

МІНІСТЕРСТВО ОСВІТИ І НАУКИ УКРАЇНИ
ХАРКІВСЬКИЙ НАЦІОНАЛЬНИЙ ЕКОНОМІЧНИЙ УНІВЕРСИТЕТ
ІМЕНІ СЕМЕНА КУЗНЕЦЯ

ЗАТВЕРДЖЕНО

на засіданні кафедри
вищої математики та економіко-математичних
методів

Протокол № 1 від 21.08.2023 р.

ПОГОДЖЕНО

Проректор з навчально-методичної роботи

Каріна НЕМАШКАЛО



ВИЩА МАТЕМАТИКА
робоча програма навчальної дисципліни (РПНД)

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Тетяна ДЕНИСОВА

Степан ЛЕБЕДЕВ

Завідувач кафедри
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економіко-математичних
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2024

INTRODUCTION

Mathematical methods of research, modeling and design play an increasingly important role in modern science and technology. This is due to the improvement of computer technology, thanks to which the possibility of successfully applying mathematics in solving specific problems has significantly increased. Mathematical sciences are closely related to the development of information computer technologies, which have penetrated almost all spheres of human activity and play a decisive role in the education of a modern competitive specialist, providing him with an apparatus for researching complex systems of any nature and the logic of building project activities. On the other hand, high-performance information technologies have turned into the most important segment of knowledge-intensive high-tech production, which can be implemented only by specialists with in-depth training in the field of mathematics and information technologies.

The course "Higher Mathematics" is a mandatory course, which is studied in accordance with the educational program for bachelors in the field of knowledge 12 "Information Technologies", specialty 121 "Software Engineering" of all forms of education.

The purpose of the course: to acquaint students with higher education with the basics of mathematical apparatus, necessary for solving theoretical and practical tasks of a professional direction; to develop the skills of mathematical research of applied problems and construction of economic and mathematical models; to form in applicants the ability to independently study literature on mathematics and applied issues; to form a complete system of theoretical and practical knowledge necessary for the professional activity of a competent specialist in the field of information technologies; to develop the skills of analytical thinking and the use of mathematical apparatus for the formalization of real processes and phenomena.

The objectives of the course are:

mastering the basic principles of building mathematical models using methods of matrix and vector analysis, coordinate method;

mastering the skills of performing the analysis of the constructed mathematical model using modern computer technology and software-mathematical complexes;

formation of a knowledge system for independent carrying out of the necessary calculations within the framework of built models in order to establish quantitative and qualitative characteristics of objects for forecasting and making optimal decisions;

acquiring experience in free operation of abstract mathematical objects and visual presentation of observation results in various fields of knowledge using geometric images;

mastering the basic principles of building mathematical models using known means of mathematical analysis;

mastering the skills of independent analysis of functional dependencies, which describe the built mathematical model, using computer equipment and application program packages;

formation of analytical and research competences in students regarding the use of mathematical analysis tools (the method of limits, differential and integral calculus, numerical and functional series, differential equations), linear and vector algebra and analytical geometry in professional activities.

The object of the course is functional dependencies between the characteristics of various phenomena and processes, in particular, economic ones, reflecting various aspects of economic decision-making.

The subject of the course is: properties of functional dependencies that describe the phenomena and processes of the surrounding world, their research by means of linear and vector algebra, analytical geometry and mathematical analysis; construction of mathematical models of real phenomena and processes in various fields of human activity.

Learning outcomes and competencies formed by the course are defined in table 1.

Table 1

Learning outcomes and competencies formed by the course

Learning outcomes	Competencies
LO05	GK01, GK02, SK02, SK03, SK08, SK14

where, LO05. Know and apply relevant mathematical concepts, methods of domain, system and object-oriented analysis and mathematical modeling for software development;

GK01. Ability to abstract thinking, analysis and synthesis;

GK02. Ability to apply knowledge in practical situations;

SK02. Ability to participate in software design, including modeling (formal description) of its structure, behavior and functioning processes;

SK03. Ability to develop architectures, modules and components of software systems;

SK08. Ability to apply fundamental and interdisciplinary knowledge to successfully solve software engineering tasks;

SK14. Ability to algorithmic and logical thinking.

COURSE CONTENT

Content module 1. Linear and vector algebra. Analytical geometry

Topic 1. Matrices and actions with them

Matrix: basic definitions, varieties (square, diagonal, triangular, single, zero, row matrix (column matrix)). Arithmetic operations (actions) on matrices and their properties. Elementary transformations and matrix equivalence.

Topic 2. Determinants of square matrices

The concept of the determinant of the 2nd, 3rd, and n-th orders. The minor and the algebraic complement of the determinant element. Properties of determinants. Methods of calculating determinants.

Topic 3. Systems of linear algebraic equations (SLAE)

3.1. Systems of n linear algebraic equations with n unknowns (SLAE- $n \times n$): definition of basic concepts (SLAE- $n \times n$, system solution, compatible, incompatible, defined, undefined, equivalent systems, homogeneous, heterogeneous).

Cramer's rule for solving SLAE- $n \times n$. Inverse matrix: definition, existence theorem and methods of finding. Solving SLAE- $n \times n$ using the inverse matrix. Solving matrix equations.

3.2. Systems of m linear algebraic equations with n unknowns (SLAE- $m \times n$): the concept of the rank of a matrix and methods of finding it (method of elementary transformations, stroke method). Compatibility criterion SLAE- $m \times n$ (Kronecker–Capelli theorem). Study of SLAE- $m \times n$ on compatibility, solution methods (Gauss, Jordan – Gauss). General, partial, basic and reference solutions. Homogeneous SLAE- $m \times n$ and their solutions.

Topic 4. Vector algebra. Linear m -dimensional spaces

4.1. Vectors: definition of basic concepts (vector, vector module, zero (unit) vector, equal (opposite) vectors, collinear (coplanar) vectors, coordinates). Forms of specifying vectors (geometric, coordinate, algebraic), projection of a vector onto an axis (geometric, algebraic), ortho vector, direction cosines).

4.2. Linear operations on vectors (sum, difference, multiplication by a scalar) and their properties. Nonlinear operations on vectors (scalar, vector, mixed product of vectors and their properties). The angle between two vectors. Criteria of orthogonality, collinearity, coplanarity of vectors. The use of vectors in geometry problems (finding the distance between two points, the area of a triangle, dividing a segment in a given ratio).

4.3. Linear m -dimensional spaces. Basic concepts. Linear dependence and independence of a system of vectors. The basis of a linear m -dimensional space. Decomposition of a vector by basis. Transition to a new basis. Eigenvalues and eigenvectors: definition, main properties. Characteristic equation. Finding eigenvalues and eigenvectors of 2nd and 3rd order matrices.

Topic 5. Analytical geometry on the plane

5.1. The concept of the equation of a line on a plane. Types of equations of a straight line on a plane: canonical, parametric, through two given points, through a given point in a given direction, with an angle coefficient, in segments on axes, normal, with a given normal vector, general. Basic problems on a straight line, mutual location of two straight lines on a plane. Angle between two straight lines. The distance from a point to a straight line.

5.2. Curves of the 2nd order: definition, general equation, conditions of belonging to the elliptic, hyperbolic, parabolic type. Central (non-central) curves of the 2nd order. Circle, ellipse, hyperbola, parabola: definition, canonical equation, parameters, eccentricity, construction. Reduction of the general equation of the 2nd order curve to the canonical form.

Topic 6. Analytical geometry in space

6.1. The concept of surface equations in space. Types of plane equations: through a given point with a given normal vector, general, through three given points,

in segments on the axes, normal. Conditions of intersection, parallelism, orthogonality, coincidence of two planes. Angle between two planes. The distance from the point to the plane. Types of equations of a straight line in space: canonical, parametric, through two given points, general. Conditions of parallelism, perpendicularity, coincidence, intersection, crossing of two straight lines. The angle between two straight lines in space. The distance between parallel (cross) lines. Analysis of the mutual arrangement of a straight line and a plane. The angle between a straight line and a plane.

6.2. General surface equation of the 2nd order. The most important surfaces of the 2nd order (cylindrical, triaxial ellipsoid, sphere, one- and two-cavity hyperboloids, conical, elliptical and hyperbolic paraboloids). Study of the shape of the surface of the 2nd order by the method of sections.

Content module 2. Differential calculus of a function of one variable

Topic 7. The limit of a function

7.1. Numerical functions: basic definitions, assignment methods. Basic elementary functions, their properties and graphs. Complex numbers: definition, geometric representation, problem forms, operations (actions) on complex numbers. The limit of a numerical sequence: definition, existence criterion, properties, geometric content. Infinitesimals: definition, properties. Infinitely large: definition, properties, connection with infinitely small. Uncertainties: definitions, types. Practical recommendations for finding boundaries.

7.2. Boundary of a function: definition, geometric meaning, one-sided boundaries, existence criteria. Basic properties of function boundaries at a point. The first and second significant boundaries and their consequences. Practical recommendations for calculating limits. Comparison of infinitesimals, application of equivalent infinitesimals to the calculation of limits.

Topic 8. Continuity of a function

8.1. Definition of continuity of a function at a point, continuity of basic elementary functions. Criterion of continuity and properties of functions continuous at a point. Discontinuities of functions and their classification, research of functions for continuity.

8.2. Continuity of a function on an interval: definition, basic theorems about continuous functions.

Topic 9. Derivative and differential of a function

9.1. Derivative of functions: definition, general order of finding, connection with continuity. Table of derivatives of basic elementary functions and basic differentiation rules. Differentiation of complex functions and functions of different forms of the task. Logarithmic differentiation. Geometric, physical and economic applications of the derivative: the equation of the tangent and normal to a curve; speed, acceleration, production cost, labour productivity.

9.2. Differential of a function: definition, geometric meaning, rules of finding, basic properties, application to approximate calculations. Derivatives and differentials of higher orders.

Topic 10. Study of functions and construction of graphs

10.1. Properties of differential functions. Fermat's, Rolle's, Cauchy's and Lagrange's theorems. Calculation of limits of functions using L'Hopital's rule. Taylor's formula and its application.

10.2. Growth and decline of functions. Extreme of functions. Study of functions for monotony and extreme. Finding the largest and smallest value of a function on a segment.

10.3. Study of functions for convexity (concavity), inflection points. Asymptotes of a curve (vertical, horizontal, inclined) and their finding. The general scheme of the study of functions and the construction of their graphs.

Content module 3. Functions of several variables

Topic 11. Functions of several variables

11.1. Definition of a function of several variables. The area of definition of the function of two variables and its graphic representation. Level lines and surfaces. Limit and continuity of a function of two variables.

11.2. Partial derivative functions of several variables, their geometric and economic meaning. Partial differentials and complete differential of a function of two variables, its application to approximate calculations of function values. Partial derivatives and differentials of higher orders. Derivative in direction. Gradient and its properties. The elasticity of the function, its economic meaning.

Topic 12. Extremes of a function of two variables

12.1. Local extremes of a function of two variables: definition of basic concepts, necessary and sufficient conditions for an extreme. The largest and smallest value of a function of two variables in a closed region.

12.2. Conditional extreme: formulation of the problem, reducing the problem of the conditional extreme of the function of two variables to the problem of the local extreme of the function of one variable. The method of Lagrange multipliers. Construction of empirical formulas by the method of least squares.

Content module 4. Integral calculus of functions of one and several variables

Topic 13. Indefinite integral

13.1. Primitive function: definition, theorem on the set of all primitives. Indefinite integral: definition, basic properties, table of basic indefinite integrals. Method of direct integration. The method of integration by replacing a variable (substitution). Method of integration by parts. The concept of integrals that "do not take".

13.2. The concept of a rational fraction. Integration of the simplest rational fractions. Integrating expressions that contain a quadratic trinomial. Decomposition of a proper fraction into the sum of the simplest. Integration of an arbitrary rational fraction.

13.3. Integration of trigonometric functions using universal trigonometric substitution. Some features of using variable substitution in integrals containing trigonometric functions. Integration of the simplest irrational functions. Integration of quadratic irrational functions using trigonometric substitutions.

Topic 14. The definite integral

14.1. Definite integral: definition, geometric content, existence theorem, basic properties. Theorem about the mean. The Newton – Leibnitz theorem. The main methods of definite integration: direct integration, method of variable replacement (substitution), integration by parts.

14.2. Geometric applications of definite integrals: calculation of the area of a flat figure, volume of a body of revolution, length of an arc of a curve, surface area. Economic applications of definite integrals: calculation of production volume, production costs. Approximate calculation of definite integral: formulas of rectangles, trapezoids and Simpson.

14.3. The concept of improper integrals with infinite limits of integration (of the 1st kind) and improper integrals of discontinuous functions (of the 2nd kind), conditions for their convergence. The Euler – Poisson integral and its application.

Topic 15. Multiple integrals

15.1. The problem of the volume of a cylindrical body. Double integral: definition, conditions of existence, main properties, geometric and physical meaning, calculation methods in Cartesian coordinates.

15.2. Polar coordinate system. Calculation of the double integral in polar coordinates. Change of variables in the double integral.

15.3. Application of double integrals: geometric (calculation of areas of figures, volumes of bodies); physical (calculation of body mass, static moments and inertia, coordinates of the center of gravity).

15.4. Triple integral: definition, existence theorem, basic properties, geometric and physical contents. Calculation of triple integrals in Cartesian, cylindrical and spherical coordinates.

Topic 16. Curvilinear integrals

16.1. Curvilinear integrals by the length of an arc: definition, sufficient conditions of existence, geometric and physical meaning, basic properties, calculation by reduction to a definite integral for various forms of specifying the path of integration.

16.2. Curvilinear integrals by coordinates: definition, sufficient conditions of existence, basic properties, physical meaning, calculation by reduction to a definite integral for various forms of the path of integration, connection with double integrals (Green's formula), conditions of independence from the form of the path of integration.

16.3. Application of curvilinear integrals: geometric (calculation of the length of the arc of the curve, area of the figure, area of the cylindrical surface, volume of the cylindrical body), physical (calculation of the mass of the arc length of the curve, static moments and moments of inertia, coordinates of the center of mass).

Content module 5. Ordinary differential equations. Rows

Topic 17. Ordinary differential equations of the 1st order

17.1. Differential equations of the 1st order: definition of basic concepts (solution, general and partial solutions), theorem of existence and uniqueness of the solution. Cauchy's problem: statement and geometric meaning.

17.2. Integration of the main types of differential equations of the 1st order: the simplest, with separable variables, homogeneous, linear, Bernoulli, in complete differentials.

Topic 18. Differential equations of higher orders

18.1. Differential equations of higher orders: definition of basic concepts, theorem of existence and uniqueness of the solution. Integration of differential equations, assuming a decrease in order: which contain only the highest derivative; do not contain derivatives up to the $(k-1)$ -th order; do not contain an explicit independent variable. Euler's equation.

18.2. Linear differential equations of the 2nd order: definition, structure of the general solution. Vronsky's determinant. Integration of homogeneous linear differential equations of the 2nd order with constant coefficients. Integration of inhomogeneous linear differential equations of the 2nd order with constant coefficients having a right-hand side of a special form.

Topic 19. Systems of linear differential equations (SLDE)

SLDR: definition of the main concepts (solution of the system, general and partial solutions), the theorem on the existence and uniqueness of the solution. Cauchy's problem. Homogeneous and inhomogeneous SLDE with constant coefficients: definition, solution by reduction to one differential equation and Euler's method.

Topic 20. Numerical series

20.1. Definition of a number series, n th partial sum, convergent and divergent number series. Properties of convergent numerical series. Harmonic series. The condition of convergence of numerical series is necessary. A series of geometric progression.

20.2. Sufficient signs of convergence of numerical series with positive terms: comparison sign, D'Alambert's sign, Cauchy's radical and integral signs.

20.3. Interchangeable series: definition, a sufficient sign of convergence. Absolute and conditional convergence. Alternating series: definition, Leibniz's test.

Topic 21. Functional series

21.1. Definition of functional series. Power series: definition, Abel's theorem, radius, interval and region of convergence. Taylor and McLaren series: definition, development of basic elementary functions into power series. Application of power series to approximate calculations of function values and definite integrals.

21.2. The concept of trigonometric Fourier series. Decomposition of periodic functions into a Fourier series. Calculation of expansion coefficients in the case of even and odd functions. The complex form of the Fourier series. Application of Fourier series.

The list of practical (seminar) studies in the course is given in table 2.

Table 2

The list of practical (seminar) studies

Name of the topic and/or task	Content
1 Semester	
Topic 1.	Practical study 1. Performing arithmetic operations (actions) with matrices.
Topic 2.	Practical study 2. Calculation of the determinants of the 2nd, 3rd and of any order.
Topic 3.	Practical study 3. Solution of square SLAE.
Topic 4.	Practical study 4. Solution of rectangular and homogeneous SLAE.
Topic 5.	Practical study 5. Performing linear and non-linear operations on vectors.
Topic 6.	Practical study 6. Solving problems using types of equations of a straight line on a plane. Reduction of the general equation of the 2nd order curve to the canonical form and its construction.
Topic 7.	Practical study 7. Solving problems using different types of equations of a straight line and a plane in space.
Topic 8.	Practical study 8. Calculation of limits of numerical sequences.
Topic 9.	Practical study 9. Calculation of the limits of functions of a continuous argument.
Topic 10.	Practical study 10. Study of functions for continuity. Classification of breakpoints.
Topic 11.	Practical study 11. Differentiation of functions of one variable. Solving problems on the application of the derivative.
Topic 12.	Practical study 12. Study of functions and construction of their graphs.
2 Semester	
Topic 11.	Practical study 1. Finding the domain of existence, level lines of functions of two variables, partial derivatives and differentials of functions of several variables. Practical study 2. Calculation of the derivative along the direction and the gradient of functions of several variables.
Topic 12.	Practical study 3. Study of the function of two variables for a local extreme. The largest and smallest value of the function in a closed region. Practical study 4. Study of the function of two variables at a conditional extreme. Construction of empirical formulas by the method of least squares.
Topic 13.	Practical study 5. Finding the original using basic methods of indefinite integration. Practical study 6. Integration of rational algebraic fractions. Integration of functions rationally dependent on trigonometric and algebraic irrationalities
Topic 14.	Practical study 7. Calculation of definite integrals. Practical study 8. Application of definite integrals. Study of improper integrals for convergence.
Topic 15.	Practical study 9. Double and triple integrals: calculations and applications.

Topic 16.	Practical study 10. Curvilinear integrals: calculation of curvilinear integrals by arc length (I kind), by coordinates (II kind) and their application.
Topic 17.	Practical study 11. Integration of differential equations of the 1st order.
Topic 18.	Practical study 12. Integration of linear differential equations of the 2nd order with constant coefficients and a right-hand side of a special form.
Topic 19.	Practical study 13. Integration of systems of linear differential equations.
Topic 20.	Practical study 14. Convergence study of constant number series. Practical study 15. Research of sign-changing and sign-alternating numerical series for convergence. Absolute and conditional convergence.
Topic 21.	Practical study 16. Development of functions into power series and Fourier series.

The list of laboratory studies in the course is given in table 3.

Table 3

The list of laboratory studies

Name of the topic and/or task	Content
1 Semester	
Topic 1.	Laboratory study 1. Actions with matrices.
Topic 2.	Laboratory study 2. Calculation of determinants.
Topic 3.	Laboratory study 3. Resolution of identified SLAE. Laboratory study 4. Solution of undefined SLAE.
Topic 4.	Laboratory study 5. Vector algebra.
Topic 5.	Laboratory study 6. Straight line on a plane. Laboratory study 7. Curves of the 2nd order.
Topic 6.	Laboratory study 8. Line and plane in space.
Topic 7.	Laboratory study 9. Calculation of limits of functions.
Topic 8.	Laboratory study 10. Study of functions for continuity.
Topic 9.	Laboratory study 11. Differentiation of functions.
Topic 10.	Laboratory study 12. Study of functions and construction of their graphs.
2 Semester	
Topic 11.	Laboratory study 1. Functions of several variables: graphical representation, contour lines, partial derivatives. Laboratory study 2. Differentials of functions of several variables, directional derivative, gradient.
Topic 12.	Laboratory study 3. Study of the function of two variables for a local extreme.
Topic 13.	Laboratory study 4. Basic methods of indefinite integration. Laboratory study 5. Integration of rational algebraic fractions and functions rationally dependent on trigonometric ones.

Topic 14.	Laboratory study 6. Calculation of definite integrals. Laboratory study 7. Application of definite integrals. Laboratory study 8. Study of improper integrals for convergence.
Topic 15.	Laboratory study 9. Calculation of double and triple integrals.
Topic 16.	Laboratory study 10. Calculation of curvilinear integrals
Topic 17.	Laboratory study 11. Integration of differential equations of the 1st order.
Topic 18.	Laboratory study 12. Integration of differential equations of the 2nd order.
Topic 19.	Laboratory study 13. Integration of systems of linear differential equations.
Topic 20.	Laboratory study 14. Research of numerical series for convergence.
Topic 21.	Laboratory study 15. Power series. Laboratory study 16. Fourier series.

The list of self-studies in the course is given in table 4.

Table 4

The list of self-studies

Name of the topic and/or task	Content
Topic 1 – 21	Study of lecture material, search, selection and review of literary sources on a given topic.
Topic 1 – 21	Preparation for practical studies.
Topic 1 – 21	Doing homework.
Topic 5, 6, 15, 16	Performing individual creative work.
Topic 1 – 21	Preparation of reports on the performance of laboratory studies.
Topic 7, 9, 17, 18, 20, 21	Performance of individual educational and research tasks.
Topic 11 – 21	Preparation for the exam

The number of hours of lectures, practical (seminar) and laboratory studies and hours of self-study is given in the technological card of the course.

TEACHING METHODS

In the process of teaching the course, in order to acquire certain learning outcomes, to activate the educational process, it is envisaged to use such teaching methods as:

- verbal: lecture (topics: 1, 3, 7, 9, 10, 13, 15, 16, 17, 19), problem lecture (topics: 2, 4, 5, 6, 10, 11, 12, 18, 20, 21), lecture-provocation (topics 8, 14), lecture-dialogue (topics: 2, 3, 10, 12, 18);
- visual (demonstration (topics 1 – 21));
- practical (practical studies (topics 1 – 21), laboratory studies (topics 1 – 21), presentations (topic 5, 6, 15, 16), work in small groups (topics: 1, 2, 3, 4, 6, 7, 8, 9, 14, 15, 19, 21); brainstorming (topics: 7, 9, 17, 18, 20, 21).

FORMS AND METHODS OF ASSESSMENT

The University uses a 100-point cumulative system for assessing the learning outcomes of students.

Current control is carried out during lectures, practical and laboratory studies and is aimed at checking the level of readiness of the applicant to perform a specific job and is evaluated by the sum of points scored:

– for courses with a form of semester control as grading: maximum amount is 100 points; minimum amount required is 60 points.

– for courses with a form of semester control as an exam: maximum amount is 60 points; minimum amount required is 35 points.

The final control with a form of semester control as grading includes current control and assessment of the student; the final control with a form of semester control as an exam includes current control and an exam.

Semester control is carried out in the form of a semester exam or grading.

The final grade in the course is determined:

– for courses with a form of grading, the final grade is the amount of all points received during the current control.

– for courses with a form of exam, the final grade is the amount of all points received during the current control and the exam grade.

During the teaching of the course, the following control measures are used:

– current control *in the first semester* involves assessment during the performance of: homeworks (estimated at 2 points (11 homeworks during the semester – the total maximum number of points – 22 points), written control works (estimated at 8 points (3 written control works during the semester – the total maximum number of points – 24 points), laboratory studies (estimated at 2 points (12 laboratory works during the semester – the total maximum number of points – 24 points), colloquiums (estimated at 11 points (2 colloquiums during the semester – the total maximum number of points – 22 points), individual creative work (maximum score – 8 points); current control *in the second semester* involves assessment during the performance of: homeworks (estimated at 2 points (7 homeworks during the semester – the total maximum number of points – 14 points), written control works (estimated at 5 points (3 written control works during the semester – the total maximum number of points – 15 points), laboratory studies (estimated at 1 points (14 laboratory works during the semester – the total maximum number of points – 14 points), colloquiums (estimated at 5 points (2 colloquiums during the semester – the total maximum number of points – 10 points), individual creative work (maximum score – 7 points).

Semester control: in the first semester – grading; in the second semester – an exam (40 points).

More detailed information on the assessment system is provided in technological card of the course.

Below is an example of an examination ticket and evaluation criteria for the course "Higher Mathematics".

The example of an examination ticket

SIMON KUZNETS KHARKIV NATIONAL UNIVERSITY OF
ECONOMICS

First (bachelor) level of higher education
Specialty 121 "Software Engineering"
Educational and professional program "Software engineering"
2nd semester
Course "Higher mathematics"

EXAMINATION TICKET №1

Task 1 (heuristic, 10 points).

For a given function $z = 4x^3 + 4y^3 - 24xy + 7$, it is necessary to: 1) find its gradient at a point $M_0(1,2)$ and its derivative in the gradient direction; 2) find its derivative at a point $M_0(1,2)$ in the vector direction $\overline{M_0M_1}$ if $M_1(4,5)$; 3) examine the function for a local extreme.

Task 2 (stereotypical, 7 points).

Find: $\int \frac{e^{2x}}{\sqrt[3]{e^{2x} - 5}} dx$.

Task 3 (diagnostic, 8 points).

Find the volume of the body formed by rotation around the axis Ox (Oy) of the figure bounded by the given lines: $y - x = 0$, $y = x^3$ ($x \geq 0$).

Task 4 (stereotypical, 7 points).

Find the partial solution (or partial integral) of the differential equation:

$$(x^2 - 6xy) \cdot y' = x^2 + xy - 5y^2, \quad y(1) = 0.$$

Task 5 (diagnostic, 8 points)

Find the region of convergence of the power series: $\sum_{n=0}^{\infty} \frac{n+1}{2n^2+9} (x+4)^n$.

Approved at the meeting of of Higher Mathematics and Economic Mathematical Methods Department
Protocol № __ від «__». _____ 20__ y.

Examinator

Stepan LEBEDIEV

Head of department

Lyudmyla MALYARETS

Criterion for evaluation of examination work

Each ticket contains five practical tasks: two tasks of the first level – stereotyped – determine the degree of assimilation of the student's initial theoretical foundations of the course; two tasks of the second level – diagnostic – reveal the applicant's ability to solve typical tasks, and one task of the third level – heuristic – aims to assess the depth of knowledge and creative capabilities of the applicant (Table 5).

Table 5

The structure of the examination ticket

Level task	Task content by level
First (stereotypical)	Tasks on finding the original given function; finding the general (or partial) solution (or integral) of a differential equation 1st or 2nd order, or systems of differential equations.
Second (diagnostic)	Problems for the study of numerical series for convergence and finding the region of convergence of power series; problems on geometric applications of the definite integral (calculation of the area of a plane figure, the volume of a body of revolution, the length of an arc, the coordinates of the centre of gravity of a plane figure).
Third (heuristic)	Tasks for calculating the gradient of a function of several variables and the derivative along the direction; study of a function of several variables at a local extreme.

Perfect performance of tasks is evaluated as follows: 1st level – 7 points; 2nd level – 8 points; 3rd level – 10 points.

The score for each task of the ticket is reduced depending on the shortcomings and mistakes made, the list of which is given in the table 6.

Table 6

Decreasing the grade for completing the task depending on the shortcomings and mistakes made

Task level	Downgrading points	Corresponding disadvantages and mistakes
First	1	The problem is solved correctly, but the mathematical notation is poor or comments are not given for all steps of the solution.
	2	The problem is partially solved: there is a significant error in the calculations that affected the correct result, or the geometric constructions were performed incorrectly
	3	A semantic error was made during the solution process: an incorrectly selected calculation formula or a geometric representation does not correspond to numerical calculations
	4	Only the initial correct considerations for solving the problem are given, but there are errors that significantly affected the process of correct solution.
	5	The solution of the problem has been started, the theoretical material has been used only at the level of initial concepts, the correct formulas for calculations have been chosen and written down, but their application has not been carried out

First	6	The solution algorithm that does not meet the condition of the problem is selected
Second	1	The problem is solved correctly, but the mathematical notation is poor or comments are not given for all steps of the solution
	2	In the process of solving the problem, the relevant facts, formulas and dependencies were correctly used, but 1-2 insignificant errors were made, which did not affect the correctness of the further solution and the final answer
	3	The problem was not fully solved: the calculation formulas were chosen correctly, but the calculation process was performed with errors, which affected the correct final result
	4	The problem was only partially solved: the calculation formulas were chosen correctly, but the final numerical result was not obtained
	5	The solution of the problem was started, but a significant error was made, which led to further erroneous calculations and the final numerical result was not obtained
	6	There are significant errors: with a correct solution algorithm, certain steps are incorrectly implemented or calculation formulas do not meet the condition of the problem
	7	The solution of the problem has been started, the theoretical material has been used only at the level of initial concepts, the correct formulas for calculations have been chosen and written down, but their application has not been carried out
Third	1	The solution of the problem is performed correctly, but the culture of mathematical notation is poor, or comments are not given for all steps of the solution
	2	The problem was solved correctly, but the comment was not given for all steps of the solution, but insignificant errors were made that did not affect the correctness of the subsequent solution and the final answer
	3	The problem was solved correctly in general, but insignificant errors were made that did not affect the correctness of the further solution and the final answer
	4	In the process of solving the problem, the relevant facts, formulas and dependencies were correctly used, but 1-2 insignificant errors were made, which did not affect the correctness of the further solution and the final answer
	5	The solution of the problem was started, but a significant error was made, which led to further erroneous calculations and an incorrect final result
	6	The solution of the problem is not fully completed: the calculation formulas and the solution algorithm are selected correctly, but the final numerical result is not obtained
	7	The solution of the problem is started, but not brought to a logical end: only individual steps of the solution algorithm are implemented, some formulas are written correctly (without further relevant calculations)
	8	The process of solving the problem has started, but a significant error was made that affected the further process of solving
	9	The course of solving the task is incorrectly stated, but its individual steps indicate the presence of some basic knowledge, the theoretical material was used only at the level of initial concepts, the correct formulas for calculations were chosen and written down, but they were not applied

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Information resources

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