the one-dimensional case can be found in [12–15], and their multidimensional analogue in [16].

Calculation of double integrals from highly oscillating functions in a regular and irregular cases is considered in [17-19]. In these works, cubature formulas are built in two cases. In the first case, the information about the function was given by traces on the lines, in the second it was given by the values of the function in nodes. In [2], the algorithm for the optimal choice of planes in the construction of the cubature formula for the approximate calculation of triple integrals is indicated.

In this work we demonstrate high accuracy of numerical calculation of integral

$$I(\omega) = \int_0^1 \int_0^1 f(x,y) \sin \omega x \sin \omega y dx dy$$

by cubature formula

$$\Phi(\omega) = \sum_{k=1}^{\ell} \sum_{\tilde{j}=1}^{\ell^2} f(x_k, \tilde{y}_{\tilde{j}}) \int_{x_{k-\frac{1}{2}}}^{x_{k+\frac{1}{2}}} \sin \omega x dx \int_{\tilde{y}_{\tilde{j}-\frac{1}{2}}}^{\tilde{y}_{\tilde{j}+\frac{1}{2}}} \sin \omega y dy +$$

$$+ \sum_{j=1}^{\ell} \sum_{\tilde{k}=1}^{\ell^2} f(\tilde{x}_{\tilde{k}}, y_j) \int_{\tilde{x}_{\tilde{k}-\frac{1}{2}}}^{\tilde{x}_{\tilde{k}+\frac{1}{2}}} \sin \omega x dx \int_{y_{j-\frac{1}{2}}}^{y_{j+\frac{1}{2}}} \sin \omega y dy - \sum_{k=1}^{l} \sum_{j=1}^{l} f(x_k, y_j) \int_{x_{k-\frac{1}{2}}}^{x_{k+\frac{1}{2}}} \sin \omega x dx \int_{y_{j-\frac{1}{2}}}^{y_{j+\frac{1}{2}}} \sin \omega y dy,$$

$$x_k = k\Delta - \frac{\Delta}{2},\ y_j = j\Delta - \frac{\Delta}{2},\ \ k, j = \overline{1,l},\ \ \Delta = \frac{1}{l},\ \tilde{x}_{\tilde{k}} = \tilde{k}\Delta_1 - \frac{\Delta_1}{2}, \tilde{y}_{\tilde{j}} = \tilde{j}\Delta_1 - \frac{\Delta_1}{2}, \tilde{k}, \tilde{j} = \overline{1, \ell^2}, \Delta_1 = \frac{1}{\ell^2}.$$

The test results for numerical calculation $I(\omega)$ were carried out using Wolfram Mathematica for $f(x,y) = \sin(x+y)$, $\omega = 20\pi$, $\omega = 60\pi$ and are shown in the Table 2. We calculated the exact values of the integrals: $I(20\pi) = -0{,}00019606576748294079$, $I(60\pi) = -0{,}00002177527592369306$.

Table 2. Calculation of $I(\omega)$ using the cubature formula $\Phi(\omega)$.

ω	l	$\left\lvert\frac{I(\omega)-\Phi(\omega)}{I(\omega)}\right\rvert$	ω	l	$\left\lvert\frac{I(\omega)-\Phi(\omega)}{I(\omega)}\right\rvert$
20π	20	$5{,}23 \cdot 10^{-7}$	60π	20	$5{,}90 \cdot 10^{-7}$
	40	$3{,}29 \cdot 10^{-8}$		40	$7{,}51 \cdot 10^{-8}$
	60	$6{,}53 \cdot 10^{-9}$		60	$6{,}53 \cdot 10^{-9}$
	80	$2{,}07 \cdot 10^{-9}$		80	$2{,}04 \cdot 10^{-9}$
	100	$8{,}49 \cdot 10^{-10}$		100	$8{,}39 \cdot 10^{-10}$
	120	$4{,}10 \cdot 10^{-10}$		120	$4{,}05 \cdot 10^{-10}$
	140	$2{,}21 \cdot 10^{-10}$		140	$2{,}20 \cdot 10^{-10}$
	160	$1{,}30 \cdot 10^{-10}$		160	$1{,}29 \cdot 10^{-10}$
	180	$8{,}08 \cdot 10^{-11}$		180	$8{,}07 \cdot 10^{-11}$
	200	$5{,}30 \cdot 10^{-11}$		200	$5{,}32 \cdot 10^{-11}$

There are known examples of the integral calculus applications in the study of social and economic problems, a review of which is given in [20]. The authors see prospects for the application of the studied methods of calculating double and triple integrals for the following well-known problem: assume D is the area of sowing of some crop and at each point $M(x,y) \in D$ the yield of this crop $q(x,y)$ is known (for example, according to space observations). Then the value $Q = \iint\limits_D q(x,y) dx dy$

is the amount of harvest that can be collected from the area D in the absence of losses. If during the calculation the change in yield over time is taken as a random variable during the calculation, then the problem can be transformed into the calculation of the triple integral.

Many problems in mathematical finance can be formulated as high-dimensional integrals. The paper [21] focuses on investigating the special features of certain typical high-dimensional finance problems, namely, option pricing and bond valuation. New regulations, stronger competitions and more volatile capital markets have increased the demand for stochastic asset-liability management models for insurance companies. The numerical simulation of such models is usually performed by Monte Carlo methods. In the article [22], the use of deterministic integration schemes, such as quasi-Monte Carlo and sparse grid quadrature methods are proposed. Numerical experiments with different asset-liability management models for portfolios of participating life insurance products demonstrate that deterministic methods often converge faster, less erratic and produce more accurate results than Monte Carlo simulation.

Among promising areas of using the approximate calculation of multidimensional integrals, we consider quantitative estimates [23] in the development of a methodology of financial stability evaluation of commercial banks [24].

6. Conclusions

Due to the rapid development and implementation of the latest information technologies in many fields of science and technology significant changes have taken place. The theory of new information operators, which is widely used in mathematical modeling, was created. The article identifies scientific areas for which using of mathematical models with new information operators will be effective. Among them we can mention the numerical calculation of multidimensional integrals. In the article we demonstrate cubature formulas for the approximate calculation of the multidimensional integrals using the sets of traces of the function on the planes and on the lines. Furthermore, calculation of double integral was studied in the case when the information about the function is given in nodes. In this work attention is paid to the application of such approximate calculation of integrals in various fields of science. The cubature formulas for approximate calculating of integrals of highly oscillating functions of many variables are built using various information about the function. Such integrals are widely used in digital signal and image processing, astronomy, physics and in optic research. We are sure that the methods proposed in the article for the approximate calculation of integrals using new information operators will also be useful in solving social and economic problems.

References

[1] Sergienko I and Lytvyn O 2018 Information Operators in Mathematical Modeling (A Review) *Cybernetics and Systems Analysis* vol **54** (1) pp 21-30
[2] Lytvyn O and Nechuiviter O 2014 Approximate Calculation of Triple Integrals of Rapidly Oscillating Functions with the Use of Lagrange Polynomial Interflatation *Cybernetics and Systems Analysis* vol **50** (3) pp 410-418
[3] Bakhvalov N 1959 On the approximate calculation of integrals *Vestnik MGU Ser. Mat. Mekh. Astron. Fiz. Khim* vol **4** pp 2–18
[4] Hammer P and Stroud A 1958 Numerical Evaluation of Multiple Integrals II *M.T.A.C.* vol **12** pp 272
[5] Carstairs A 2015 Numerical Solutions to Two-Dimensional Integration Problems *Thesis Georgia State University* https://scholarworks.gsu.edu/math_theses/151
[6] Ghorpade S and Limaye B 2010 A Course in Multivariable Calculus and Analyis *Springer* pp 346-361
[7] Yu C and Sheu S 2014 Using Mean Value Theorem to Solve Some Double Integrals Turkish *Journal of Analysis and Number Theory* vol **2** no 3 pp 75-79
[8] Aziz I and Khan W 2011 Quadrature rules for numerical integration based on Haar wavelets and hybrid functions *Computers & Mathematics with Applications* 61 (9) pp 2770-2781

[9] Islam S, Aziz I and Haq F 2010 A comparative study of numerical integration based on Haar wavelets and hybrid functions *Comput. Math. Appl.* 59 pp 2026-2036

[10] Alimin N, Rasedee A, Sathar M, Ahmedov A and Asbullah M 2019 Efficient Quadrature Rules for Numerical Integration Based on Linear Legendre Multi-Wavelets *Journal of Physics: Conference Series*, vol **1366** (2019) 012092

[11] Nechuiviter O, Chorna O, Darahan K, Pidlisnyi O and Chornyi S 2019 New informational operators in problems of numerical integration of functions of two variables *Bulletin of the National Technical University "KhPI" Ser. Mathematical modeling in engineering and technology* vol **8** (1333) pp 232-239

[12] Iserles A 2004 On the numerical quadrature of highly-oscillating integrals I: Fourier transforms *IMA J. Numer. Anal*. 24 pp 365–391

[13] Milovanovic G and Stanic M 2014 Numerical Integration of Highly Oscillating Functions *Analytic Number Theory, Approximation Theory and Special Functions* pp 613-649.

[14] Olver S 2008 Numerical Approximation of Highly Oscillatory Integrals *PhD thesis Cambridge: University of Cambridge*

[15] Gao J and Iserles A 2016 Error analysis of the extended Filon-type method for highly oscillatory integrals *Tech. Re-ports Numerical Analysis (NA2016/03) DAMPT: University of Cambridge*

[16] Iserles A and Norsett S 2007 From high oscillation to rapid approximation III: Multivariate expansions *Tech. Reports Numerical Analysis (NA2007/01) DAMPT: University of Cambridge*

[17] Lytvyn O and Nechuiviter O 2010 Methods in the Multivariate Digital Signal Processing with *Using Spline-interlineation IASTED International Conferences on Automation, Control and Information Technology: proceedings* pp 90–96

[18] Mezhuyev V, Lytvyn O M, Nechuiviter O, Pershyna Y, Lytvyn O O, Keita K 2017 Cubature formula for approximate calculation of integrals of two-dimensional irregular highly oscillating functions *U.P.B. Sci. Bull., Series A* vol **80** (3) pp 169–182

[19] Lytvyn O, Nechuiviter O, Pershyna I and Mezhuyev V 2019 Input Information in the Approximate Calculation of Two-Dimensional Integral from Highly Oscillating Functions (Irregular Case) *Recent Developments in Data Science and Intelligent Analysis of Information : XVIII International Conference on Data Science and Intelligent Analysis of Information : proceedings Kyiv* pp 365–373

[20] Iarmosh O 2015 The applications of integral calculus in socio-economic processes modeling in view of data problems *Socio-economic research bulletin* vol **1** (56) pp 281-287

[21] Wang X and Sloan I 2005 Why are high-dimensional finance problems often of low effective dimension? *SIAM J. Sci. Comput*. vol **27** (1): pp 159–183

[22] Gerstner T, Griebel M and Holtz M 2009 Efficient Deterministic Numerical Simulation of Stochastic Asset-Liability Management Models in Life Insurance *Insurance: Mathematics and Economics* vol **44** (3) pp 434-446

[23] Cherniak O, Trishch R, Kim N and Ratajczak S 2020 Quantitative assessment of working conditions in the workplace *Engineering Management in Production and Services* vol **12** pp 99-106

[24] Brauers W, Ginevičius R and Podviezko A 2014 Development of a methodology of evaluation of financial stability of commercial banks. *Panoeconomicus* vol **61** pp 349-367