MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE

SIMON KUZNETS KHARKIV NATIONAL UNIVERSITY OF ECONOMICS

MATHEMATICS
(GEOMETRY AND VECTORS)

Textbook
for students of the preparatory department

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All the themes of the course of geometry and vectors are outlined on the basis of the educational program of secondary school of Ukraine. Methods for solving problems in all themes of geometry and vectors are considered in detail, and solutions of typical examples are given. Numerous tasks for individual solution are offered.

For international students of the preparatory department.

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Contents

Introduction

This textbook is intended for foreign students of the preparatory department of Simon Kuznets Kharkiv National University of Economics.

In the process of mastering the course of elementary mathematics, students acquire the skills in solving problems envisaged by the school program in mathematics in Ukraine.

The purpose of the textbook is to remind and expand the basic information on elementary mathematics in terminology and symbolism adopted in Ukraine.

The textbook covers typical examples on all topics, according to the program of the preparatory department. The presentation of the material is accompanied by a detailed explanation of a large number of typical examples; it also demonstrates skills and techniques for solving different problems.

Each topic begins with a brief theoretical introduction and a list of the necessary formulas and contains a sufficient number of well-solved typical examples of various degrees of complexity. Tasks for individual work are given, with the answers placed at the end of the textbook.

This is a concise edition, which presents basic notions, formulas, rules, ratios, theorems, and methods on each of the topics in brief form.

The textbook is intended for a wide audience of high school graduates (not specialized in mathematics) as well as graduate and postgraduate students.

The first section of the textbook contains basics of plane and solid geometry (definitions, formulas, ratios, basic laws, properties, statements).

The second section of the textbook contains the elements of vectors (definitions, formulas, basic ratios, properties, statements).

A compact and clear presentation of the material allows the reader to get quick help on (or revise) the desired topic. Special attention is paid to issues that many high school graduates and students may find difficult to understand.

When selecting the material, the authors have given a pronounced preference to practical aspects; namely, to formulas, ratios, identities and laws that most frequently occur in economics and university education.

For the convenience of a wider audience with different mathematical backgrounds, the authors tried to avoid special terminology whenever possible.

Therefore, some of the topics and methods are outlined in a schematic and somewhat simplified manner, which is sufficient for them to be used successfully in most cases. Many parts were written so that they could be read independently. The material within subsections is arranged in increasing order of complexity. This allows the reader to get to the heart of the matter quickly.

The material of the textbook can be roughly categorized into the following two groups according to meaning:

1. The main text containing a concise, coherent survey of the most important definitions, formulas, rations, statements and laws.

2. For the reader's better understanding of the topics and methods under study, numerous examples are given throughout the book.

A list of tasks and tests for individual work on the topics of interest are offered to the reader. The material is selected in such a way that knowledge of the course of elementary mathematics is sufficient for studying it.

1. Geometry

Solving the geometric tasks begins with the construction of a drawing. The drawing should be performed as close as possible to the conditions of the task, satisfying both mathematical requirements and aesthetic criteria. A correctly executed drawing often gives a "key" to solving the task which allows you to define a clear plan for solving the task.

Do not overload the task drawing with unnecessary details, especially in the initial solving. Each extra line distracts the attention. So, in many tasks, there is no need to draw an inscribed or circumscribed circle, but it is enough to indicate (or justify) the positions of their centers and one radius (most often perpendicular to the tangent). This instruction should be used when performing additional constructions.

When solving a number of tasks, especially those that are solved by algebraic techniques, the use of a drawing is not necessary.

Starting to solve the task, it is necessary to use the properties of the data and the required elements included in the task, remember the theorems to which the data and the required elements of the problem are connected, and the reason: the triangle is isosceles, therefore ...; the circle is inscribed in a trapezoid, therefore ...; the radius is drawn in the point of tangency, therefore ..., and so on.

However, it should be remembered that the way of solving is not always obvious, and the enumeration of the elements of the theory related to a given task allows you to define the steps of solving.

1.1. Plane geometry

1.1.1. A triangle

A triangle (for example, ΔABC) is a plane figure bounded by three straight line segments (sides) connecting three noncollinear points A, B, C which are called vertices. The smaller angle between the two rays issuing from a vertex and passing through the other two vertices is called an interior angle of the triangle (for example, α, β and γ). The angle adjacent to an interior angle is called an external angle of the triangle.

The basic elements of a triangle are sides and angles.

A triangle is completely defined by any of the following sets of its parts:

a) its three sides;

b) two sides and their included angle;

c) two angles and their included side.

Depending on the angles, a triangle is said to be:

a) *acute* if all three angles are acute;

b) *right* (or right-angled) if one of the angles is right;

c) *obtuse* if one of the angles is obtuse.

Depending on the relation between the side lengths, a triangle is said to be:

a) *regular* (or *equilateral*) if all sides have the same length;

b) isosceles if two of the sides are of equal length;

c) *scalene* if all sides have different lengths.

Congruence tests for triangles:

1) if two sides of a triangle and their included angle are congruent to the corresponding parts of another triangle, the triangles are congruent;

2) if two angles of a triangle and their included side are congruent to the corresponding parts of another triangle, the triangles are congruent;

3) if three sides of a triangle are congruent to the corresponding sides of another triangle, the triangles are congruent.

When repeating the material on the topic "triangle", you should pay attention to the most important properties of a triangle:

1. The angles at the base are equal in an isosceles triangle.
2. The altitude drawn to the base is also the bisector and the median in an isosceles triangle.
3. There is a larger angle opposite the larger side of a triangle.
4. The length of each side of a triangle is less than the sum of the lengths of the other two sides and greater than their difference.
5. In any triangle the sum of the values of their interior angles is $180°$.
6. If the acute angle in a right-angled triangle is $30°$, the length of the leg opposite to it is half the length of the hypotenuse.
7. The external angle of a triangle equals the sum of two interior ones that are not adjacent to it.
8. Three medians of a triangle intersect in a single point lying strictly inside the triangle. This point cuts the medians in the ratio 2 : 1 (counting from the corresponding vertices).

9. An angle bisector of a triangle is a line segment between the vertex and the point of the opposite side dividing the angle at that vertex into two equal parts.

10. Three angle bisectors intersect in a single point lying strictly inside the triangle. This point is equidistant from all sides and is called the incenter (the center of the incircle of the triangle).

11. The bisector of the interior angle of a triangle divides the opposite side into parts proportional to the adjacent sides of the triangle.

12. The midline of a triangle (the segment joining the midpoints of the two sides of a triangle) is parallel to the third side and half as long.

13. Metric ratios in a right-angled triangle are as shown in (Fig. 1.1).

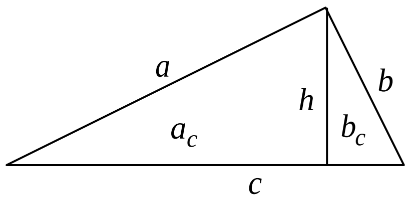


Fig. 1.1. **The drawing to property 13**

14. The law of sines (Fig. 1.2) is described by the formula

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R,$$

where R is the radius of the inscribed circle.

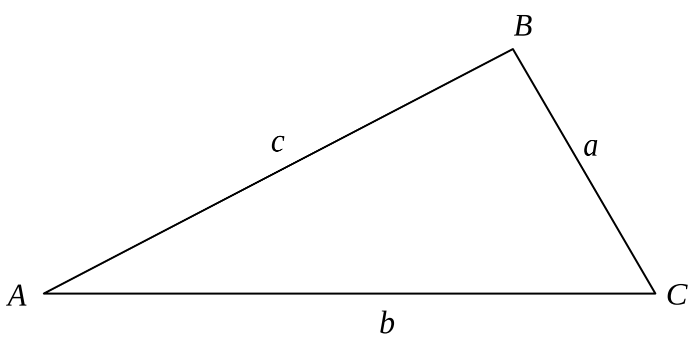


Fig. 1.2. **The drawing to properties 14 and 15**

15. The law of cosines (Fig. 1.2) is described by the formula

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A.$$

16. An altitude of a triangle is a straight line passing through the vertex perpendicular to the straight line containing the opposite side.

17. The three altitudes of a triangle intersect in a single point, called the *orthocenter* of the triangle.

Triangles are said to be *similar* if their corresponding angles are equal and their corresponding sides are proportional.

Similarity tests for triangles:

1) if all three pairs of the corresponding sides in a pair of triangles are in proportion, the triangles are similar;

2) if two pairs of corresponding angles in a pair of triangles are congruent, the triangles are similar;

3) if two pairs of corresponding sides in a pair of triangles are in proportion and the included angles are congruent, the triangles are similar.

Example 1. Determine the angles of a triangle if one of them is $\frac{2}{3}$ of the second one and $\frac{4}{5}$ of the third one.

Solution. Let's draw a figure to this example. Let's denote the angles of $\Delta\, ABC$ as α, β and γ (Fig. 1.3).

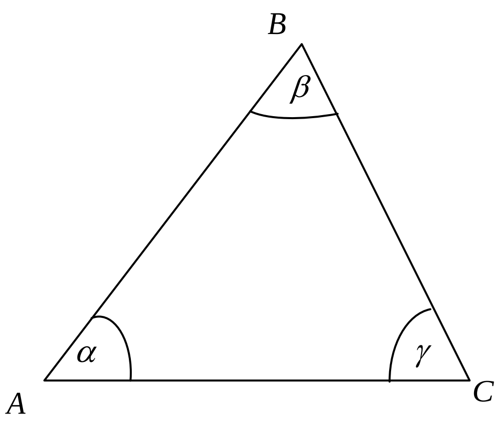


Fig. 1.3. **The drawing to example 1**

According to the condition of this example we have:

$$\beta = \frac{3}{2}\alpha,\ \gamma = \frac{5}{4}\alpha.$$

Since $\alpha + \beta + \gamma = 180^\circ$, $\alpha = 48^\circ$, and, therefore, $\beta = \frac{3}{2} \cdot 48^\circ$, $\gamma = \frac{5}{4} \cdot 48^\circ = 60^\circ$.

Answer: The angles of the triangle are $48^\circ, 72^\circ, 60^\circ$.

Example 2. In a triangle, an altitude and a median, drawn from one vertex, divide the angle at this vertex into three equal parts. Prove that the angles of the triangle are equal to $30^\circ, 60^\circ, 90^\circ$.

Solution. Let's draw a figure to this example. Let $\angle ABC$ be given as the triangle in (Fig. 1.4).

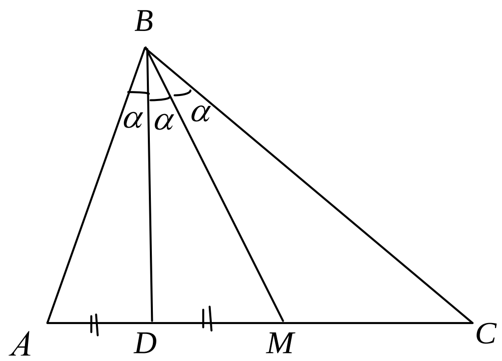


Fig. 1.4. **The drawing to example 2**

Let's draw the altitude BD and the median BM. According to the condition $\angle ABD = \angle DBM$, then $AB = BM$ and $AD = DM$. Let $DM = x$. Then $MC = 2x$. Since BM is the bisector of the angle DBC, then $\frac{DM}{MC} = \frac{DB}{BC}$ or $\frac{x}{2x} = \frac{BD}{BC}$, therefore $BC = 2 \cdot BD$, then, BD lies opposite to the angle of 30°, i.e. $\angle C = 30^\circ$, and $\angle DBC = 60^\circ$, therefore $\angle\alpha = 30^\circ$, $\angle B = 90^\circ, \angle A = 60^\circ$, Q.E.D.

Example 3. The legs of a right-angled triangle are related as 3 : 2, and the altitude divides the hypotenuse into segments, one of which is 2 m larger than the other. Find the hypotenuse.

Solution. Let's draw a figure to this example. Let $\angle ABC$ be given as the triangle $\angle ABC = 90°$ (Fig. 1.5).

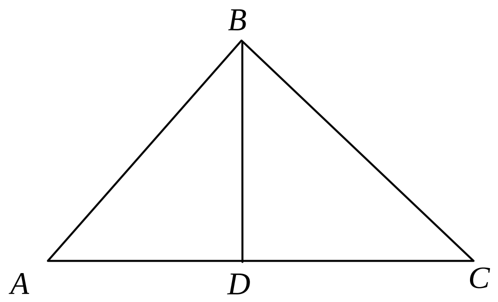


Fig. 1.5. **The drawing to example 3**

Let's draw the altitude $BD \perp AC$. Let $AD = x$, then

$$DC = x + 2,\ AB^2 = AD \cdot AC, BC^2 = AC \cdot DC,$$
$$\text{or } AB^2 = x \cdot (2x+2), BC^2 = (x+2)\cdot(2x+2).$$

According to the condition $BC : AC = 3 : 2$, we get $BC^2 : AC^2 = 9 : 4$. Then $\frac{(x+2)(2x+2)}{x(2x+2)} = \frac{9}{4}, \frac{(x+2)}{x} = \frac{9}{4}$,so $x = 1.6$, $AC = 2 \cdot 1.6 + 2 = 5.2$ m.

Answer: 5.2 m.

1.1.2. Quadrilaterals

Quadrilaterals are presented by a parallelogram and its types, which are a rectangle, a square and a rhombus. Their particular properties, in addition to the general properties of the parallelogram, are as follows:

a) the diagonals of a rectangle are equal;

b) the diagonals of a rhombus are mutually perpendicular and divide its corners in half;

c) the diagonals of a square are equal; at the point of intersection they are divided in half, mutually perpendicular and divide the angles of the square in half.

When solving tasks with a parallelogram, it is often useful to apply the following theorem: the sum of the squares of the diagonals of a parallelogram is equal to the sum of the squares of all its sides.

When studying the trapezoid, remember the basic definition of the trapezoid, its elements, as well as the definitions of an isosceles trapezoid and a rectangular trapezoid.

As you know, in an isosceles trapezoid: a) the angles at the base are equal; b) the diagonals are equal.

The midline of a trapezoid (the segment joining the midpoints of the legs) is parallel to the bases and equal to their half-sum.

Example 4. Prove that in an isosceles trapezoid, the perpendicular drawn from the vertex of the smaller base to the larger one divides it into parts, the larger of which is equal to the length of the midline of the trapezoid.

Solution. Let's draw a figure to this example. Let $ABCD$ be a given by the trapezoid in Fig. 1.6.

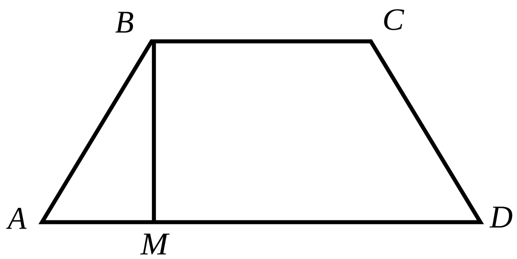


Fig. 1.6. **The drawing to example 4**

Let's denote $AD = a$, $BC = b$. Let's draw the altitude $BM \perp AD$.

It's obvious that $AM = \dfrac{a-b}{2}$.

Then $MD = AD - AM = a - \dfrac{a-b}{2} = \dfrac{a+b}{2}$, Q.E.D.

Example 5. In a parallelogram $ABCD$, the bisector of an angle divides the opposite side into segments $AM = 8$ cm and $MD = 12$ cm. Find the perimeter of the parallelogram (Fig. 1.7).

Solution. Let's draw a figure to this example. Let BM be the bisector of the angle B (Fig. 1.7).

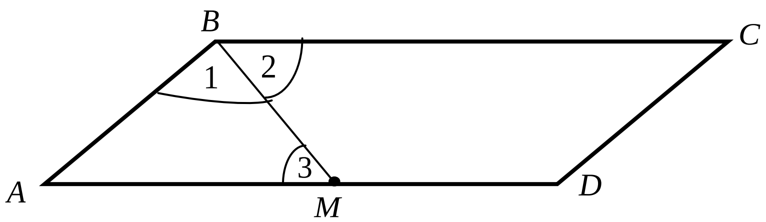


Fig. 1.7. **The drawing to example 5**

Therefore, $\angle 1 = \angle 2$. But $\angle 2 = \angle 3$ as the alternate interior angles and then $\angle 1 = \angle 3$, therefore $AB = AM = 8$ cm.

The perimeter equals $2 \cdot (8 + 20) = 56$ cm.

Answer: 56 cm.

Example 6. In a rhombus, the altitude and the smaller diagonal form an angle of $15°$. Find the altitude of the rhombus if its perimeter is 32 cm.

Solution. Let's draw a figure to this example. Let BM be the altitude of the rhombus $ABCD$, $\angle DBM = 15°$ (Fig. 1.8).

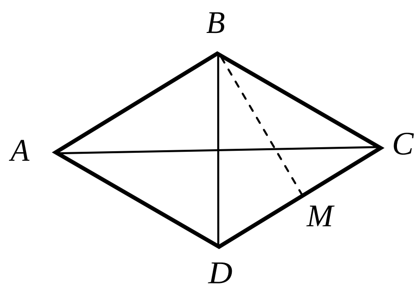


Fig. 1.8. **The drawing to example 6**

Then $\angle BDM = 75°$, $\angle ADC = 150°$. The angle $\angle BCD = 30°$, $BC = \dfrac{32}{4} = 8$ cm. From ΔMBC we have $BM = \dfrac{1}{2} BC = 4$ cm.

Answer: 4 cm.

Example 7. In a trapezoid, the angles at one of the bases are equal to $20°$ and $70°$, and the segment joining the midpoints of the bases is 2. Find the base of the trapezoid if its midline is 4.

Solution. Let's draw a figure to this example. Let's continue the sides of the trapezoid $ABCD$ to the intersection at the point O (Fig. 1.9).

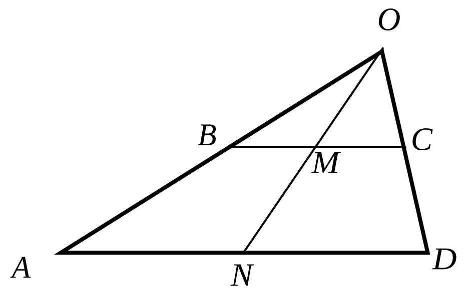


Fig. 1.9. **The drawing to example 7**

Let $\angle A = 20°$, $\angle D = 70°$ be angles, then in $\angle AOD$ we have $\angle O = 90°$.

In $\angle AOD$ we draw a median ON. Since $\angle BOC \approx \angle AOD$, this segment OM is the median of $\angle BOC$.

Then according to the condition of this example we have $MN = 2$. Let $AD = a$, $BC = b$, then $AN = \frac{a}{2}$. Similarly, in $\angle BOM$,$OM = \frac{b}{2}$. Therefore, $MN = \frac{a-b}{2}$.

We have:

$$\frac{a-b}{2} = 2, \frac{a+b}{2} = 4 \qquad \text{or} \qquad \begin{cases} a-b=4, \\ a+b=8, \end{cases}$$

therefore $a = 6, b = 2$.

Answer: $a = 6, b = 2$.

1.1.3. A circle and a disk

The second basic figure of plane geometry is a disk and its outline is a circle.

Let's note the basic theoretical statements of this topic.

1. Through any three points that do not lie on one straight line, you can draw a circle and, moreover, only one.

2. The diameter perpendicular to the chord divides this chord and both arcs formed by it in half.

3. The arcs between the parallel chords are equal.

4. The tangent to the circle is perpendicular to the diameter passing through the point of the tangency.

5. The segments of two tangents drawn to the circle from one point are equal.

Example 8. The ends of a diameter are 16 cm and 6 cm from the tangent. Determine the length of the diameter.

Solution. Let's draw a figure to this example (Fig. 1.10).

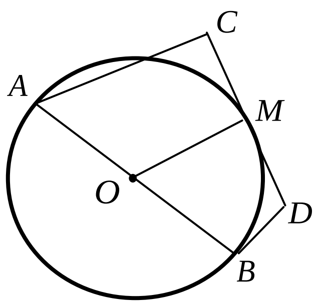


Fig. 1.10. **The drawing to example 8**

Let M be the point of tangency.

Then $OM \perp CD$, $DB = 6$ cm, $AC = 16$ cm. $OM = \dfrac{AC + DB}{2} = 11$ cm as the midline of the trapezoid $ABCD$. $AB = 2 \cdot OM = 22$ cm.

Answer: 22 cm.

1.1.4. Metric ratios in a circle

The properties of a circle are as follows.

1. If two chords are drawn through a point taken inside the circle, then the product of the segments of each chord is equal to the product of the segments of the diameter passing through the same point (Fig. 1.11), i.e.

$$AO : OB = MO : ON \, .$$

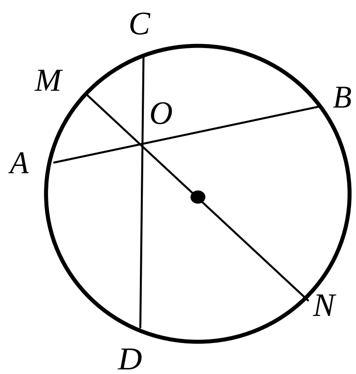


Fig. 1.11. **The drawing to property 1**

2. If a tangent and a secant are drawn from a point taken outside the circle, the product of the secant and its external part is equal to the square of the tangent (Fig. 1.12), i.e.

$$AB^2 = BD \cdot BC.$$

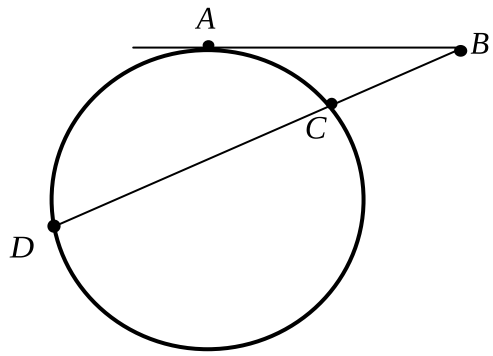


Fig. 1.12. **The drawing to property 2**

Example 9. A secant of 12 cm length and a tangent, whose length is $\frac{2}{3}$ of the interior secant segment, are drawn from the interior point to a circle. Find the length of the tangent.

Solution. Let's draw a figure to this example (Fig. 1.13).

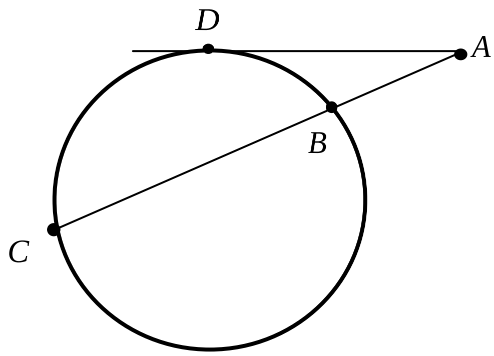


Fig. 1.13. **The drawing to example 9**

Let AD be the tangent and AC be the secant. Then, according to the condition of this example, we have $AC = 12$ cm; $AD = \frac{2}{3}BC$. Let $BC = x$, then $AB = 12 - x$, $AD = \frac{2}{3}x$.

We get: $AD^2 = AC \cdot AB$ or $\frac{4}{9}x^2 = 12(12 - x)$, therefore $x_1 = 9$, $x_2 < 0$. Then $AD = \frac{2}{3} \cdot 9 = 6$ cm.

Answer: 6 cm.

1.1.5. Inscribed and circumscribed triangles and quadrilaterals

This topic is very important in plane geometry. Let's consider the basic concepts of this topic.

1. Any triangle has a single circumscribed circle around it; the center of this circle coincides with the point of intersection of the middle perpendiculars to the sides of the triangle. The radius of the circumscribed circle is found by the formula

$$R = \frac{abc}{4S},$$

where a, b, c are the sides of the triangle, S is the area of the triangle.

For an equilateral triangle with side a the radius is calculated as

$$R = \frac{a\sqrt{3}}{3}.$$

Obviously, the center of the circle circumscribed around a right-angled triangle lies in the middle of the hypotenuse.

2. As you know, the three bisectors of a triangle intersect at one point and this point is called the center of the circle inscribed in the triangle. The radius r of this circle can be found by the formula

$$r = \frac{S}{p},$$

where S is the area of the triangle, and p is its semiperimeter.

For an equilateral triangle with side a the radius is calculated as

$$r = \frac{a\sqrt{3}}{6}.$$

3. A circle can be circumscribed around a quadrilateral if and only if the sum of its opposite angles is equal to $180°$.

It's obvious that:

a) a circle can be circumscribed around any rectangle;

b) a circle can be circumscribed around an isosceles trapezoid.

4. A circle can be inscribed in a quadrilateral if the sums of all opposite sides are equal.

From this statement the next properties follow:

a) out of all parallelograms, only a rhombus and a square can be inscribed in a circle;

b) a circle can be circumscribed in a trapezoid if the sum of its bases is equal to the sum of its legs.

Example 10. A right-angled triangle with the hypotenuse of 26 cm is inscribed around a circle with a radius of 4 cm. Find the perimeter of the triangle.

Solution. Let's draw a figure to this example (Fig. 1.14).

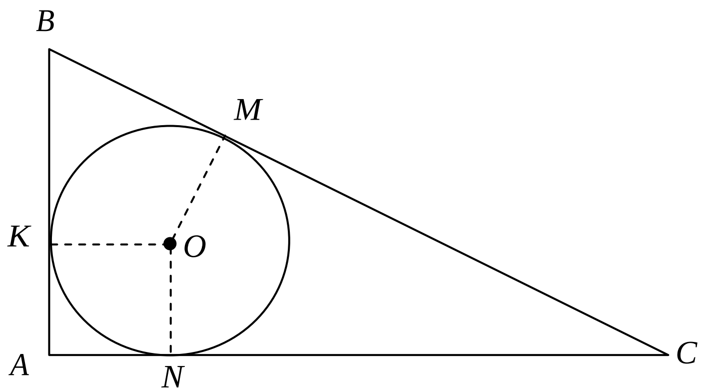


Fig. 1.14. **The drawing to example 10**

Let $BM = x$, then $MC = 26 - x$.

According to the property of tangents to the circle we have $MC = NC$, $BM = BK$, $AK = AN = 4$.

The perimeter of the triangle ABC equals

$$BC + (BK + NC) + (AK + AN) = 26 + 26 + 8 = 60 \text{ cm.}$$

Answer: 60 cm.

Example 11. An isosceles trapezoid with a leg equal to 8 cm and an angle of 60° at the base is inscribed around a circle. Find the bases of the trapezoid.

Solution. Let's draw a figure to this example (Fig. 1.15). In the $\angle ABM$, BM is the altitude, $\angle ABM = 30°$.

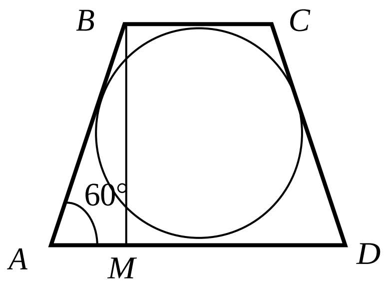


Fig. 1.15. **The drawing to example 11**

Then $AM = \frac{1}{2}AB = 4$ cm. Let $BC = x$, then $AD = 8 + x$. Since the circle is inscribed in the trapezoid, $DC + AD = 2AB$ or $x + 8 + x = 16$, therefore $x = 4$. So, $DC = 4$ cm, and $AD = 12$ cm.

Answer: 4 cm, 12 cm.

1.1.6. Areas of figures

The areas of triangles are calculated by the formulas given in Fig. 1.16.

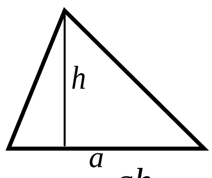
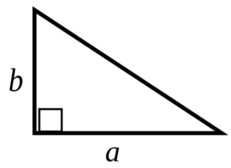
The area of an arbitrary triangle	 $S = \frac{ah}{2}$
The area of a right-angled triangle	 $S = \frac{ab}{2}$

Fig. 1.16. **The areas of triangles**

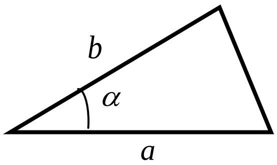
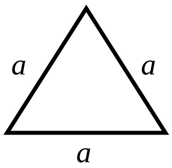
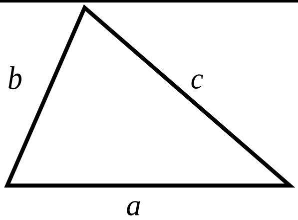
The area of an arbitrary triangle	 $S = \frac{ab}{2}\sin\alpha$
The area of an equilateral triangle	 $S = \frac{\sqrt{3}a^2}{4}$
The area of an arbitrary triangle	 $S = \sqrt{p(p-a)(p-b)(p-c)},$ $p = \frac{a+b+c}{2}$ (Heron's formula)
The area of an arbitrary triangle	$S = \frac{abc}{4R}, S = r \cdot p,$ where a, b, c are the sides of a triangle, r and R are the radii of the inscribed and circumscribed circles; $p = \frac{a+b+c}{2}$

Fig. 1.16 (the end)

The areas of quadrilaterals are calculated by the formulas given in Fig. 1.17.

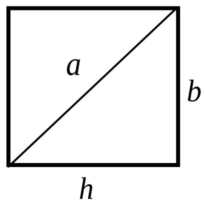
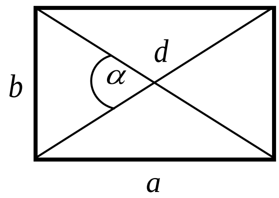
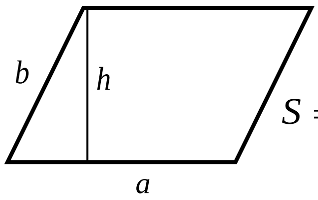
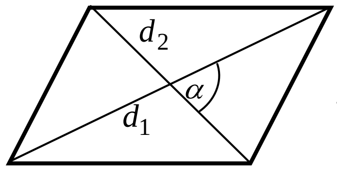
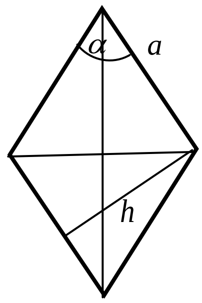
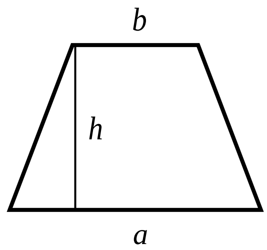
The area of a square	 $S=\frac{a+b}{2}h$ $S=\frac{1}{2}d^2$
The area of a rectangle	 $S=ab$ $S=\frac{d^2}{2}\sin\alpha$
The area of a parallelogram	 $S=ah$ $S=ab\cdot\sin\alpha$
The area of a parallelogram	 $S=\frac{1}{2}d_1d_2\sin\alpha$
The area of a rhombus	 $S=\frac{1}{2}d_1d_2$ $S=ah$ $S=a^2\sin\alpha$
The area of a trapezoid	 $S=\frac{a+b}{2}h$

Fig. 1.17. **The areas of quadrilaterals**

The area of a disk is calculated as $S=\pi R^2$ (Fig. 1.18).

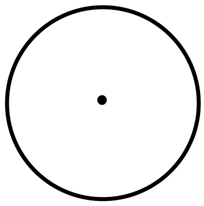


Fig. 1.18. **The drawing of a disk**

The areas of a circular sector and a circular segment are calculated as (Fig. 1.19):

$$S = \frac{1}{2} R^2 \alpha, \qquad\qquad S = \frac{1}{2}\left(R^2 \alpha - R^2 \sin \alpha\right).$$

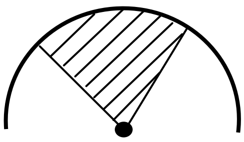


Fig. 1.19. **The drawing of a circular sector**

Example 12. Find the area of the right-angled triangle, if one of its legs is equal to 8 cm, and the radius of the inscribed circle is 3 cm (Fig. 1.20).

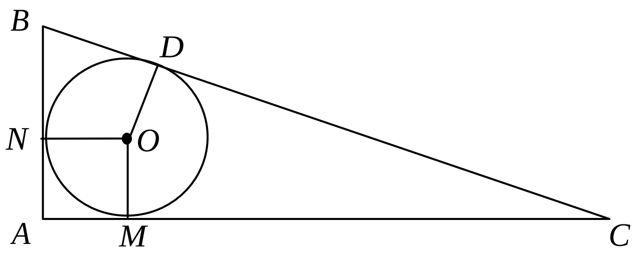


Fig. 1.20. **The drawing to example 12**

Solution. Let's draw a figure to this example. Let ABC be the given triangle (Fig. 1.20), in which $\angle A = 90°$, $AB = 8$ cm, point O is the center of the inscribed circle, and ON, OM, OD are its radii.

According to the property of tangents to the circle we have $BD = BN$, $CD = CM, AN = AM = 3$. $BN = 8 - 3 = 5, BD = 5$.

Let $MC = DC = x$. According to the Pythagorean theorem we have:

$$8^2 + (3 + x)^2 = (5 + x)^2,$$

then $AC = 3 + 12 = 15$.

The sought area is calculated as $S_\Delta = \frac{1}{2} AB \cdot AC = \frac{1}{2} \cdot 8 \cdot 15 = 60$ cm^2.

Answer: 60 cm^2.

Example 13. The lengths of the sides of a triangle are proportional to the numbers 5, 12 and 13. The largest side of the triangle exceeds the smallest one by 1.6 m. Find the area of the triangle.

Solution. Let one part be x, then the sides of the triangle are $a = 5x, b = 12x, c = 13x$. According to this condition $c - a = 1.6; 13x - 5x = 1.6;$ then $x = 0.2$ m. So, $a = 1$ m, $b = 2.4$ m, $c = 2.6$ m.

The sought area is calculated as $S_\Delta = \frac{ab}{2} = \frac{1 \cdot 2.4}{2} = 1.2$ m^2.

Answer: 1.2 m^2.

Example 14. The perimeter of a rhombus is 48, and the sum of the lengths of the diagonals is 26. Find the area of the rhombus (Fig. 1.21).

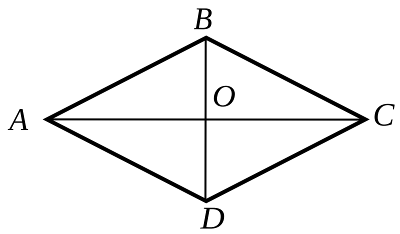


Fig. 1.21. **The drawing to example 14**

Solution. Let's draw a figure to this example. Let $ABCD$ be the given rhombus (Fig. 1.21). Since the perimeter of the rhombus equals 48, its side according to the condition $AC + BD = 26$ cm of this example will be $BO = OC = 13$. Let $BO = x$, then $OC = 13 - x$. The sought area is $S = 2x(13 - x)$. Since the diagonals are mutually perpendicular, let's use the Pythagorean theorem for $\angle BOC$:

$$x^2 + (13 - x)^2 = 12^2.$$

Let's transform this equation, allocating the perfect square of a sum of the left side:

$$x^2+2x(13-x)+(13-x)^2-2x(13-x)=144,$$

or $$(x+13-x)^2-2x(13-x)=144,$$

or $$(x+13-x)^2-S=144.$$

Therefore $S=25$ cm^2.

Answer: 25 cm^2.

Example 15. In an isosceles trapezoid, the difference of the bases is 16 cm, and the perimeter is 52 cm. Find the area of the trapezoid if the leg and its altitude are related as $5:3$.

Solution. Let's draw a figure to this example (Fig. 1.22). Let $AD=a$, $BC=b$, $AB=CD=c, BM=h$.

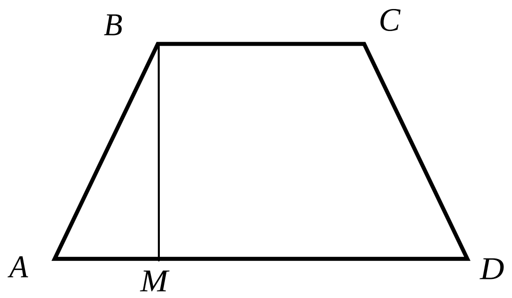


Fig. 1.22. **The drawing to example 15**

According to the condition, $a+b+2c=52, a-b=15$, $c:h=5:3$.

$$S=\frac{a+b}{2}\cdot h.$$

Let's denote one part as x , then $c=5x, h=3x,$ and

$$AM=\sqrt{25x^2-9x^2}=4x.$$

On the other hand, $AM=\dfrac{a-b}{2}=\dfrac{16}{2}=8$.

We have: $4x=8, x=2, c=10, h=6.$

From this condition $a+b+2c=52$ we get: $a+b=52-2\cdot 10=32$.

Then $S=\frac{32}{2}\cdot 6=96$ cm^2.

Answer: 96 cm^2.

Example 16. The angles of a triangle are related as 2 : 3 : 7. The smallest side is a. Find the area of a circle circumscribed around the triangle.

Solution. Let's draw a figure to this example (Fig. 1.23).

Let's denote one part as x. Then

$$\angle A=2x, \angle B=3x, \angle C=7x; \qquad 2x+3x+7x=180°,$$

therefore $x=15°$ and $\angle A=30°, \angle B=45°, \angle C=105°$.

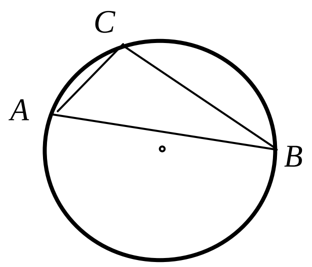


Fig. 1.23. **The drawing to example 16**

According to the condition $BC=a$ as a side opposite a smaller angle.

Let's use the law of sines and get $\frac{BC}{\sin A}=2R,$ so $R=a$.

Then the area of the inscribed disk is: $S=\pi a^2$.

Answer: πa^2.

Questions for self-assessment

1. In which case is the angle $\angle ABC$ in the triangle ΔABC obtuse?
2. Can a cosine of any angle in the triangle ΔABC be negative?
3. What is the form of a triangle in which the cosines of all angles are positive?
4. Which polygon is called convex?
5. What is the sum of the angles of a convex n-gon?

6. What is the sum of the exterior angles of a convex n-gon, taken one at each vertex?
7. Into how many triangles is a convex n-gon divided by the diagonal drawn from one vertex?
8. What segment is called the diagonal of a polygon?
9. How many diagonals are there in a convex polygon?
10. What should the interior angles in a polygon be for it to be convex?
11. Which polygon is called inscribed in a circle?
12. What polygon is called circumscribed around a circle?
13. Which polygon is called regular?
14. What is the sum of the angles of a regular polygon?
15. What is the value of the angle of a regular n-gon?
16. Is it possible to circumscribe a circle around any polygon?
17. Is it possible to inscribe a circle in any polygon?
18. How are the radii of inscribed and circumscribed circles related?
19. How is the radius of an inscribed circle calculated if the side of a regular n-gon is equal to a?
20. How is the radius of a circumscribed circle calculated if the side of a right n-gon is equal to a?
21. What is the sum of the exterior angles of a triangle?
22. What is the side of a hexagon where the radius of the inscribed circle is equal to R?
23. What is the side of a square whose diagonal is d?
24. Are the diagonals of any rectangle perpendicular?
25. What should a rectangle be so that its diagonals be perpendicular?
26. What is the angle of an equilateral triangle?
27. How is the height of an equilateral triangle with the side a calculated?
28. What line is called the bisector of the angle?
29. What segment is called the median of the angle?
30. For which of the triangles do the altitude and the median coincide?
31. How is the length of a bisector calculated?
32. How is the length of a median calculated?
33. How is the length of an altitude calculated?
34. What is a triangle called if its two angles are equal?
35. Formulate the Pythagorean theorem.
36. Formulate the law of cosines.

37. Formulate the law of sines.
38. Is the statement correct: "Either side of any triangle is less than the sum of the other two sides"?
39. What is the sum of all angles of a triangle?
40. What is the sum of the acute angles of a right-angled triangle?
41. Complete the sentence: "The leg opposite to the angle of 30º is equal to…"
42. Are triangles equal if their respective sides and two adjacent angles are equal?
43. Are two triangles equal if their respective sides are equal?
44. Are two triangles equal if three of their respective sides are equal?
45. Are two triangles equal if their two corresponding sides are equal?
46. Can a triangle have two right angles?
47. Complete the sentence: "The leg opposite to the angle α is equal to…".
48. Complete the sentence: "The leg adjacent to the angle α is equal to…".
49. What is the midline of a triangle?
50. How is the midline of a triangle calculated?
51. How is the area of a triangle calculated if its base and the altitude drawn to this base are known?
52. How is the area of a triangle calculated if all its sides are known?
53. How is the area of a triangle calculated if you know its two sides and the angle between them?
54. If a quadrilateral has two pairs of parallel sides, it is called …
55. If a quadrilateral has a pair of parallel sides, it is called …
56. Which quadrilateral is called a right-angled trapezoid?
57. What is the middle line of a trapezoid?
58. How is the middle line of a trapezoid calculated?
59. How is the area of a trapezoid calculated?
60. How is the area of a parallelogram calculated?
61. How is the area of a rhombus calculated?
62. How is the area of a square calculated?
63. How is the area of a rectangle calculated?
64. How is the area of any quadrilateral calculated if you know its diagonals and the angle between them?
65. What is the length of a circle of the radius R?
66. What is the area of a circle of the radius R?

1.2. Stereometry (solid geometry)

Stereometry (solid geometry) deals with the study of the properties of spatial figures, all the points of which cannot be located on the same plane.

A well-executed drawing plays an important role in the successful solution of tasks in stereometry, since the ambiguity of the drawing is often the main difficulty in solving the task.

A stereometric drawing should create to a certain extent, the same impression that the figure being depicted makes. When drawing geometric figures and bodies, simplified drawings are used.

In this case, it is recommended to remember the conventionality of the stereometric drawing: equal segments can have different lengths, a right angle can be drawn as right, obtuse, and acute; visible lines are drawn in solid, invisible in dotted lines; parallel lines in the drawing should be parallel, the altitude of the body is drawn parallel to the edge of the notebook.

Every detail of the drawing must be theoretically justified. For example, if it is known that all lateral faces of a pyramid are inclined to the base plane at the same angle, the apex of this pyramid is projected into the center of the circle inscribed in the base. For example, an isosceles trapezoid is the base of such a pyramid, then the apex of the pyramid cannot project into the intersection of the trapezoid diagonals, which is a common mistake.

If the edges of a pyramid have the same angle of inclination to the plane of the base, the apex of the pyramid is projected to the center of the circle inscribed around the base of the polygon.

If a triangle or a polygon is the base of a pyramid, the presence of one of the indicated cases gives an additional information to solving the task.

If the condition an angle between two planes is given, it is necessary to prove that the angle indicated in the figure is indeed the angle between the perpendiculars drawn to the line of the intersection of the indicated planes.

If the configuration of two geometric bodies is given in the task statement, it is necessary to justify their mutual position in the figure. An essential point in solving tasks for the combination of a polyhedron with a ball is to determine (substantiate) the position of the center of the ball. In this case, they are guided by the following statements:

if a ball is inscribed in the pyramid, its center will be the point of the intersection of the bisector planes of all dihedral angles of the pyramid;

if a ball is circumscribed around the pyramid, its center will be the point of the intersection of all planes drawn through the middle of the pyramid's edges perpendicular to these edges;

if it is necessary to construct a section of some geometric body by a plane, it is necessary to describe the process of constructing this section and justify its shape.

When a formula of the sought value of the geometric task has been obtained, it is necessary to investigate whether, for all real values of the parameters included in this formula, an answer is obtained, expressed by a real positive number in units of the corresponding dimension.

When a geometric task, it is necessary to indicate which theorem or formula is used in this solving.

1.2.1. Polyhedra

In solid geometry (or stereometry), only some types of convex polyhedra are considered. These are prisms (right and oblique) and their particular types or parallelepipeds, pyramids, truncated pyramids.

Let's distinguish two basic types of polyhedron problems, i.e. problems for calculating such elements as lengths, areas, volumes, linear and dihedral angles.

Let's use basic formulas for calculation of the area of the lateral surface S_{lat}, the area of the total surface S_{total} and the volume V of the indicated polyhedra.

For an oblique prism, basic formulas are

$$S_{lat} = P_{\perp} \cdot l; \qquad S_{total} = S_{lat} + 2S_{base}; \qquad V = S_{\perp} \cdot l,$$

where $P_{\perp}$ and $S_{\perp}$ are the perimeter and the area of the perpendicular section of the prism;

l is the lateral edge of the prism.

For a right prism, basic formulas are:

$$S_{lat} = P_{base} \cdot H; \qquad S_{total} = S_{lat} + 2S_{base}; \qquad V = S_{base} \cdot H,$$

where P_{base} and S_{base} are the perimeter and the area of the bases of the prism;

H is the altitude of the prism equal to the length of its lateral edge.

For a pyramid, basic formulas are:

$$V = \frac{1}{3} S_{base} \cdot H \,; \qquad\qquad S_{total} = S_{lat} + 2S_{base} \,,$$

where S_{base} is the area of the bases of the prism;

H is the altitude of the prism.

In a general case, S_{lat} is defined as a sum of areas of lateral faces.

For a regular pyramid, the area of the lateral surface S_{lat} is calculated as

$$S_{lat} = \frac{1}{2} P_{base} \cdot h_{lat} \,,$$

where P_{base} is the perimeter of the base of the pyramid;

h_{lat} is the apothem.

If all the lateral faces of a pyramid are inclined to the base plane at the same angle α,

$$S_{lat} = \frac{S_{base}}{\cos\alpha} .$$

For a truncated pyramid the volume is calculated as

$$V = \frac{1}{3} H\left(S_1 + \sqrt{S_1 S_2} + S_2\right) ,$$

where S_1, S_2 are the areas of the bases of the truncated pyramid;

H is its altitude.

For a regular truncated pyramid the area of the lateral surface S_{lat} is calculated as

$$S_{lat} = \frac{1}{2}\left(P_1 + P_2\right) \cdot h_{lat} \,,$$

where P_1, P_2 are the perimeters of the bases;

h_{lat} is the altitude of the lateral face of the truncated pyramid.

Example 17. The legs of a right-angled triangle are 6 and 8 cm. At what distance from the plane of the triangle is point M being 13 cm away from each vertex of the triangle?

Solution. Let point M be equidistant from the vertices of ΔABC. Let's draw a figure to this example (Fig. 1.24).

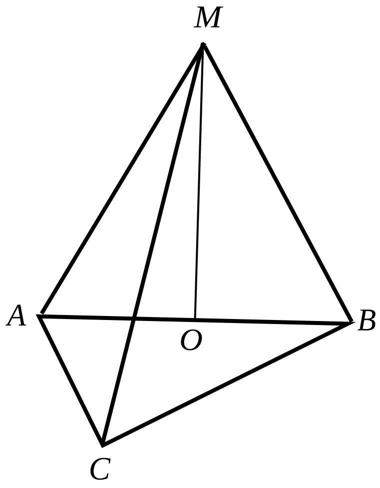


Fig. 1.24. **The drawing to example 17**

Therefore, $AM = BM = CM$. From the equality of the inclined lines the equality of their projections follows as: $AO = BO = CO$, i.e. point O is equidistant from the vertices of ΔABC.

Therefore, it is the center of the disk, inscribed around this triangle, and it lies inside in the hypotenuse:

$$AB = \sqrt{36+64} = 10,\ OB = 5;\ MO = \sqrt{169-25} = 12.$$

Answer: 12 cm.

Example 18. In a regular quadrangular prism, the diagonal of the prism is 5 cm and the diagonal of the lateral face is 4 cm. Calculate the altitude of the prism.

Solution. Let's draw a figure to this example (Fig. 1.25).

According to the condition of this example we have $B_1D = 5$, $C_1D = 4$.

In $\Delta B_1C_1D_1\ \angle C_1 = 90°$ according to the theorem of three perpendiculars. Therefore

$$B_1C_1 = \sqrt{25-16} = 3.\ DC = B_1C_1 = 3,$$

because this prism is regular.

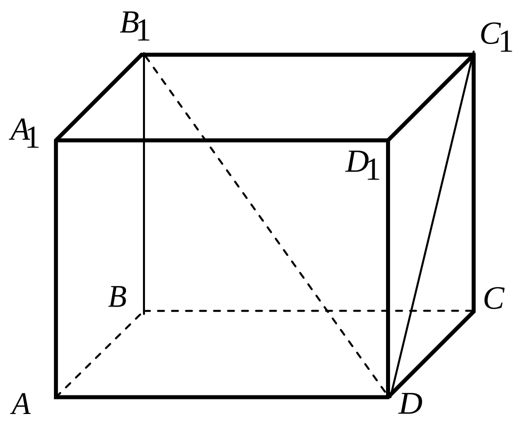


Fig. 1.25. **The drawing to example 18**

From ΔC_1CD:

$$C_1C = \sqrt{C_1D^2 - CD^2} = \sqrt{16-9} = \sqrt{7}.$$

Answer: $\sqrt{7}$ cm.

Example 19. The altitude of a rectangular parallelepiped is equal to h, and its diagonal forms an angle α with the base. The angle between the diagonals of the parallelepiped is β. Find the volume of the parallelepiped.

Solution. Let's draw a figure to this example (Fig. 1.26).

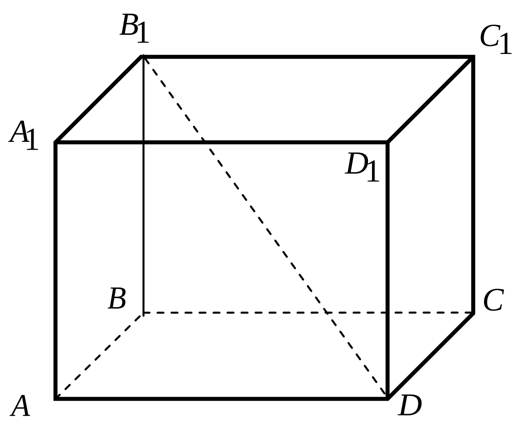


Fig. 1.26. **The drawing to example 19**

Using ΔB_1BD we have $BD = h \cdot \mathrm{ctg}\alpha$. Then

$$S_{base} = \frac{1}{2}BD^2 \sin\beta = \frac{1}{2}h^2 \mathrm{ctg}^2\alpha \cdot \sin\beta$$

and

$$V = S_{base} \cdot h = \frac{1}{3} h^3 \mathrm{ctg}^2 \alpha \cdot \sin\beta .$$

Answer: $\frac{1}{3} h^3 \mathrm{ctg}^2 \alpha \cdot \sin\beta$.

Example 20. Calculate the total surface area of a regular quadrangular pyramid, whose altitude is $H = 3.1$ m, the apothem forms the angle of $60°$ with the base.

Solution. Let's draw a figure to this example (Fig. 1.27).

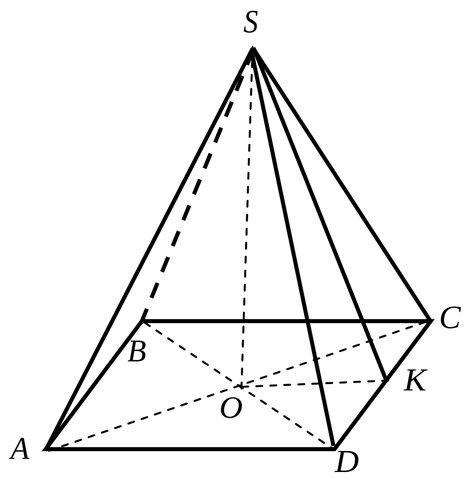


Fig. 1.27. **The pyramid in example 20**

The area of the total surface is calculated as

$$S_{total} = S_{lat} + S_{base} = 4 \cdot DC \cdot \frac{SK}{2} + DC^2 = 2 \cdot DC \cdot SK + DC^2 .$$

From ΔSOK we have: $SK = \dfrac{SO}{\cos 60°} = \dfrac{6.2}{\sqrt{3}}$, $OK = \dfrac{SK}{2} = \dfrac{3.1}{\sqrt{3}}$,

(OK lies opposite the angle of $30°$); $DC = 2 \cdot OK = \dfrac{6.2}{\sqrt{3}}$.

$$S_{total} = 2 \cdot \frac{6.2}{\sqrt{3}} \cdot \frac{6.2}{\sqrt{3}} + \left(\frac{6.2}{\sqrt{3}}\right)^2 = 38.44 \text{ m}^2 .$$

Answer: 38.44 m^2.

Example 21. The base of the pyramid is a right-angled triangle with the hypotenuse C and the angle α. The lateral edges of the pyramid are inclined to the base at the angle β. Find the volume of the pyramid.

Solution. Let's draw a figure to this example (Fig. 1.28).

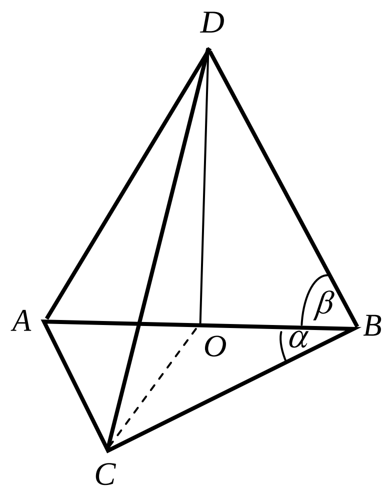


Fig. 1.28. **The drawing to example 21**

Since the edges are equally inclined to the plane of the base, the apex of the pyramid is projected into the center of the circle circumscribed around the base, i.e. to the middle of the hypotenuse.

We have: $R = \frac{C}{2}$. From ΔBOD we have: $H = R \cdot \mathrm{tg}\beta = \frac{C}{2} \cdot \mathrm{tg}\beta$.

Let's calculate the area of the base and the volume:

$$S_{base} = \frac{1}{2} \cdot AC \cdot BC = \frac{1}{2} \cdot c \sin\alpha \cdot c\cos\alpha = \frac{1}{4} c^2 \sin 2\alpha.$$

$$V = \frac{1}{3} \cdot \frac{1}{4} c^2 \sin 2\alpha \frac{c}{2} \cdot \mathrm{tg}\beta = \frac{1}{24} c^3 \sin 2\alpha \cdot \mathrm{tg}\beta.$$

Answer: $\frac{1}{24} c^3 \sin 2\alpha \cdot \mathrm{tg}\beta$.

Example 22. The sides of the bases of a regular quadrangular truncated pyramid are equal to 6 and 8 cm. A dihedral angle at the edge of the base is $60°$. Find the lateral surface area of the truncated pyramid.

Solution. Let's draw a figure to this example (Fig. 1.29).

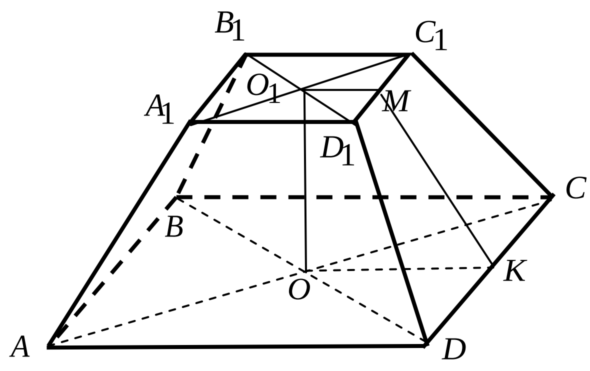


Fig. 1.29. **The drawing to example 22**

According to the condition of this example we have $AD = 8$ cm, $A_1D_1 = 6$ cm, $\angle MKO = 60°$.

The area of the lateral surface is calculated as

$$S_{lat} = 4 \cdot \frac{DC + D_1C_1}{2} \cdot MK .$$

In the trapezoid OO_1MK: $O_1M = 3$, $OK = 4$, therefore, $EK = 1$, $MK = 2 \cdot EK = 2$. Then

$$S_{lat} = 4 \cdot \frac{8 + 6}{2} \cdot 2 = 56 \text{ cm}^2.$$

Answer: 56 cm^2.

1.2.2. Round bodies. The cylinder. The cone. The ball

When studying this topic, you should pay attention to the concepts of the generator and the directrix. The generator and the directrix are located in different planes. Any generator of a right circular cylinder, as well as a segment joining the centers of its bases, can be the altitude of the cylinder.

Special consideration should be given to the sections of the cylinder and the cone made by planes parallel to the axis. In the section of the cylinder, rectangles are obtained, the largest of which is the axial section, the base of which is the diameter, and the altitude is the altitude of the cylinder. If the axial section of the cylinder is a square, the cylinder is equilateral.

The axial section of the cone is an isosceles triangle. If the axial section of the cone is an equilateral triangle, the cone is equilateral.

The axial section of the cone is an isosceles trapezoid formed by the diameters of the lower and upper bases and the generators of the truncated cone.

Any section of a cylinder and a cone parallel to the base is a disk.

Let's use basic formulas for calculation of the area of the lateral surface S_{lat}, the area of the total surface S_{total} and the volume V of round bodies.

For a cylinder, basic formulas are:

$$S_{lat} = 2\pi Rh;\ \ S_{total} = 2\pi R(H+R);\ \ V = \pi R^2 H,$$

where R is the radius of the base;

H is the altitude of the cylinder.

For a cone, basic formulas are:

$$S_{lat} = \pi Rl;\ \ S_{total} = \pi R(R+l);\ \ V = \frac{1}{3}\pi R^2 H,$$

where R, is the radius of the base;

H and l are the altitude and the generator of the cone.

For a truncated cone, basic formulas are:

$$S_{lat} = \pi Rl;\ \ S_{total} = \pi R^2 + \pi r^2 + \pi l(R+r);$$

$$V = \frac{1}{3}\pi H\left(R^2 + Rr + r^2\right),$$

where R, r are the radii of the lower and upper bases;

H and l are the altitude and the generator of the cone.

For a sphere and a ball, basic formulas are:

$$S = 4\pi R^2;\ \ V = \frac{4}{3}\pi R^3,$$

where R is the radius of the ball.

Example 23. Determine the angle between the diagonals of the axial section of the cylinder if the unfolding of the lateral surface is a square.

Solution. Let's draw a figure to this example (Fig. 1.30).

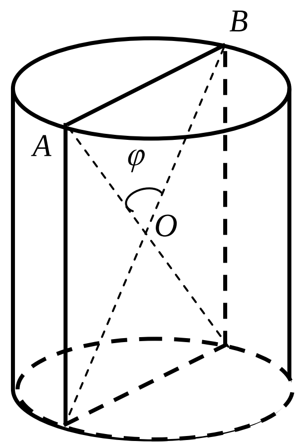


Fig. 1.30. **The drawing to example 23**

Let's denote the sought angle AOB as φ. According to the condition of this example we have $H = 2\pi R$,

$$\operatorname{tg}\frac{\varphi}{2} = R : \frac{H}{2} = \frac{1}{\pi}, \text{ therefore } \varphi = 2 \cdot \operatorname{arctg}\frac{1}{\pi}.$$

Answer: $2 \cdot \operatorname{arctg}\frac{1}{\pi}$.

Example 24. In an equilateral cone, the radius of the base is R. Find the area of the cross-section drawn through two generators with a $30°$ angle between them.

Solution. Let's draw a figure to this example (Fig. 1.31).

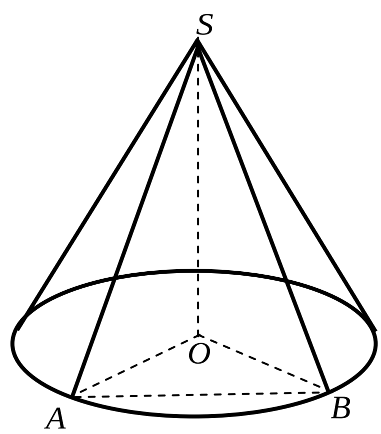


Fig. 1.31. **The drawing to example 24**

According to the condition of this example we have $\angle AS = 30°$, $S_{ASB} = \frac{1}{2} AS^2 \cdot \sin 30° = \frac{AS^2}{4}$.

But $AS = 2R$ (for an equilateral cone), then

$$S_{\sec} = \frac{4R^2}{4} = R^2.$$

Answer: R^2.

Example 25. A cone with the altitude of 20 cm is given. A plane parallel to the base is drawn through the middle of the altitude. A cross-sectional area is 30 cm^2. Determine the volume of the resulting truncated cone.

Solution. Let's draw a figure to this example (Fig. 1.32).

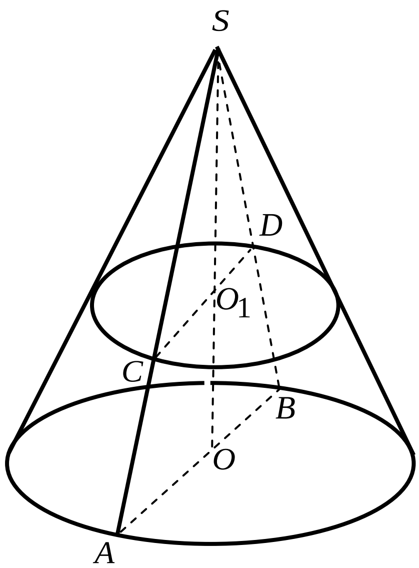


Fig. 1.32. **The drawing to example 25**

According to the condition of this example we have the altitude H of the truncated cone which equals 10 cm.

$$CD = \frac{1}{2} AB \qquad \text{or} \qquad 2r = \frac{1}{2} 2R, R = 2r.$$

Since $\pi r^2 = 30$, $r = \sqrt{\frac{30}{\pi}}$, and $R = 2\sqrt{\frac{30}{\pi}}$.

$$V = \frac{1}{3}\pi \cdot 10\left(4 \cdot \frac{30}{\pi} + 2 \cdot \frac{30}{\pi} + \frac{30}{\pi}\right) = 700\,\text{cm}^2.$$

Answer: 700 cm^2.

Example 26. A right-angled triangle with a leg a and an adjacent angle of $60°$ rotates around the hypotenuse. Find the volume of the body of revolution.

Solution. Let's draw a figure to this example (Fig. 1.33).

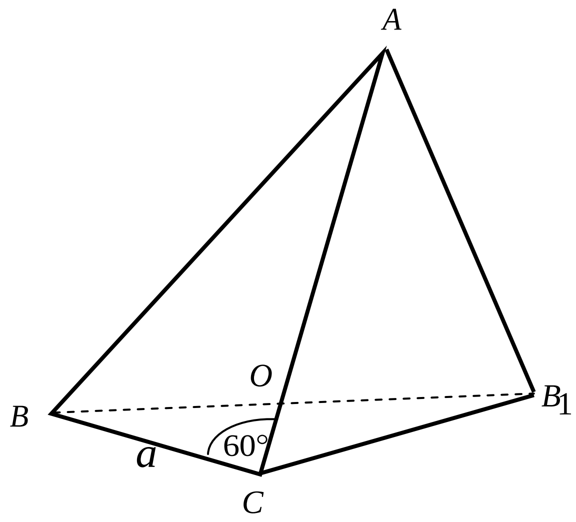


Fig. 1.33. **The drawing to example 26**

Let ABC be the given triangle, in which $\angle C = 60°$, $\angle B = 90°$, $BC = a$.

As a result of the rotation of ΔABC around the hypotenuse AC, two cones are formed with common bases and altitudes AO and OC. The volume of the body of revolution is calculated as

$$V_{rot} = \frac{1}{3}\pi \cdot BO^2 \cdot CO = \frac{1}{3}\pi \cdot BO^2 \cdot (AO + OC) = \frac{1}{3}\pi \cdot BO^2 \cdot AC =$$
$$= \frac{1}{3}\pi \cdot (a \cdot \sin 60°)^2 \cdot 20 = \frac{1}{2}\pi \cdot a^3.$$

Answer: $\frac{1}{2}\pi \cdot a^3$.

Tasks for individual work

1. A triangle

Task 27. In an isosceles triangle, the base and the leg are related as $3:5$, the sum of their lengths is 32 cm. Find the perimeter of the triangle.

Task 28. The perimeter of a right-angled triangle is 17 cm. Find the lengths of its sides if the hypotenuse is 2 cm longer than the leg, and the acute angles are equal.

Task 29. Find the perimeter of an isosceles triangle if the length of the bisector of the angle at the base is 15 cm and it is 75 % of the length of the leg and $3/5$ of the length of the base.

Task 30. Determine the values of the angles of a triangle if they are related as 2 : 3 : 4. Define the type of triangle depending on the angles.

Task 31. In the triangle ABC, the bisectors of the interior angles B and C at the intersection form an angle of $116°$. Find the angle A.

Task 32. The angle at the base of an isosceles triangle is $130\ \%$ of the angle at the vertex. Determine the angles of the triangle.

Task 33. In a right-angled triangle, the value of one of the acute angles is $60°$, and the sum of the lengths of the hypotenuse and the smaller leg is 1.8 m. Determine the length of the hypotenuse.

Task 34. In an isosceles triangle, the leg length equals 26 cm, and the length of the altitude is 13 cm. Find the value of the obtuse angle between the bisectors of the angles at the base.

Task 35. The values of the angles at the base of a triangle are equal to $65°$ and $37°$. Determine the value of the angle between the bisector and the altitude drawn from the vertex of the triangle.

Task 36. The values of the angles of a triangle are related as 1 : 2 : 3. The sum of the lengths of its smaller and larger sides is 7.2 cm. Find the length of the larger side of the triangle.

Task 37. In a right-angled triangle ABC, $\angle A = 30°$, $AC = 3.5$ cm. Find the lengths of the segments formed by the perpendicular drawn from the vertex of the right angle to divide the hypotenuse.

Task 38. In an isosceles triangle, the value of the angle between the base and the altitude drawn to the leg is equal to $48°$. Find the values of the angles of the triangle.

2. Quadrilaterals

Task 39. The lengths of the sides of a parallelogram are related as 2 : 5, and its perimeter equals 9.8 cm. Determine the lengths of the sides of the parallelogram.

Task 40. In a parallelogram, the bisector of an angle divides the opposite side into 12 and 8 cm segments. Find the perimeter of the parallelogram.

Task 41. The diagonal of a rectangle divides its angle in the ratio of 1 : 2. Determine the length of the diagonal of the rectangle if the sum of the lengths of both diagonals and the smaller sides is 24 cm.

Task 42. In a rectangle, the bisector divides the opposite side in the ratio of 1 : 3. Find the lengths of the sides of the rectangle if its perimeter is 56 cm.

Task 43. The diagonal of a rectangle is twice as long as its side. Find the value of the acute angle between the diagonals.

Task 44. The values of the angles formed by the side of the rhombus with its diagonals are related as 5 : 4. Find the values of the angles of the rhombus.

Task 45. Find the values of the angles of a rhombus, if the altitude drawn from the vertex of the obtuse angle divides the opposite side in half.

Task 46. In a rhombus, the altitude and the smaller diagonal form the value of the angle equal to 15°. Find the altitude of the rhombus if its perimeter is 32 cm.

Task 47. Find the perimeter of a rhombus, with the altitude of 10 cm, and the obtuse angle five times greater than the acute one.

Task 48. In an isosceles trapezoid, the smaller base is 34 cm, the leg is 20 cm, and the angle between them equals 120°. Find the bigger base.

Task 49. In an isosceles trapezoid, the bases are 25 and 16 cm. The diagonal of the trapezoid divides its acute angle in half. Determine the perimeter of the trapezoid.

Task 50. In an isosceles trapezoid, the altitude drawn from the vertex of the obtuse angle equal to 120° divides the base into 13 and 21 cm segments. Find the perimeter of the trapezoid.

Task 51. In an isosceles trapezoid, the distance from the end of the larger base to the opposite leg is half the length of this base. Determine the values of the angles of the trapezoid.

Task 52. The smaller base of an isosceles trapezoid is equal to the leg, and the diagonal is perpendicular to the leg. Determine the values of the angles of the trapezoid.

Task 53. In an isosceles trapezoid with an acute angle of $60°$, the midline is 26 cm, and the leg is 14 cm. Find the lengths of the bases of the trapezoid.

Task 54. In an isosceles trapezoid, the altitude is 10 cm, and the diagonals are mutually perpendicular. Find the length of the midline of the trapezoid.

Task 55. The trapezoid diagonal divides the midline into two segments, with the lengths related as 3 : 4. Find the lengths of the bases of the trapezoid if one segment is 5 cm longer than the other.

3. A circle

Task 56. The chord intersects the diameter at the angle of $30°$ and divides it into two segments of 2 and 6 cm. Find the distance of the chord from the center.

Task 57. In a circle, at a distance of 2 cm from the center, two mutually perpendicular chords of 10 cm length are drawn. Into what parts does one chord divide the other?

Task 58. a) In a circle with the radius of 16 cm, a chord is drawn that forms the arc of $120°$. Find the distance from the center to the chord. b) Two disks are given, their common interior tangents are mutually perpendicular. The lengths of the chords connecting the tangency points are 3 and 5 cm. Determine the distance between the centers.

4. Inscribed and circumscribed triangles and quadrangles

Task 59. One of the acute angles of a right-angled triangle is equal to $40°$. Determine the acute angle between the radius of the circumscribed circle, drawn at the vertex of the right angle, and the hypotenuse.

Task 60. The side of the rhombus is 8 cm, and the acute angle is $30°$. Determine the radius of the inscribed circle.

Task 61. Determine the diameter of a circle circumscribed around a rectangle if one of its sides is equal to a and forms an angle of $60°$ with the diagonal.

Task 62. A quadrilateral is inscribed around a circle, with the lengths of three consecutive sides equal to 10, 15, 16 cm. Find the perimeter of the quadrilateral.

Task 63. An isosceles trapezoid is inscribed around a circle, its perimeter is 18 cm. The angle of the trapezoid is equal to 30°. Determine the diameter of the circle.

Task 64. An isosceles trapezoid with a side of 8 cm and an angle of 60° at the base is inscribed around a circle. Calculate the base of the trapezoid.

Task 65. An isosceles trapezoid with an angle of 30° is inscribed around a circle. Its midline is 1 m. Determine the radius of the circle.

5. Areas of figures

Task 66. Find the sides of a rectangle if they are related as 4 : 9 and the area is 36 cm^2.

Task 67. One of the adjacent sides of a rectangle is 8 cm larger than the other; the adjacent sides are related as 3 : 5. Calculate the area of the rectangle.

Task 68. The altitudes of a parallelogram are related as 2 : 3, its perimeter is 40 cm, and the acute angle is 30°. Determine the area of the parallelogram.

Task 69. The sides of a parallelogram are 6 and 8 cm, the acute angle between them is 30°. Find the area of the parallelogram.

Task 70. The sides of a parallelogram are 10 cm and 16 cm, and the angles adjacent to one side are related as 1 : 5. Find the area of the parallelogram.

Task 71. Determine the lower altitude of the triangle, whose sides are 25, 29 and 36 cm.

Task 72. Find the area of a right-angled triangle if one of its legs is 15 cm, and the altitude drawn to the hypotenuse is 12 cm.

Task 73. Find the area of a right-angled triangle if one of its legs is 8 cm, and the radius of the inscribed circle is 3 cm.

Task 74. Find the area of a right-angled triangle if the radius of the circumscribed circle is 5 cm, and the altitude, drawn to the hypotenuse, is 3 cm.

Task 75. By what percentage will the area of a triangle increase if its base is increased by 50 %?

Task 76. The area of a rhombus is 120 %, its altitude is 8 cm. Find the perimeter of the rhombus.

Task 77. The side of a rhombus is 20 cm. Find the area of the rhombus if one of its angles is $150°$.

Task 78. The area of a rhombus is 144 cm^2, and one of its diagonals is 18 cm. Find the length of the other diagonal.

Task 79. Determine the area of an isosceles trapezoid whose bases are 42 and 54 cm, and the angle at the larger base is $45°$.

Task 80. In an isosceles trapezoid, the lower base is 44 cm, the leg is 17 cm, and the diagonal is 39 cm. Find the area of the trapezoid.

Task 81. Find the area of an isosceles trapezoid inscribed about a circle if its leg is 8 cm and the acute angle is $30°$.

Task 82. The bases of a rectangular trapezoid are 60 and 24 cm, the smaller diagonal is 40 cm. Find the area of the trapezoid and its larger diagonal.

Task 83. Determine the area of an isosceles trapezoid whose bases are 51 and 69 cm, and the leg is 41 cm.

Task 84. Calculate the area of an isosceles trapezoid whose bases are 12 and 20 cm, and the diagonals are mutually perpendicular.

Task 85. Determine the area of a trapezoid with parallel sides of 60 and 20 cm and non-parallel sides of 13 and 37 cm.

6. The circumference and the area of a disk

Task 86. An isosceles trapezoid with an angle of $30°$ is inscribed around a circle, the midline of the trapezoid is 8 cm. Find the circumference.

Task 87. By how many centimeters will the circumference increase if its diameter equal to 20 cm is increased by 25 %.

Task 88. Determine the radius of a circle, if it is known that it is 107 cm longer than its diameter.

Task 89. A disk is circumscribed about an isosceles trapezoid, whose bases are 6 and 8 cm, and the altitude is 7 cm. Find the area of the disk.

Task 90. A rectangle with the sides of 12 and 16 cm is circumscribed in a circle. Find the circumference and the area of the disk.

Task 91. In a disk, the chord forms the arc of $120°$, and it is 7 cm from the center. Find the circumference and the area of the disk.

Task 92. By what percentage will the area of the disk increase if its radius is increased by 50 %.

Task 93. Find the area of a disk if it is less than the area of the circumscribed square by 86 cm^2.

7. A parallelepiped and a prism

Task 94. The diagonal of a regular quadrangular prism is 7 cm. The diagonal of the lateral face is 5 cm. Find the altitude of the prism.

Task 95. The diagonal of a regular quadrangular prism is inclined to the lateral face at an angle of 30°. Determine the angle of its inclination to the base.

Task 96. The altitude of the rectangular parallelepiped is 8 dm, the lengths of the two sides of the base and the diagonal of the parallelepiped form an arithmetic progression with a difference of 5 dm. Determine the sides of the base and its diagonal.

Task 97. The diagonal of a cube is $2\sqrt{3}$. Find the total surface area of the cube.

Task 98. The sides of the base of a rectangular parallelepiped are 6 and 8 cm, and the area of the diagonal section is 180 cm^2. Find the total surface area of the parallelepiped.

Task 99. In a right parallelepiped, the sides of the base are 3 and 8 cm and form an angle of 60°. The larger diagonal of the parallelepiped is 49 cm. Calculate the lateral surface area of the parallelepiped.

Task 100. The base of a right parallelepiped is a parallelogram with the sides of 1 and 4 cm and an acute angle of 60°. The larger diagonal of the parallelepiped is 5 cm. Calculate its volume.

Task 101. The measurements of a rectangular parallelepiped are related as 2 : 7 : 26, its diagonal is 81 cm. Find the volume of the parallelepiped.

Task 102. The base of a right parallelepiped is a rhombus, the diagonals of which are related as 5 : 16. The diagonals of the parallelepiped are 26 and 40 cm. Find the volume of the parallelepiped.

Task 103. The base of a right prism is an isosceles trapezoid whose parallel sides are 5 and 11 cm, the altitude of the base is 4 cm. The lateral edge of the prism is 6 cm. Find the volume of the prism.

8. A pyramid

Task 104. The base of a pyramid is a rectangle with the sides of 12 and 16 cm; each edge of the pyramid is 26 cm. Find the altitude of the pyramid.

Task 105. The base of a pyramid is a triangle with the sides of 20, 21 and 29 cm. The lateral faces of the pyramid form angles of 45° at the base plane. Find the altitude of the pyramid.

Task 106. The base of a pyramid is an isosceles triangle with a base of 6 cm and an altitude of 9 cm; all lateral edges are equal to 13 cm. Find the altitude of the pyramid.

Task 107. Find the lateral surface area of a regular triangular pyramid if its altitude is 9 cm, and the apothem is 18 cm.

Task 108. In a regular quadrangular pyramid, the lateral surface area is 240 cm^2, and the total surface area is 384 cm^2. Find the side of the base and the altitude of the pyramid.

Task 109. The base of the pyramid is a square with a side of 16 cm, and the two lateral faces are perpendicular to the plane of the base. The altitude of the pyramid is 12 cm. Find S_{total}.

Task 110. The base of the pyramid is a rhombus with the diagonals of 6 and 8 cm. The altitude of the pyramid is 1 m. All dihedral angles at the base are equal. Find S_{total}.

Task 111. The apothem of a regular triangular pyramid is equal to k, and it forms an angle α with the base. Find S_{total}.

Task 112. The base of a pyramid is a right-angled triangle with a leg of 4 cm and an adjacent angle of 30°. The lateral edge forms an angle of 60° with the base. Find the volume of the pyramid.

Task 113. The base of the pyramid is a rhombus with a side of 15 cm. Each face of the pyramid forms an angle of 45° with the base. Find the volume of the pyramid if $S_{lat} = 300$ cm^2.

Questions for self-assessment

1. What figure is called a prism?
2. What prism is right?
3. What is the base of a prism?
4. What figure is the lateral face of a prism?
5. How is the area of the lateral surface of a prism calculated?
6. How is the area of the total surface of a prism calculated?
7. How is the volume of a prism calculated?
8. What figure is called a parallelepiped?
9. What parallelepiped is right?
10. What is the base of a parallelepiped?
11. What figure is the lateral face of a parallelepiped?
12. How is the area of the lateral surface of a parallelepiped calculated?
13. How is the area of the total surface of a parallelepiped calculated?
14. How is the volume of a parallelepiped calculated?
15. What figure is called a pyramid?
16. What pyramid is regular?
17. What is the base of a pyramid?
18. What figure is the lateral face of a pyramid?
19. How is the area of the lateral surface of a pyramid calculated?
20. How is the area of the total surface of a pyramid calculated?
21. How is the volume of a pyramid calculated?
22. What figure is called a pyramid?
23. What pyramid is truncated?
24. What figure is the lateral face of a truncated pyramid?
25. How is the area of the lateral surface of a truncated pyramid calculated?
26. How is the area of the total surface of a truncated pyramid calculated?
27. How is the volume of a truncated pyramid calculated?
28. What figure is called a cylinder?
29. What is the base of a cylinder?
30. What figure is the lateral face of a cylinder?
31. How is the area of the lateral surface of a cylinder calculated?
32. How is the area of the total surface of a cylinder calculated?
33. How is the volume of a cylinder calculated?
34. What figure is called a cone?
35. What is the base of a cone?

36. What figure is the lateral face of a cone?
37. How is the area of the lateral surface of a cone calculated?
38. How is the area of the total surface of a cone calculated?
39. How is the volume of a pyramid calculated?
40. What figure is called a sphere?
41. What figure is called a ball?
42. How is the area of the surface of a sphere calculated?
43. How is the volume of a ball calculated?

2. Vectors

2.1. The Cartesian coordinate system on the plane and in space

If a one-to-one correspondence between the points on the plane and the numbers (pairs of numbers) is specified, one says that a *coordinate system* is introduced on the plane.

A *rectangular Cartesian coordinate system on the plane* is determined by a scale segment for measuring lengths and two mutually perpendicular axes. The point of intersection of the axes is usually denoted by the letter O and is called the *origin*, while the axes themselves are called the *coordinate axes*. As a rule, one of the coordinate axes is horizontal and the right sense is positive. This axis is called the *abscissa axis* and is denoted by the letter X or by OX . On the vertical axis, which is called the *ordinate axis* and is denoted by Y or OY , the upward sense is usually positive (Fig. 2.1). The coordinate system introduced above is often denoted by XY or OXY .

The abscissa axis divides the plane into the *upper* and *lower* half-planes, while the ordinate axis divides the plane into the *right* and *left* half-planes. The two coordinate axes divide the plane into four parts, which are called *quadrants* and numbered as shown in Fig. 2.1.

Let's take an arbitrary point A on the plane and project it onto the coordinate axes, i.e. draw perpendiculars to the axes OX and OY through A.

The points of intersection of the perpendiculars with the axes are denoted by A_X and A_Y , respectively (Fig. 2.1). The numbers $x = OA_X$ and $y = OA_Y$, where OA_X and OA_Y are the respective values of the segments $\overrightarrow{OA_X}$ and $\overrightarrow{OA_Y}$ on the abscissa and ordinate axes, are called the *coordinates of the point* A in the rectangular Cartesian coordinate system.

The number X is the first coordinate, or the *abscissa*, of the point A, and Y is the second coordinate, or the *ordinate*, of the point A. One says that the point A has the coordinates (X,Y) and uses the notation $A(X,Y)$.

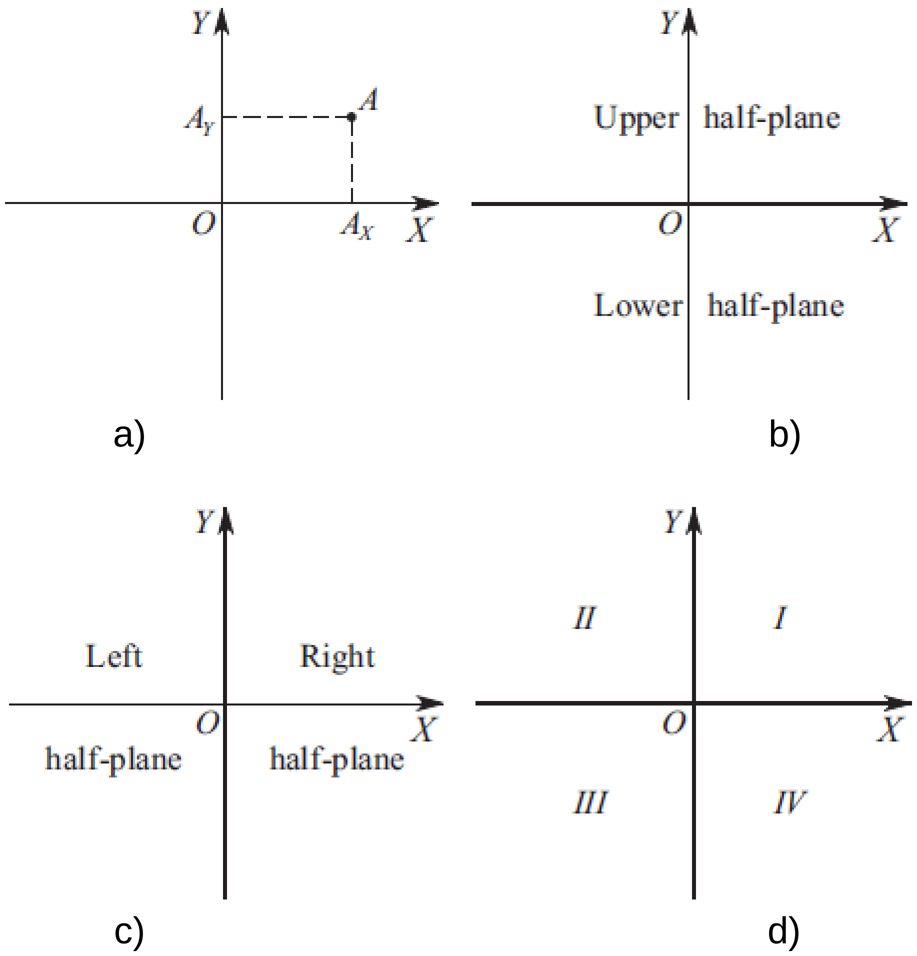


Fig. 2.1. **The rectangular Cartesian coordinate system**

The *rectangular Cartesian coordinate system in space* is determined by a scale segment for measuring lengths and three pairwise perpendicular directed straight lines OX, OY and OZ (the *coordinate axes*) concurrent at a single point O (the *origin*). The three coordinate axes divide the space into eight parts called *octants*.

We choose an arbitrary point M in space and project it onto the coordinate axes, i.e. draw the perpendiculars to the axes OX, OY and OZ through M. We denote the points of intersection of the perpendiculars with the axes by x, y and z, respectively. These numbers (Fig. 2.2) are the signed lengths of the segments of the axes OX, OY and OZ, respectively, and they are called the *coordinates of the point* M in the rectangular Cartesian coordinate system.

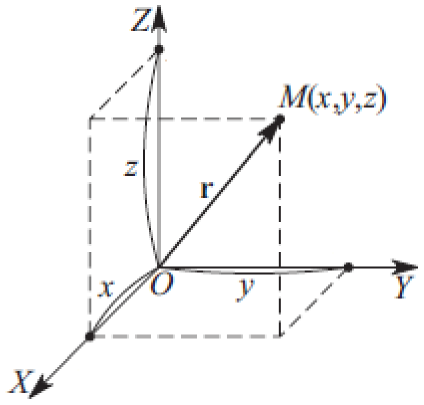


Fig. 2.2. **A point in the rectangular Cartesian coordinate system in space**

The number x is called the first coordinate or the *abscissa* of the point M, the number y is called the second coordinate or the *ordinate* of the point M, and the number z is called the third coordinate or the *applicate* of the point M. Usually one says that the point M has the coordinates (x, y, z), and the notation $M(x, y, z)$ is used.

2.2. Formulas of division of a segment in the given ratio

Let's assume that the point $M(x_M, y_M, z_M)$ divides a segment between the points $M_1(x_{M_1}, y_{M_1}, z_{M_1})$ and $M_2(x_{M_2}, y_{M_2}, z_{M_2})$ in the ratio λ, that is $\lambda = \dfrac{|M_1M|}{|MM_2|}$.

In this case the following formulas should be used to find the coordinates of the point M:

$$x_M = \frac{x_{M_1} + \lambda \cdot x_{M_2}}{1+\lambda}, \quad y_M = \frac{y_{M_1} + \lambda \cdot y_{M_2}}{1+\lambda}, \quad z_M = \frac{z_{M_1} + \lambda \cdot z_{M_2}}{1+\lambda}.$$

In the particular case, if the point M bisects the segment M_1M_2 $(\lambda = 1)$,

$$x_M = \frac{x_{M_1} + x_{M_2}}{2}, \quad y_M = \frac{y_{M_1} + y_{M_2}}{2}, \quad z_M = \frac{z_{M_1} + z_{M_2}}{2}.$$

2.3. Vectors and basic vector operations

A segment bounded by points A and B is called a *directed segment* if its initial point and endpoint are chosen. Such a segment with initial point A and endpoint B is denoted by $\overrightarrow{AB}$ (Fig. 2.3).

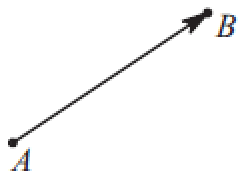


Fig. 2.3. **A vector**

A directed segment with initial point A and endpoint B is called the *vector* $\overrightarrow{AB}$ or $\vec{a}$. A nonnegative number equal to the length of the segment AB joining the points A and B is called the *length* $|\overrightarrow{AB}|$ of the *vector* $\overrightarrow{AB}$. The vector $\overrightarrow{BA}$ is said to be *opposite to the vector* $\overrightarrow{AB}$, i.e. $\overrightarrow{AB} = -\overrightarrow{BA}$.

Let the point $A(x_A, y_A, z_A)$ be an initial point of the vector $\overrightarrow{AB}$ and the point $B(x_B, y_B, z_B)$ be its endpoint. The coordinates of the vector $\overrightarrow{AB}$ are defined as $\overrightarrow{AB} = (x_B - x_A, y_B - y_A, z_B - z_A)$.

To each point M of three-dimensional space one can assign its position vector. The directed segment $\overrightarrow{OM}$ is called the *position vector of the point* M. The position vector determines the vector $\vec{r}\ (\vec{r} = \overrightarrow{OM})$ whose coordinates are its projections on the axes OX, OY and OZ, respectively.

An arbitrary vector $\vec{a} = (a_x, a_y, a_z)$ can be represented as

$$\vec{a} = a_x \cdot \vec{i} + a_y \cdot \vec{j} + a_z \cdot \vec{k},$$

where a_x, a_y, a_z are projections of the vector $\vec{a}$ on the axes OX, OY and OZ, respectively; $\vec{i} = (1,0,0)$, $\vec{j} = (0,1,0)$, $\vec{k} = (0,0,1)$ are the unit vectors with the same directions as the coordinate axes OX, OY and OZ.

Each of the vectors $\vec{i}, \vec{j}, \vec{k}$ is perpendicular (orthogonal) to both others. These vectors form a so-called orthonormalized basis. The projections a_x, a_y, a_z are coordinates of the vector in the orthonormalized basis.

The distance between the initial point and the endpoint of a vector is called its length or module and designated by $|\vec{a}|$ or $|\overrightarrow{AB}|$.

The module of the vector $\vec{a}$ is calculated according to the following formula:

$$|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}\,.$$

The module of the vector $\overrightarrow{AB}$ is calculated according to the following formula:

$$|\overrightarrow{AB}| = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}\,.$$

Direction cosines are cosines of the angles between the vector $\vec{a}$ and positive directions of the corresponding coordinate axes and they are defined as follows:

$$\cos\alpha = \frac{a_x}{|\vec{a}|} = \frac{a_x}{\sqrt{a_x^2 + a_y^2 + a_z^2}}; \quad \cos\beta = \frac{a_y}{|\vec{a}|} = \frac{a_y}{\sqrt{a_x^2 + a_y^2 + a_z^2}};$$

$$\cos\gamma = \frac{a_z}{|\vec{a}|} = \frac{a_z}{\sqrt{a_x^2 + a_y^2 + a_z^2}}\,.$$

They are related to the equality

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1.$$

Example 114. Calculate the length of the vector $\vec{a}=(-2;3;-6)$.

Solution. The vector length is calculated by the formula:

$$|\vec{a}|=\sqrt{a_x{}^2+a_y{}^2+a_z{}^2}\text{, i.e. }|\vec{a}|=\sqrt{4+9+36}=\sqrt{49}=7.$$

Answer: 7.

Example 115. Find point A if the initial point of the vector coincides with point A and its endpoint coincides with $B(3;-1;1)$.

Solution. We have $\vec{a}=\overrightarrow{AB}=(3-x_A;-1-y_A;1-z_A)$. Let's equate it to vector $\vec{a}=(5;3;-2)$, i.e. we get:

$$\begin{aligned}
&3-x_A=5, \quad x_A=-2;\\
&-1-y_A=3, \quad y_A=-4;\\
&1-z_A=-2, \quad z_A=3.
\end{aligned}$$

Answer: $A(-2;-4;3)$.

Example 116. Find the direction cosines of the vector $\overrightarrow{AB}$ if the points $A(1;\,2;\,0)$ and $B(3;1;-2)$ are given.

Solution. The coordinates of the vector $\overrightarrow{AB}$ are calculated in this way:

$$\overrightarrow{AB}=(3-1;\ 1-2;-2-0)=(2;-1;-2)$$

Its length is:

$$\left|\overrightarrow{AB}\right|=\sqrt{(x_B-x_A)^2+(y_B-y_A)^2+(z_B-z_A)^2}=\sqrt{2^2+(-1)^2+(-2)^2}=$$
$$=\sqrt{9}=3.$$

The direction cosines are:

$$\cos\alpha=\frac{a_x}{|\vec{a}|}=\frac{2}{3},\qquad \cos\beta=\frac{a_y}{|\vec{a}|}=-\frac{1}{3},\qquad \cos\gamma=\frac{a_z}{|\vec{a}|}=-\frac{2}{3}.$$

Answer: ${}^2/_3;-{}^1/_3;-{}^2/_3$.

Two vectors are said to be *collinear (parallel)* if they lie on the same straight line or on parallel lines. Three vectors are said to be *coplanar* if they lie in the same plane or in parallel planes. Two vectors should be considered *equal* if they are *collinear,* equally directed and have equal lengths.

A vector $\vec{0}=(0{,}0{,}0)$, whose initial point and endpoint coincide, is called a *zero vector* (*a null vector*).

The length of the zero vector is equal to zero $\left(|\vec{0}|=0\right)$, and the direction of the zero vector is assumed to be arbitrary. A vector $\vec{e}$ of unit length is called a *unit vector*.

Basic vector operations:

1. The *sum* $\vec{a}+\vec{b}$ *of vectors* $\vec{a}$ *and* $\vec{b}$ is defined as the vector directed from the initial point of $\vec{a}$ to the endpoint of $\vec{b}$ under the condition that $\vec{b}$ is applied at the endpoint of $\vec{a}$.

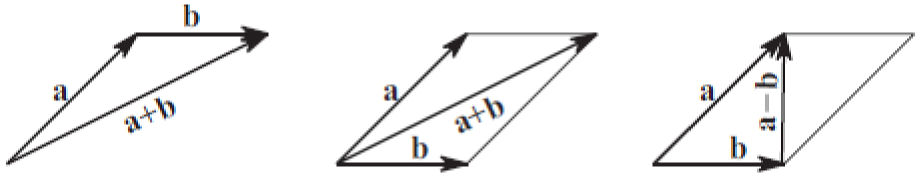
The rule for addition of vectors, which is contained in this definition, is called the *triangle rule of vectors* (Fig. 2.4, a). The sum $\vec{a}+\vec{b}$ can also be found using the *parallelogram rule* (Fig. 2.4, b). The *difference* $\vec{a}-\vec{b}$ *of vectors* $\vec{a}$ *and* $\vec{b}$ is defined as follows: $\vec{b}+\left(\vec{a}-\vec{b}\right)=\vec{a}$ (Fig. 2.4, c).

A sum or difference of vectors is determined according to the formulas:

$$\vec{a}\pm\vec{b}=\left(a_x\pm b_x, a_y\pm b_y, a_z\pm b_z\right) \quad \text{or}$$

$$\vec{a}\pm\vec{b}=(x_1\pm x_2; y_1\pm y_2; z_1\pm z_2).$$

2. The *product* $\lambda\vec{a}$ *of* the *vector* $\vec{a}$ *by* the *number* λ is defined as the vector whose length is equal to $|\lambda\vec{a}|=|\lambda|\cdot|\vec{a}|$ and whose direction coincides with that of the vector $\vec{a}$ if $\lambda>0$ or is opposite to the direction of the vector $\vec{a}$ if $\lambda<0$.



a) b) c)

Fig. 2.4. **The sum of vectors: the triangle rule (a) and the parallelogram rule (b). The difference of vectors (c)**

A product (multiplication) of a vector by a number is determined according to the formula:

$$\alpha \cdot \vec{a} = (\alpha \cdot a_x, \alpha \cdot a_y, \alpha \cdot a_z).$$

Remark. If $\vec{a} = 0$ or $\lambda = 0$, the absolute value of the product is zero, i.e. it is the zero vector. In this case, the direction of the product $\lambda\vec{a}$ is undetermined.

Main properties of operations with vectors:

1. $\vec{a} + \vec{b} = \vec{b} + \vec{a}$ (commutativity).
2. $\vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c}$ (associativity of addition).
3. $\vec{a} + \vec{0} = \vec{a}$ (existence of the zero vector).
4. $\vec{a} + (-\vec{a}) = \vec{0}$ (existence of the opposite vector).
5. $\lambda(\vec{a} \pm \vec{b}) = \lambda\vec{a} \pm \lambda\vec{b}$ (distributivity with the ratio to addition or difference of vectors).
6. $(\lambda \pm \mu)\vec{a} = \lambda\vec{a} \pm \mu\vec{a}$ (distributivity with the ratio to addition or difference of constants).
7. $\lambda(\mu\vec{a}) = (\lambda\mu)\vec{a}$ (associativity of product).
8. $1 \cdot \vec{a} = \vec{a}$ (multiplication by unity).

Projection of a vector onto the axis. A straight line with a unit vector $\vec{e}$ lying on it determining the positive sense of the line is called an *axis*. The *projection* $pr_{\vec{e}}\vec{a}$ of a vector $\vec{a}$ onto the axis (Fig. 2.5) is defined as the directed segment on the axis whose signed length is equal to the scalar product of $\vec{a}$ by the unit vector $\vec{e}$, i.e. is determined by the formula

$$pr_{\vec{e}}\vec{a} = \vec{a} \cdot \cos\varphi,$$

where φ is the angle between the vectors $\vec{a}$ and $\vec{e}$.

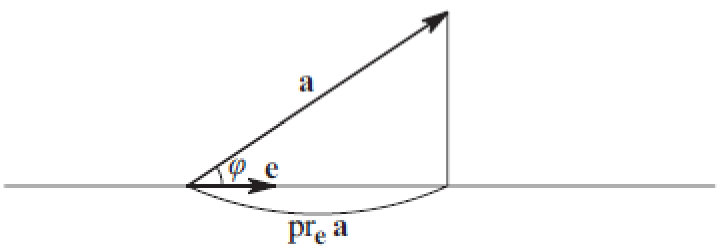


Fig. 2.5. **Projection of a vector onto the axes**

Example 117. Two vectors $\vec{a}=(3,-2,-6)$ and $\vec{b}=(-2,1,0)$ are given. Determine the projections on the coordinate axes of the following vectors: 1) $\vec{a}+\vec{b}$; 2) $\vec{a}-\vec{b}$; 3) $2\vec{a}$; 4) $2\vec{a}-3\vec{b}$.

Solution. According to the rule of vector addition and vector multiplication by a number we have:

$$\vec{a}+\vec{b}=\left(a_x+b_x, a_y+b_y, a_z+b_z\right)=(3+(-2),-2+1,-6+0)=(1,-1,-6);$$

$$\vec{a}-\vec{b}=\left(a_x-b_x, a_y-b_y, a_z-b_z\right)=(3-(-2),-2-1,-6-0)=(5,-3,-6);$$

$$2\cdot\vec{a}=\left(2\cdot a_x,2\cdot a_y,2\cdot a_z\right)=(2\cdot 3,2\cdot(-2),2\cdot(-6))=(6,-4,-12);$$

$$2\vec{a}-3\vec{b}=\left(2a_x+3b_x,2a_y+3b_y,2a_z+3b_z\right)=$$

$$=(2\cdot 3-3\cdot(-2),2\cdot(-2)-3\cdot 1,2\cdot(-6)-3\cdot 0)=(12,-7,-12).$$

Answer: $(1;\ -1;\ -6), (5;\ -3;\ -6), (6;\ -4;\ -12), (12;\ -7;\ -12)$.

Example 118. Three vectors $\vec{a}=(2;-1;3)$, $\vec{b}=(0;2;-1)$, $\vec{c}=(1;2;3)$ are given. Find the vector $\vec{d}=3\vec{a}-2\vec{b}+\vec{c}$ and its length $\left|\vec{d}\right|$.

Solution. Let's find the coordinates of the vectors:

$3\vec{a}=(6;-3;9)$ and $2\vec{b}=(0;4;-2)$.

According to the vector addition rule, we have:

$$\vec{d}=(6-0+1;-3-4+2;9+2+3)=(7;-5;14).$$

Further $\left|\vec{d}\right|=\sqrt{7^2+(-5)^2+14^2}=\sqrt{270}$.

Vectors parallel to the same straight line are called collinear. If the vectors $\vec{a}=(x_1;y_1;z_1)$ and $\vec{b}=(x_2;y_2;z_2)$ are collinear,

$$\frac{x_1}{x_2}=\frac{y_1}{y_2}=\frac{z_1}{z_2}.$$

Answer: $(7;\ -5;\ 14)$, $\sqrt{270}$.

Example 119. At what values of α and β are both vectors $\vec{a} = (2; -8; \alpha)$ and $\vec{b} = (-3; \beta; 9)$ collinear?

Solution. From the condition of collinearity of two vectors, we have the relation:

$$\frac{2}{-3} = \frac{-8}{\beta} = \frac{\alpha}{9},$$

whence we get:

$$\beta = \frac{-3 \cdot (-8)}{2} = 12, \qquad \alpha = \frac{2 \cdot 9}{-3} = -6.$$

Answer: $\alpha = -6$; $\beta = 12$.

2.4. A scalar or dot product of two vectors

A scalar or dot product of two vectors is defined as the product of their absolute values times the cosine of the angle between the vectors (Fig. 2.6):

$$\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos \varphi.$$

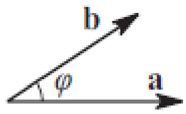


Fig. 2.6. **A scalar product of two vectors**

If the angle between the vectors $\vec{a}$ and $\vec{b}$ is acute, $\vec{a} \cdot \vec{b} > 0$; if the angle is obtuse, $\vec{a} \cdot \vec{b} < 0$; if the angle is right, $\vec{a} \cdot \vec{b} = 0$. So, we can write the scalar product as

$$\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos \varphi = |\vec{a}| \cdot pr_{\vec{a}} \vec{b} = |\vec{b}| \cdot pr_{\vec{b}} \vec{a}.$$

Remark. The scalar product of the vector $\vec{a}$ by the vector $\vec{b}$ is also denoted by $(\vec{a}, \vec{b})$ or $\vec{a}\vec{b}$.

The *angle* φ *between the vectors* is determined by the formula

$$\cos\varphi = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|} = \frac{a_x\cdot b_x + a_y\cdot b_y + a_z\cdot b_z}{\sqrt{a_x{}^2 + a_y{}^2 + a_z{}^2}\sqrt{b_x{}^2 + b_y{}^2 + b_z{}^2}}.$$

The *properties of a scalar product* are:

1. $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$ (commutativity).

2. $\vec{a}\cdot\left(\vec{b}\pm\vec{c}\right) = \vec{a}\cdot\vec{b}\pm\vec{a}\cdot\vec{c}$ (distributivity with the ratio to addition of vectors). This property holds for any number of summands.

3. If the vectors $\vec{a}$ and $\vec{b}$ are collinear, $\vec{a}\cdot\vec{b} = \pm|\vec{a}|\cdot|\vec{b}|$. (The sign "+" is taken if the vectors $\vec{a}$ and $\vec{b}$ have the same sense, and the sign "−" is taken if the senses are opposite.)

4. $(\lambda\vec{a})\cdot\vec{b} = \lambda\left(\vec{a}\cdot\vec{b}\right)$ (associativity with the ratio to a scalar factor).

5. $\vec{a}\cdot\vec{a} = |\vec{a}|^2$. The scalar product $\vec{a}\cdot\vec{a}$ is denoted by $|\vec{a}|^2$ (the *scalar square* of the vector $\vec{a}$).

6. The length of a vector is expressed via the scalar product by the formula

$$|\vec{a}| = \sqrt{\vec{a}\cdot\vec{a}} = \sqrt{|\vec{a}|^2}.$$

7. Two nonzero vectors $\vec{a} = (x_1; y_1; z_1)$ and $\vec{b} = (x_2; y_2; z_2)$ are perpendicular if and only if $\vec{a}\vec{b} = 0$, i.e. $x_1x_2 + y_1y_2 + z_1z_2 = 0$.

8. The scalar products of basis vectors are

$$\vec{i}\cdot\vec{j} = \vec{i}\cdot\vec{k} = \vec{j}\cdot\vec{k} = 0,\ \ \vec{i}\cdot\vec{i} = \vec{j}\cdot\vec{j} = \vec{k}\cdot\vec{k} = 1.$$

9. If vectors are given by their coordinates, $\vec{a} = \left(a_x, a_y, a_z\right)$ and $\vec{b} = \left(b_x, b_y, b_z\right)$,

$$\vec{a}\cdot\vec{b} = \left(a_x\vec{i} + a_y\vec{j} + a_z\vec{k}\right)\cdot\left(b_x\vec{i} + b_y\vec{j} + b_z\vec{k}\right) = a_x\cdot b_x + a_y\cdot b_y + a_z\cdot b_z.$$

10. The Cauchy – Schwarz inequality and the Minkowski inequality are:

$$\left|\vec{a}\cdot\vec{b}\right| \le \left|\vec{a}\right|\cdot\left|\vec{b}\right| \quad \text{and} \quad \left|\vec{a}+\vec{b}\right| \le \left|\vec{a}\right| + \left|\vec{b}\right|.$$

Example 120. Find $\left(3\vec{a}-5\vec{b}\right)\left(2\vec{a}+7\vec{b}\right)$ if $\left|\vec{a}\right|=3$; $\left|\vec{b}\right|=1$, $\vec{a}\perp\vec{b}$.

Solution. Let's find this product:

$$\left(3\vec{a}-5\vec{b}\right)\left(2\vec{a}+7\vec{b}\right)=6\vec{a}^{\,2}+21\vec{a}\cdot\vec{b}-10\vec{b}\cdot\vec{a}-35\vec{b}^{\,2}=$$

$$=6\left|\vec{a}\right|^2-35\left|\vec{b}\right|^2=6\cdot 9-35\cdot 1=19, \text{ because } \vec{a}\cdot\vec{b}=0.$$

Answer: 19.

Example 121. Vectors $\vec{a}=(1;-2;2)$ and $\vec{b}=(2;-2;-1)$ are given. Find $2\vec{a}^{\,2}-4\vec{a}\cdot\vec{b}+5\vec{b}^{\,2}$.

Solution. Let's find $\vec{a}^{\,2}$, $\vec{a}\cdot\vec{b}$ and $\vec{b}^{\,2}$:

$$\vec{a}^{\,2}=1+(-2)^2+2^2=9,$$

$$\vec{a}\cdot\vec{b}=1\cdot 2-2\cdot(-2)+2\cdot(-1)=4,$$

$$\vec{b}^{\,2}=2^2+(-2)^2+(-1)^2=9.$$

So, $2\vec{a}^{\,2}-4\vec{a}\cdot\vec{b}+5\vec{b}^{\,2}=2\cdot 9-4\cdot 4+5\cdot 9=47$.

Answer: 47.

Example 122. At what value m is the vector $\vec{a}=(4;m;-6)$ perpendicular to the vector $\vec{b}=(m;2;-7)$?

Solution. According to the condition of vector perpendicularity, we have the equality:

$$4\cdot m+m\cdot 2-6\cdot(-7)=0, \text{ whence } 6m=-42, \text{ and } m=-7.$$

Answer: −7.

Tasks for individual work

Task 123. Vectors $\vec{a}$ and $\vec{b}$ are mutually perpendicular, and $|\vec{a}| = 5$, $|\vec{b}| = 12$. Find $|\vec{a} + \vec{b}|$ and $|\vec{a} - \vec{b}|$.

Task 124. Vectors $\vec{a} = (-3;4;-1)$, $\vec{b} = (-1;2;3)$ and $\vec{c} = (-4;-2;1)$ are given. Find the vectors $\vec{m} = 4\vec{a} + 3\vec{b} - 2\vec{c}$, $\vec{n} = -5\vec{a} + 4\vec{b} + \vec{c}$ and their lengths.

Task 125. Points $A(2;-6;-3)$ and $B(4;-1;-7)$ are given. Find the vector $\overrightarrow{AB}$, its length and direction cosines.

Task 126. Define the values α and β if the vectors $\vec{a} = (-3;\alpha;9)$ and $\vec{b} = (2;-8;\beta)$ are collinear.

Task 127. It is known that $|\vec{a}| = 5$, $|\vec{b}| = 6$. Find the scalar (or dot) product of the vectors $\vec{a}$ and $\vec{b}$, if the angle between them equals:
1) $45°$; 2) $60°$.

Task 128. Find $(2\vec{a} - 2\vec{b})(2\vec{a} + 7\vec{b})$, if $|\vec{a}| = 3$; $|\vec{b}| = 1$ and $\vec{a} \perp \vec{b}$.

Task 129. Find the scalar (or dot) product of the vectors $\vec{a}$ and $\vec{b}$, the angle between them, if $\vec{a} = (2;-5;4)$ and $\vec{b} = (-1;2;7)$ are given.

Task 130. Define the value m of the vector $\vec{a} = (4;m;-6)$ which is perpendicular to the vector $\vec{b} = (m;2;-7)$.

Task 131. The vertices of the quadrangle are given as $A(1;-2;2)$, $B(1;4;0)$, $C(-4;1;1)$, $D(-5;-5;3)$. Prove that its diagonals AC and BD are perpendicular.

Task 132. Two vectors $\vec{a} = -\vec{i} + 2\vec{j} - \vec{k}$ and $\vec{b} = 2\vec{i} - \vec{j} + 2\vec{k}$ are given. Determine the projections on the coordinate axes of the following vectors: 1) $\vec{a} + \vec{b}$; 2) $\vec{a} - \vec{b}$; 3) $-4\vec{a}$; 4) $-3\vec{a} + 2\vec{b}$.

Task 133. Find projections of the vector $\vec{a} = \overrightarrow{AB} + \overrightarrow{CD}$ on the coordinate axis if $A(2,3,1)$, $B(4,1,-2)$, $C(6,3,7)$, $D(-5,-4,2)$.

Task 134. Find the scalar product of the vectors $\vec{a} = \vec{i} + 2\vec{j} + 2\vec{k}$ and $\vec{b} = -3\vec{i} + 4\vec{k}$ and the angle between them.

Task 135. At what value of *m* are the vectors $\vec{a} = m\vec{i} + 3\vec{j} + 4\vec{k}$ and $\vec{b} = 4\vec{i} + m\vec{j} - 7\vec{k}$ perpendicular?

Task 136. Find $\left(2\vec{a} + 4\vec{b}\right)\cdot\left(2\vec{a} - \vec{b}\right)$ if $|\vec{a}| = 3$, $\left|\vec{b}\right| = 2$, $\vec{a} \perp \vec{b}$.

Task 137. Find such a value α for which the vectors $\vec{a} = \alpha\vec{i} - 7\vec{j} + 5\vec{k}$ and $\vec{b} = 3\vec{i} + \alpha\vec{j} + 4\vec{k}$ are mutually perpendicular.

Task 138. The vertices of the triangle are given as $A(-1;-2;4)$, $B(3;2;-1)$, $C(-4;-2;0)$. Find the angle at the vertex C.

Questions for self-assessment

1. The definition of coordinates of a vector.
2. The length of a vector.
3. The sum and difference rule for vectors.
4. The definition of collinear vectors.
5. The condition of collinear vectors.
6. The definition of perpendicular vectors.
7. The condition of perpendicular vectors.
8. Formulas of coordinates of a middle point.
9. The definition of an opposite vector.
10. The definition of direction cosines.
11. The definition of the projection of a vector onto an axis.
12. The definition of the projection of a vector on a vector.
13. The definition of a scalar product of vectors.
14. The definition of an angle between vectors using a scalar product.
15. Properties of a scalar product of vectors.

3. Tests

Test 1. Planimetry. Basic concepts

Level 1

1. The segment AB is divided into two unequal parts. The distance between the middles of these parts is equal to a. Find the length AB.

A a	**B** $1.5a$	**C** $2a$	**D** $2.5a$	**E** $3a$

2. Find the value of the angle between the bisectors of two adjacent angles.

A $90°$	**B** $135°$	**C** $180°$	**D** $225°$	**E** $270°$

3. Find the value of the angle between the bisectors of two vertical angles.

A $60°$	**B** $90°$	**C** $120°$	**D** $150°$	**E** $180°$

4. One of the two angles obtained at the intersection of two lines is 440 times larger than the other. Find the larger angle.

A $144°$	**B** $133°$	**C** $122°$	**D** $111°$	**E** $100°$

5. One of the adjacent angles is 25 % of the other one. Find this angle.

A $12°$	**B** $18°$	**C** $24°$	**D** $30°$	**E** $36°$

6. At what angle does the clock turn by one minute?

A $15'$	**B** $30'$	**C** $45'$	**D** $60'$	**E** $75'$

7. Find the largest number of the points of intersection of four lines.

A 0	**B** 2	**C** 4	**D** 6	**E** 8

8. Two lines form $40°$ and $60°$ angles with the third one. What angle do the given lines form?

A $60°$	**B** $70°$	**C** $80°$	**D** $90°$	**E** $100°$

9. A ray is drawn from the vertex of the angle MON. It contains $120°$. A point A lying on the ray inside the angle is equidistant from the sides of the angle. Find the angle MOA.

A $30°$	**B** $45°$	**C** $60°$	**D** $75°$	**E** $90°$

10. The sides of an angle MON are intersected by two parallel lines $BC \parallel AD$ at the points A, D, B, C. It is known that $AB = 1$ m, $CD = 1.5$ m, $BO = 8$ m. Find CO.

A 10.5 m	**B** 12 m	**C** 13.5 m	**D** 15 m	**E** 16.5 m

11. A triangle whose perimeter is 24 cm is divided by its altitude into two triangles whose perimeters are 14 and 18 cm. Find the length of this altitude.

A 6 cm	**B** 5.5 cm	**C** 5 cm	**D** 4.5 cm	**E** 4 cm

12. In an isosceles triangle, the base relates to the side as 4 : 7. The difference in their lengths is 6 cm. Find the perimeter of the triangle.

A 36 cm	**B** 37 cm	**C** 38 cm	**D** 39 cm	**E** 40 cm

13. The perimeter of a right-angled triangle is 17 cm. Find the length of the hypotenuse if it is greater than the leg by 2 cm, and the acute angles are equal to each other.

A 8.5 cm	**B** 8 cm	**C** 7.5 cm	**D** 7 cm	**E** 6.5 cm

14. Find the values of the angles of a triangle if they are related as 2 : 3 : 4.

A $30°, 50°, 100°$	**B** $45°, 60°, 75°$	**C** $50°, 60°, 70°$	**D** $35°, 45°, 100°$	**E** $40°, 60°, 80°$

15. The value of the angle at the base of an isosceles triangle is 130 % of the value of the angle at the vertex. Find the angles of the triangle.

A $55°, 50°, 80°$	**B** $65°, 65°, 50°$	**C** $70°, 70°, 40°$	**D** $55°, 55°, 70°$	**E** $45°, 45°, 90°$

16. In a right-angled triangle, the acute angle is equal to $45°$. Find the length of the hypotenuse if the sum of its length and the altitude drawn to it, is 12 cm.

A	6 cm	**B**	7.5 cm	**C**	8 cm	**D**	8.5 cm	**E**	9 cm

17. In an isosceles triangle, the angle between the base and the altitude, which is drawn to the side, is equal to $48°$. Find the angles of the triangle.

A	$42°, 42°$, $96°$	**B**	$36°, 36°$, $108°$	**C**	$46°, 46°$, $88°$	**D**	$40°, 40°$, $100°$	**E**	$44°, 44°$, $92°$

18. The sides of a parallelogram are related as 2 : 5, and its perimeter is 9.8 cm. Find the sides of the parallelogram.

A	1.2 cm, 3 cm	**B**	1 cm, 2.5 cm	**C**	1.4 cm, 3.5 cm	**D**	1.6 cm, 4 cm	**E**	1.5 cm, 3.75 cm

19. The diagonal of a rectangle divides its angle in the ratio 1 : 2. Find the diagonal of the rectangle if the sum of the lengths of both diagonals and the smaller sides is 24 cm.

A	6 cm	**B**	6.5 cm	**C**	7 cm	**D**	7.5 cm	**E**	8 cm

20. In a rectangle, find the angle between the smaller side and the diagonal if it is $30°$ more than the angle between the diagonals resting on the same side.

A	$55°$	**B**	$60°$	**C**	$65°$	**D**	$70°$	**E**	$75°$

21. The angles of a rhombus, formed by the sides of the rhombus with its diagonals, are related as 5 : 4. Find the angles of the rhombus.

A	$75°, 105°$	**B**	$80°, 100°$	**C**	$70°, 110°$	**D**	$65°, 115°$	**E**	$60°, 120°$

22. The altitude of a rhombus is 1 cm, and the perimeter is 8 cm. Find the obtuse angle of the rhombus.

A	$150°$	**B**	$145°$	**C**	$140°$	**D**	$135°$	**E**	$130°$

23. In an isosceles right-angled triangle with a 4 cm leg, a square is inscribed that has a common angle with the triangle. Find the perimeter of the square.

A 6.5 cm	**B** 7 cm	**C** 7.5 cm	**D** 8 cm	**E** 8.5 cm

24. In an isosceles trapezoid, the larger base is 2.7 m, and the side is 1 m. The angle between them is $60°$. Find the smaller base.

A 1.8 m	**B** 1.7 m	**C** 1.6 m	**D** 1.5 m	**E** 1.4 m

25. The altitude of an isosceles trapezoid is 5 cm, the midline is 18 cm, the acute angle $45°$. Find the base of the trapezoid.

A 9 cm, 27 cm	**B** 10 cm, 26 cm	**C** 11 cm, 25 cm	**D** 12 cm, 24 cm	**E** 13 cm, 23 cm

Level 2

26. Calculate the value of each of the adjacent angles if one of them is 50 % larger than the other.

A 72°,108°	**B** 60°,120°	**C** 80°,100°	**D** 36°,144°	**E** 40°,140°

27. Calculate the angle whose value is $\frac{3}{7}$ of the adjacent, one.

A 33°	**B** 126°	**C** 21°	**D** 54°	**E** 72°

28. An angle and two adjacent ones are $230°$. Find this angle.

A 100°	**B** 110°	**C** 120°	**D** 130°	**E** 140°

29. Calculate the values of the adjacent angles, if $\frac{1}{3}$ of one of them is 0.2 of the other.

A 30°,50°	**B** 67,5°, 112,5°	**C** 42°,60°	**D** 61,5°, 102,5°	**E** 45°,75°

30. Find the adjacent angles whose difference is $56°$.

A $44°,100°$	**B** $56°,112°$	**C** $58°,114°$	**D** $60°,116°$	**E** $62°,118°$

31. Two parallel lines form internal one-sided angles with the secant, the values of which relate as $4 : 5$. Find these angles.

A $24°,30°$	**B** $48°,60°$	**C** $80°,100°$	**D** $36°,45°$	**E** $32°,40°$

32. Two angles with perpendicular sides are given. One of them is four times smaller than the other. Find these angles.

A $12°,48°$	**B** $18°,72°$	**C** $24°,96°$	**D** $30°,120°$	**E** $36°,144°$

33. Find two angles with parallel sides if their values are related as $7 : 11$.

A $35°,55°$	**B** $49°,77°$	**C** $63°,99°$	**D** $70°,110°$	**E** $42°,66°$

34. Perpendiculars to the angle sides are drawn through the vertex of the angle equal to $40°$. Determine the obtuse angle between the perpendiculars.

A $95°$	**B** $110°$	**C** $125°$	**D** $140°$	**E** $165°$

35. The sum of two internal intersecting angles at two parallel lines and a secant is equal to $150°$. Find these angles.

A $60°,90°$	**B** $75°,75°$	**C** $70°,80°$	**D** $85°,65°$	**E** $50°,100°$

36. Find the perimeter of an isosceles triangle, if the length of the bisector of the angle at the base is 15 cm and makes 75 % of the length of the side and $\frac{3}{5}$ of the length of the base.

A 61 cm	**B** 62 cm	**C** 63 cm	**D** 64 cm	**E** 65 cm

37. In a triangle ABC, the bisectors of the interior angles B and C form an angle of $116°$ at the intersection. Find the value of the angle A.

A $52°$	**B** $53°$	**C** $54°$	**D** $55°$	**E** $56°$

38. Find the values of the angles of a triangle, if the value of one of them is $\frac{2}{3}$ of the value of the second one and $\frac{4}{5}$ of the value of the third angle.

A	$42°, 66°, 72°$	**B**	$44°, 68°, 78°$	**C**	$46°, 70°, 64°$	**D**	$48°, 72°, 60°$	**E**	$50°, 60°, 70°$

39. In a right-angled triangle, one of the acute angles is equal to $60°$, and the sum of the lengths of the smaller leg and the hypotenuse is 1.8 m. Find the length of the hypotenuse.

A	1.1 m	**B**	1.2 m	**C**	1.3 m	**D**	1.4 m	**E**	1.5 m

40. In an isosceles triangle, the side is 26 cm and the altitude is 13 cm. Find the obtuse angle between the bisectors of the angles at the base.

A	$160°$	**B**	$155°$	**C**	$150°$	**D**	$145°$	**E**	$140°$

41. The angles at the base of the triangle are equal to $65°$ and $37°$. Find the angle between the bisector and the altitude that are drawn from the vertex of the triangle.

A	$14°$	**B**	$15°$	**C**	$16°$	**D**	$13°$	**E**	$12°$

42. The exterior angle of a triangle is equal to $104°$. The internal angles that are not adjacent to it relate as 3 : 5. Find the angles of the triangle.

A	$30°, 74°, 76°$	**B**	$33°, 71°, 76°$	**C**	$36°, 68°, 76°$	**D**	$39°, 65°, 76°$	**E**	$42°, 62°, 76°$

43. The smaller diagonal of a parallelogram is $15\sqrt{3}$ cm, and the acute angle is $60°$. The diagonal of the parallelogram divides the obtuse angle into parts with the ratio 1 : 3. Find the sides of the parallelogram.

A	16 cm, 32 cm	**B**	15 cm, 30 cm	**C**	14 cm, 28 cm	**D**	13 cm, 26 cm	**E**	12 cm, 24 cm

44. The altitude and the smaller diagonal of a rhombus form an angle of $15°$. Find the altitude of the rhombus if its perimeter is 32 cm.

A	6 cm	**B**	5.5 cm	**C**	5 cm	**D**	4.5 cm	**E**	4 cm

45. A right-angled rectangle is inscribed in an isosceles right-angled triangle so that its two vertices are on the hypotenuse and two are on the legs. Find the sides of the rectangle if they are related as 5 : 2, and the length of the hypotenuse is 45 cm.

A 25, 10 cm	**B** 25, 10 cm and 18.75, 7.5 cm	**C** 30, 20 cm	**D** 20, 8 cm and 24, 10 cm	**E** 18.75, 7.5 cm

46. A square is given whose side is 1 m, and the diagonal is the side of the second square. Find the diagonal of the second square.

A 2.4 m	**B** 2.2 m	**C** 2 m	**D** 1.8 m	**E** 1.6 m

47. In an isosceles trapezoid, the bases are 25 and 16 cm. The diagonal of the trapezoid divides its acute angle in half. Find the perimeter of the trapezoid.

A 69 cm	**B** 70 cm	**C** 71 cm	**D** 72 cm	**E** 73 cm

48. The smaller base of an isosceles trapezoid is equal to the side, and the diagonal is perpendicular to the side. Find the angles of the trapezoid.

A 60°,120°	**B** 30°,150°	**C** 45°,135°	**D** 50°,130°	**E** 55°,125°

49. The altitude of an isosceles trapezoid is 10 cm, and the diagonals are mutually perpendicular. Find the midline of the trapezoid.

A 7 cm	**B** 8 cm	**C** 9 cm	**D** 10 cm	**E** 11 cm

50. The perimeter of a triangle is 27 cm, and the lengths of its sides relate as 3 : 2 : 4. Find the lengths of the sides of the triangle formed by the midlines of the given triangle.

A 3, 2, 4 cm	**B** 4.5, 3, 6 cm	**C** 3.6, 2.4, 4 cm	**D** 3.3, 2.2, 4.4 cm	**E** 3.9, 2.6, 5.2 cm

Level 3

51. The angles ABC and CBD are adjacent. The acute angle CBD is equal to α. Determine the acute angle between the perpendicular drawn from the point B to the line AD and the bisector of the angle CBD.

52. Two angles with parallel sides are given, one of which is larger than the other by $90°$. Find the larger angle in degrees.

53. Two angles with perpendicular sides are given. One of them is smaller than the other by $70°$. Find the larger angle.

54. Two parallel lines intersect the third one. The bisector of the inner angle at one of the parallel lines intersects the other at an angle of $42°$. Find the obtuse angle between the bisector and the third line.

55. The difference between two internal one-sided angles at two parallel lines and the secant is equal to $30°$. Find the acute angle.

56. Two parallel lines intersect the sides of an angle MON at points A, B, C, D $(BC \parallel AD)$. Find the length of CD, if it is known that, $AB : BO =$ $= 17 : 9$, $CD - CO = 1.6$ m.

57. Find an angle that is n times larger than its adjacent angle.

58. Two angles with a common vertex form an extended angle. Find the smaller of the angles if it is known that 40 % of one angle is 60 % of the other.

59. The angle A is divided into two angles α and β. Find the difference between these angles if we know that $\angle A = 65°12'$ and $\alpha : \beta = 5 : 3$.

60. A $344°$ angle is divided by two rays, emanating from its vertex, into three angles whose values are inversely proportional to the numbers, 8, 4 and $1\frac{3}{7}$. Find the larger of these angles.

61. The bisector of the angle at the vertex form a $104°$ angle with the base $104°$ and is equal to one of the sides. Find the angles of the triangle.

62. In the triangle ABC, the external angle at the vertex B is three times greater than the angle A and greater than the angle C. Find the angles of the triangle.

63. In a rectangle, the bisector divides the opposite side in the ratio 1 : 3. Find the lengths of the sides of the rectangle if its perimeter is 56 cm.

64. In a trapezoid $ABCD$, the sides AB and CD are extended to the mutual intersection at the point M. Find the length CD if $AB - BM$ = 17 : 9 and $CD - CM$ = 1.6 m.

65. In the triangle ABC, the bisector of the angle A divides the side BC into parts whose ratio is 2 : 3. Find the lengths of the sides that contain the angle A if their sum is 38 cm.

66. The sides of three angles are respectively parallel. Find the value of each of them, provided that one of the angles is equal to the sum of the other two.

67. Find the interior angles of a triangle, knowing that one of them is equal to half the sum of the other two, and the other is equal to one third of the sum of the other two angles.

68. The bisector of an angle at the vertex of a triangle forms an angle of $82°$ with the base, and an angle of $55°$ with the bisector of one of the angles at the base. Find the angles of the triangle.

69. In an isosceles triangle, the altitude drawn to the side divides in half the angle between the base and the bisector of the angle at the base. Find the angles of the triangle.

70. Two altitudes of a parallelogram intersect one of the diagonals at angles $65°$ and $78°$. Find the angles of the parallelogram.

71. Two altitudes of a rhombus drawn from the vertices of obtuse angles, intersect and are divided in relation 1 : 2. Find the angles of the rhombus.

72. The lengths of the sides of a triangle are 11 cm, 13 cm and 12 cm. Find the length of the median drawn to the larger side.

73. The median BD is drawn in the ABC triangle. It is known that, $BD = AB \cdot \frac{\sqrt{3}}{4}$, and $\angle DBC = \frac{\pi}{2}$. Find the angle ABD.

74. Find the angles of a triangle in which the altitude and the median, which are drawn from one vertex, divide the angle at this vertex into three equal parts.

75. A parallelogram with an acute angle of $60°$ is given. Find the ratio of the lengths of the sides of the parallelogram if the ratio of the squares of the lengths of the diagonals is equal to $\frac{1}{3}$.

Test 2. Triangles

Level 1

1. The segments AB and CD intersect at the point O, which is the middle point of each of them. It is known that $\angle AOC = 80^\circ$. Indicate which of the triangles is equal to $\Delta\ AOD$.

A ΔAOC	**B** ΔBOC	**C** ΔDOB	**D** ΔCOA	**E** ΔCOB

2. Perpendicular segments MN and KP intersect at the point O, which is the middle point of the segment KR. It is known that $MO = 10$ cm, $ON = 9$ cm. Indicate which of the triangles is equal to ΔMOK.

A ΔKON	**B** ΔNOK	**C** ΔPOM	**D** ΔMOP	**E** ΔNOP

3. In the triangle ABC, the segment AK (point K belongs to the side BC) intersects the median BD at a point O so that $BO = OD$. A straight line DM parallel to the segment AK (point M belongs to BC) is drawn through the point D. Indicate which of the segments is equal to BK.

A BO	**B** KM	**C** MC	**D** OD	**E** MD

4. In the triangle ABC, points K, L, M belong to the sides AB, BC, AC, respectively, and the segment KL is parallel to the side AC, the segment LM is parallel to AB. It is known that $BK : KA = m : n$. Find the ratio of $LM : AB$.

A $\frac{m}{n}$	**B** $\frac{n}{m}$	**C** $\frac{m}{m+n}$	**D** $\frac{n}{m+n}$	**E** 1

5. Determine the type of triangle with sides 1, 2, 3.

A rectangular	**B** acute	**C** obtuse	**D** isosceles	**E** another answer

6. Indicate which two triangles will be similar if the sides of the triangles are equal to:

A 4, 5, 6 and 8, 10, 12	**B** 5, 6, 7 and 10, 14, 18	**C** 4, 5, 6 and 6, 10, 14	**D** 6, 7, 8 and 12, 14, 18	**E** 4, 5, 6 and 4, 8, 12

7. What is the acute angle between two altitudes of an equilateral triangle?

A $90°$	**B** $45°$	**C** $60°$	**D** $120°$	**E** $75°$

8. From an arbitrary point on the side of a right-angled triangle, the perpendiculars on the other two sides are drawn. Find the angle between these perpendiculars.

A $90°$	**B** $45°$	**C** $60°$	**D** $120°$	**E** $75°$

9. In a right-angled triangle, the acute angle is $45°$, and the hypotenuse is c. Find the altitude drawn from the vertex of the right angle.

A $\frac{c}{3}$	**B** $\frac{2c}{3}$	**C** $\sqrt{2}c$	**D** $\frac{c}{\sqrt{2}}$	**E** $\frac{c}{2}$

10. The hypotenuse of a right-angled triangle is equal to c. The bisector of the angle external to one of the acute angles of the triangle forms an angle of $60°$ with the extension of the leg. Find this leg.

A $\frac{c}{3}$	**B** $\frac{2c}{3}$	**C** $\sqrt{2}c$	**D** $\frac{c}{\sqrt{2}}$	**E** $\frac{c}{2}$

11. A right-angled triangle ABC is given, in which AC = 8 m, BC = 15 m, AB = 17 m. The sides AC, CB and AB were reduced three times, and a new triangle $A_1B_1C_1$ was obtained. Find $\cos\angle B_1$.

A $\frac{8}{15}$	**B** $\frac{8}{17}$	**C** $\frac{15}{17}$	**D** $\frac{5}{8}$	**E** $\frac{5}{17}$

12. In a right-angled triangle ABC the hypotenuse AB = 20 mm, the leg BC = 12 mm. Find the perimeter of the triangle.

A 32 mm	**B** 48 mm	**C** 38 mm	**D** 36 mm	**E** 40 mm

13. The altitude of a triangle is equal to $\sqrt{2}$ cm. Calculate the distance from the vertex of the triangle to the line that runs parallel to the base of the triangle and divides its area in half.

A $\frac{\sqrt{3}}{2}$ cm	**B** $\frac{\sqrt{2}}{2}$ cm	**C** $\sqrt{2}-1$ cm	**D** 1 cm	**E** $\frac{\sqrt{2}}{3}$ cm

14. In a right-angled triangle, the hypotenuse is equal to c, and the acute angle is equal to α. Find the area of the triangle.

A $\frac{c^2}{4}\sin\alpha$	**B** $\frac{c^2}{4}\sin 2\alpha$	**C** $\frac{c^2}{2}\sin\alpha$	**D** $\frac{c^2}{2}\sin 2\alpha$	**E** $c^2\sin 2\alpha$

15. In a right-angled triangle, the altitude is drawn from the vertex of the right angle. How many pairs of similar triangles were formed?

A 1	**B** 2	**C** 3	**D** 4	**E** 5

16. In an isosceles triangle, the angle at the base is $45°$, and the base is 9 cm higher than the altitude. Find the base of the triangle.

A 20 cm	**B** 18 cm	**C** 22 cm	**D** $9\sqrt{2}$ cm	**E** 15 cm

17. The angle at the vertex of an isosceles triangle is equal to β. Express the value of the angle α at the base.

A $180° - 2\beta$	**B** $\frac{1}{2}(90° + \beta)$	**C** $90° + \frac{\beta}{2}$	**D** $90° - \frac{\beta}{2}$	**E** $\frac{1}{2}(90° - \beta)$

18. In the isosceles triangle ABC with the base AB, a bisector BD is drawn. It is known that $\angle ADB = 120°$. Find the angle at the apex.

A 20°	**B** 40°	**C** 60°	**D** 80°	**E** 100°

19. The base of an isosceles triangle is 12 cm, and the side is 10 cm. Find the area of the triangle.

A 96 cm²	**B** 48 cm²	**C** 120 cm²	**D** 60 cm²	**E** 72 cm²

20. The angle at the vertex B of the isosceles triangle ABC is equal to 2β, and the side BC of the triangle is equal to a. Find the base of the triangle.

A $2a\cos\beta$	**B** $a\sin 2\beta$	**C** $2a\sin\beta$	**D** $a\sin\beta$	**E** $a\cos 2\beta$

21. The bisectors of two angles of a triangle intersect at an angle of 121°. Find the value of the third angle of the triangle.

A 62°	**B** 70°	**C** 68°	**D** 59°	**E** 61°

22. In a triangle, the bisector of the angle at the vertex forms an angle of $98°$ with the base and is equal to one of the sides. Find the angle at the vertex of the triangle from which the bisector is drawn.

A $30°$	**B** $32°$	**C** $34°$	**D** $33°$	**E** $22°$

23. In a triangle, the median is twice the side to which it is drawn. Find the value of the largest angle of the triangle.

A $80°$	**B** $75°$	**C** $110°$	**D** $90°$	**E** $100°$

24. The angles at the base of a triangle are $65°$ and $37°$. Find the value of the angle between the altitude and the bisector which is drawn to the base of the triangle.

A $22°$	**B** $20°$	**C** $18°$	**D** $16°$	**E** $14°$

25. It is known that in the triangle ABC, $BC = 2\sqrt{2}$ cm, $\angle B = 105°$, $\angle C = 30°$. Find AB.

A $\frac{\sqrt{2}}{2}$ cm	**B** $2\sqrt{2}$ cm	**C** 2 cm	**D** 4 cm	**E** $4\sqrt{2}$ cm

Level 2

26. On the side of a right-angled triangle, choose a point that is at a distance of 3 cm and 4 cm from the ends of the side vertices; from this point perpendiculars are drawn to the other two sides. Calculate the sum of the distances from the bases of these perpendiculars to the third vertex of the triangle.

A 9.5 cm	**B** 10.5 cm	**C** 12.5 cm	**D** 20.5 cm	**E** 27.75 cm

27. On the side of a right-angled triangle, a point is given that is at a distance of 3 cm and 4 cm from the ends of the side (vertices); from this point perpendiculars are drawn to the other two sides. Calculate the length of the segment that connects the bases of the perpendiculars.

A $\sqrt{9.5}$cm	**B** $\sqrt{10.5}$cm	**C** $\sqrt{12.5}$cm	**D** $\sqrt{20.5}$cm	**E** $\sqrt{27.75}$cm

28. The area of an equilateral triangle constructed on the hypotenuse of a right-angled triangle is twice the area of the latter. Determine the angles of the right-angled triangle.

A 30°, 30°, 90°	**B** 30°, 60°, 90°	**C** 45°, 45°, 90°	**D** 60°, 60°, 90°	**E** 30°, 45°, 90°

29. In a right-angled triangle, one of the angles is $30°$, and the opposite leg is equal to 3.5 cm. Calculate the difference in the lengths of the segments into which a perpendicular dropped from the vertex of the right angle divides the hypotenuse.

A $7\sqrt{2}$ cm	**B** 3.5 cm	**C** $12\sqrt{3}$ cm	**D** 7 cm	**E** 4.5 cm

30. Find the area of a right-angled triangle, one of the legs of which is 15 cm, and the altitude dropped on the hypotenuse is 12 cm.

A 75 cm²	**B** 100 cm²	**C** 125 cm²	**D** 150 cm²	**E** 200 cm²

31. The medians of a right-angled triangle drawn to the legs relate as $\sqrt{2}:1$. Find the angles of the triangle.

A $\frac{\pi}{2}, \mathrm{arctg}2, \frac{\pi}{2} - \mathrm{arctg}2$	**B** $\frac{\pi}{2}, \mathrm{arctg}\sqrt{\frac{2}{7}}, \frac{\pi}{2} - \mathrm{arctg}\sqrt{\frac{2}{7}}$	**C** $\frac{\pi}{2}, \frac{\pi}{3}, \frac{\pi}{6}$
D $\frac{\pi}{2}, \frac{\pi}{4}, \frac{\pi}{4}$	**E** $\frac{\pi}{2}, \mathrm{arctg}\frac{7}{2}, \frac{\pi}{2} - \mathrm{arctg}\frac{7}{2}$	

32. In a right-angled triangle, the legs are related as 3 : 4, and the altitude divides the area of the triangle into parts, the difference between which is 84 dm². Find the area of the triangle.

A 100 dm²	**B** 120 dm²	**C** 150 dm²	**D** 170 dm²	**E** 300 dm²

33. In a right-angled triangle, a point on the hypotenuse, which is equidistant from both legs, divides the hypotenuse into segments 40 cm and 30 cm long. Find the larger leg.

A 54 cm	**B** 60 cm	**C** 56 cm	**D** 40 cm	**E** 55 cm

34. In a right-angled triangle, the bisector of the right angle divides the hypotenuse at the ratio $p : q$. In what ratio is the hypotenuse divided by the base of the altitude drawn to it?

A $\frac{p^2}{q}$	**B** $\frac{q^2}{p}$	**C** $\frac{p}{q^2}$	**D** $\frac{p^2}{q^2}$	**E** $\frac{q}{p}$

35. Find the area of an isosceles triangle if its base is 12 cm and the altitude lowered to the base is a segment connecting the middle of the base and the side.

A $7\sqrt{2}$ cm²	**B** 3.5 cm²	**C** $12\sqrt{3}$ cm²	**D** 7 cm²	**E** 4.5 cm²

36. The perimeter of an isosceles triangle is 50 cm. The side of the triangle is 1 cm larger than the base. Find the area of the triangle.

A 100 cm²	**B** 120 cm²	**C** 150 cm²	**D** 170 cm²	**E** 300 cm²

37. In an isosceles triangle, the base is 30 cm and the altitude is 20 cm. Determine the length of the altitude that is lowered to the side.

A 16 cm	**B** 18 cm	**C** 20 cm	**D** 22 cm	**E** 24 cm

38. In an isosceles triangle, the angle at the base contains 72°, and the bisector of this angle has the length m. Find the lengths of the sides of the triangle.

A $\frac{m}{2}$, $m(\sqrt{5}+1)$	**B** m, $\frac{m(\sqrt{5}+1)}{2}$	**C** $2m$, $m\sqrt{5}$	**D** m, $\frac{m(\sqrt{5}-1)}{2}$	**E** $2m$, $\frac{m(\sqrt{5}+1)}{2}$

39. The altitude AD, which is lowered to the side BC of the isosceles triangle ABC, divides it into triangles ABD and ADC with an area of 4 cm² and 2 cm², respectively. Find the sides of the triangle if AC is its base.

A $\frac{2\sqrt{6}}{\sqrt[4]{5}}; \frac{6}{\sqrt[4]{5}}; \frac{6}{\sqrt[4]{5}}$	**B** $\frac{2\sqrt{6}}{\sqrt[4]{5}}; \frac{2\sqrt{6}}{\sqrt[4]{5}}; \frac{6}{\sqrt[4]{5}}$	**C** $\frac{2\sqrt{5}}{\sqrt[4]{6}}; \frac{5}{\sqrt[4]{6}}; \frac{5}{\sqrt[4]{6}}$	**D** $\frac{2\sqrt{5}}{\sqrt[4]{6}}; \frac{2\sqrt{5}}{\sqrt[4]{6}}; \frac{5}{\sqrt[4]{6}}$	**E** $\frac{6}{2\sqrt[4]{5}}; \frac{\sqrt{6}}{\sqrt[4]{5}}; \frac{\sqrt{6}}{\sqrt[4]{5}}$

40. Two sides of a triangle are $6\sqrt{2}$ cm and 10 cm, respectively, and the angle opposite the second of them is 45°. Find the area of the triangle.

A 12 cm²	**B** 21 cm²	**C** 32 cm²	**D** 23 cm²	**E** 42 cm²

41. The sides of a triangle are 13, 14 and 15 cm. Calculate the altitude of the triangle, which is lowered to the side equal to 14 cm.

A 12 cm	**B** 21 cm	**C** 32 cm	**D** 23 cm	**E** 42 cm

42. In a right-angled triangle ABC $\angle C = 90°$, CD is the altitude, $AC =$ $= 4$ cm, $BD = 6$ cm. Find AD.

A 2 cm	**B** 3 cm	**C** 4 cm	**D** 5 cm	**E** 6 cm

43. The sides of a triangle a, b, c satisfy the condition: $(b + c + a)(b + c - a) = 3bc$. Find the angle between the sides b and c.

A 30°	**B** 45°	**C** 60°	**D** 90°	**E** 120°

44. In the triangle ABC, the angle A is twice as large as the angle B. Find the third side if $AC = b$ and $AB = c$.

A $\sqrt{c(b+c)}$	**B** $\sqrt{b(c-b)}$	**C** $\sqrt{c(b-c)}$	**D** $\sqrt{b(b-c)}$	**E** $\sqrt{b(b+c)}$

45. In the triangle ABC, the angle A is twice as large as the angle B, and the sides opposite to these angles are 12 cm and 8 cm, respectively. Find the length of the third side.

A 8 cm	**B** 9 cm	**C** 10 cm	**D** 11 cm	**E** 12 cm

46. The sides of a triangle are equal to $\sqrt{34}, \sqrt{45}, \sqrt{61}$. Find the area of the triangle.

A $\frac{3}{2}\sqrt{161}$	**B** $\frac{3}{4}\sqrt{161}$	**C** $\frac{\sqrt{161}}{2}$	**D** $\frac{\sqrt{161}}{3}$	**E** $\frac{2}{3}\sqrt{161}$

47. The medians of a triangle are equal to $\sqrt{73}, \sqrt{52.5}$ cm. Find the area of the triangle.

A 24 cm²	**B** 36 cm²	**C** 60 cm²	**D** 48 cm²	**E** 72 cm²

48. It is known that in the triangle ABC, $BC = 2\sqrt{2}$ cm, $\angle A = 105°$, $\angle C = 30°$. Find AC.

A	B	C	D	E
$\frac{\sqrt{2}}{2}\left(\sqrt{3}-1\right)\text{cm}$	$2\sqrt{2}\left(\sqrt{3}-1\right)\text{cm}$	$2\sqrt{2}\left(\sqrt{3}+1\right)\text{cm}$	$\sqrt{3}-1\text{cm}$	$4\sqrt{2}\,\text{cm}$

49. In the triangle ABC, the bisector AD divides the side CC in the ratio $BD : CD = 2 : 1$. In what ratio does the median CE divide this bisector?

A $1 : 1$	**B** $2 : 1$	**C** $3 : 1$	**D** $1 : 2$	**E** $3 : 2$

Level 3

50. In the triangle ABC, the point E belongs to the median BD, and $BE = 3ED$. The line AE intersects the side BC at the point M. Find the ratio of the areas of the triangles AMC and ABC.

51. Inside a right-angled triangle, a point is chosen so that its distances to the vertices of the triangle are equal to $\sqrt{52}, \sqrt{52}$ and $2\sqrt{3}$ cm. Calculate the area of the triangle.

52. From the vertex A of an equilateral triangle ABC, a ray is drawn that intersects the side BC, and some point M is chosen on it. It is known that $\angle AMB = 20°$ and $\angle AMC = 30°$. Find the angle MAB.

53. In a right-angled triangle, the medians drawn to the legs are equal to $\sqrt{52}$ and $\sqrt{73}$ cm. Find the radius of the circle circumscribed around the triangle.

54. A circle inscribed in a right-angled triangle divides the hypotenuse in the ratio 3 : 6. Find the perimeter of the triangle if the center of the circle is distant from the vertex of the right angle by $\sqrt{8}$ cm.

55. In a right-angled triangle, the hypotenuse is a, and the bisector of one of the acute angles is $\frac{a}{\sqrt{3}}$. Find the legs.

56. The altitude and bisector of a right-angled triangle, which are drawn from the vertex of the right angle, are equal to 3 and 4. Find the area of the triangle.

57. The median AM and the bisector CD of a right-angled triangle ABC ($\angle B = 90°$) intersect at the point O. Find the area of the triangle if CO = 9 cm, OD = 5 cm.

58. In an isosceles triangle, the side is one and a half times larger than the base a. Find the length of the segment that connects the points of intersection of the bisectors of the angles at the base with the sides.

59. The base of an isosceles triangle is one and a half times smaller than the side b. Altitudes are drawn to the sides of the triangle. Calculate the length of the segment whose ends coincide with the bases of the altitudes.

60. The isosceles triangle ABC ($\angle C = 90°$) and the triangle DEF are located so that the point D lies on the side AB, and the point E is on the extension of the side AB behind the point A. The segment KL is the midline in both triangles, and the area of the quadrilateral $DKLB$ is $\frac{5}{8}$ of the area of the triangle ABC. Find the angle DEF.

61. A triangle with a perimeter of 24 cm is inscribed in a triangle with the sides of 10, 17 and 21 cm so that one of its sides lies on the larger side. Find the difference between the lengths of the larger and smaller sides of the rectangle.

62. A circle of a 3 cm radius is inscribed in a triangle. Calculate the perimeter of the triangle if one of its sides is divided by the point of tangency into the segments of lengths 4 and 3 cm.

63. In a triangle with the sides of 10, 17 and 21 cm, a rectangle with a perimeter of 24 cm is inscribed so that one of its sides lies on the larger side. Find the difference between the lengths of the larger and smaller sides of the rectangle.

64. Determine the sides of a triangle if the median and altitude drawn from the vertex of one angle divide this angle into three equal parts. The median is 10 cm.

65. Two sides of a triangle are 6 cm and 8 cm, respectively. The medians drawn to these sides are perpendicular. Find the area of the triangle.

66. The altitude, base and sum of the sides of a triangle are 12 cm, 14 cm and 28 cm, respectively. Find the sides.

67. In a triangle ABC with the area of 90 cm^2, the bisector AD divides the side BC into segments BD and CD, and $BD : CD = 2 : 3$. The segment

BL intersects AD at the point E and divides the side AC into segments AL and CL such that $AL : CL = 1 : 2$. Find the area of the quadrilateral $EDCL$.

68. In the triangle ABC, the bisectors AD and BE intersect at the point O. Find the ratio of the area of the quadrilateral $DOEC$ to the area of the triangle ABC, if $AC : AB : BC = 4 : 3 : 2$.

69. In the triangle ABC, the points K and N are the midpoints of the sides AB and AC, respectively. A line is drawn through the vertex B, which intersects the side AC at the point F, and the segment KN at the point L so that $KL : LN = 3 : 2$. Determine the area of the quadrilateral $AKLF$ if the area of the triangle ABC is 40.

70. Point X divides the side AB of the triangle ABC in the ratio 1 : 2. The point Y lies on the side AC, and the segment BY is divided by the segment XC in the ratio 5 : 2. In what ratio does the point Y divide the side AC?

71. In the triangle ABC, the bisector BE and the median AD are perpendicular and have the same length, which is 4. Find the sides of the triangle.

72. In the triangle ACC ($AC = 14$, $AC = 15$, $CC = 13$) straight parallel lines AC and CC, which intersect CC and AC at points M and N, are drawn through the base of the altitude CH. The line MN intersects the continuation of the side AC at point D. Find the length of the segment BD.

73. In the acute triangle ABC, point M at the altitude AD, and point N at the altitude BP are taken so that the angles BMC and ANC are straight. The distance between the points M and N is equal to $4+2\sqrt{3}$, $\angle MCN = 30°$. Find the bisector CL of the triangle CMN.

74. In the triangle KLM, the length of the side KL is 27, the length of the bisector of the line KN is 24, and the length of the segment MN is 8. Determine the perimeter of the triangle KMN.

75. The segments that connect the bases of the altitudes of an acute-angled triangle are 5, 12, and 13. Find the area of the triangle.

Test 3. Quadrilaterals

Level 1

1. The diagonal of the square is d . Find the side of the square.

A $2\sqrt{d}$	**B** $\frac{d}{2}$	**C** $\sqrt{2d}$	**D** $\frac{\sqrt{2}}{2}d$	**E** $\sqrt{2}d$

2. The area of a square is S. Find the diagonal of the square.

A $\sqrt{2}S$	**B** $\sqrt{2S}$	**C** $2\sqrt{S}$	**D** $\frac{\sqrt{S}}{2}$	**E** $\sqrt{\frac{S}{2}}$

3. The diagonal of a square is 4 m. Its side is equal to the diagonal of another square. Find the side of the last square.

A 2 m	**B** $2\sqrt{2}$ m	**C** 4 m	**D** $\frac{\sqrt{2}}{2}$ m	**E** $4\sqrt{2}$ m

4. A square is inscribed in an isosceles right-angled triangle so that its two vertices are on the hypotenuse and the other two are on the legs. Find the side of the square if it is known that the hypotenuse is 3 cm.

A 1.5 cm	**B** 2 cm	**C** 1 cm	**D** 2.5 cm	**E** 3 cm

5. The diagonal of a rectangle is twice as large as one of its sides. Calculate the value of the acute angle between the diagonals of the rectangle.

A 45°	**B** 65°	**C** 40°	**D** 80°	**E** 60°

6. The smaller side of the rectangle is 8.5 cm; the bisector of its angle divides the larger side into two equal segments. Calculate the perimeter of the rectangle.

A 49 cm	**B** 51 cm	**C** 60 cm	**D** 55 cm	**E** 53 cm

7. In a rectangle, the point of intersection of the diagonals is 4 cm farther from the smaller side than from the larger side. The perimeter of the rectangle is 56 cm. Find the sides of the rectangle.

A	**B**	**C**	**D**	**E**
14 cm, 14 cm	13 cm, 15 cm	11 cm, 17 cm	12 cm, 16 cm	10 cm, 18 cm

8. Two mutually perpendicular chords are drawn from one point of the circle, which are 6 and 10 cm away from the center. Find their lengths.

A	**B**	**C**	**D**	**E**
6 cm, 10 cm	6 cm, 12 cm	12 cm, 20 cm	10 cm, 20 cm	10 cm, 12 cm

9. In a right-angled triangle, each leg of which is equal to 6 cm, a rectangle is inscribed that has a common angle with the triangle. Find the perimeter of the rectangle.

A 12 cm	**B** 18 cm	**C** 24 cm	**D** 20 cm	**E** 30 cm

10. The perimeter of a rhombus is 0.8 dm, and its altitude is 0.1 dm. Calculate the obtuse angle of the rhombus.

A 150°	**B** 140°	**C** 120°	**D** 110°	**E** 100°

11. The altitude of a rhombus divides its side in half. Calculate the angle between the altitudes of the rhombus, which are drawn from the vertex of the obtuse angle.

A 20°	**B** 30°	**C** 40°	**D** 50°	**E** 60°

12. The angles formed by the diagonals of the rhombus on one of its sides are related as 4 : 5. Find the acute angle of the rhombus.

A 50°	**B** 60°	**C** 70°	**D** 80°	**E** 90°

13. In a rhombus, one of the diagonals is equal to the side. Find the angles of the rhombus.

A	**B**	**C**	**D**	**E**
45° and 135°	60° and 120°	30° and 150°	90° and 90°	80° and 100°

14. The perimeter of a parallelogram is equal to P. One of the sides is equal to a. Find the adjacent side of the parallelogram.

A $\frac{P}{2}-a$	**B** $\frac{P}{2}+a$	**C** $P-\frac{a}{2}$	**D** $P+\frac{a}{2}$	**E** $P-2a$

15. The acute angle of a parallelogram is equal to $40°$. Find the acute angle between the altitudes of the parallelogram, which are drawn from the vertex of the obtuse angle.

A $20°$	**B** $30°$	**C** $40°$	**D** $50°$	**E** $60°$

16. How many parallelograms can be constructed with vertices at three given points that do not lie on one line?

A 0	**B** 1	**C** 2	**D** 3	**E** 4

17. The leg of an isosceles triangle is 5 m. From a point on the base of this triangle, two lines are drawn parallel to the sides. Find the perimeter of the obtained parallelogram.

A 8 m	**B** 9 m	**C** 10 m	**D** 11 m	**E** 12 m

18. The perimeter of the parallelogram $ABCD$ is 10 cm. Find the length of the diagonal BD, if it is known that the perimeter of the triangle ABD is 8 cm.

A 5 cm	**B** 4 cm	**C** 3 cm	**D** 2 cm	**E** 1 cm

19. In an isosceles trapezoid, the smaller base is a, the leg is b, and the angle between them is $120°$. Find the larger base of the trapezoid.

A $b+2a$	**B** $b-2a$	**C** $a+b$	**D** $b-a$	**E** $a-b$

20. One of the bases of a trapezoid is a, and the middle line is m. Express the value of the second base b.

A $b=m+a$	**B** $b=2m-a$	**C** $b=2a-m$	**D** $b=2a+m$	**E** $b=m-2a$

21. What are the angles of an isosceles trapezoid equal to if it is known that the difference of opposite angles is equal to $40°$?

A	**B**	**C**	**D**	**E**
$70°$ and $110°$	$60°$ and $120°$	$45°$ and $135°$	$30°$ and $150°$	$20°$ and $160°$

22. In an isosceles trapezoid, the altitude drawn from the vertex of the obtuse angle divides the larger base into segments of 6 and 30 cm. Find the smaller base of the trapezoid.

A 30 cm	**B** 28 cm	**C** 26 cm	**D** 24 cm	**E** 22 cm

23. The bases of a trapezoid are related as 2 : 3, and the middle line is 5 m. Find the sum of the bases of the trapezoid.

A 7 m	**B** 8 m	**C** 9 m	**D** 10 m	**E** 12 m

24. The smaller base of an isosceles trapezoid is equal to the side, and the diagonal is perpendicular to the side. Find the angles of the trapezoid.

A	**B**	**C**	**D**	**E**
$70°$ and $110°$	$60°$ and $120°$	$45°$ and $135°$	$30°$ and $150°$	$20°$ and $160°$

25. The diagonals of a quadrilateral are a and b. Find the perimeter of the quadrilateral whose vertices are the midpoints of the sides of this quadrilateral.

A $a+b$	**B** $a-b$	**C** $2a-b$	**D** $2a+b$	**E** $2a+2b$

Level 2

26. The sides of a square are divided in the ratio of 3 : 2, and each vertex is adjacent to one large and one small segment. The dividing points are joined in series by segments. Find the area of the resulting quadrilateral if the side of this square is 5 cm.

A 12 cm^2	**B** 13 cm^2	**C** 14 cm^2	**D** 15 cm^2	**E** 16 cm^2

27. A second square is inscribed in a first square, the vertices of which lie on the sides of the first square, and the sides are inclined at an angle of $30°$ to the sides of the first square. What part of the area of this square is the area inscribed?

A $3+2\sqrt{3}$	**B** $2+3\sqrt{2}$	**C** $2(2-\sqrt{3})$	**D** $2(\sqrt{3}+2)$	**E** $3(2-\sqrt{3})$

28. On the side NP of the square $MNPQ$, point A is taken; on the side PQ, point B is taken so that $NA : AP = PB : BQ = 2 : 3$. The point L is the point of intersection of the segments MA and NB. In what ratio does the point L divide the segment MA?

A 5 : 4	**B** 25 : 4	**C** 5 : 2	**D** 25 : 3	**E** 4 : 3

29. A rectangle is inscribed in a square with the area of 18 cm^2 so that one vertex of the rectangle lies on each side of the square. The lengths of the rectangle are related as 1 : 2. Find the area of the rectangle.

A $4\sqrt{2}$ cm²	**B** 4 cm²	**C** 9 cm²	**D** 8 cm²	**E** $9\sqrt{2}$ cm²

30. The side of a rectangle and its diagonal are related as 3 : 5, and the other side is 8 cm. Calculate the area of the rectangle.

A 48 cm²	**B** 50 cm²	**C** 52 cm²	**D** 54 cm²	**E** 56 cm²

31. The sides of the rectangle are related as 1 : 2. Parallel lines are drawn through both pairs of its opposite vertices, which form a rectangle with the sides of 10 and 11 m. Find the area of the given rectangle.

A 48 m²	**B** 50 m²	**C** 52 m²	**D** 54 m²	**E** 56 m²

32. In the rectangle $ABCD$, on the sides $AB = 6$ and $BC = 8$, points M and N are taken so that the segment MN is parallel to the segment AC. It is known that the perimeter of the polygon $AMNCD$ and the perimeter of the triangle MBN are related as 7 : 3. Find the length of the segment MN.

A 4	**B** $\frac{7}{6}$	**C** 5	**D** $\frac{10}{3}$	**E** $\frac{14}{3}$

33. In the rectangle $ABCD$, the sides $AB = a$, $AD = b$ $(a > b)$ are given. On the side AB point E is taken so that $\angle CED = \angle AED$. Find the length of the segment AE.

A $a-\sqrt{a^2-b^2}$	**B** $a+\sqrt{a^2-b^2}$	**C** $a-\sqrt{a^2+b^2}$	**D** $a+\sqrt{a^2+b^2}$	**E** $a-\sqrt{b^2-a^2}$

34. Find the area of a rhombus whose perimeter is 120 m, and one of the diagonals is 36 m.

A 800 m^2	**B** 864 m^2	**C** 784 m^2	**D** 860 m^2	**E** 900 m^2

35. A rhombus with an acute angle α is given. What part of the rhombus is the area of a circle inscribed in its area?

A $\frac{\pi}{3}\sin\alpha$	**B** $\frac{\pi}{4}\sin\alpha$	**C** $\frac{\pi}{3}\sin 2\alpha$	**D** $\frac{\pi}{3}\cos\alpha$	**E** $\frac{\pi}{4}\cos\alpha$

36. The lengths of the smaller diagonal, the side and the larger diagonal of a rhombus form a geometric progression. Find the angles of the rhombus.

A	**B**	**C**	**D**	**E**
$\mathrm{arctg}\left(2-\sqrt{3}\right)$, $\pi-\mathrm{arctg}\left(2-\sqrt{3}\right)$	$\mathrm{arctg}\sqrt{3}$, $\pi-\mathrm{arctg}\sqrt{3}$	$2\mathrm{arctg}\left(2-\sqrt{3}\right)$, $\pi-2\mathrm{arctg}\left(2-\sqrt{3}\right)$	$\mathrm{arctg}\left(2+\sqrt{3}\right)$, $\pi-\mathrm{arctg}\left(2+\sqrt{3}\right)$	$\mathrm{arctg}2$, $\pi-\mathrm{arctg}2$

37. The obtuse angle of a rhombus is 5 times greater than its acute angle. How many times is the side of the rhombus greater than the radius of the inscribed circle?

A 2	**B** 3	**C** 4	**D** 5	**E** 6

38. One of the diagonals of a parallelogram is its altitude. Find the smaller diagonal of the parallelogram if its perimeter is 50 cm and the difference between the sides is 1 cm.

A 5 cm	**B** 15 cm	**C** 25 cm	**D** 30 cm	**E** 35 cm

39. The area of a parallelogram is 480 cm^2, its perimeter is 112 cm. The distance between the larger sides is 12 cm. Find the distance between the smaller sides.

A 5 cm	**B** 15 cm	**C** 25 cm	**D** 30 cm	**E** 35 cm

40. In the parallelogram $ABCD$, the value of the angle BCD is equal to $\frac{\pi}{3}$, the length of the side AB is equal to a. The bisector of the angle BCD intersects the side AD at the point N. Find the area of the triangle NCD.

A $\frac{a^2\sqrt{3}}{4}$	**B** $\frac{a^2\sqrt{3}}{2}$	**C** $\frac{a^2}{4}$	**D** $\frac{a^2\sqrt{2}}{4}$	**E** $\frac{a^2\sqrt{2}}{2}$

41. Find the area of a parallelogram if its diagonals are 3 and 5 cm, and the acute angle of the parallelogram is $60°$.

A $4\sqrt{2}$ cm²	**B** $2\sqrt{3}$ cm²	**C** $\sqrt{2}$ cm²	**D** $4\sqrt{3}$ cm²	**E** $2\sqrt{2}$ cm²

42. On the BC side of the parallelogram $ABCD$ the point E is taken. The segments AE and BD intersect at the point F so that $AF : FE = 7 : 3$. Find in what ratio the line AE divides the area of the parallelogram $ABCD$.

A $7:3$	**B** $3:10$	**C** $9:49$	**D** $3:14$	**E** $3:11$

43. The bases of an isosceles trapezoid are 7 and 13 cm, and its area is 30 cm^2. Find the acute angle at the vertex.

A $\frac{\pi}{4}$	**B** $\frac{\pi}{6}$	**C** $\frac{\pi}{3}$	**D** $40°$	**E** $70°$

44. A right-angled trapezoid is divided diagonally into two triangles: an equilateral triangle with side a and a right-angled triangle. Calculate the area of the trapezoid.

A $\frac{a^2\sqrt{3}}{2}$	**B** $\frac{a^2\sqrt{2}}{2}$	**C** $\frac{3a^2\sqrt{2}}{4}$	**D** $\frac{3a^2\sqrt{3}}{8}$	**E** $\frac{a^2\sqrt{3}}{4}$

45. The area of an isosceles trapezoid is equal to S, the angle between its diagonals, opposite to the side, is equal to α. Find the altitude of the trapezoid.

A $S\sqrt{\mathrm{tg}\frac{\alpha}{2}}$	**B** $\sqrt{S\cdot\mathrm{tg}\alpha}$	**C** $\mathrm{tg}\alpha\sqrt{S}$	**D** $\sqrt{S\cdot\mathrm{tg}\frac{\alpha}{2}}$	**E** $\sqrt{S\cdot\mathrm{tg}2\alpha}$

46. In a trapezoid with the bases a and b, a straight line parallel to the bases is drawn through the point of intersection of the diagonals. Find the length of the segment of this line that intersects the sides of the trapezoid.

A $\frac{ab}{a+b}$	**B** $\frac{2ab}{a+b}$	**C** $\frac{ab}{a-b}$	**D** $\frac{a+b}{a-b}$	**E** $\frac{a-b}{a+b}$

47. In an isosceles trapezoid $ABCD$, the lengths of the side and the smaller base are 2 cm, and BD is perpendicular to AB. Find the area of the trapezoid.

A $3\sqrt{3}$ cm²	**B** $3\sqrt{2}$ cm²	**C** $2\sqrt{3}$ cm²	**D** $4\sqrt{3}$ cm²	**E** $4\sqrt{2}$ cm²

48. In a convex quadrilateral, the lengths of the diagonals are 2 and 4 cm. Find the area of the quadrilateral if it is known that the lengths of the segments joining the midpoints of the opposite sides are equal.

A 3 cm²	**B** 4 cm²	**C** 5 cm²	**D** 6 cm²	**E** 7 cm²

49. A quadrilateral $ABCD$ is circumscribed around a circle with center O. Find the sum of the angles AOB and COD.

A 60°	**B** 90°	**C** 120°	**D** 135°	**E** 180°

50. In a convex quadrilateral $ABCD$, the length of the segment joining the midpoints of the sides AB and CD is 1 m. The lines BC and AD are perpendicular. Find the lengths of the segment that joins the midpoints of the diagonals AC and BD.

A 0.25 m	**B** 0.5 m	**C** 0.75 m	**D** 1 m	**E** 1.25 m

Level 3

51. A straight line is drawn through the vertex of a square, which is perpendicular to the diagonal, to the intersection with the extensions of the sides of the square passing through the vertex opposite to the one chosen above. Find the distance between the radii of the circles inscribed in the formed triangle and the given square if its diagonal is d cm.

52. A square is circumscribed around a right-angled triangle with side b so that one of their vertices is common. Find the diagonal of the square.

53. The square $ABCD$ and the circle are tangent so that the circle touches the line AC at point C, and the center of the circle lies on the same side of the line AC as the point D. Tangents to the circle drawn from point D form an angle of $120°$. Find the ratio of the area of the square to the area of the circle bounded by the given circle.

54. The centers of four circles are located at the vertices of a square with side a. The radii of these circles are equal to a. Determine the area of their common part.

55. A triangle with a base equal to a is inscribed in a square, whose one side lies at the base of the triangle. The area of the square is equal to $\frac{1}{6}$ part

of the area of the triangle. Determine the altitude of the triangle and the side of the square.

56. In a rectangle with sides a and b, bisectors of all angles are drawn to the intersection. Find the area of the quadrilateral formed by the bisectors.

57. In a triangle with base b and altitude h, a rectangle is inscribed, whose side is parallel to the altitude and relates to the adjacent side of the rectangle as $n : m$. Find the side of the rectangle parallel to the altitude of the triangle.

58. A circle whose diameter is equal to $\sqrt{10}$, passes through adjacent vertices A and B of the rectangle $ABCD$. The length of the tangent drawn from point C to the circle is 3, $AB = 1$. Find all possible values that can take the length of the side BC.

59. On the extended side AD of the rhombus $ABCD$ the point K is taken as the point D. The straight lines AC and BK intersect at the point Q. It is known that $AK = 14$ and the points A, C and Q lie on a circle of radius 6, whose center belongs to the segment AK. Find the length of the segment BK.

60. In a rhombus whose diagonals are 15 dm and 20 dm, two altitudes are drawn from the vertex of the obtuse angle, and their ends are connected. Determine the altitude of the triangle thus formed, which is drawn from the vertex of the obtuse angle of the rhombus.

61. The altitude of the rhombus is h, and the smaller diagonal is $1.25h$. Find the area of the rhombus.

62. A rhombus with diagonals of 6 and 12 cm is inscribed in a rectangle with the sides parallel to the diagonals. The area of the rectangle is equal to $\frac{1}{3}$ of the area of the rhombus. Find the perimeter of the rectangle.

63. From the vertex B of the obtuse angle of the rhombus $ABCD$ the altitudes BM and BN are drawn. A circle of radius 1 cm is inscribed in the quadrilateral $BMDN$. Find the side of the rhombus if $\angle ABC = 2\operatorname{arctg}2$.

64. The sum of the diagonals of a parallelogram is 10 cm, and the sum of the perimeters of four triangles into which the parallelogram is intersected by its diagonals is 38 cm. Find the diagonals of the parallelogram if its base is twice as large as the side.

65. In the parallelogram $ABCD$, bisectors of angles C and D are drawn. The distances from the point of intersection of these bisectors to the sides AB and CD are 12 cm and 5 cm. Find the altitudes of the parallelogram.

66. Inside a parallelogram, there are two identical circles of radius 6, each of which touches the side of the parallelogram, both bases and another circle. The side is divided by the point of contact in the ratio 9 : 4. Find the area of the parallelogram.

67. The sides of a parallelogram are 4 cm and 6 cm. From the middle of the larger side, the parallel side is visible at an angle of $45°$. Find the area of the parallelogram.

68. Inside the parallelogram $ABCD$, a point K is taken so that the triangle CKD is right. It is known that the distance from the point K to the lines AD, AB and BC are 3, 6 and 5, respectively. Find the perimeter of the parallelogram.

69. In the trapezoid $ABCD$ with the lengths of the bases AD = 12 cm, BC = 8 cm point M is taken on the ray BC so that AM divides the trapezoid into two equal figures. Find the segment CM.

70. The lengths of the sides AB and CD of the trapezoid $ABCD$ are 8 cm and 10 cm, respectively, and the length of the base BC equals 2 cm. The bisector of the angle ADC passes through the middle of the side AB. Find the area of the trapezoid.

71. In the trapezoid $ABCD$ $\angle BAD = 90°$, $\angle ADC = 30°$. The circle whose the center lies on the segment AD, touches the lines AB, BC and CD. Find the area of the trapezoid if it is known that the radius of the circle is equal to R.

72. The sum of the squares of the parallel sides of a trapezoid is 288. Determine the length of the segment parallel to these sides, which divides the area of the trapezoid in half.

73. In an isosceles trapezoid $ABCD$, the length of the base AD is 14 and the length of the base BC is 2. The circle touches the sides AB, BC and CD, and the side is divided by the point of contact in the ratio 1 : 9, counting from the smaller base. Find the radius of the circle.

74. An arbitrary quadrilateral is cut by its diagonals into four triangles; the areas of three of them are 10, 20, and 30 cm^2, and each is less than the area of the fourth triangle. Find the area of the quadrilateral.

75. A quadrilateral is inscribed in a circle whose the sides are a, b, c and d. Calculate the ratio of the diagonals of this quadrilateral.

Test 4. The circle

Level 1

1. From a point taken on a circle, a diameter and a chord equal to the radius are drawn. Find the angle between them.

A $30°$	**B** $90°$	**C** $45°$	**D** $60°$	**E** $75°$

2. The chord intersects the diameter at an angle of $30°$ and divides it into two segments 2 and 6 cm long. Find the distance from the center of the circle to the chord.

A 0.5 cm	**B** 1 cm	**C** 1.5 cm	**D** 2 cm	**E** 2.5 cm

3. The chord divides the circle in the ratio 5 : 13. Find the inscribed angles that rest on this chord.

A $30°$, $150°$	**B** $40°$, $140°$	**C** $60°$, $120°$	**D** $70°$, $110°$	**E** $50°$, $130°$

4. The value of the inscribed angle is $28°$ less than the central angle, which rests on the same arc. Find the values of these angles.

A $56°$, $28°$	**B** $60°$, $32°$	**C** $66°$, $38°$	**D** $80°$, $52°$	**E** $64°$, $36°$

5. The angles of a triangle relate as 3 : 4 : 5. At what values of angles are each of its sides visible from the center of the circumscribed circle?

A $60°$, $80°$, $220°$	**B** $70°$, $100°$, $190°$	**C** $90°$, $120°$, $150°$	**D** $100°$, $120°$, $140°$	**E** $110°$, $150°$, $100°$

6. In the triangle ABC, $\angle C = 90°$, $\angle B = 30°$, point O is the center of the inscribed circle, OA = 12 cm. Find the radius of the inscribed circle.

A 4 cm	**B** 4.5 cm	**C** 5 cm	**D** 5.5 cm	**E** 6 cm

7. A quadrilateral is circumscribed around a circle. The lengths of its three consecutive sides are 10, 15 and 16 cm. Find the perimeter of the quadrilateral.

A 40 cm	**B** 46 cm	**C** 52 cm	**D** 58 cm	**E** 64 cm

8. The smaller side of the rectangle is 1 m, the acute angle between the diagonals is $60°$. Find the radius of the circumscribed circle.

A 1 m	**B** 1.25 m	**C** 1.5 m	**D** 1.75 m	**E** 2 m

9. The chord of a circle tightens the arc of $120°$ and is 7 cm from the center. Find the length of the circle.

A 20π cm	**B** 22π cm	**C** 26π cm	**D** 28π cm	**E** 30π cm

10. A rectangle whose sides are 12 and 16 cm is inscribed in a circle. Find the area of the circle.

A 80π cm^2	**B** 90π cm^2	**C** 100π cm^2	**D** 110π cm^2	**E** 120π cm^2

11. From the point M two tangents are drawn to a circle whose radius is 8 cm. A and B are the points of contact, $\angle AMB = 60°$. Find the distance from the center of the circle to the chord AB.

A 3 cm	**B** 3.5 cm	**C** 4 cm	**D** 4.5 cm	**E** 5 cm

12. The interior angles of a triangle relate as 3 : 2 : 3. Find the values of the arcs of the circumscribed circle, contracted by the sides of the triangle.

A 72°, 144°, 144°	**B** 60°, 150°, 150°	**C** 80°, 140°, 140°	**D** 90°, 135°, 135°	**E** 100°, 130°, 130°

13. Points A, B and C divide a circle into parts whose arcs relate as 3 : 4 : 5. At what angles do the tangents to the circle, constructed at points A, B, C intersect?

A 30°, 60°, 90°	**B** 40°, 80°, 60°	**C** 50°, 60°, 70°	**D** 24°, 48°, 88°	**E** 36°, 54°, 90°

14. The chords AB and AC of a circle are mutually perpendicular, and each has a length a. Find their distance from the center of the circle.

A a	**B** $\frac{1}{3}a$	**C** $\frac{1}{4}a$	**D** $\frac{2}{3}a$	**E** $\frac{1}{2}a$

15. OA and OB are the radii of a circle. A line passing through the midpoints of these radii is a quarter of the radius away from the center. Determine the length of the arc AB.

A 130°	**B** 120°	**C** 110°	**D** 100°	**E** 90°

16. The side of a triangle inscribed in a circle tightens the arc of 108°. The values of the angles on this side relate as 2 : 5. Find the angles of the triangle.

A 40°, 60°, 80°	**B** 36°, 54°, 90°	**C** 50°, 60°, 70°	**D** 24°, 48°, 88°	**E** 30°, 60°, 90°

17. Chords AB and AC tighten the arcs of 58° and 94°. Determine the angle between these chords.

A 105°	**B** 104°	**C** 102°	**D** 101°	**E** 100°

18. The bases of a trapezoid tighten 68° and 168° arcs of the circle circumscribed around it. Determine the angle between the diagonals of the trapezoid.

A 60°	**B** 62°	**C** 64°	**D** 50°	**E** 62° or 50°

19. The smaller base of a trapezoid is equal to the radius of the circle circumscribed around it. The diagonal bisects the angle at the other base. Find the angle between the diagonals of the trapezoid.

A 50°	**B** 55°	**C** 60°	**D** 65°	**E** 70°

20. The vertices of a quadrilateral divide a circle into arcs whose the values relate as 3 : 4 : 5 : 6. Find the acute angle between the diagonals of the quadrilateral.

A 80°	**B** 75°	**C** 70°	**D** 65°	**E** 60°

21. An isosceles trapezoid is circumscribed around a circle with the perimeter of 18 cm. The angle of the trapezoid is equal to 30°. Find the diameter of the circle.

A 2.25 cm	**B** 2 cm	**C** 1.75 cm	**D** 1.5 cm	**E** 1.25 cm

22. An isosceles trapezoid with a side of 8 cm and an angle of $60°$ at the base is circumscribed around a circle. Calculate the bases of a trapezoid.

A 10, 6 cm	**B** 8, 6 cm	**C** 12, 4 cm	**D** 10, 8 cm	**E** 12, 6 cm

23. In a right-angled triangle, the median, which is drawn from the vertex of a right angle, is 5 cm. A circle is circumscribed around this triangle. Find its length.

A 6π cm	**B** 7π cm	**C** 8π cm	**D** 9π cm	**E** 10π cm

24. A rhombus with an angle 30^{o} is circumscribed around a circle whose area is equal to Q. Find the area of the rhombus.

A $\frac{6Q}{\pi}$	**B** $\frac{8Q}{\pi}$	**C** $\frac{4Q}{\pi}$	**D** $\frac{5Q}{\pi}$	**E** $\frac{2Q}{\pi}$

25. A circle with a radius of 4 cm is inscribed in a right-angled triangle on whose side a square is constructed. Find the radius of the circle circumscribed around this square.

A $3\sqrt{3}$ cm	**B** $1.5\sqrt{3}$ cm	**C** $3\sqrt{6}$ cm	**D** $2\sqrt{6}$ cm	**E** $\sqrt{6}$ cm

Level 2

26. The diameter intersects with the chord and divides it into two segments 3 and 6 cm long. The distance from the center of the circle to the chord is 1.5 cm. Find the acute angle between the chord and the diameter.

A 45°	**B** 50°	**C** 55°	**D** 60°	**E** 65°

27. From the point of a circle two mutually perpendicular chords are drawn. The segment connecting their midpoints is 12 cm. Find the radius of the circle.

A 8 cm	**B** 9 cm	**C** 10 cm	**D** 11 cm	**E** 12 cm

28. The chord divides a circle into two parts, one of which is $125\ \%$ of the other. Find the values of the inscribed angles that rest on this chord.

A 90°, 90°	**B** 100°, 80°	**C** 110°, 70°	**D** 120°, 60°	**E** 130°, 50°

29. The chord tightens the arc of $46°$. Find the values of the angles formed by the chord with the tangents to the circle, drawn through its ends.

A $19°$	**B** $20°$	**C** $21°$	**D** $22°$	**E** $23°$

30. Around a circle whose radius is 4 cm, a right-angled triangle is circumscribed whose hypotenuse is 26 cm. Find the perimeter of the triangle.

A 40 cm	**B** 50 cm	**C** 60 cm	**D** 70 cm	**E** 80 cm

31. In a right-angled triangle, the median drawn from the vertex of the right angle is 5 cm. Find the length of the circle circumscribed around the triangle.

A 7π cm	**B** 8π cm	**C** 9π cm	**D** 10π cm	**E** 11π cm

32. Around a circle with a diameter of 15 cm an isosceles trapezoid with a 17 cm side is circumscribed. Find the bases of the trapezoid.

A 7 and 30 cm	**B** 6 and 28 cm	**C** 8 and 26 cm	**D** 10 and 24 cm	**E** 9 and 25 cm

33. The side of a rhombus is 8 cm, and the acute angle is $30°$. Find the radius of the inscribed circle.

A 2 cm	**B** 2.5 cm	**C** 3 cm	**D** 3.5 cm	**E** 4 cm

34. A circle is circumscribed around an isosceles trapezoid with the bases 6 and 8 cm, and the altitude of 7 cm. Find the area of the circle.

A 4π cm^2	**B** 9π cm^2	**C** 16π cm^2	**D** 25π cm^2	**E** 36π cm^2

35. How many centimeters will the length of the circle increase if its diameter, which is equal to 20 cm, increase by 25 %?

A 4π cm	**B** 5π cm	**C** 6π cm	**D** 7π cm	**E** 8π cm

36. In a triangle with the angles whose ratio is 4 : 5 : 6 a circle is inscribed that touches the sides at points K, L, M. Find the angles of the triangle KLM.

A $50°$, $70°$, $60°$	**B** $40°$, $60°$, $80°$	**C** $52°$, $48°$, $80°$	**D** $56°$, $48°$, $76°$	**E** $54°$, $60°$, $66°$

37. The acute angles of a right-angled triangle relate as 2 : 3. Determine the values of the arcs of the circle circumscribed around this triangle, into which it is divided by the vertices of this triangle.

A 72°, 108°, 180°	**B** 86°, 144°, 120°	**C** 108°, 162°, 90°	**D** 84°, 126°, 150°	**E** 60°, 90°, 210°

38. The base of a trapezoid is the diameter of the circle circumscribed around it. The diagonal bisects the angle at this base. Determine the angles that the diagonal forms with the sides of the angle at a smaller base.

A 40°, 80°	**B** 45°, 90°	**C** 50°, 70°	**D** 30°, 90°	**E** 20°, 100°

39. The radii of two circles are 27 and 13 cm, and the distance between the centers is 50 cm. Find the length of their common outer tangent.

A 49 cm	**B** 48 cm	**C** 47 cm	**D** 46 cm	**E** 45 cm

40. In an isosceles triangle, the base is 30 cm and the side is 39 cm. Find the radius of the inscribed circle.

A 8 cm	**B** 9 cm	**C** 10 cm	**D** 11 cm	**E** 12 cm

41. The perimeter of a right-angled triangle inscribed in a circle is 36 cm. Find the perimeter of the square inscribed in the same circle.

A $8\sqrt{6}$ cm	**B** $14\sqrt{6}$ cm	**C** $16\sqrt{6}$ cm	**D** $12\sqrt{6}$ cm	**E** $10\sqrt{6}$ cm

42. A circle is inscribed in a square whose side is equal to a; and another circle is circumscribed around this square. Find the lengths of these circles.

A $\pi a, \pi a\sqrt{2}$	**B** $\pi a, 2\pi a$	**C** $\frac{\pi a}{2}, \pi a$	**D** $\frac{\pi a}{2}, \pi a\sqrt{2}$	**E** $\frac{\pi a}{4}, \frac{\pi a}{2}$

43. How many centimeters is the length of a circle whose radius is 1 dm, longer than the perimeter of a regular hexagon inscribed in this circle.

A 2 cm	**B** 2.2 cm	**C** 2.4 cm	**D** 2.6 cm	**E** 2.8 cm

44. A circle is inscribed in a regular hexagon with a side of 10 cm. Find its length.

A $50\pi\sqrt{3}$ cm	**B** $10\pi\sqrt{3}$ cm	**C** $15\pi\sqrt{3}$ cm	**D** $20\pi\sqrt{3}$ cm	**E** $25\pi\sqrt{3}$ cm

45. An isosceles trapezoid with an angle of $30°$ is circumscribed around a circle. The middle line of the trapezoid is 8 cm. Find the length of the circle.

A 3.2π cm	**B** 3.6π cm	**C** 3.8π cm	**D** 4π cm	**E** 4.2π cm

46. Find the area of a circle circumscribed around an isosceles triangle whose base is 16 cm and the altitude is 4 cm.

A 110π cm	**B** 100π cm	**C** 90π cm	**D** 80π cm	**E** 70π cm

47. In a right-angled triangle, the length of two large sides is 37 and 35 cm. Find the radius of the inscribed circle.

A 4 cm	**B** 4.25 cm	**C** 4.5 cm	**D** 4.75 cm	**E** 5 cm

48. A rectangle with the ratio of the lengths of the sides $5:12$ is inscribed in a circle with a radius of 39 cm. Find the area of the rectangle.

A 2124 cm^2	**B** 2150 cm^2	**C** 2160 cm^2	**D** 2170 cm^2	**E** 2180 cm^2

49. Find the area of a rectangle whose perimeter is 34 cm and the radius of the circumscribed circle is 7 cm.

A 46 cm^2	**B** 46.5 cm^2	**C** 47 cm^2	**D** 47.5 cm^2	**E** 48 cm^2

50. Around a circle whose radius is R, a rhombus with an acute angle of $30°$ is circumscribed. Calculate the area of the rhombus.

A $12R^2$	**B** $4R^2$	**C** $10R^2$	**D** $6R^2$	**E** $8R^2$

Level 3

51. Common internal tangents of two circles are mutually perpendicular. The lengths of the chords connecting the points of contact are 3 and 5 cm. Find the distance between the centers of the circles.

52. The chord of a circle is equal to 10 cm. A tangent to the circle is drawn through one end of the chord, and a secant parallel to the tangent is drawn through the other end. Find the radius of the circle if the inner segment of the secant is 12 cm.

53. Through the ends of the chord, which divides the circle in the ratio 2 : 7, two tangents to their mutual intersection are made. Find the angle between the tangents.

54. The diameter AB and the chord intersect at the point M. It is known that, $\angle MAC = 30^\circ$ $\angle ACM = 20^\circ$. Find the angle $\angle AMD$.

55. A circle is inscribed in a right-angled triangle. The point of tangency divides the hypotenuse into segments of length 5 and 12 cm. Find the area of the triangle.

56. A square and a right-angled triangle are inscribed in a circle. The perimeter of the square is 32 cm. Find the area of the triangle.

57. The bases of an isosceles trapezoid are 9 and 21 cm, and the altitude is 8 cm. Find the radius of the circle circumscribed around the trapezoid.

58. By what percentage will the area of a circle increase if its radius is increased by 50 %?

59. A rhombus with an angle of 30° is circumscribed around a circle whose area is equal to 2π cm^2. Find the area of the rhombus.

60. A perpendicular drawn from the vertex A of the rectangle $ABCD$ to the diagonal BD divides it in the ratio 1 : 3. The continuation of the perpendicular intersects the circumscribed circle at the point M. Find the values of the arcs BM and MD.

61. Points A, B, C divide a circle into parts whose arcs relate as 3 : 4 : 5. At what angles do the tangents to the circle, constructed at points A, B and C, intersect?

62. In the triangle ABC, on the side BC, a point M is taken so that $BM = 2MC$, moreover $\angle AMB = 60^\circ$. Knowing that $\angle BAC = 60^\circ$, find the values of the angles B and C.

63. The center of the inscribed circle is connected by segments with the vertices of a triangle. The areas of the formed parts are 9 cm^2, 10 cm^2 and 17 cm^2. Find the radius of this circle.

64. The lengths of a triangle sides are 13 cm, 30 cm and 37 cm. Find the radius of a circle centered on the middle side that touches the other sides.

65. In an isosceles trapezoid, the bases are 14 and 50 cm, and the side is 30 cm. Find the radius of the circle circumscribed around the trapezoid.

66. A trapezoid is inscribed in a circle, in which the lengths of the bases are 104 and 32 cm, and the distance between them is 24 cm. Find the radius of the circumscribed circle.

67. Find the lengths of the diagonals of a rhombus, if their ratio is equal, and the radius of a circle inscribed in the rhombus is 24 cm.

68. In the triangle ABC, $AB = 36$ cm, $BC = 24$ cm, $AC = 30$ cm. A straight line, parallel to AC, cuts off a trapezoid in which you can inscribe a circle. Find the perimeter of the trapezoid.

69. The perimeter of a right-angled triangle is 24 cm, its area is 24 cm^2. Find the area of the circumscribed circle.

70. The chord of a circle is equal to 10 cm. A tangent to the circle is drawn through one end of the chord, and a secant is drawn through the other end, which is parallel to the tangent. Find the radius of the circle if the inner segment of the section is 12 cm.

71. From point A outside a circle, a secant and a tangent are drawn to it. The distance from point A to the point of contact is 16 cm, and the distance from point A to one of the points of intersection of the section with the circle is 32 cm. Find the radius of the circle if the section is 5 cm away from its center.

72. A diamond with diagonals of 12 cm and 6 cm is inscribed in the intersection of two equal circles. Find the radius of the circles.

73. From the outer point of a circle, a 12 cm section and a tangent whose length is the inner segment of the section are drawn. Find the length of the tangent.

74. A right-angled triangle is inscribed in a circle with a diameter $\sqrt{12}$. At its altitude, as on the side, a right-angled triangle is built, in which a new circle is inscribed. Find the radius of this circle.

Test 5. Stereometry. Basic concepts

Level 1

1. In the triangle ABC, the angle B is right and the leg BC is 8 cm. A perpendicular AD is drawn from the vertex A to the plane of the triangle. Find the distance from the point D to the leg BC if the distance between the points D and C is 10 cm.

A 6 cm	**B** $2\sqrt{41}$ cm	**C** 7 cm	**D** 5 cm	**E** 6.5 cm

2. The legs of a right-angled triangle are 18 and 32 cm. From the point N that divides the hypotenuse in half, a perpendicular NK is drawn whose length is 12 cm. Find the distance from point K to the larger leg.

A 16 cm	**B** 10 cm	**C** 17 cm	**D** 15 cm	**E** 20 cm

3. From the point A, which is 12 cm away from a plane, an inclined line AB is drawn to this plane whose length is 37 cm. Find the projection of AB on this plane.

A 33 cm	**B** 30 cm	**C** 25 cm	**D** 40 cm	**E** 35 cm

4. From the center of a circle with a radius of 18 cm, a perpendicular to its plane is drawn. Find the distance from the end of this perpendicular to the points of the circle if the length of the perpendicular is 80 cm.

A 84 cm	**B** 82 cm	**C** 85 cm	**D** 86 cm	**E** $16\sqrt{2}$ cm

5. From the center O of a circle whose radius is 3 dm, a perpendicular OC to its plane is drawn. A tangent is drawn to the circle at point A, and on this tangent, a segment AC is marked at the point of tangency whose length is 2 dm. Find the length of the inclined line AC, if OC = 6 dm.

A 8 dm	**B** 6.5 dm	**C** $6\sqrt{2}$ dm	**D** 7.5 dm	**E** 7 dm

6. From the vertex D of the rectangle $ABCD$ whose sides are $AB = 9$ cm and $BC = 8$ cm, a perpendicular $DF = 12$ cm is drawn to the plane of the rectangle. Find the distance from the point F to the vertex B.

A $16\sqrt{2}$ cm	**B** 16 cm	**C** 17 cm	**D** 15 cm	**E** $14\sqrt{3}$ cm

7. From a point outside a plane, an inclined line 20 cm long is drawn to this plane, which forms an angle of $45°$ with this plane. Find the distance from the given point to the plane.

A $5\sqrt{2}$ cm	**B** $10\sqrt{2}$ cm	**C** 20 cm	**D** 10 cm	**E** 15 cm

8. From the ends of the segment $AB = 26$ cm, which is outside the plane α, perpendiculars AC and BD are drawn to this plane. Find the length

of the projection of the segment AB on the plane α if $AC=$ 32 cm and $BD=$ 22 cm.

A 24 cm	**B** 25 cm	**C** 26 cm	**D** $22\sqrt{2}$ cm	**E** 23 cm

9. From the ends of the segment AC, which is outside the plane α, perpendiculars AC = 80 cm and BD = 60 cm are drawn to this plane. Find the distance of the middle point of the segment AB from the plane α.

A 72 cm	**B** 80 cm	**C** 75 cm	**D** 60 cm	**E** 70 cm

10. Outside the plane α there is a segment AC, which is parallel to the plane α and has a length of 2 dm. A segment AB_1 which joins the end A with the projection of point B (the other end of the segment), forms an angle of $30°$ with the plane α. Find the length of the segment AB_1.

A $\frac{4\sqrt{3}}{3}$ dm	**B** $4\sqrt{3}$ dm	**C** 4 dm	**D** $4\sqrt{2}$ dm	**E** $\frac{4\sqrt{2}}{3}$ dm

11. From the point M of the line AB, which is parallel to the plane α and is 0.5 m from it, two equal lines inclined to this plane and perpendicular to the line AB are drawn. Find the length of these inclined lines if the angle between them is equal to $120°$.

A 1 m	**B** 0.5 m	**C** 0.75 m	**D** 1.2 m	**E** 1.5 m

12. Point M is at a distance of 15 cm from the plane α. The distance from the projection of point M on the plane α to the line p lying in this plane is 20 cm. Find the distance from point M to the line p.

A 20 cm	**B** 25 cm	**C** 24 cm	**D** $5\sqrt{7}$ cm	**E** $7\sqrt{5}$ cm

13. The area of a polygon lying in the plane α is equal to 2 dm^2. Find the area of the projection of this polygon on the plane β if the angle between the planes α and β is equal to $45°$.

A $\frac{\sqrt{2}}{2}$ dm^2	**B** 2 dm^2	**C** $\sqrt{3}$ dm^2	**D** $\sqrt{2}$ dm^2	**E** $2\sqrt{2}$ dm^2

14. The distance of the point M, located inside a dihedral angle, from each of its faces is equal to 2 dm. Find the distance of the point M from the edge of the dihedral angle if the angle between the perpendiculars, which are drawn from the point M on its face, is equal to $120°$.

A 3.5 dm	**B** 1 dm	**C** 4 dm	**D** $\sqrt{3}$ dm	**E** 2 dm

15. Inside a right dihedral angle, a point is taken at a distance of 12 cm and 16 cm from its faces. Find the distance of this point from its edge.

A 15 cm	**B** 20 cm	**C** $4\sqrt{7}$ cm	**D** 21 cm	**E** $14\sqrt{5}$ cm

16. The dihedral angle is equal to $45°$. A point is given on one face whose distance from the second face is 15 cm. Find the distance of this point from the edge.

A 15 cm	**B** 20 cm	**C** $10\sqrt{2}$ cm	**D** $7.5\sqrt{2}$ cm	**E** $15\sqrt{2}$ cm

17. On the face of a dihedral angle, two points are taken. The distance from the second face is 10 cm and 16 cm. The distance of the second point from the edge of the dihedral angle is 32 cm. Find the distance of the first point from the edge.

A 25 cm	**B** $20\sqrt{3}$ cm	**C** 20 cm	**D** 15 cm	**E** $20\sqrt{2}$ cm

18. In a trihedral angle, two linear angles are $60°$. On their common edge, from the vertex, a segment of 4 dm is taken. Find its projection on the plane of the third plane angle which is equal to $90°$.

A 3 dm	**B** $3\sqrt{2}$ dm	**C** 2 dm	**D** $2\sqrt{3}$ dm	**E** $2\sqrt{2}$ dm

19. From the point M, taken outside a plane, an inclined line is drawn to the plane at an angle of $30°$ which is 18 dm. Find the distance from the point M to the plane.

A 9 dm	**B** $9\sqrt{3}$ dm	**C** $9\sqrt{2}$ dm	**D** 10 dm	**E** 8 dm

20. From the point M, located at a distance of 12 cm from a plane, an inclined line is drawn to the plane at an angle of $60°$. Find the length of this inclined line.

A $12\sqrt{2}$ cm	**B** 24 cm	**C** $8\sqrt{3}$ cm	**D** 15 cm	**E** $9\sqrt{2}$ cm

21. The ends of a segment are located at a distance of 30 cm and 44 cm from a plane. The projection of this segment on the plane is 48 cm. Find the length of this segment.

A 52 cm	**B** 50 cm	**C** 48 cm	**D** $40\sqrt{2}$ cm	**E** $40\sqrt{3}$ cm

22. In a cube $ABCDA_1B_1C_1D_1$ a section is drawn through the edges BB_1 and D_1D. Find the value of the dihedral angle formed by this cross-section with the face A_1D_1DA.

A $60°$	**B** $30°$	**C** $45°$	**D** $\operatorname{arctg}\sqrt{2}$	**E** $\arcsin\frac{1}{4}$

23. From the point M, which is outside a plane, two inclined lines are drawn to the plane: MA and MB. The length of the line MB is 12 cm, and its projection on the plane is $6\sqrt{3}$ cm. Find the projection of the inclined line MA, if $MA = 10$ cm.

A 6 cm	**B** 7 cm	**C** $6\sqrt{2}$ cm	**D** 8 cm	**E** $5\sqrt{3}$ cm

24. Planes α and β are mutually perpendicular. A line is drawn on the plane β, which is parallel to the line of intersection of the planes α and β and it is 3 cm away from it. A point M is taken on the plane α, which is 4 cm away from the line of intersection of the planes. Find the distance of the point M to the line AB.

A 5 cm	**B** $\sqrt{7}$ cm	**C** 6 cm	**D** $4\sqrt{3}$ cm	**E** $4\sqrt{2}$ cm

25. The inclined line to the plane α is 8 cm, the projection of this inclined line is $4\sqrt{3}$ cm. Find the angle of inclination of this inclined line to the plane.

A $\arcsin\frac{1}{4}$	**B** $\operatorname{arctg}\frac{\sqrt{3}}{2}$	**C** $45°$	**D** $60°$	**E** $30°$

Level 2

26. From point A two inclined lines AB and AC are drawn to the plane α, the length of each is 2 cm; the angle between the inclined lines

is equal to $60°$, and the angle between their projections BD and DC is equal to $90°$. Find the distance of point A from the plane α.

A 1.5 cm	**B** 1 cm	**C** $\sqrt{2}$ cm	**D** $\frac{\sqrt{2}}{2}$ cm	**E** $\sqrt{3}$ cm

27. From some point A a perpendicular AO is drawn to a plane. The length of AO is 4 cm. Two inclined lines are also drawn from point A, which form angles of $60°$ with the perpendicular, and an angle of $90°$ between them. Find the distance between the bases of the inclined lines on the plane.

A $8\sqrt{2}$ cm	**B** $4\sqrt{2}$ cm	**C** 4 cm	**D** $16\sqrt{2}$ cm	**E** 8 cm

28. From point A, two equal inclined lines are drawn to some plane. The segment joining the bases of the inclined lines is 6 cm and it forms an angle of $60°$ with the inclined line, and an angle of $30°$ with its projection. Find the distance of point A from the plane.

A $6\sqrt{2}$ cm	**B** $3\sqrt{2}$ cm	**C** $2\sqrt{6}$ cm	**D** $\sqrt{6}$ cm	**E** $3\sqrt{6}$ cm

29. Point A is 10 cm away from each vertex of a right-angled triangle. The hypotenuse of a right-angled triangle is 12 cm. Find the distance of point A from the plane of the triangle.

A 8.5 cm	**B** 7 cm	**C** 6.5 cm	**D** 6 cm	**E** 8 cm

30. A point lying outside the plane of a right angle is 7 cm away from its vertex and 5 cm from each side. Find the distance of the point from the plane of the right angle.

A 1 cm	**B** 2 cm	**C** 1.5 cm	**D** $\sqrt{5}$ cm	**E** $2\sqrt{2}$ cm

31. The base of an isosceles triangle is 4 cm, the angle at the vertex is $90°$; a plane is drawn through the base of the triangle, which forms an angle of $30°$ with each side. Find the distance of this plane from the vertex of the triangle.

A $\sqrt{6}$ cm	**B** $\sqrt{2}$ cm	**C** $\sqrt{3}$ cm	**D** 2 cm	**E** $2\sqrt{2}$ cm

32. Find the distance of a point from a plane if the distance of this point from two points lying on the plane is 51 cm and 30 cm, and the projections of the corresponding inclined lines on this plane relate as $5:2$.

A $22\sqrt{3}$ cm	**B** $20\sqrt{2}$ cm	**C** 25 cm	**D** 24 cm	**E** 22 cm

33. From a point outside a plane, a perpendicular to this plane whose length is 12 cm and an inclined line whose length is 16 cm are drawn. Find the projection of the perpendicular to the inclined line.

A 6 cm	**B** 11 cm	**C** 8 cm	**D** 10 cm	**E** 9 cm

34. The legs of the right-angled triangle ABC are 12 dm and 16 dm. From the vertex of the right angle C a perpendicular $CM = 28$ dm is drawn to the plane of the triangle. Find the distance from the point M to the hypotenuse.

A 36 dm	**B** 32.8 dm	**C** 29.6 dm	**D** 30 dm	**E** 29.5 dm

35. From the point M, which is outside two parallel planes, two lines are drawn, intersecting the planes at points A, B and A_1, B_1, respectively. Find the length of the segment AA_1 if $BB_1 = 28$ cm, and $MA : AB = 5 : 2$.

A 20.5 cm	**B** 15 cm	**C** 25 cm	**D** 20 cm	**E** 18 cm

36. One of the sides of a rhombus lies on the plane α, and the opposite side is 16 cm from the plane α. The projections of the diagonals of the rhombus on the plane are 32 cm and 8 cm. Find the length of the side of the rhombus.

A 20 cm	**B** 24 cm	**C** 21 cm	**D** 19 cm	**E** 18 cm

37. Two segments are between parallel planes. The projections of these segments on the plane are 1 dm and 7 dm. Find the sum of the lengths of these segments if their difference is 4 dm.

A 11 dm	**B** 12 dm	**C** 10 dm	**D** 13 dm	**E** $12\sqrt{2}$ dm

38. A plane parallel to the hypotenuse at a distance of 20 cm from it is drawn through the vertex of the right angle of a triangle. Projections of the legs on this plane are 60 cm and 1 m. Find the hypotenuse.

A 115 cm	**B** 110 cm	**C** 125 cm	**D** 100 cm	**E** 120 cm

39. A dihedral angle is equal to $120°$. The point inside it is 6 cm away from each face. Find the distance between the bases of the perpendiculars drawn from the point on each face.

A 5 cm	**B** $6\sqrt{3}$ cm	**C** $6\sqrt{2}$ cm	**D** 6 cm	**E** 9 cm

40. The distance of the point M lying on one face of a dihedral angle from the second face is 12 cm. At the point M there is a perpendicular to the face where the point M is, to the intersection with the second face. Find the length of this perpendicular if the dihedral angle is equal to $60°$.

A 24 cm	**B** 12 cm	**C** $6\sqrt{2}$ cm	**D** $12\sqrt{3}$ cm	**E** 20 cm

41. Find the segment AC, located between the faces of a right dihedral angle, if the projections of this segment on the face are 25 cm and 21 cm and the projection on the edge is 15 cm.

A 25 cm	**B** $27\sqrt{2}$ cm	**C** 30 cm	**D** $\sqrt{1291}$ cm	**E** 29 cm

42. Through the hypotenuse of an isosceles right-angled triangle, a plane is drawn at an angle of $30°$ to the plane of the triangle. Find the distance of this plane from the vertex of the right angle if the hypotenuse is 40 cm.

A $9\sqrt{2}$ cm	**B** 11 cm	**C** 9 cm	**D** 8 cm	**E** 10 cm

43. From a point on the edge of a dihedral angle, two lines are drawn on the faces, each inclined at an angle of $45°$ to the edge. Find the angle between the lines.

A $60°$	**B** $45°$	**C** $30°$	**D** $90°$	**E** $arc\sin\frac{\sqrt{3}}{4}$

44. The ends of the segment $AB = 75$ cm are on two faces of a right dihedral angle. These ends are located at a distance of $AC = 50$ cm and $BD = 55$ cm from its edge. Find the length of the projection of the segment AB on the edge of the dihedral angle.

A $8\sqrt{2}$ cm	**B** 12 cm	**C** 10 cm	**D** 11 cm	**E** $9\sqrt{3}$ cm

45. A point that is 26 cm away from each vertex of a right-angled triangle is 24 cm from a plane. One of the legs of the triangle is 16 cm. Find the length of the second leg.

A $8\sqrt{3}$ cm	**B** $10\sqrt{2}$ cm	**C** 15 cm	**D** 10 cm	**E** 12 cm

46. From a point outside a plane, two inclined lengths $2\sqrt{3}$ are drawn to this plane at an angle of $30°$. Their projections form an angle of $120°$ between them. Find the distance between the bases of the slopes.

A $3\sqrt{3}$	**B** $3\sqrt{2}$	**C** $1.5\sqrt{3}$	**D** $4\sqrt{2}$	**E** $4\sqrt{3}$

47. Find the angle at which the diagonal of a cube is inclined to its lateral face.

A $\arcsin\frac{1}{4}$	**B** $30°$	**C** $60°$	**D** $45°$	**E** $\mathrm{arctg}\frac{\sqrt{2}}{2}$

48. The base AC of the isosceles triangle ABC lies in the plane, and its vertex B is at a distance of $OB = 6$ cm from the plane. What is the dihedral angle formed by the plane α of the triangle and the plane if $AC = 10$ cm, $AB = BC = 13$ cm?

A $\arcsin\frac{1}{4}$	**B** $\mathrm{arctg}\,\sqrt{2}$	**C** $45°$	**D** $60°$	**E** $30°$

49. A segment 15 cm long intersects a plane. Its ends are located at a distance of 3 cm and 6 cm from the plane. Find the projection of this segment on the plane.

A 10 cm	**B** 12 cm	**C** $7\sqrt{3}$ cm	**D** $15\sqrt{2}$ cm	**E** 8 cm

50. In the triangle ABC angle B is right and leg $BC = a$. From the vertex, a perpendicular AD is drawn to the plane of the triangle. Find the distance from the point D to the leg BC if the distance between the points D and C is equal to 1.

A $\sqrt{1^2 - a^2}$	**B** $\sqrt{1^2 + a^2}$	**C** $\sqrt{1^2 - \frac{a^2}{4}}$	**D** $\sqrt{21^2 - a^2}$	**E** $\frac{a+1}{2}$

Level 3

51. In an isosceles triangle, the base and the altitude are 4 cm. This point is at a distance of 6 cm from the plane of the triangle and at the same distance from all its vertices. Find this distance.

52. A plane α parallel to the smaller diagonal of a rhombus is drawn through one of the vertices of the rhombus. The larger diagonal of the rhombus is equal to d. The projection of the rhombus on the plane α is a square with the side α. Find the side of the rhombus.

53. The smaller base of a trapezoid lies in the plane α, which is separated from the larger base of the trapezoid at a distance of 10 cm; the bases of the trapezoid relate as 3 : 5. Find the distance of the point of intersection of the diagonals of the trapezoid from the plane α.

54. The sides of a triangle are 10 cm, 17 cm and 21 cm. A perpendicular to its plane 15 cm long is drawn from the vertex of the greater angle of this triangle. Find the distance from the end of this perpendicular, which lies outside the plane of the triangle, to the larger side of the triangle.

55. A slope is drawn through the vertex of a square to its plane, which forms an angle α with each side of the square passing through this vertex. Find the angle between this slope and the diagonal of the square.

56. From a point outside a plane, two slopes are drawn to it, which form an angle α with each other and an angle β with the plane. Find the angle between their projections on the given plane.

57. From the point O of the intersection of the diagonals of a rhombus to its plane a perpendicular OD is placed. Find the distance $\frac{\partial^2 \Omega}{\partial u \partial v}$ of the point from the side of the rhombus if the acute angle α of the rhombus is greater than the diagonal d, and ED forms an angle φ with the plane of the rhombus.

58. A plane is drawn through the side of a rhombus, which forms angles α and 2α with diagonals. Find the acute angle of the rhombus.

59. From a point D lying outside a plane, three slopes are drawn, each of which forms an angle 60° with the given plane. The bases of the slopes A, B and C are connected by segments. Find the sides of the triangle ABC if the distance from the point D to the plane is equal to a, the angles ADB, BDC, CDA are equal to each other.

60. The base AC of the trapezoid $ABCD$ lies on the plane α, and the base DC is separated from the plane α by 40 cm. At what distance from the plane is the point M of intersection of the diagonals of this trapezoid if $AB:DC=5:3$?

61. Two mutually perpendicular planes and a line intersecting them are given. The segment of this line lying between the planes is 20 cm. The projections of the segment on the planes are 16 cm and 15 cm. Find the length of the projection of the segment on the line of intersection of the planes.

62. A plane is formed through the hypotenuse of a right-angled triangle, which forms a dihedral angle $60°$ with the plane of the triangle. Find the distance from the vertex of the right angle of the triangle to the plane α if the legs of this triangle are 6 cm and 8 cm.

63. From the vertex of a triangular angle, all flat angles of which are straight, a segment is drawn inside the angle. The projection of this segment on the edges of the triangular angle is 25 cm, 50 cm and 50 cm. Find the length of this segment.

64. Point M is outside the plane of the isosceles triangle ABC $(AB=AC)$ and at the same distance from its vertices; the side of the triangle is equal to b, and the angle at its vertex is equal to β. Find the distance from the point M to the plane of the triangle if the segment MB is inclined to the plane of the triangle at an angle α.

65. The sides of a triangle are 27 cm, 30 cm and 51 cm. A perpendicular 10 cm long is drawn from the vertex of the smaller angle of the triangle to its plane. Find the distance from the end of the perpendicular (outside the plane) to the hypotenuse.

66. The diagonals of a rhombus are 60 cm and 80 cm. A perpendicular 45 cm long is drawn at the point of intersection of the diagonals to the plane of the rhombus. Find the distance from the end of this perpendicular (outside the plane) to the side of the rhombus.

67. Point M is located at a distance of 11 cm from each side of an isosceles trapezoid with the bases of 16 cm and 30 cm. Find the distance of the projection of point M on the plane of the trapezoid to the sides of the trapezoid.

68. A segment intersects a plane, its ends are 3 cm and 12 cm from the plane. Find the distance of the middle of the segment from the plane.

69. A segment of the length α rests with its ends on the face of a right dihedral angle, forming an angle α with each of them. Find the projection of this segment on the edge of the dihedral angle.

70. Two equal squares have a common side; their planes form a dihedral angle α. Diagonals are drawn from the common vertex in each of the squares. Find the angle between these diagonals.

71. Two isosceles triangles have a common base, and their planes form an angle $60°$. The common base is 16 cm, the side of one triangle is 17 cm, and the sides of the second one form a right angle. Find the distance between the vertices of the triangles.

72. AB is a segment parallel to the plane α; AC and BD are two levels inclined to the plane α, which are drawn perpendicular to the segment AB and in different directions from it. The segment AB has a length of 2 cm and is 7 cm from the plane, and the segments of the AC and BD are 8 cm. Find the distance CD.

73. The segment AB is parallel to the plane α and it is located at a distance of 5 cm from it. On the plane, a point M is given, the distance from which to the line AM is 6 cm. Find the length of the segment AC if MA and MB form angles $30°$ and $45°$ with the plane α, respectively.

74. From the center O of a right-angled triangle ABC, a perpendicular OM is drawn to the plane of this triangle. Find the shortest distance between the lines AM and BC if the side of the triangle is a and the segment OM is equal to b.

75. Two equal rectangles have a common side and are inclined to each other at an angle of $45°$. Find the ratio of the areas of two figures into which the projection of the side of one rectangle divides the other.

Test 6. Polyhedra

Level 1

1. One of the faces of a polyhedron is a hexagon. What is the smallest number of edges that this polyhedron can have?

A 6	**B** 12	**C** 18	**D** 24	**E** 15

2. How many edges has a prism with a minimum number of faces?

A 9	**B** 6	**C** 3	**D** 12	**E** 8

3. The diagonal of the base of a regular quadrangular prism is 8 dm, and the diagonal of the lateral face is 2 dm. Find the diagonal of the prism.

A 5 dm	**B** 6 dm	**C** $6\sqrt{2}$ dm	**D** 7.5 dm	**E** $5\sqrt{3}$ dm

4. Each of the edges of a regular hexagonal prism is equal to 5 cm. Find the largest diagonal of the prism.

A 15 cm	**B** 25 cm	**C** $5\sqrt{2}$ cm	**D** 10 cm	**E** $5\sqrt{5}$ cm

5. Find the ratio of the area of the diagonal section of a regular quadrilateral prism to the area of its lateral face.

A $\frac{1}{2}$	**B** 2	**C** $\sqrt{2}$	**D** $2\sqrt{2}$	**E** $\frac{\sqrt{2}}{2}$

6. The area of the lateral face of a regular hexagonal prism is equal to Q. Find the area of its largest diagonal section.

A $2Q$	**B** $Q\sqrt{3}$	**C** Q	**D** $4Q$	**E** $2\sqrt{3}Q$

7. The area of the lateral face of a regular hexagonal prism is equal to $\sqrt{12}$. Find the area of its smallest diagonal section.

A $4\sqrt{3}$	**B** 12	**C** 6	**D** $2\sqrt{3}$	**E** $3\sqrt{3}$

8. The sides of the base of a right parallelepiped are $\sqrt{18}$ and 7 cm, the angle between them is equal to $135°$. The lateral edge is 12 cm. Find the smaller diagonal of the parallelepiped.

A 13 cm	**B** 19 cm	**C** $\sqrt{253}$ cm	**D** $12\sqrt{2}$ cm	**E** 26 cm

9. The sides of the base of a right parallelepiped are 4 and 5 dm, the angle between them is equal to $30°$. Find the cross-sectional area of the parallelepiped that intersects all its lateral edges and forms an angle of $45°$ with the base plane.

A $5\sqrt{6}$ dm^2	**B** $10\sqrt{6}$ dm^2	**C** $5\sqrt{2}$ dm^2	**D** $10\sqrt{2}$ dm^2	**E** 20 dm^2

10. The lateral edge of a right prism is 6 dm. The base of this prism is a right-angled triangle with legs of 3 and 2 cm. Find the cross-sectional areas of the prism with planes passing through each of the legs and forming angles of $60°$ with the base plane.

A $3\ \text{cm}^2$	**B** $6\ \text{cm}^2$	**C** $1.5\ \text{cm}^2$	**D** $2\sqrt{3}\ \text{cm}^2$	**E** $1.5\sqrt{3}\ \text{cm}^2$

11. The side of the base of a regular triangular prism is 4 cm. Find the cross-sectional area of the prism formed by a plane passing through the middle points of the two sides of the base, forming an angle of $45°$ with its base and intersecting only one lateral edge of the prism.

A $\sqrt{6}\ \text{cm}^2$	**B** $\frac{\sqrt{6}}{2}\ \text{cm}^2$	**C** $\sqrt{2}\ \text{cm}^2$	**D** $3\sqrt{6}\ \text{cm}^2$	**E** $\frac{3\sqrt{6}}{2}\ \text{cm}^2$

12. The edge of a cube is equal to 3. Find the cross-sectional area of the cube with a plane passing through the side of the base if the angle between this plane and the base is equal to $30°$.

A $3\sqrt{3}$	**B** $6\sqrt{3}$	**C** $\frac{9}{2}$	**D** 18	**E** $2\sqrt{3}$

13. The area of the face of a cube is 9 cm^2. Find its diagonal.

A $9\ \text{cm}$	**B** $3\ \text{cm}$	**C** $27\ \text{cm}$	**D** $3\sqrt{3}\ \text{cm}$	**E** $3\sqrt{5}\ \text{cm}$

14. The side of the base of a regular hexagonal pyramid is equal to a, the altitude of the pyramid is h. Find the area of the larger diagonal section.

A $\frac{1}{2}ah$	**B** ah	**C** $2ah$	**D** $\frac{1}{3}ah$	**E** $3ah$

15. The altitude of a regular quadrangular truncated pyramid is $2\sqrt{2}$ cm, the sides of the bases are 8 and 12 cm. Find the area of the diagonal section.

A $40\sqrt{2}\ \text{cm}^2$	**B** $20\ \text{cm}^2$	**C** $80\ \text{cm}^2$	**D** $40\ \text{cm}^2$	**E** $20\sqrt{2}\ \text{cm}^2$

16. The dihedral angle at the base of a regular hexagonal pyramid is equal to α. Find the angle of inclination of the lateral edge of the pyramid to its base.

A	B	C	D	E
$\arccos\frac{2}{\sqrt{3}}\text{tg}\alpha$	$\arccos\frac{\sqrt{3}}{2}\text{tg}\alpha$	$\arcsin\frac{\sqrt{3}}{2}\text{tg}\alpha$	$\text{arctg}\frac{2}{\sqrt{3}}\text{tg}\alpha$	$\text{arctg}\frac{\sqrt{3}}{2}\text{tg}\alpha$

17. The sides of the base of a right parallelepiped are 8 and 9 cm. The diagonal of the parallelepiped is 17 cm. Find the altitude of the parallelepiped.

A 12 cm	**B** $\sqrt{272}$ cm	**C** $3\sqrt{34}$ cm	**D** 16 cm	**E** $2\sqrt{34}$ cm

18. In a regular quadrangular truncated pyramid, the diagonals drawn in one diagonal section are mutually perpendicular. Find the altitude of the pyramid if the diagonal of the pyramid is 5 dm.

A $5\sqrt{2}$ dm	**B** $\frac{5\sqrt{3}}{2}$ dm	**C** $3\sqrt{2}$ dm	**D** $\frac{5\sqrt{2}}{2}$ dm	**E** $\frac{3\sqrt{2}}{2}$ dm

19. A plane is drawn through the side of the base of a regular triangular prism at an angle $30°$ to the base. The plane intersects the opposite lateral edge. Find the cross-sectional area if the side of the base is 14 cm.

A 98 cm^2	**B** 24.5 cm^2	**C** 49 cm^2	**D** 196 cm^2	**E** $98\sqrt{3}$ cm^2

20. In a regular hexagonal pyramid, the side of the base is 10 cm, the dihedral angle at the base is $30°$. Find the apothem of the pyramid.

A $10\sqrt{3}$cm	**B** 7.5 cm	**C** 10 cm	**D** $\frac{10\sqrt{3}}{3}$ cm	**E** 11 cm

21. The lateral edge of a right quadrangular pyramid is 4 cm and it is inclined to the plane of the base at an angle of $45°$. Find the side of the base of the pyramid.

A $3\sqrt{2}$ cm	**B** 8 cm	**C** $2\sqrt{2}$ cm	**D** 6 cm	**E** 4 cm

22. The side of the base of a regular quadrangular pyramid is 2 cm, the linear angle at the apex is $60°$. Find the altitude of the pyramid.

A $2\sqrt{3}$ cm	**B** $2\sqrt{2}$ cm	**C** 2 cm	**D** $\sqrt{2}$ cm	**E** $\sqrt{3}$ cm

23. In a regular hexagonal pyramid, a dihedral angle at the base is $30°$. The distance from the center of the base to the lateral face is 9 cm. Find the altitude of the pyramid.

A $6\sqrt{3}$ cm	**B** 18 cm	**C** $18\sqrt{3}$ cm	**D** $\frac{3\sqrt{3}}{2}$ cm	**E** $\frac{9\sqrt{3}}{2}$ cm

24. The altitude of a pyramid is divided into four equal parts. The planes, which are parallel to the base, pass through the points of division. The area of the base of the pyramid is 64 cm^2. Find the area of the upper section.

A 24 cm^2	**B** 32 cm^2	**C** 8 cm^2	**D** 16 cm^2	**E** 4 cm^2

25. Find the diagonal of a right parallelepiped if its dimensions are 3, 4 and 12 cm.

A 19 cm	**B** $\sqrt{119}$cm	**C** 13 cm	**D** $\sqrt{137}$cm	**E** 15 cm

Level 2

26. Find the larger diagonal of a right prism whose base is a rhombus with the side of $\sqrt{2}$ cm and an acute angle of $60°$, and the larger diagonal of this prism is inclined to the plane of the base at an angle of $45°$.

A 2 cm	**B** $\sqrt{3}$ cm	**C** $\sqrt{6}$ cm	**D** 4 cm	**E** $2\sqrt{3}$ cm

27. Find a smaller diagonal of a right prism whose base is a rhombus with a side of 6 cm and an acute angle of $60°$, and a larger diagonal of this prism is inclined to the plane of the base at an angle of $45°$.

A 12 cm	**B** $6\sqrt{6}$ cm	**C** 6 cm	**D** $6\sqrt{3}$ cm	**E** 24 cm

28. The side of the base of a right triangular prism is 8 cm. Find the cross-sectional area of the prism formed by a plane passing through the

middle points of the two sides of the base, forming an angle of $30°$ with its base and intersecting its two lateral edges.

A $24\sqrt{3}\ \text{cm}^2$	**B** $24\ \text{cm}^2$	**C** $18\ \text{cm}^2$	**D** $6\sqrt{3}\ \text{cm}^2$	**E** $6\sqrt{6}\ \text{cm}^2$

29. Find the area of the diagonal section of a regular quadrangular pyramid if the side of the base is equal to a and the lateral edge forms an angle φ with the plane of the base.

A $\frac{a^2}{2}\sin\phi$	**B** $a^2\text{ctg}\phi$	**C** $a^2\text{tg}\phi$	**D** $\frac{a^2}{2}\text{ctg}\phi$	**E** $\frac{a^2}{2}\text{tg}\phi$

30. The side of the base of a regular hexagonal pyramid is 2 cm, the altitude is 4 cm. Find the area of the smaller diagonal section.

A $\sqrt{51}\ \text{cm}^2$	**B** $2\sqrt{51}\ \text{cm}^2$	**C** $3\sqrt{5}\ \text{cm}^2$	**D** $6\sqrt{5}\ \text{cm}^2$	**E** $4\sqrt{15}\ \text{cm}^2$

31. The altitude of a regular quadrangular truncated pyramid is 16 cm, the sides of the bases are 24 and 40 cm. Find the diagonal of the truncated pyramid.

A 36 cm	**B** 24 cm	**C** 48 cm	**D** $8\sqrt{14}$ cm	**E** $8\sqrt{15}$cm

32. Each edge of a right quadrilateral prism is equal to a, the acute angle of its base is equal to α. Find the cross-sectional area of the prism passing through the opposite sides of the bases.

A $a^2\sqrt{1+\sin^2\alpha}$	**B** $a^2\sqrt{1+\cos^2\alpha}$	**C** $a^2\sin\alpha$	**D** $a^2\cos\alpha$	**E** $\frac{a^2}{\cos\alpha}$

33. In a regular quadrangular truncated pyramid, the sides of the bases are 20 and 14 cm. Find the lateral edge of this pyramid if it forms an angle of $30°$ with the base.

A $\frac{3\sqrt{6}}{2}$ cm	**B** $3\sqrt{6}$ cm	**C** $2\sqrt{6}$ cm	**D** $\frac{17\sqrt{6}}{2}$ cm	**E** $6\sqrt{2}$ cm

34. The sides of the bases of a regular triangular truncated pyramid are 15 and 9 cm, the dihedral angle with a larger base is equal to $60°$. Find the altitude of the pyramid.

A $\frac{3}{2}$ cm	**B** $\frac{3}{2}\sqrt{3}$ cm	**C** 3 cm	**D** 1 cm	**E** $\frac{9}{2}$ cm

35. In a right parallelepiped, the lateral edge is 10 cm, the sides of the base are 23 and 11 cm, and the diagonals of the base are related as $2:3$. Find the area of the smaller diagonal section.

A $100\sqrt{2}\ \text{cm}^2$	**B** $150\sqrt{2}\ \text{cm}^2$	**C** $300\ \text{cm}^2$	**D** $200\ \text{cm}^2$	**E** $100\ \text{cm}^2$

36. The base of a right prism is a right-angled triangle whose hypotenuse is 2 dm and the acute angle is $\alpha = 30°$. A plane is drawn through the leg adjacent to the angle α, which forms an angle $\varphi = 60°$ with the plane of the base and intersects the opposite lateral edge. Find the cross-sectional area.

A $\frac{\sqrt{3}}{2}\ \text{dm}^2$	**B** $4\sqrt{3}\ \text{dm}^2$	**C** $\sqrt{3}\ \text{dm}^2$	**D** $1.5\ \text{dm}^2$	**E** $2\ \text{dm}^2$

37. In a right triangular pyramid, a dihedral angle at the base is α. Find the angle of inclination of the lateral edge to the base.

A	B	C	D	E
$\operatorname{arctg}\left(\frac{1}{2}\operatorname{tg}\alpha\right)$	$\operatorname{arctg}\left(\frac{3}{4}\operatorname{tg}\alpha\right)$	$\operatorname{arcctg}\left(\frac{1}{2}\operatorname{tg}\alpha\right)$	$\operatorname{arcctg}\left(\frac{3}{4}\operatorname{tg}\alpha\right)$	α

38. The base of a pyramid is a square. One of the lateral faces is perpendicular to the plane of the base, two adjacent lateral faces are inclined to the plane of the base at an angle φ. Find the angle of inclination to the plane of the base of the fourth lateral face.

A	B	C	D	E
$\operatorname{arctg}(2\operatorname{tg}\varphi)$	$\operatorname{arcctg}\left(\frac{1}{2}\operatorname{ctg}\varphi\right)$	$\operatorname{arcctg}\left(\frac{1}{2}\operatorname{tg}\varphi\right)$	φ	$\operatorname{arctg}\left(\frac{1}{2}\operatorname{tg}\varphi\right)$

39. In a regular hexagonal pyramid, a dihedral angle at the base is $60°$. The distance from the center of the base to the lateral face is 3 cm. Find the length of the lateral edge.

A $4\sqrt{3}\ \text{cm}$	**B** $8\sqrt{3}\ \text{cm}$	**C** $2\sqrt{13}\ \text{cm}$	**D** $\frac{\sqrt{145}}{2}\ \text{cm}$	**E** $12\ \text{cm}$

40. The base of a pyramid is a triangle with the sides of 26, 28 and 30 cm. The lateral edge, opposite to the average side of the base, is

perpendicular to the base and forms an angle of $45°$ with the lateral face. Find the altitude of the pyramid.

A $24\sqrt{3}$ cm	**B** 12 cm	**C** $12\sqrt{3}$ cm	**D** 24 cm	**E** 48 cm

41. The base of a pyramid is an equilateral triangle. Two side edges of the with length b are inclined to the base at an angle α. The face, which includes these edges, is inclined to the base at an angle φ. Calculate the area of the base of the pyramid if b = 4 dm, $\alpha = 30°$, $\varphi = 45°$.

A $4\sqrt{3}$ dm^2	**B** $32\sqrt{3}$ dm^2	**C** $8\sqrt{2}$ dm^2	**D** $8\sqrt{2}$ dm^2	**E** $16\sqrt{2}$ dm^2

42. The lateral edge of a regular truncated triangular pyramid is 5 cm, the sides of the bases are 9 and 6 cm. Find the altitude of the pyramid.

A $\sqrt{22}$ cm	**B** $\frac{4\sqrt{3}}{3}$ cm	**C** 4.5 cm	**D** $\frac{\sqrt{102}}{3}$ cm	**E** $\frac{5\sqrt{3}}{2}$ cm

43. The base of a pyramid is a rhombus whose side is 12 cm, and the acute angle is $60°$; two faces of the pyramid adjacent to the acute angle of the base are perpendicular to it, and the other two faces are inclined to the base at an angle of $30°$. Determine the altitude of the pyramid.

A 18 cm	**B** 6 cm	**C** $2\sqrt{3}$ cm	**D** $4\sqrt{3}$ cm	**E** 9 cm

44. The area of the diagonal section of a regular quadrangular pyramid is equal to m^2, and the angle between the lateral edge and the plane of the base is α. Find the altitude of the pyramid.

A $2m\sqrt{\mathrm{ctg}\alpha}$	**B** $\frac{m}{2}\sqrt{\mathrm{tg}\alpha}$	**C** $m\sqrt{\mathrm{tg}\alpha}$	**D** $m\sqrt{\mathrm{ctg}\alpha}$	**E** $2m\sqrt{\mathrm{tg}\alpha}$

45. Two equal lateral faces of a triangular pyramid are perpendicular to the plane of the base. It is known that one of the angles of the base is equal to $60°$, and the equal lateral edges form an angle α with the base. Find the linear angle at the apex of the third lateral face.

A	B	C	D	E
$2\arcsin\left(\frac{1}{2}\cos\alpha\right)$	$2\arcsin(\cos\alpha)$	$\arcsin\left(\frac{1}{2}\cos\alpha\right)$	$\arcsin(\cos\alpha)$	$\frac{\alpha}{2}$

46. All faces of a pyramid are right-angled triangles. Find its dihedral angles.

A $\frac{\pi}{3}$	**B** $\arccos\frac{1}{3}$	**C** $\arccos\frac{2}{3}$	**D** $\arcsin\frac{1}{3}$	**E** $2\arccos\frac{2}{3}$

47. In a right parallelepiped, the sides of the base are equal to a and b, and the acute angle is equal to $60°$. The larger diagonal of the base is equal to the smaller diagonal of the parallelepiped. Calculate the altitude of the parallelepiped if $a = 10$ cm, $b = 5$ cm.

A 12 cm	**B** $5\sqrt[4]{3}$ cm	**C** 5 cm	**D** $10\sqrt[4]{3}$ cm	**E** 10 cm

48. The base of a pyramid is a square. One edge of the pyramid is perpendicular to the plane of the base, and the opposite edge forms an angle of $30°$ with the plane of the base and has a length of 10 cm. Find the length of the last other lateral edges.

A	B	C	D	E
5 cm; $\frac{5\sqrt{10}}{2}$ cm	$5\sqrt{3}$ cm; $\frac{5\sqrt{10}}{2}$ cm	$5\sqrt{3}$ cm; $\frac{5\sqrt{14}}{2}$ cm	5 cm; $\frac{5\sqrt{14}}{2}$ cm	5 cm; $5\sqrt{3}$ cm

49. The base of a pyramid is a right-angled triangle with the side a. One of the lateral edges is perpendicular to the plane of the base, and the other two are inclined to the plane of the base at equal angles β. Calculate the area of the largest lateral face of the pyramid if $a = 2$ cm and $\beta = 60°$.

A $\sqrt{14}$ cm^2	**B** $\sqrt{15}$ cm^2	**C** $\sqrt{5}$ cm^2	**D** $\frac{\sqrt{2}}{2}$ cm^2	**E** $\sqrt{13}$ cm^2

50. The base of a right prism is a trapezoid, inscribed in a semicircle of radius R so that its larger base coincides with the diameter, and the smaller one has the arc 2α. Find the altitude of the prism if the diagonal of the face passing through the side of the base is inclined to the base at an angle α (calculate for $\alpha = 30°$ and R = 18 cm).

A $6\sqrt{3}$ cm	**B** $18\sqrt{3}$ cm	**C** $3\sqrt{3}$ cm	**D** 18 cm	**E** 54 cm

Level 3

51. The side of the base of a right quadrangular prism is equal to a, the altitude of the prism is equal to H. Find the area of the section drawn through the middle points of the two adjacent sides of the base and the center of symmetry of the prism.

52. A trihedral angle, in which all linear angles are right, intersects two parallel planes so that equilateral triangles are formed in the sections, with the sides of 6 and 10 cm. Find the altitude of the truncated pyramid which is formed.

53. In a regular triangular truncated pyramid, the sides of the bases are 5 and 2 cm, the altitude is 1 cm. A plane parallel to the opposite lateral edge is drawn through the side of the smaller base. Find the area of the formed section.

54. The base of a prism is an equilateral triangle. One of the lateral edges forms equal angles with adjacent sides of the base and an angle of 45° with the plane of the base. The area of the lateral face, which is opposite to this edge, is equal to Q. Find the cross-sectional area passing through this edge and the altitude of the base of the prism.

55. The side of the base of a right triangular pyramid is equal to a, the lateral edge is b. Find the shortest distance between two opposite edges.

56. The base of a pyramid is a triangle, one side of which is equal to c, and the adjacent angles are α and β. All lateral edges of the pyramid are inclined to the base at an angle φ. Find the altitude of the pyramid.

57. The base of a pyramid is a rhombus, whose altitude is equal to b. The distance from the apex of the pyramid to each side of the base is a greater than the altitude of the pyramid. Find the altitude of the pyramid.

58. In a regular quadrangular pyramid, the distance from the center of the base to the lateral face is equal to m, the lateral edge is n. Find the side of the base of the pyramid.

59. The dihedral angle between two side faces of a regular triangular pyramid with the base ABC and the apex S is equal to 120°. The distance from the vertex B of the base to the edge AS is equal to b. Find the apothem of the pyramid.

60. The dihedral angle between two side faces of a regular quadrangular pyramid is equal to α. The side of the base is equal to a. Find the lateral edge of the pyramid.

61. The dihedral angle between two lateral faces of a regular quadrangular pyramid is equal to $120°$. The side of the base is 6. Find the altitude of the pyramid.

62. In a right triangular pyramid, the linear angle at the apex is 2α. Find the dihedral angle at the lateral edge.

63. The base of a pyramid is an isosceles trapezoid whose parallel sides are equal to a and b. Each of the dihedral angles at the base of the pyramid is equal to α. Find the altitude of the pyramid.

64. The side of the base of a regular triangular pyramid is equal to a. The dihedral angle at the lateral edge is equal to $90°$. Find the altitude of the pyramid.

65. The base of a pyramid is a square. Two lateral edges are equal to each other, and the face to which these edges belong forms an angle $\alpha > 90°$ with the base. The opposite face forms an angle β with the base. Find the angles that are formed by the other two lateral faces with the base.

66. A plane is drawn through the apex of a regular quadrangular pyramid at an angle β to the base of this pyramid, in which the side of the base is equal to a, and the linear angle at the apex is α, this plane is parallel to the side of the base. Find the cross-sectional area.

67. The base of a right quadrangular prism is a rhombus with an acute angle α. At what angle to the plane of the base should the plane be drawn so that the cross section be a square with vertices on the lateral edges of the prism?

68. The altitude of a regular truncated hexagonal pyramid is equal to h, the lateral edge is a. Find the apothem.

69. The base of a pyramid is a right-angled triangle with the hypotenuse c and an acute angle α. All lateral edges are inclined to the base at an angle β. Find the linear angles at its apex.

70. A plane is drawn through the apex of a regular triangular pyramid and the middle points of the two sides of the base. Determine the cross-sectional area if the side of the base of the pyramid is equal to a and the angle forming the cross-section with the base plane is equal to α.

71. In a regular triangular pyramid, the side of the base is equal to a. The angles between the edges at its apex are equal to each other, and each is equal to α $(\alpha \le 90°)$. Find the cross-sectional area drawn through the base side perpendicular to the opposite lateral edge.

72. The dihedral angle at the lateral edge of a regular hexagonal pyramid is equal to φ. Find the linear angle at the apex of the pyramid.

73. The base of a right prism is a right-angled triangle ABC, which has $\angle C = 90°$, $\angle A = \alpha$ and the leg $AC = b$. The diagonal of the lateral face of the prism passing through the hypotenuse AB forms an angle β with the lateral face passing through the leg AC. Find the area of the lateral face passing through the hypotenuse AB.

74. The lateral edges of a triangular pyramid have the same length l. Two linear angles at the apex of the pyramid are equal to α, and the third one is β. Find the altitude of the pyramid.

75. From the middle point of the altitude of a regular quadrangular pyramid a perpendicular to the lateral edge of the length h, as well as a perpendicular to the lateral face of the length b are drawn. Find the altitude of the pyramid.

Test 7. Solids of revolution

Level 1

1. The radius of the base of a cylinder is 4 cm, the diagonal of the axial section is 10 cm. Find the altitude of the cylinder.

A 6 cm	**B** 8 cm	**C** 5 cm	**D** 4 cm	**E** 7 cm

2. The radius of the base of a cone is 3 cm, its altitude is 4 cm. Find the generator of this cone.

A 8 cm	**B** 7 cm	**C** 4 cm	**D** 6 cm	**E** 5 cm

3. The diagonal of the axial section of a cylinder is 26 cm, the altitude of the cylinder is 24 cm. Find the area of the base of the cylinder.

A 100π cm^2	**B** 25π cm^2	**C** 36π cm^2	**D** 7π cm^2	**E** 8π cm^2

4. The diameter of the base of a cone is 40 cm, its altitude is 20 cm. Find the angle between the generator and the plane of the base.

A 45°	**B** 30°	**C** 60°	**D** 90°	**E** 120°

5. The radii of the lower and upper bases of the truncated cone are 3 cm and 6 cm, and the altitude is 4 cm. Find the generator of the cone.

A 10 cm	**B** 8 cm	**C** 6 cm	**D** 4 cm	**E** 5 cm

6. The radii of the lower and upper bases of a truncated cone are 3 dm and 7 dm, and its generator is 5 dm. Find the area of the axial section of the cone.

A 10 m^2	**B** 25 m^2	**C** 30 m^2	**D** 15 m^2	**E** 20 m^2

7. A ball with a radius of 41 dm intersects a plane at a distance of 9 dm from the center. Find the cross-sectional area.

A 4π m^2	**B** 8π m^2	**C** 16π m^2	**D** 20π m^2	**E** 24π m^2

8. A cylinder is inscribed in a ball of radius R. Find the altitude of the cylinder.

A R	**B** $0.75R$	**C** $1.5R$	**D** $2R$	**E** $3R$

9. Three points A, B and C are taken on a large circle of a ball with a radius of 5 m. A straight line AB passes through the center of the ball. Find the length of AB.

A 5 m	**B** 5.75 m	**C** 6 m	**D** 8 m	**E** 10 m

10. The edge of a cube is equal to a. Find the radius of the inscribed ball.

A a	**B** $\frac{1}{2}a$	**C** $\frac{1}{4}a$	**D** $2a$	**E** $1.5a$

11. The area of the base of a cylinder is related to the area of the axial section as π : 4. Find the angle between the diagonals of the axial section.

A 100°	**B** 120°	**C** 60°	**D** 75°	**E** 90°

12. The ratio of the area of the base of a cone to the area of the axial section is equal to π. Find the angle of inclination of the generator to the base.

A 40°	**B** 45°	**C** 50°	**D** 55°	**E** 60°

13. The altitude of a cone is 4, the radius of the base is 3; the lateral surface of the cone is located on a plane. Find the angle of the resulting sector.

A $180°$	**B** $190°$	**C** $200°$	**D** $216°$	**E** $220°$

14. The generator of a truncated cone is equal to $2a$ and forms an angle of 60 with the base. The radius of one base is twice the radius of the other. Find each radius.

A $a, 2a$	**B** $\frac{a}{2}, a$	**C** $\frac{a}{4}, \frac{a}{2}$	**D** $\frac{a}{3}, \frac{2}{3}a$	**E** $\frac{3}{2}a, 3a$

15. A plane passes through the apex of a cone and forms an angle of 45 with its base. This plane cuts off a quarter of the circle of the base. The altitude of the cone is 10 cm. Find the cross-sectional area.

A $100\sqrt{3}\,\text{cm}^2$	**B** $90\sqrt{3}\,\text{cm}^2$	**C** $80\sqrt{3}\,\text{cm}^2$	**D** $90\sqrt{2}\,\text{cm}^2$	**E** $100\sqrt{2}\,\text{cm}^2$

16. The altitude of a cone is 20, the radius of the base is 25. Find the cross-sectional area drawn through the apex if its distance from the center of the base of the cone is 12.

A 540	**B** 520	**C** 500	**D** 480	**E** 60

17. The altitude of a cone is equal to the radius R of the base. A plane is drawn through its apex, which cuts off an arc of $60°$ from the circle of the base. Find the cross-sectional area.

A $\frac{\sqrt{7}}{4}R^2$	**B** $\frac{\sqrt{6}}{4}R^2$	**C** $\frac{\sqrt{5}}{4}R^2$	**D** $\frac{\sqrt{3}}{4}R^2$	**E** $\frac{\sqrt{2}}{4}R^2$

18. A plane is drawn through the middle of the radius of a ball perpendicular to the radius. Find the ratio of the area of the obtained section to the area of the large disk.

A $\frac{3}{4}$	**B** $\frac{3}{5}$	**C** $\frac{1}{2}$	**D** $\frac{1}{4}$	**E** $\frac{2}{3}$

19. The radius R of a ball is given. A plane is drawn through the end of the radius to form an angle of $60°$ to it. Find the cross-sectional area.

A $\frac{1}{3}\pi R^2$	**B** $\frac{1}{2}\pi R^2$	**C** $\frac{1}{4}\pi R^2$	**D** $\frac{2}{3}\pi R^2$	**E** $\frac{3}{4}\pi R^2$

20. The altitude of a cone is 8 m, the generator is 10 m. Find the radius of the ball inscribed in the cone.

A 2.2 m	**B** 2.4 m	**C** 2.6 m	**D** 2.8 m	**E** 3 m

21. The radius of a ball is 5 cm. A cone with a base radius of 4 cm is inscribed in the ball. Find the altitude of the cone.

A 8 cm	**B** 2 cm	**C** 8 or 2 cm	**D** 7.5 or 2.5 cm	**E** 5 cm

22. The area of the axial section of a cylinder is 8 m^2, the area of the base is 12 m^2. Calculate the cross-sectional area that is parallel to the axis of the cylinder and 1 m distant from it.

A ≈ 8 m	**B** ≈ 7.5 m	**C** ≈ 7 m	**D** ≈ 6.5 m	**E** ≈ 6 m

23. A plane cuts off an arc of $60°$ from the circle and passes parallel to the axis of the cylinder The length of the axis is 20 cm, its distance from the cross-sectional plane is 6 cm. Find the cross-sectional area.

A $100\sqrt{3}$ cm^2	**B** $120\sqrt{3}$ cm^2	**C** $80\sqrt{3}$ cm^2	**D** $100\sqrt{2}$ cm^2	**E** 120 cm^2

24. The radius of the base of a cylinder is 37 dm, its altitude is 24 dm. At what distance from the axis of the cylinder is a section that has the shape of a square?

A 38 dm	**B** 37 dm	**C** 36 dm	**D** 35 dm	**E** 34 dm

25. The altitude of a cylinder is 6 dm, the radius of the base is 5 dm. The ends of the segment AB lie on the circles of both bases. The distance between the segment AB and the axis of the cylinder is 3 dm. Find the length of AB.

A 8 dm	**B** 9 dm	**C** 10 dm	**D** 11 dm	**E** 12 dm

Level 2

26. The area of the base of an equilateral cylinder is equal to $\frac{\pi Q}{4}$. Find the area of its axial section.

A πQ	**B** $\sqrt{Q}$	**C** $\frac{Q}{2}$	**D** Q	**E** $\frac{3Q}{2}$

27. The axial section of the cylinder is a rectangle whose area is 48 cm^2. The length of the base of the cylinder is 12π cm. Find the altitude of the cylinder.

A 8 cm	**B** 6 cm	**C** 2 cm	**D** 10 cm	**E** 4 cm

28. The axial section of a cone is an isosceles triangle with the side of $8\sqrt{3}$ cm, and the angle at the apex of $120°$. Find the length of the base of the cone.

A 24π cm	**B** 26π cm	**C** 20π cm	**D** 16π cm	**E** 18π cm

29. The axial cross-sectional area of a cone is 48 cm^2. Its generator forms an angle of $45°$ with the base. Find the area of the base of the cone.

A 48π cm^2	**B** 16π cm^2	**C** 24π cm^2	**D** 32π cm^2	**E** 12π cm^2

30. The radii of the lower and upper bases of the truncated cone are 18 and 30 cm, the generator is 20 cm. Find the distance from the center of the smaller base to the circle of the lower base.

A 36 cm	**B** 38 cm	**C** 34 cm	**D** 30 cm	**E** 32 cm

31. The radii of the lower and upper bases of a truncated cone and its generator are related as 1 : 4 : 5. The altitude of the cone is 8 cm. Find the area of the axial section.

A 48 cm^2	**B** 54 cm^2	**C** 32 cm^2	**D** 80 cm^2	**E** 24 cm^2

32. A ball is inscribed in an equilateral cone whose generator is equal to l. Find the radius of the ball.

A $\frac{l\sqrt{3}}{12}$	**B** $\frac{l\sqrt{3}}{6}$	**C** $\frac{l\sqrt{3}}{4}$	**D** $\frac{l\sqrt{3}}{3}$	**E** $\frac{l\sqrt{3}}{2}$

33. The base of a right prism is a right-angled triangle with legs of 6 and 8 cm. The lateral edge of the prism is 10 cm. Find the area of the axial section of the cylinder circumscribed around the prism.

A $90\ \text{cm}^2$	**B** $100\ \text{cm}^2$	**C** $80\ \text{cm}^2$	**D** $60\ \text{cm}^2$	**E** $120\ \text{cm}^2$

34. A ball is circumscribed around a cylinder with a radius of 2 cm and an altitude of 3 cm. Find its radius.

A 4 cm	**B** 5 cm	**C** 2.5 cm	**D** 3.5 cm	**E** 4.5 cm

35. The length of the generator of a cone is equal to l and forms an angle of $30°$ with the base. A ball is circumscribed around the cone. Find the radius of the circumscribed ball.

A l	**B** $\sqrt{3}l$	**C** $\sqrt{2}l$	**D** $\frac{3}{2}l$	**E** $2l$

36. A square $ABCD$ with the side of a cm rotates around the side AB. In the formed cylinder a plane passes the middle of CC_1 parallel to the axis AB and perpendicular to CC_1. Find the area of the formed section.

A $a^2\sqrt{2}\ \text{cm}^2$	**B** $\frac{a^2\sqrt{3}}{2}\ \text{cm}^2$	**C** $a^2\sqrt{3}\ \text{cm}^2$	**D** $a^2\ \text{cm}^2$	**E** $\frac{a^2\sqrt{2}}{2}\ \text{cm}^2$

37. The diagonal of the axial section of a cylinder has a length d and is inclined to the plane of the base at an angle α. Find the area of the base of the cylinder.

A $\frac{1}{4}\pi d^2\cos^2\alpha$	**B** $\frac{1}{3}\pi d^2\cos^2\alpha$	**C** $\frac{1}{2}\pi d^2\cos^2\alpha$	**D** $\pi d^2\cos^2\alpha$	**E** $\frac{2}{3}\pi d^2\cos^2\alpha$

38. In a cylinder, whose radius of the base is R and the altitude is h, two mutually perpendicular diameters of the base AB and CD and generators BB_1 and CC_1 are drawn. Find the length of the segment MC_1 where M is the middle of the side B_1D.

A	B	C	D	E
$\frac{2}{3}\sqrt{100R^2-h^2}$	$\frac{1}{3}\sqrt{100R^2-h^2}$	$\frac{1}{2}\sqrt{100R^2-h^2}$	$\frac{1}{4}\sqrt{100R^2-h^2}$	$\sqrt{100R^2-h^2}$

39. The radius of the base of an equilateral cone is R. Find the cross-sectional area that is drawn through two generators with a $30°$ angle between them.

A $\frac{2}{3}R^2$	**B** $\frac{R^2}{4}$	**C** $\frac{3}{2}R^2$	**D** $\frac{R^2}{2}$	**E** R^2

40. The altitude of a cone is H. The angle between the altitude and the generator is $60°$. Find the cross-sectional area that is drawn through two mutually perpendicular generators.

A $2.5H^2$	**B** $2H^2$	**C** $1.5H^2$	**D** H^2	**E** $\frac{H^2}{2}$

41. A semicircle facing a conical surface is given. Find the angle between the generator and the altitude of the cone.

A $20°$	**B** $30°$	**C** $40°$	**D** $50°$	**E** $60°$

42. The radius of a sector is 3 m, its angle is $120°$. The sector was rolled into a conical surface. Find the radius of the base of the cone.

A 2 m	**B** 1.75 m	**C** 1.5 m	**D** 1.25 m	**E** 1 m

43. The areas of the lower and upper bases of a truncated cone are 4 and 16 m^2. A plane, parallel to the base, is drawn through the middle of the altitude. Find the cross-sectional area.

A 11 m^2	**B** 10 m^2	**C** 9 m^2	**D** 8 m^2	**E** 7 m^2

44. The areas of the lower and upper bases of a truncated cone are 4 and 25 m^2. The altitude is divided into three equal parts, and planes are drawn through the dividing points parallel to the bases. Find the cross-sectional areas.

A 9, 16 m^2	**B** 8, 20 m^2	**C** 10, 18 m^2	**D** 9, 18 m^2	**E** 8, 18 m^2

45. The diameter of a ball is 25 m. On its surface, there is a point A and a circle all of whose points are 15 cm away from A. Find the radius of this circle.

A 12.5 cm	**B** 12 cm	**C** 11.5 cm	**D** 11 cm	**E** 10.5 cm

46. A ball intersects a plane which is 24 cm from the center of the ball. Find the radius of the ball if the length of the circle of the resulting cross section is $\frac{3}{5}$ of the length of the large circle.

A 29 cm	**B** 30 cm	**C** 31 cm	**D** 32 cm	**E** 33 cm

47. The radii of the spheres are 17 and 25 dm. The length of the line of intersection of the spheres is 3π m. Find the distance between the centers of the spheres.

A 39.86 dm	**B** 40 dm	**C** 41.89 dm	**D** 42 dm	**E** 42.12 dm

48. Two mutually perpendicular sections of a ball have a common chord of 16 dm. Find the radius of the ball if the areas of these sections are 185π dm^2 and 320π dm^2.

A 25 dm	**B** 24 dm	**C** 23 dm	**D** 22 dm	**E** 21 dm

49. In the base of a cone, a square whose side is equal to a is inscribed. The plane passing through the apex of the cone and the side of the square, in the intersection with the surface of the cone forms a triangle with a $60°$ angle at its apex. Find the altitude of the cone.

A $\frac{2}{3}a$	**B** $\frac{3}{4}a$	**C** $\frac{\sqrt{2}}{2}a$	**D** $\frac{\sqrt{3}}{2}a$	**E** $\frac{a}{2}$

50. The radii of two balls are 25 and 29 dm, and the distance between their centers is 36 dm. Find the length of the line along which their surfaces are intersected.

A 4π m	**B** 3.5π m	**C** 3π m	**D** 2.5π m	**E** 2π m

Level 3

51. The area of the base of a cylinder is related to the area of the axial section as $\pi:2$. Find the acute angle between the diagonals of the axial section.

52. A plane which is parallel to the axis of a cylinder cuts off a $60°$ arc from the circle of the base. The altitude of the cylinder is 15 cm, the distance

of the cutting plane from the axis of the cylinder is 3 cm. Find the cross-sectional area.

53. A section is drawn through two generators of a cone with a 60° angle between them. Its area is equal to $4\sqrt{3}$ cm^2. Find the angle between the plane of the section and the plane of the base of the cone if the section cuts off the arc of 90° from the circle.

54. The radius of the base of a cone is 6 cm, its altitude is 12 cm. Find the area of the section drawn parallel to the axis of the cone at a distance of 2 cm from it.

55. A truncated cone is inscribed around a ball with a radius of 6 cm. The radii of the lower and upper bases of the cone are related as 4 : 9. Find the area of the axial section of the cone.

56. Two planes are drawn through one of the points of a ball of the radius R. The first plane is tangent to the ball, the second one is drawn at an angle of 30° to the first plane. Find the cross-sectional area.

57. The altitude of a cone is 20 cm, the generator is 25 cm. Find the radius of the hemisphere inscribed in the cone whose base lies at the base of the cone.

58. A cylinder is inscribed around a right prism. Its base is a right-angled triangle with a hypotenuse of 20 cm and an acute angle of 30°. The diagonal of the lateral face of the prism, which contains the leg opposite the angle of 30°, is inclined to the plane of the base at an angle of 45°. Find the altitude of the cylinder.

59. A ball is inscribed in an equilateral cylinder with a radius of 5 cm. Calculate the cross-sectional area that is perpendicular to the axis of the cylinder and placed 3 cm from the center of the ball.

60. A regular quadrangular pyramid with a lateral edge of 20 cm is inscribed around a cone. The angle between the lateral edge of the pyramid and its altitude is equal to 30°. Find the radius of the base of the cone.

61. A straight line parallel to the generator l is drawn through the middle of the altitude of a cone. Find the length of the segment of the straight line that is inside the cone.

62. The radius of the base of a cone is R, the altitude is H. A cube is circumscribed in the cone. Find the edge of the cube.

63. The areas of the lower and upper bases of a truncated cone are 1 and 49 m^2. The area of the parallel section is equal to their half-sum. What parts does this section divide the altitude into?

64. In a truncated cone, the altitude is 10 cm, and the radii of the lower and upper bases are 8 and 18 cm. At what distance from the smaller base is a parallel section whose area is the average proportional between the areas of the bases?

65. The diameter of a ball is divided into 10 equal parts by nine points. The planes perpendicular to the diameter are drawn through the first and fourth points. How many times is the area of one section larger than the area of the other?

66. The angle between the generator and the plane of the base of a cone is equal to α. Find the generator of the cone, if a chord of the length c in its base forms an arc φ.

67. A rectangle with sides $5\sqrt{3}$ and 8 cm rotates around the larger side. In the formed cylinder, a plane is drawn through the middle of the base radius and perpendicular to it. Find the area of the formed section.

68. In a truncated cone, the generator of the length l forms an angle α with the plane of the larger base, and it is perpendicular to the diagonal of the axial section. Find the radii of the bases of the truncated cone.

69. A ball with a radius of 5 cm touches all sides of a triangle whose lengths are 13, 14 and 15 cm. Find the distance from the center of the ball to the plane of the triangle.

70. The diagonals of a rhombus are 15 and 20 cm. A ball touches all sides of the rhombus. The radius of the ball is 10 cm. Find the distance from its center to the plane of the rhombus.

71. A ball touches all sides of an isosceles trapezoid, in which the leg is $8\sqrt{2}$ dm and the obtuse angle is $135°$. Find the radius of the ball if the distance from the center of the ball to the plane of the trapezoid is 12 m.

72. A ball is inscribed in a truncated cone in which the area of one base is four times larger than the area of the second base. Find the angle between the generator of the cone and the plane of the larger base.

73. The altitude of a cone is 20 cm, its generator is 25 cm. Find the radius of the inscribed hemisphere whose base lies at the base of the cone.

74. A truncated cone is circumscribed around a ball. The radius of the bases of the cone are r and R. Find the radius of the ball.

75. A plane which is parallel to the axis of a cylinder cuts off the arc 2α from the circle. The diagonal of the obtained section forms an angle φ with the plane of the base. Find the cross-sectional area if the radius of the cylinder base is equal to R.

Test 8. The area of the total surface and the volume of polyhedra and solids of revolution

Level 1

1. The diagonal of a cube is $2\sqrt{3}$ cm. Find the area of the total surface of the cube.

A 72 cm^2	**B** 24 cm^2	**C** 36 cm^2	**D** 12 cm^2	**E** 18 cm^2

2. The area of the diagonal section of a cube is equal to Q. Find the area of the total surface of the cube.

A $3Q\sqrt{2}$	**B** $2Q\sqrt{2}$	**C** $6Q$	**D** $4Q$	**E** $2Q\sqrt{3}$

3. The sides of the base of a rectangular parallelepiped are 6 and 8 cm, and the area of the diagonal section is 200 cm^2. Find the area of the total surface of the parallelepiped.

A 610 cm^2	**B** 560 cm^2	**C** 600 cm^2	**D** 608 cm^2	**E** 656 cm^2

4. In a right parallelepiped, the sides of the base are 3 and 8 cm and form an angle of $60°$. The larger diagonal of the parallelepiped is 49 cm. Find the area of the lateral surface of the parallelepiped.

A 1056 cm^2	**B** 1000 cm^2	**C** $192\sqrt{13}\text{cm}^2$	**D** 2096 cm^2	**E** 980 cm^2

5. Find the area of the lateral surface of a prism whose base is a rhombus with diagonals of 16 and 22 cm, the lateral faces of the prism are squares.

A 1092 cm^2	**B** 740 cm^2	**C** 370 cm^2	**D** 1100 cm^2	**E** 750 cm^2

6. The altitude of a regular quadrangular prism is 6 cm, the side of the base is 3 cm. Find the area of the total surface of this prism.

A 80 cm^2	**B** 120 cm^2	**C** 110 cm^2	**D** 100cm^2	**E** 90 cm^2

7. The side of the base of a regular quadrangular pyramid is 8 cm, and the altitude of the pyramid is 3 cm. Find the area of the total surface of the pyramid.

A 144 cm^2	**B** 224 cm^2	**C** 96 cm^2	**D** 84 cm^2	**E** 124 cm^2

8. The side of the base of a regular triangular pyramid is 4 cm, and the altitude of the pyramid is 5 cm. Find the volume of the pyramid.

A $\frac{0\sqrt{3}}{3}$ cm^3	**B** $\frac{20\sqrt{3}}{3}$ cm^3	**C** $20\sqrt{3}$ cm^3	**D** $\frac{20}{3}$ cm^3	**E** $10\sqrt{3}$ cm^3

9. The apothem of a regular hexagonal pyramid is 5 cm. The side of the base is 3 cm. Find the area of the lateral surface of the pyramid.

A $\frac{45}{2}$ cm^2	**B** 80 cm^2	**C** 90 cm^2	**D** $\frac{15}{2}$ cm^2	**E** 45 cm^2

10. The apothem of a regular quadrangular pyramid is 6 cm. The side of the base is 3 cm. Find the area of the total surface of the pyramid.

A 40 cm^2	**B** 66 cm^2	**C** 33 cm^2	**D** 90 cm^2	**E** 45 cm^2

11. Find the volume of a right parallelepiped whose base is a rhombus with a side of 3 cm and an acute angle of $30°$. The diagonal of the lateral face forms an angle of $60°$ with the lateral edge.

A $81\sqrt{3}$ cm^2	**B** $\frac{27\sqrt{3}}{2}$ cm^2	**C** $\frac{9\sqrt{3}}{2}$ cm^2	**D** $\frac{27}{2}$ cm^2	**E** $\frac{9\sqrt{2}}{2}$ cm^2

12. In a regular quadrangular pyramid, the area of the lateral surface is 380 cm^2, and the area of the total surface is 524 cm^2. Find the side of the base of the pyramid.

A 12 cm	**B** 10 cm	**C** 14 cm	**D** 8 cm	**E** $10\sqrt{2}$ cm

13. The total surface area of a cube is 54 cm^2. Find its diagonal.

A 4 cm	**B** $3\sqrt{2}$ cm	**C** $4\sqrt{3}$ cm	**D** $3\sqrt{6}$ cm	**E** $3\sqrt{3}$ cm

14. The radii of the bases of a truncated cone and its generator are related as 1 : 4 : 5, the altitude is 8 cm. Find the lateral surface of the cone.

A 50π cm^2	**B** 120π cm^2	**C** 100π cm^2	**D** 75π cm^2	**E** 110π cm^2

15. The axial section of the cylinder is a square with the area S. Find the volume of the cylinder.

A $\frac{\pi S\sqrt{S}}{4}$	**B** $\frac{\pi S\sqrt{S}}{2}$	**C** $2\pi S\sqrt{S}$	**D** $\pi S\sqrt{S}$	**E** $\frac{\pi S\sqrt{S}}{8}$

16. Find the area of the total surface of a rectangular parallelepiped if its dimensions are 20, 15 and 16 cm.

A 1420 cm^2	**B** 1720 cm^2	**C** 1120 cm^2	**D** 1000 cm^2	**E** 1160 cm^2

17. The generator of a cone is 2 dm, the angle at the apex of its axial section is $60°$. Find the area of the lateral surface of the cone.

A $\pi\text{ dm}^2$	**B** $4\pi\text{ dm}^2$	**C** $2\pi\text{ dm}^2$	**D** $\frac{2}{3}\pi\text{ dm}^2$	**E** $\frac{\pi}{3}\text{ dm}^2$

18. The altitude of a cone is $4\sqrt{2}$ cm, the angle at the apex of its axial section is $90°$. Find the area of the total surface of the cone.

A	B	C	D	E
$32\pi\left(1+\sqrt{2}\right)\text{cm}^2$	$32\sqrt{2}\pi\text{ cm}^2$	$16\pi\left(1+\sqrt{2}\right)\text{cm}^2$	$16\sqrt{2}\pi\text{ cm}^2$	$64\pi\left(1+\sqrt{2}\right)\text{cm}^2$

19. Find the area of the lateral surface of a regular quadrangular pyramid if the plane of the base is 150 cm^2, and all the lateral faces are inclined to the plane of the base at an angle of $60°$.

A 75 cm^2	**B** 300 cm^2	**C** $150\sqrt{3}\text{ cm}^2$	**D** $75\sqrt{3}\text{ cm}^2$	**E** 150 cm^2

20. The dimensions of a rectangular parallelepiped are 60, 100 and 36 cm. Find the edge of the cube equal to it.

A $120\sqrt{15}\text{ cm}$	**B** $60\sqrt{15}\text{ cm}$	**C** 60 cm	**D** 120 cm	**E** 30 cm

21. Two cylinders are obtained by rotating a rectangle around each of its sides a and b. Find the ratio of the volumes of these cylinders.

A $b^2 : 2a^2$	**B** $2a : b$	**C** $2b : a$	**D** $b : a$	**E** $b^2 : a^2$

22. Find the volume of an equilateral cone if its altitude is 3 cm.

A 3π cm^3	**B** π cm^3	**C** 6π cm^3	**D** $\frac{3}{2}\pi$ cm^3	**E** $\frac{\pi}{6}$ cm^3

23. The base of a pyramid is a right-angled triangle with a smaller leg a and an adjacent angle β. Find the volume of the pyramid if its altitude is equal to the greater leg of the base.

A $\frac{2a^3}{3}\operatorname{tg}^2\beta$	**B** $\frac{a^3}{3}\operatorname{tg}\beta$	**C** $\frac{a^3}{6}\operatorname{tg}\beta$	**D** $\frac{a^3}{3}\operatorname{tg}^2\beta$	**E** $\frac{a^3}{6}\operatorname{tg}^2\beta$

24. The base of a regular pyramid is a triangle whose side is 5 cm, the altitude of the pyramid is 12 cm. Find the volume of the pyramid.

A $15\sqrt{3}$ cm^3	**B** $25\sqrt{3}$ cm^3	**C** $50\sqrt{3}$ cm^3	**D** $\frac{25\sqrt{3}}{2}$ cm^3	**E** $\frac{50\sqrt{3}}{3}$ cm^3

25. The volume of a cube is 27 m^3. Find the area of the total surface.

A 12 m^2	**B** 36 m^2	**C** 54 m^2	**D** 48 m^2	**E** 64 m^2

Level 2

26. In a regular triangular pyramid, the lateral edge is perpendicular to the lateral face and is equal to b. Find the area of the lateral surface of the pyramid.

A $3b^2$	**B** $\frac{3}{4}b^2$	**C** $\frac{b^2}{4}$	**D** $\frac{3}{2}b^2$	**E** $3\sqrt{2b^2}$

27. The areas of two mutually perpendicular lateral faces of a triangular prism are 40 and 30 m^2. The lateral edge of the prism is 5 m. Find the area of the lateral surface of the prism.

A 120 m^2	**B** $80\sqrt{7}$ m^2	**C** 100 m^2	**D** 110 m^2	**E** 150 m^2

28. The diagonals of a right parallelepiped form angles $30°$ and $45°$ with the plane of the base, and the sides of the bases are 6 and 8 cm. Find the area of the lateral surface of the parallelepiped.

A 140 cm^2	**B** $140\sqrt{2}$ cm^2	**C** $150\sqrt{3}$ cm^2	**D** 150 cm^2	**E** $140\sqrt{6}$ cm^2

29. The base of a right prism is an isosceles trapezoid whose bases are 22 and 42 cm, and the leg is 26 cm. The area of the diagonal section of the prism is 400 cm^2. Find the area of the total surface of the prism.

A 2696 cm^2	**B** 3000 cm^2	**C** 2500 cm^2	**D** 2605 cm^2	**E** $2100\sqrt{2}\text{ cm}^2$

30. In a regular quadrangular prism, the diagonal is equal to l and forms an angle of $30°$ with the lateral face. Find the area of the lateral surface of the prism.

A $l^2\sqrt{3}$	**B** $2l^2$	**C** $4\sqrt{2}l^2$	**D** $4\sqrt{3}l^2$	**E** $l^2\sqrt{2}$

31. Find the area of the lateral surface of a regular hexagonal prism if its smaller diagonal is equal to l and forms an angle α with the plane of the base.

A $l^2\cos 2\alpha$	**B** $\sqrt{3}l^2\cos\alpha$	**C** $\sqrt{2}l^2\cos\alpha$	**D** $\sqrt{2}l^2\sin\alpha$	**E** $\sqrt{3}l^2\sin 2\alpha$

32. The smaller diagonal of a right parallelepiped whose base is a rhombus, forms an angle α with the plane of the base. The side of the rhombus is equal to a, the acute angle is β. Find the area of the lateral surface of the parallelepiped.

A	B	C	D	E
$8a^2\text{tg}\,\alpha\sin\dfrac{\beta}{2}$	$4a^2\text{tg}\,\alpha\sin\dfrac{\beta}{2}$	$8a^2\text{ctg}\,\alpha\sin\beta$	$4a^2\text{ctg}\,\alpha\cos\beta$	$8a^2\text{tg}\,\alpha\cos\dfrac{\beta}{2}$

33. The base of a right parallelepiped is a rhombus whose diagonals are related as 5 : 16. The diagonals of the parallelepiped are 26 and 40 cm. Find the volume of the parallelepiped.

A 3760 cm^3	**B** 3840 cm^3	**C** 4000 cm^3	**D** 3540 cm^3	**E** 4110 cm^3

34. The diagonal of a regular quadrilateral prism is equal to d and forms an angle β with the lateral face. Find the volume of the prism.

A	B	C	D	E
$d^3\sin^2\beta\sqrt{\cos 2\beta}$	$d^3\sin\beta\sqrt{\cos\beta}$	$d^3\cos\beta\sqrt{\sin\beta}$	$d^3\text{tg}\beta\sqrt{\cos 2\beta}$	$d^3\cos\beta\sin^2\beta$

35. Find the volume of a regular hexagonal prism if its larger diagonal is 4 cm and forms an angle of $30°$ with the base.

A 12 cm^3	**B** $18\sqrt{3}\text{ cm}^3$	**C** 64 cm^3	**D** $27\sqrt{3}\text{ cm}^3$	**E** $9\sqrt{3}\text{ cm}^3$

36. The side of the base of a right parallelepiped is 2 cm, the angle between this side and the diagonal of the base is $30°$. The diagonal of the parallelepiped is inclined to the plane of the base at an angle of $45°$. Find the volume of the parallelepiped.

A $\frac{8}{3}$ cm^3	**B** $\frac{16}{3}$ cm^3	**C** $16\sqrt{3}$ cm^3	**D** 16 cm^3	**E** $8\sqrt{2}$ cm^3

37. The base of a right parallelepiped is a rhombus with the side of 4 cm and the acute angle of $60°$. The larger diagonal of the parallelepiped forms an angle of $30°$ with the plane of its base. Find the volume of the parallelepiped.

A $48\sqrt{3}$ cm^3	**B** 96 cm^3	**C** $32\sqrt{3}$ cm^3	**D** $\frac{128}{3}\sqrt{3}$ cm^3	**E** $64\sqrt{3}$ cm^3

38. Find the area of the lateral surface of a right triangular pyramid if its altitude is 9 cm, and the apothem is 18 cm.

A 1458 cm^2	**B** 2916 cm^2	**C** 729 cm^2	**D** 486 cm^2	**E** 1500 cm^2

39. In a regular quadrangular pyramid, the area of the lateral surface is 240 cm^2, and the area of the total surface is 384 cm^2. Find the altitude of the pyramid.

A 10 cm	**B** 8 cm	**C** 12 cm	**D** 7 cm	**E** 16 cm

40. In a regular triangular pyramid, the apothem is 6 cm and forms an angle of $60°$ with the base. Find the area of the total surface of the pyramid.

A $9\sqrt{6}$ cm^2	**B** $81\sqrt{2}$ cm^2	**C** 81 cm^2	**D** $27\sqrt{3}$ cm^2	**E** $81\sqrt{3}$ cm^2

41. The base of a pyramid is a right-angled triangle with a leg of 2 cm and an adjacent angle of $30°$. The lateral edges form an angle of $60°$ with the base. Find the volume of the pyramid.

A $\frac{4\sqrt{3}}{9}$ cm^3	**B** $\frac{4\sqrt{3}}{81}$ cm^3	**C** $\frac{4\sqrt{2}}{9}$ cm^3	**D** $\frac{\sqrt{3}}{10}$ cm^3	**E** 5 cm^3

42. The base of a pyramid is a rhombus with the side of 15 cm. Each face of the pyramid forms a $45°$ angle with the base. Find the volume of the pyramid if the area of the lateral surface is 300 cm^2.

A $400\sqrt{2}\,\text{cm}^3$	**B** $450\ \text{cm}^3$	**C** $500\ \text{cm}^3$	**D** $500\sqrt{2}\,\text{cm}^3$	**E** $450\sqrt{3}\ \text{cm}^3$

43. The base of a pyramid is a rectangle with the area of 1 m^2. Two lateral faces are perpendicular to the base, and the other two are inclined to it at angles of $30°$ and $60°$. Find the volume of the pyramid.

A $\frac{\sqrt{3}}{2}\ \text{m}^3$	**B** $\frac{2}{3}\ \text{m}^3$	**C** $\frac{\sqrt{3}}{3}\ \text{m}^3$	**D** $\frac{1}{3}\ \text{m}^3$	**E** $\frac{\sqrt{2}}{3}\ \text{m}^3$

44. The base of the pyramid is a rhombus with the side of 3 cm and the acute angle of $30°$. The lateral faces are inclined to the plane of the base at an angle of $45°$. Find the volume of the pyramid.

A $\frac{9}{8}\ \text{cm}^3$	**B** $\frac{27}{8}\ \text{cm}^3$	**C** $\frac{9\sqrt{3}}{4}\ \text{cm}^3$	**D** $\frac{3\sqrt{3}}{4}\ \text{cm}^3$	**E** $\frac{8}{9}\ \text{cm}^3$

45. A right-angled triangle with a leg a and the adjacent angle of $60°$ rotates around the hypotenuse. Find the volume of the solid of revolution.

A πa^3	**B** $\frac{1}{3}\pi a^3$	**C** $\frac{1}{2}\pi a^3$	**D** $2\pi a^3$	**E** $\frac{3}{2}\pi a^3$

46. The base of a right parallelepiped is a parallelogram with the sides of 1 and 4 cm and the acute angle of $60°$. The larger diagonal of the parallelepiped is 5 cm. Find its volume.

A $4\sqrt{3}\,\text{cm}^3$	**B** $2\sqrt{3}\,\text{cm}^3$	**C** $4\sqrt{2}\,\text{cm}^3$	**D** $4\ \text{cm}^3$	**E** $3\sqrt{3}\,\text{cm}^3$

47. Three edges of a right parallelepiped are related as 2 : 7 : 26, its diagonal is 81 cm. Find the volume of the parallelepiped.

A $9000\ \text{cm}^3$	**B** $9828\ \text{cm}^3$	**C** $7284\ \text{cm}^3$	**D** $8382\ \text{cm}^3$	**E** $4000\ \text{cm}^3$

48. The base of a right prism is an isosceles trapezoid, the parallel sides of which are 5 and 11 cm, the altitude of the base is 4 cm. The lateral edge of the prism is 6 cm. Find the volume of the prism.

A $198\ \text{cm}^3$	**B** $200\ \text{cm}^3$	**C** $384\ \text{cm}^3$	**D** $96\ \text{cm}^3$	**E** $192\ \text{cm}^3$

49. The base of a right prism is a right-angled triangle with an angle of $30°$. The altitude of the prism is 10 cm. The diagonal of the lateral face adjacent to the hypotenuse forms an angle of $60°$ with the base. Find the volume of the prism.

A $\frac{125}{3}\text{cm}^3$	**B** $\frac{125\sqrt{2}}{3}\text{cm}^3$	**C** $125\sqrt{3}\,\text{cm}^3$	**D** $\frac{125\sqrt{3}}{2}\text{cm}^3$	**E** $\frac{125\sqrt{3}}{3}\text{cm}^3$

50. The base of a prism is a triangle with the sides of 6, 10 and 14 cm. The lateral edge is 8 cm, and it is inclined to the plane of the base at an angle of $60°$. Determine the volume of the prism.

A 180 cm^3	**B** 90 cm^3	**C** 200 cm^3	**D** 190 cm^3	**E** 210 cm^3

Level 3

51. The base of a pyramid is a square with the side of 16 cm; its two lateral faces are perpendicular to the plane of the base. The altitude of the pyramid is 12 cm. Find the area of the total surface of the pyramid.

52. The base of a pyramid is a right-angled triangle whose legs are 3 and 4 cm. Each lateral face is inclined to the plane of the base at an angle of $60°$. Find the area of the total surface of the pyramid.

53. The base of a pyramid is a rhombus with the diagonals of 6 and 8 dm. The altitude of the pyramid is 1 dm. All dihedral angles at the base are equal. Find the area of the total surface.

54. The base of a pyramid is a triangle with the sides of 6, 10 and 14 cm. The lateral faces are inclined to the plane of the base at an angle of $60°$. Find the area of the total surface of the pyramid.

55. The base of a pyramid is an isosceles trapezoid whose parallel sides are 3 and 5 cm, and the leg is 7 cm. The altitude of the pyramid passes through the point of intersection of the diagonals of the base, and the larger lateral edge is 10 cm. Find the volume of the pyramid.

56. A truncated cone is circumscribed around a ball of the radius R whose generator forms an angle α with the base. Find the area of the lateral surface of this cone.

57. A ball of the radius R is inscribed in a truncated cone. The generator of the cone is inclined to the plane of the base at an angle α. Find the volume of the truncated cone.

58. The base of a pyramid is an isosceles triangle; its leg is a, and the angle between the sides is equal to α. One of the lateral faces of the pyramid, which passes through the base of the isosceles triangle, is perpendicular to the plane of the base, the other two faces form an angle φ with the base. Find the lateral surface of this pyramid.

59. The diagonals of a right parallelepiped are 9 and $\sqrt{33}$ cm. The perimeter of its base is 18 cm, the lateral edge is 4 cm. Find the volume of the parallelepiped.

60. The diagonals of a right parallelepiped are 9 and $\sqrt{33}$ cm. The perimeter of its base is 18 cm, the lateral edge is 4 cm. Find the area of the lateral surface of the parallelepiped.

61. The diagonals of a right parallelepiped are 9 and $\sqrt{33}$ cm. The perimeter of its base is 18 cm, the lateral edge is 4 cm. Find the area of the total surface of the parallelepiped.

62. The apothem of a regular hexagonal pyramid is equal to m. The dihedral angle at the base is equal to α. Find the area of the total surface of the pyramid.

63. The base of a right prism is an isosceles triangle in which the angle α between the equal sides is equal to a. Diagonals of equal lateral faces are drawn from the apex of the upper base, the angle between the diagonals is equal to β. Find the volume of the prism.

64. A triangle with the sides of 13, 14 and 15 cm rotates around the side of 14 cm. Find the volume of the solid of revolution.

65. An isosceles triangle with an angle of $120°$ at the vertex and a side of 20 cm rotates around the base. Find the area of the total surface.

66. A circle is inscribed in a cone whose generator forms an angle α with the plane of the base. Find the volume of the cone if the volume of the ball is equal to V.

67. In a truncated cone, the diagonals of the axial section are mutually perpendicular, and the generator forms an angle α with the plane of the lower base, and it is equal to l. Find the area of the total surface of the truncated cone.

68. In a regular truncated quadrangular pyramid, the dihedral angle at the base is equal to β, and the sides of the bases a and b. Find its lateral surface.

69. A ball is inscribed in a cone. The difference between the generator of the cone and the radius of its base is d. Determine the volume of the cone if the angle between the generator and the plane of the base of the cone is equal to α.

70. The generator of a cone forms an angle α with its axis. Determine the ratio of the volume of this cone to the volume of the ball circumscribed around it.

71. A cone is inscribed in a ball of the radius R; the generator of the cone forms an angle α with the plane of the base. Calculate the volume of the cone.

72. Find the altitude of a regular quadrangular pyramid whose total surface is equal to S, and the linear angle of the lateral face at the apex is equal to α.

73. In a truncated cone, the length of the diagonal of the axial section is equal to a, the generator forms an angle α with the plane of the base and is equal to b. Find the lateral surface of this cone.

74. A ball of the radius R is inscribed in a pyramid whose base is a rhombus with an acute angle α. The lateral faces of the pyramid are inclined to the plane of the base at an angle φ. Find the volume of the pyramid.

75. A truncated cone is inscribed in a ball of the radius R. The bases of the truncated cone cut off from the ball two segments with arcs α and β in the axial section. Find the area of the lateral surface of the truncated cone.

76. Find the volume of a cone if its base has a chord, which is equal to a, with the arc α, and the altitude of the cone forms an angle β with the generator.

Test 9. Vectors

Level 1

1. The coordinates of the vectors $\vec{a} = (1;-2;-5)$, $\vec{b} = (6;2;-1)$ are given. Find the vector $\vec{c} = 4\vec{a} + \vec{b}$.

A	B	C	D	E
$(-10;6;1)$	$(-6;10;21)$	$(10;-6;-21)$	$(-10;-6;21)$	$(10;6;1)$

2. The coordinates of the vector are given as $x = 4$; $y = -12$. Find its third coordinate z under the condition $|\vec{a}| = 13$.

A	B	C	D	E
± 3	± 4	± 2	± 1	0

3. Determine the initial point of the vector $\vec{a} = (2; -3; -1)$ if its endpoint coincides with the point $(1; -1; 2)$.

A	B	C	D	E
$(1; -2; 3)$	$(-1; 2; 3)$	$(-1; -2; 3)$	$(-1; 2; -3)$	$(2; 3; -1)$

4. The vector module is $|\vec{a}| = 2$ and the angles are $\alpha = 45°$, $\beta = 60°$, $\gamma = 120°$. Find the projections of the vector $\vec{a}$ on the coordinate axes.

A	B	C	D	E
$(1; -\sqrt{2}; 1)$	$(\sqrt{2}; 1; -1)$	$(-1; \sqrt{2}; 1)$	$(\sqrt{3}; 1; 1)$	$(\sqrt{2}; -1; 1)$

5. Find the ort $\vec{a}_0$ of the vector $\vec{a} = (2; -6; 3)$.

A	B	C	D	E
$\left(\frac{2}{7}; \frac{6}{7}; \frac{3}{7}\right)$	$\left(-\frac{2}{7}; -\frac{6}{7}; \frac{3}{7}\right)$	$\left(\frac{2}{7}; -\frac{6}{7}; \frac{3}{7}\right)$	$\left(-\frac{2}{7}; -\frac{6}{7}; \frac{3}{7}\right)$	$\left(\frac{2}{7}; \frac{3}{7}; -\frac{6}{7}\right)$

6. Two vectors are given as $\vec{a} = (3; -2; 6)$ and $\vec{b} = (-2; 1; 0)$. Determine the projections on the coordinate axes of the vector $\vec{c} = 2\vec{a} + 3\vec{b}$.

A	B	C	D	E
$(1; 0; 12)$	$(-1; 0; 12)$	$(0; 1; -12)$	$(0; -1; 12)$	$(0; -2; -12)$

7. Find the direction cosines of the vector $\overrightarrow{AB}$ if $A(-1; 2; 4)$, $B(1; 6; 0)$.

A	B	C	D	E
$\left(\frac{1}{3}; -\frac{2}{3}; \frac{2}{3}\right)$	$\left(\frac{2}{7}; -\frac{6}{7}; \frac{3}{7}\right)$	$\left(\frac{1}{3}; \frac{2}{3}; -\frac{2}{3}\right)$	$\left(-\frac{1}{3}; \frac{2}{3}; \frac{2}{3}\right)$	$\left(-\frac{2}{3}; \frac{1}{3}; \frac{2}{3}\right)$

8. Find the value p at which the vectors $\vec{a} = (3; p; -1)$ and $\vec{b} = (p; -2; 5)$ are perpendicular.

A ± 5	**B** ± 2	**C** 5	**D** 2	**E** 4

9. Find the projection of the vector $\vec{a}$ on the vector $\vec{b}$ if vectors $\vec{a}=(1;1;-2)$, $\vec{b}=(2;-1;-2)$ are given.

A $\frac{3}{5}$	**B** $\frac{7}{3}$	**C** $\frac{4}{5}$	**D** $\frac{5}{3}$	**E** $-\frac{5}{3}$

10. Find the work A of force $\vec{F}=(2;3;-4)$ when moving a material point from position $M(-2;1;5)$ to position $N(-1;6;3)$.

A 23	**B** 21	**C** 12	**D** 22	**E** 25

11. Three points $A(-4;1)$, $B(-1;5)$, $C(3;3)$ are given. Find the cosine of the angle formed by the vector $\overrightarrow{AB}$ with the vector $\overrightarrow{AC}$.

A $\frac{6}{\sqrt{85}}$	**B** $\frac{9}{\sqrt{85}}$	**C** $\frac{4}{\sqrt{17}}$	**D** $\frac{29}{5\sqrt{53}}$	**E** $\frac{4}{\sqrt{65}}$

12. Calculate the sides of a triangle with vertex coordinates $A(-2;1;-1)$, $B(1;2;-2)$, $C(2;-1;-3)$.

A	B	C	D	E
$(\sqrt{11};\ \sqrt{11};\ 11)$	$(\sqrt{11};\ 2\sqrt{11};\ 11)$	$(\sqrt{11};\ \sqrt{11};\ 2\sqrt{6})$	$(\sqrt{6};\ \sqrt{6};\ 6)$	$(11;\ \sqrt{11};\ 2\sqrt{6})$

13. Find the length of the vector $\overrightarrow{OA}$ and the angle α formed by the vector and the positive direction of the axis OX if $O(0;0)$, $A(-2;-2)$ are given.

A $(2\sqrt{2};\ \frac{\pi}{2})$	**B** $(2\sqrt{2};\ \frac{3\pi}{4})$	**C** $(2\sqrt{2};\ \frac{5\pi}{4})$	**D** $(2\sqrt{2};\ \frac{7\pi}{4})$	**E** $(\sqrt{2};\ \frac{\pi}{4})$

14. Vectors $\vec{a}$ and $\vec{b}$ form an angle $\varphi=\frac{2}{3}\pi$. Calculate $(3\vec{a}+2\vec{b})^2$ if $|\vec{a}|=3$ and $|\vec{b}|=4$.

A -61	**B** 73	**C** 72	**D** 61	**E** -72

15. Vectors $\vec{a}$ and $\vec{b}$ are mutually perpendicular; vector $\vec{c}$ forms angles $\frac{\pi}{3}$ with them. Find $(\vec{a}+\vec{b}+\vec{c})^2$ if $|\vec{a}|=3$, $|\vec{b}|=4$, $|\vec{c}|=8$.

A 62	**B** 123	**C** 184	**D** -145	**E** 162

16. Determine the value α at which the vectors $\vec{a}+\alpha\vec{b}$, $\vec{a}-\alpha\vec{b}$ will be mutually perpendicular if $|\vec{a}|=3$, $|\vec{b}|=5$.

A $\alpha=\pm\frac{3}{5}$	**B** $\alpha=\pm\frac{5}{3}$	**C** $\alpha=\pm\frac{4}{3}$	**D** $\alpha=\pm\frac{3}{4}$	**E** $\alpha=\pm\frac{1}{2}$

17. Vectors $\vec{a}$ and $\vec{b}$ form an angle $\varphi=\frac{2}{3}\pi$. Find $(3\vec{a}-2\vec{b})(\vec{a}+2\vec{b})$ if $|\vec{a}|=3$, $|\vec{b}|=4$ are known.

A 62	**B** -61	**C** -62	**D** 36	**E** -64

18. Three vectors $\vec{a}=(2;3;-5)$, $\vec{b}=(3;0;1)$ and $\vec{c}=(4;-3;2)$ are given. Find the length of the vector $\vec{d}=3\vec{a}+\vec{b}-\vec{c}$.

A $5\sqrt{7}$	**B** $7\sqrt{5}$	**C** $5\sqrt{15}$	**D** $5\sqrt{17}$	**E** $7\sqrt{15}$

19. Define the values of x at which the vectors $\vec{a}=(x;3;4)$ and $\vec{b}=(5;6;3)$ are perpendicular.

A -3	**B** 2	**C** 5	**D** 3	**E** -6

20. Determine the values α and β at which the vectors $\vec{a}=(3;-\alpha;4)$ and $\vec{b}=(6;2;\beta)$ are collinear.

A	**B**	**C**	**D**	**E**
$\alpha=2$, $\beta=4$	$\alpha=3$, $\beta=6$	$\alpha=-1$, $\beta=8$	$\alpha=1$, $\beta=8$	$\alpha=1$, $\beta=-8$

21. Find the modules of the sum and the difference of the vectors $\vec{a}=(3;-5;8)$ and $\vec{b}=(-1;1;-4)$.

A	**B**	**C**	**D**	**E**
$|\vec{a}+\vec{b}|=8$ $|\vec{a}-\vec{b}|=12$	$|\vec{a}+\vec{b}|=3$ $|\vec{a}-\vec{b}|=14$	$|\vec{a}+\vec{b}|=6$ $|\vec{a}-\vec{b}|=4$	$|\vec{a}+\vec{b}|=6$ $|\vec{a}-\vec{b}|=14$	$|\vec{a}+\vec{b}|=8$ $|\vec{a}-\vec{b}|=14$

22. Vectors $\vec{a} = \vec{i} + 2\vec{j} + \alpha\vec{k}$ and $\vec{b} = 4\vec{i} + \beta\vec{j} + 20\vec{k}$ are given. Find the values of α and β at which the vectors are collinear.

A	B	C	D	E
$\alpha = 5, \beta = 8$	$\alpha = 8, \beta = 5$	$\alpha = 4, \beta = 4$	$\alpha = 2, \beta = 3$	$\alpha = 1, \beta = 6$

23. In a triangle, vertices $A(3;7;-4)$; $B(2;-1;1)$; $C(1;3;0)$ are given. Determine the length of the midline parallel to the side AC.

A 6	**B** 3	**C** 2	**D** 4	**E** 5

24. In a triangle, vertices $A(0;2;-3)$, $B(1;4;-2)$, $C(3;2;-4)$ are given. Find the length of the median AM.

A $\sqrt{3}$	**B** 5	**C** $\sqrt{5}$	**D** $\sqrt{6}$	**E** 6

25. Find the distance from the point $C(-1;0;2)$ to the middle of the segment AB if $A(4;-1;1)$ and $B(-2;3;5)$ are given.

A $\sqrt{5}$	**B** $\sqrt{6}$	**C** $\sqrt{7}$	**D** 6	**E** 7

Level 2

26. The vector $\vec{a}$ forms angles $\alpha = 60°$, $\beta = 120°$ with the coordinate axes OX and OY. Find its coordinates x, y, z under the condition $|\vec{a}| = 2$.

A	B	C	D	E
$(1;\ -1;\ \sqrt{2})$	$(1;\ -1;\ \pm\sqrt{2})$	$(-1;\ 1;\ \pm\sqrt{2})$	$(\pm\sqrt{2};\ 1;\ -1)$	$(1;\ -\sqrt{2};\ 1)$

27. $|\vec{a}| = 10$, $|\vec{b}| = 20$, $|\vec{a} + \vec{b}| = 26$ are given. Find $|\vec{a} - \vec{b}|$.

A 15	**B** 18	**C** 22	**D** 17	**E** 13

28. $|\vec{a}| = 6$, $|\vec{b}| = 7$, $|\vec{a} - \vec{b}| = 1$ are given. Find $|\vec{a} + \vec{b}|$.

A 9	**B** 11	**C** 7	**D** 13	**E** 19

29. The l-axis forms equal acute angles with the coordinate axes. Find the ort of this axis.

A	B	C	D	E
$\left(\frac{2}{\sqrt{3}};\frac{2}{\sqrt{3}};\frac{2}{\sqrt{3}}\right)$	$\left(\frac{\sqrt{3}}{2};\frac{\sqrt{3}}{2};\frac{\sqrt{3}}{2}\right)$	$\left(\frac{1}{\sqrt{3}};\frac{1}{\sqrt{3}};\frac{1}{\sqrt{3}}\right)$	$\left(\frac{\sqrt{2}}{2};\frac{\sqrt{2}}{2};\frac{\sqrt{2}}{2}\right)$	$(1;\ 1;\ 1)$

30. Vectors $\vec{a}$ and $\vec{b}$ form an angle of $60°$. It is known that $|\vec{a}|=1$, $|\vec{b}|=2$. Find the length of the vector $\vec{c}=5\vec{a}-2\vec{b}$.

A	B	C	D	E
$\sqrt{20}$	$\sqrt{23}$	5	$\sqrt{21}$	6

31. Find the angle φ between the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ if $A(-1;\ 4;\ -2)$, $B(3;\ 1;\ -2)$, $C(1;\ 3;\ 0)$ are given.

A	B	C	D	E
$\varphi=\arccos\frac{10}{11}$	$\varphi=\arccos\frac{9}{11}$	$\varphi=\arccos\frac{9}{15}$	$\varphi=\arccos\frac{11}{15}$	$\varphi=\arccos\frac{8}{15}$

32. The vertices of a triangle ABC are given as $A(-1;-2;4)$, $B(-4;-2;0)$, $C(3;-2;1)$. Find its interior angle at the vertex B.

A $60°$	**B** $30°$	**C** $45°$	**D** $120°$	**E** $90°$

33. Find the coordinates of the vector $\vec{b}$ which is collinear to the vector $\vec{a}=(1;-2;-5)$ under the condition $\vec{a}\cdot\vec{b}=-10$.

A	B	C	D	E
$\left(\frac{1}{3};-\frac{2}{3};\frac{5}{3}\right)$	$\left(\frac{1}{3};\frac{2}{3};-\frac{5}{3}\right)$	$\left(-\frac{1}{3};\frac{2}{3};\frac{5}{3}\right)$	$\left(-\frac{1}{3};-\frac{2}{3};\frac{5}{3}\right)$	$\left(\frac{1}{3};-\frac{2}{3};-\frac{5}{3}\right)$

34. Find the scalar product of vectors $\vec{m}$ and $\vec{n}$ if $\vec{m}=3\vec{a}+\vec{b}$, $\vec{n}=\vec{a}-4\vec{b}$, $|\vec{a}|=2$, $|\vec{b}|=3$, $\varphi=\left(\widehat{\vec{a},\vec{b}}\right)=\frac{\pi}{3}$ are given.

A -49	**B** 57	**C** -67	**D** -57	**E** 67

35. Vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ in pairs form angles of $60°$ with each other. Find the modulus of the vector $\vec{p} = \vec{a} + \vec{b} + \vec{c}$ if $|\vec{a}| = 4$, $|\vec{b}| = 2$, $|\vec{c}| = 6$.

A $\|\overline{p}\| = 15$	**B** $\|\overline{p}\| = 20$	**C** $\|\overline{p}\| = 100$	**D** $\|\overline{p}\| = 14$	**E** $\|\overline{p}\| = 10$

36. Find a point D such that the vector $\overrightarrow{AD}$ has the same length and the same direction as the vector $\overrightarrow{BC}$ if $A(2;-1)$, $B(4;2)$, $C(-2;5)$ are given.

A $(4;-2)$	**B** $(-4;2)$	**C** $(2;-4)$	**D** $(-4;-2)$	**E** $(2;4)$

37. Calculate $(\vec{a} - \vec{b})^2$ if $|\vec{a}| = 2\sqrt{2}$, $|\vec{b}| = 4$, $\widehat{\vec{a},\vec{b}} = 135°$.

A 10	**B** 20	**C** 40	**D** 80	**E** 0

38. Vectors $\vec{a} = (4;-2;4)$ and $\vec{b} = (4;-2;-4)$ are given. Find the angle φ between the vectors $\vec{c}$ and $\vec{\alpha}$ if $\vec{c} = \frac{1}{2}\vec{a}$ and $\overline{\alpha} = 2\vec{a} + \vec{b}$.

A	B	C	D	E
$\arccos\frac{19}{21}$	$\arccos\frac{17}{21}$	$\arccos(-\frac{17}{21})$	$\arccos\frac{15}{21}$	0

39. Find the angle between the diagonals of a parallelogram constructed on the vectors $\vec{a} = (2;1;0)$ and $\vec{b} = (0;-2;1)$.

A $\frac{\pi}{4}$	**B** $\frac{\pi}{2}$	**C** $\frac{3\pi}{4}$	**D** 0	**E** $\frac{\pi}{3}$

40. Two vectors $\vec{a} = (2;-3;6)$ and $\vec{b} = (1;2;-2)$ have the same initial point. Find the coordinates of the vector $\vec{c}$ that is directed along the bisector of the angle between the vectors $\vec{a}$ and $\vec{b}$ under the condition $|\vec{c}| = 3\sqrt{42}$.

A	B	C	D	E
$(-3;-15;-12)$	$(3;-15;12)$	$(-3;15;12)$	$(-3;15;-12)$	$(3;-15;-12)$

41. The vector $\vec{c}=(16;-15;12)$ is given. Find the coordinates of the vector $\vec{d}$ that is parallel to the vector $\vec{c}$ and has the opposite direction if $|\vec{d}|=75$.

A	**B**	**C**	**D**	**E**
$(-48;45;-12)$	$(-48;45;-36)$	$(-32;45;-4)$	$(45;-12;48)$	$(48;-45;36)$

42. In the parallelogram $ABCD$, $\overrightarrow{CB}=(2;-1;4)$; $\overrightarrow{CD}=(-3;2;1)$; $A(5;-3;2)$ are given. So, the sum of the coordinates of the point C is equal to

A 1	**B** -1	**C** 2	**D** -2	**D** 3

43. Find the vector $\vec{a}$ that is collinear to the vector $\vec{b}=(1;-3;1)$ and satisfies the condition $\vec{a}\cdot\vec{b}=22$.

A $(-2;6;-2)$	**B** $(2;-6;2)$	**C** $(2;-3;1)$	**D** $(-2;-6;-2)$	**E** $(2;6;2)$

44. Vectors $\vec{a}$ and $\vec{b}$ form an angle of $120°$ and $|\vec{a}|=3$, $|\vec{b}|=5$. Find $|\vec{a}-\vec{b}|$
.

A 13	**B** 17	**C** 5	**D** 7	**E** 3

45. Find the angle between the vectors $2\vec{a}$ and $\frac{\vec{b}}{2}$ if $\vec{a}=(-4;2;4)$, $\vec{b}=(\sqrt{2};-\sqrt{2};0)$.

A $\frac{\pi}{4}$	**B** $\frac{3\pi}{4}$	**C** $\frac{\pi}{2}$	**D** $\frac{\pi}{8}$	**E** $\frac{\pi}{6}$

46. Find $(2\vec{a}+3\vec{b})(4\vec{a}-6\vec{b})$ if $\vec{a}$ and $\vec{b}$ are mutually perpendicular orts.

A 10	**B** -5	**C** -10	**D** 5	**E** -6

47. Find the cosine of the angle between the vectors $\vec{a}-\vec{b}$ and $\vec{a}+\vec{b}$ if $\vec{a}=(1;2;1)$, $\vec{b}=(2;-1;0)$.

A $\frac{1}{11}$	**B** $\frac{1}{13}$	**C** $-\frac{1}{15}$	**D** $\frac{1}{10}$	**E** $\frac{1}{17}$

48. Points $A(-2;3;-4)$, $B(3;2;5)$, $C(1;-1;2)$, $D(3;2;-4)$ are given. Find the projection of the vector $\overrightarrow{AB}$ on the vector $\overrightarrow{CD}$.

A $\frac{47}{7}$	**B** $-\frac{47}{7}$	**C** $\frac{43}{7}$	**D** $-\frac{42}{7}$	**E** -7

49. Three vectors $\vec{a}=(-2;1;1)$, $\vec{b}=(1;5;0)$, $\vec{c}=(4;4;-2)$ are given. Find the projection of the vector $(3\vec{a}-2\vec{b})$ on the vector $\vec{c}$.

A 11	**B** -11	**C** 6	**D** -5	**E** 5

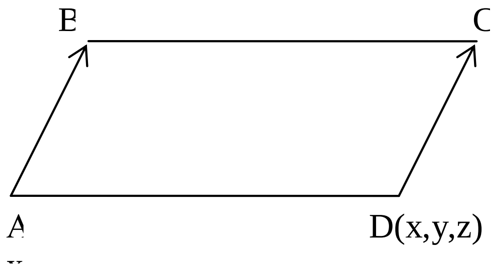
50. Two vectors $\vec{a}=(x;1;-1)$ and $\vec{b}=(1;0;1)$ are given. Define the value of x under the condition $(\vec{a}+3\vec{b})^2=(\vec{a}-2\vec{b})^2$.

A -2	**B** -1	**C** 0	**D** 1	**E** 2

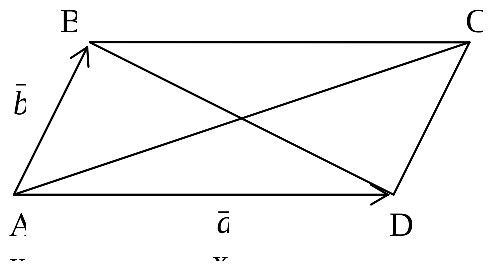
Level 3

51. Three nonzero vectors $\vec{a},\vec{b},\vec{c}$ are not collinear with each other in pairs. Find their sum if the vector $\vec{a}+\vec{b}$ is collinear to vector $\vec{c}$ and the vector $\vec{b}+\vec{c}$ is collinear to vector $\vec{a}$.

52. Points $A(1;3;-1)$, $B(2;1;2)$, $C(1;-2;1)$ are the vertices of a parallelogram. Find the coordinates of the vertex D.



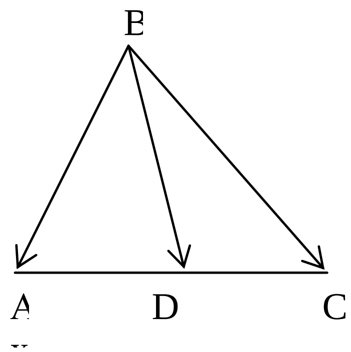
53. Find the acute angle between the diagonals of a parallelogram which is constructed on the vectors $\vec{a}=(3;1;1)$ and $\vec{b}=(1;-1;2)$.



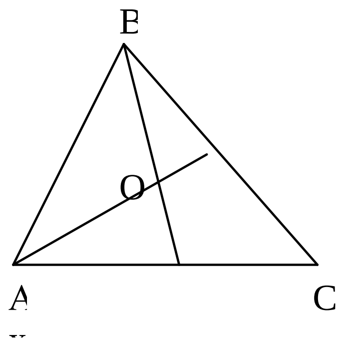
54. Find the projection of the vector $\vec{a}=2\vec{m}-\vec{n}$ on the vector $\vec{b}=3\vec{m}+2\vec{n}$ if $|\vec{m}|=2$, $|\vec{n}|=3$, $\left(\widehat{\vec{m},\vec{n}}\right)=\frac{2\pi}{3}$ are given.

55. The vector $\vec{a}$ is collinear to the vector $\vec{b}=(-12;16;15)$, and it forms an obtuse angle with the OY axis. It is known that $|\vec{a}|=100$. Find the coordinates of the vector $\vec{a}$.

56. On the side AC of the triangle ABC the point D lies such that $\frac{CD}{CA}=k$. Prove that $\overrightarrow{BD}=k\cdot\overrightarrow{BA}+(1-k)\cdot\overrightarrow{BC}$.



57. In the triangle ABC the medians intersect at the point O. Find the sum of the vectors $\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}$.



58. Points A, B, C are the vertices of the triangle ABC. It is known that $\overrightarrow{AB}=\vec{a}$, $\overrightarrow{AC}=\vec{b}$. Find the decomposition of the vector $\overrightarrow{AO}$ into the vectors $\vec{a}$ and $\vec{b}$ where the point O is the point of intersection of the medians of the triangle ABC.

59. Vectors $\vec{a}=(1;3;5)$, $\vec{b}=(2;1;4)$, $\vec{c}=(3;4;1)$ are given. Find the vector $\vec{x}$ that satisfies the conditions $(\overline{x}\overline{a})=20$, $(\overline{x}\overline{b})=15$, $(\overline{x}\overline{c})=-5$.

60. Vectors are given as $\overrightarrow{OA}=\vec{a}$ and $\overrightarrow{OB}=\vec{b}$ for which $|\vec{a}|=2$, $|\vec{b}|=6$, $\left(\widehat{\vec{a},\vec{b}}\right)=60^{\circ}$. Find the angle between the median $\overrightarrow{OM}$ and the side $\overrightarrow{OA}$ of the triangle OAB.

61. A point M moving from a point $A(-1;-4)$ on a straight line at an angle $\frac{\pi}{4}$ in the positive direction of the OX axis passed a distance $7\sqrt{2}$ to the point B. Determine the coordinates of the point B.

62. In a rectangle, vertices $A(0;0)$, $B(12;0)$, $C(12;6)$, $D(0;6)$ are given. Vectors $\overrightarrow{AB}=\vec{a}$ and $\overrightarrow{AD}=\vec{b}$ are diagonals. The point R is the intersection of the diagonals. Express the vectors $\overrightarrow{RC}, \overrightarrow{BD}, \overrightarrow{RB}$ through the vectors $\vec{a}$ and $\vec{b}$.

63. Vectors $\vec{a}=\overrightarrow{OA}=(3;1)$, $\vec{b}=\overrightarrow{OB}=(1;2)$ (O is the origin) are given. Express the vector $\vec{m}=\overrightarrow{OM}$ through $\vec{a}$ and $\vec{b}$ if $M(2;2)$ is given.

64. Construct a parallelogram on the vectors $\overrightarrow{OA}=(1;1;0)$ and $\overrightarrow{OB}=(0;-3;1)$ and determine the diagonals $\overrightarrow{OC}$ and $\overrightarrow{AB}$ of the parallelogram and their lengths.

65. Vectors $\overrightarrow{AB}=(2;6;-4)$ and $\overrightarrow{AC}=(4;2;-2)$ coincide with the sides of a triangle ABC. Determine the coordinates of the vectors that are attached to the vertices of the triangle and coincide with its medians AM, BN, CP.

66. Prove the validity of the identity $(\vec{a}+\vec{b})^2+(\vec{a}-\vec{b})^2=2(\vec{a}^2+\vec{b}^2)$ and define its geometric meaning.

67. In the parallelogram $OABC$ the vertices $O(0;0)$, $A(3;6)$, $B(8;6)$ are given. Find the ratio of the lengths of the diagonals OB and AC, and find the equations of the sides of the parallelogram.

68. Find a unit vector which is collinear to the vector directed along the bisector of the angle BAC of the triangle ABC if its vertices $A(1;1;1)$, $B(3;0;1)$, $C(0;3;1)$ are given.

69. Two nonzero vectors $\vec{a}$ and $\vec{b}$ are given such that $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$. Prove that $\vec{a}\perp\vec{b}$.

70. Prove that for any triangle ABC the inequality $\cos A+\cos B+\cos C\le\frac{3}{2}$ holds.

71. Three points $A(2;1)$, $B(3;-1)$, $C(-4;0)$ are given, which are the vertices of an isosceles trapezoid $ABCD$. Find the coordinates of the point D if $\overrightarrow{AB}=k\cdot\overrightarrow{CD}$.

72. Calculate the lengths of the diagonals AC and BD of the parallelogram $ABCD$ if $A(1;-3;0)$, $B(-2;4;1)$, $C(-3;1;1)$ are given.

73. There are vectors $\vec{p}$ and $\vec{q}$ on which a parallelogram is constructed. Using them, determine the vector that coincides with the altitude of the parallelogram which is perpendicular to the side $\vec{p}$.

74. Find the cosine of the angle between the vectors $\vec{p}$ and $\vec{q}$, which satisfy the system of equations: $\begin{cases}2\vec{p}+\vec{q}=\vec{a}\\ \vec{p}+2\vec{q}=\vec{b}\end{cases}$ if it is known that in a rectangular coordinate system the vectors $\vec{a}$ and $\vec{b}$ have the form $\vec{a}=(1;1)$, $\vec{b}=(1;-1)$.

75. Three consecutive vertices $A(-3;-2;0)$, $B(3;-3;1)$ and $C(5;0;2)$ of a parallelogram are given. Find its fourth vertex D and the angle between the vectors $\overrightarrow{AC}$ and $\overrightarrow{BD}$.

Key to the tasks for individual work

Geometry

1. A triangle

Task 27. 52 cm. **Task 28.** 5; 5; 7 cm. **Task 29.** 65 cm. **Task 30.** $40°$; $60°$; $80°$. **Task 31.** $52°$. **Task 32.** $65°$; $65°$; $50°$. **Task 33.** 1.2 m. **Task 34.** $150°$. **Task 35.** $14°$. **Task 36.** 4.6 m. **Task 37.** 1.75; 5.25 cm. **Task 38.** $42°$; $42°$; $96°$.

2. Quadrilaterals

Task 39. 1.4 cm, 3.5 cm. **Task 40.** 56 or 64 cm. **Task 41.** 8 cm. **Task 42.** 16.12 cm or 5.6; 22.4 cm. **Task 43.** $60°$. **Task 44.** $80°$; $100°$. **Task 45.** $60°$; $120°$. **Task 46.** $150°$. **Task 47.** 4 cm. **Task 48.** 54 cm. **Task 49.** 73 cm. **Task 50.** 94 cm. **Task 51.** $30°$; $120°$. **Task 52.** $60°$; $120°$. **Task 53.** 19 cm; 33 cm. **Task 54.** 10 cm. **Task 55.** 30 cm; 40 cm.

3. A circle

Task 56. 1 cm. **Task 57.** 3; 7 cm. **Task 58.** 8 cm.

4. Inscribed and circumscribed triangles and quadrangles

Task 59. $80°$. **Task 60.** 2 cm. **Task 61.** $2a$. **Task 62.** 52 cm. **Task 63.** 2.25 cm. **Task 64.** 12; 4 cm. **Task 65.** 0.25 m.

5. Areas of figures

Task 66. 4; 9 cm. **Task 67.** 240 cm^2. **Task 68.** 48 cm^2. **Task 69.** 24 cm^2. **Task 70.** 80 m^2. **Task 71.** 20 cm. **Task 72.** 150 cm^2. **Task 73.** 60 cm^2. **Task 74.** 15 cm. **Task 75.** 20 %. **Task 76.** 60 cm. **Task 77.** 2 dm^2. **Task 78.** 16 cm. **Task 80.** 40 cm^2. **Task 81.** 32 cm^2. **Task 82.** 1344 cm^2; 68 cm. **Task 83.** 24 dm^2. **Task 84.** 256 cm^2. **Task 85.** 480 cm^2.

6. The circumference and the area of a disk

Task 86. 4π cm. **Task 87.** 5π cm. **Task 88.** 25 cm. **Task 89.** 25 cm^2. **Task 90.** 20π cm; 20π cm. **Task 91.** 28π cm. 196π cm^2. **Task 92.** 125 %. **Task 93.** $\approx 10\pi$ cm^2.

7. A parallelepiped and a prism

Task 94. 1 cm. **Task 95.** 45 %. **Task 96.** 11; 16; 21 dm. **Task 97.** 24 cm^2. **Task 98.** 600 cm^2. **Task 99.** 1056 cm^2. **Task 100.** $4\sqrt{3}$ cm^3. **Task 101.** 9828 cm^3. **Task 103.** 3840 cm^3.

8. A pyramid

Task 104. 24 cm. **Task 105.** 6 cm. **Task 106.** 12 cm. **Task 107.** 1450 cm^2. **Task 108.** 12;8 cm. **Task 109.** 18 cm^2. **Task 110.** 50 m^2. **Task 111.** $6k^2\sqrt{3}\cos\alpha\cdot\cos^2\dfrac{\alpha}{2}$. **Task 112.** $\dfrac{a^3\sqrt{3}}{18}$. **Task 113.** 500 cm^2.

Vectors

Task 123. $\left|\vec{a}+\vec{b}\right|=13$; $\left|\vec{a}-\vec{b}\right|=13$. **Task 124.** $\vec{m}=(-7;26;3)$; $|\vec{m}|=\sqrt{734}$; $\vec{n}=(7;-14;18)$; $|\vec{n}|=\sqrt{569}$. **Task 125.** $\overrightarrow{AB}=(2;5;-4)$; $\left|\overrightarrow{AB}\right|=3\sqrt{5}$; $\cos\left(\overrightarrow{AB};\vec{i}\right)=\dfrac{2}{\sqrt{45}}$; $\cos\left(\overrightarrow{AB};\vec{j}\right)=\dfrac{5}{\sqrt{45}}$; $\cos\left(\overrightarrow{AB};\vec{k}\right)=\dfrac{-4}{\sqrt{45}}$. **Task 126.** $\alpha=12$; $\beta=-6$. **Task 127.** 1) $15\sqrt{2}$; 2) 15. **Task 128.** 22. **Task 129.** $\vec{a}\cdot\vec{b}=16$; $\varphi=\arccos\dfrac{16}{9\sqrt{30}}$. **Task 130.** $m=-7$. **Task 131.** The diagonals are perpendicular. **Task 132.** $\vec{a}+\vec{b}=\vec{i}+\vec{j}+\vec{k}$, $\vec{a}-\vec{b}=3\vec{i}-3\vec{j}+3\vec{k}$, $-4\vec{a}=-4\vec{i}+8\vec{j}-4\vec{k}$, $-3\vec{a}+2\vec{b}=\vec{i}+4\vec{j}+\vec{k}$. **Task 133.** $\vec{a}=(-9;-9;-8)$. **Task 134.** $\vec{a}\cdot\vec{b}=5$, $\arccos\dfrac{1}{3}$. **Task 135.** $m=4$. **Task 136.** 2. **Task 137.** $\alpha=5$. **Task 138.** $\angle C=\arccos\dfrac{17}{5\sqrt{66}}$.

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Навчальний посібник
для слухачів підготовчого відділення
(англ. мовою)

Самостійне електронне текстове мережеве видання

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