МІНІСТЕРСТВО ОСВІТИ І НАУКИ УКРАЇНИ
ХАРКІВСЬКИЙ НАЦІОНАЛЬНИЙ ЕКОНОМІЧНИЙ УНІВЕРСИТЕТ ІМЕНІ СЕМЕНА КУЗНЕЦЯ

ЗАТВЕРДЖЕНО
на засіданні кафедри економіко-математичного моделювання
Протокол № 7 від 18.11.2024 р.

ПОГОДЖЕНО
Проректор з навчально-методичної роботи

Каріна НЕМАШКАЛО

ТЕОРІЯ ЙМОВІРНОСТЕЙ ТА МАТЕМАТИЧНА СТАТИСТИКА

робоча програма навчальної дисципліни (РПНД)

Галузь знань	**07 «Управління та адміністрування»**
Спеціальність	**075 «Маркетинг»**
Освітній рівень	**перший (бакалаврський)**
Освітня програма	**«Маркетинг»**
Статус дисципліни	**обов'язкова**
Мова викладання, навчання та оцінювання	**англійська**

Розробник:
канд. техн. наук, доцент Євгенія МІСЮРА

Завідувач кафедри економіко-математичного моделювання Людмила МАЛЯРЕЦЬ

Гарант програми Олена НЕБИЛИЦЯ

Харків
2024

MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE
SIMON KUZNETS KHARKIV NATIONAL UNIVERSITY OF ECONOMICS

APPROVED
at the meeting of economic and mathematical modeling

Protocol № 7 of 18.11.2024

AGREED
Vice-rector for educational and methodical work

Karina NEMASHKALO

Probability Theory and Mathematical Statistics
Program of the course

Field of knowledge	**07 Management and administration**
Specialty	**075 Marketing**
Study cycle	**first (bachelor)**
Study programme	**Marketing**
Course status	**mandatory**
Language	**English**

Developers:
PhD (Technics),
Associate Professor

Ievgeniia MISIURA

Head of Economic and mathematical modelling Lyudmyla MALYRETS

Head of Study Programme Olena NEBYLITSIA

Kharkiv

2024

INTRODUCTION

Probability theory and mathematical statistics are used in various fields of science and technology, but one of the most important areas of their use is economics. Without the help of probability theory, the issues of organization and planning, which are related to the need to take into account random events, cannot be solved, and the study of certain phenomena by mathematical statistics makes it possible to solve many questions posed by science and practice (correct organization of the technological process, the most appropriate planning and others).

The main purpose of teaching is forming future specialists' basic mathematical knowledge for solving theoretical and practical problems in professional activity of competent specialists in a service sphere, skills of analytical thinking and skills of using mathematical knowledge for formation of real processes and developments, and for solving economic problems.

The main tasks that should be solved in the process of teaching the course are: giving students knowledge of the basic parts of probability probability and mathematical statistics; definitions, theorems, rules; proving of the main theorems; mastering the fundamentals of a methodology of a mathematical investigation of the applied economic problems; independent broadening of knowledge, development of logical and algorithmic thinking; the obtainment of primary skills for independent-learning of mathematical and applied Bibliography by students.

The subject of the course "Probability theory and mathematical statistics" is the fundamentals of probability theory and mathematical statistics.

The object of study of the course is a system of mass phenomena, trials and experiments, the results of which are certain random events, as well as the study of the results of these phenomena.

A modern tendency in higher education is a reorientation of students of higher educational institutions from a process of education to a result, from knowledge to skills, forming definite competences.

The learning outcomes and competencies formed by the course are defined in Table 1.

Table 1

Learning outcomes and competencies formed by the course

Learning outcomes	Competencies
LO2	GC3
LO4	GC8
LO6	GC3

where GC3. Ability to abstract thinking, analysis and synthesis.
GC8. Ability to conduct research at an appropriate level.
LO2. Analyze and forecast market phenomena and processes based on the application of fundamental principles, theoretical knowledge and applied skills of marketing

activities.
LO4. Collect and analyze the necessary information, calculate economic and marketing indicators, substantiate management decisions based on the use of the necessary analytical and methodical tools.
LO6. Determine the functional areas of the marketing activity of the market entity and their relationships in the management system, calculate the relevant indicators that characterize the effectiveness of such activity.

COURSE CONTENT

Content module 1: Probability Theory.

Topic 1. Empirical and logical foundations of probability theory

Events and the space of elementary events. Operations on elementary events. Elements of combinatorics (permutations, placement, combinations without repetition). Classical, statistical and geometric definition of probability. Axiomatics of Kolmogorov.

Topic 2. Basic theorems of probability theory, their economic interpretation

The sum of events and addition theorems of probabilities. Conditional probability and multiplication theorems of probabilities. The probability of the opposite event. The probability of occurrence of at least one event. Probability of hypotheses. Complete group of events. Formula of a total probability. Bayes formula. The application of the total probability formula and Bayes formula in economics and finance.

Topic 3. Scheme of independent trials

Bernoulli’s scheme. Bernoulli's formula. Probability distribution. The most probable number of occurrences of an event in a series of independent trials. Local and integral theorems of Moivre – Laplace, Poisson's theorem.

Topic 4. Random variables and their economic interpretation

One-dimensional discrete random variable, distribution function, its properties. The basic numerical characteristics of a random variable (mathematical expectation, variance, root-mean-square deviation), their properties. Initial and central theoretical moments. Economic interpretation.

Topic 5. Distribution laws and numerical characteristics of a random variable

Basic laws of the distribution of a discrete random variable. Binomial, geometric, hypergeometric laws of distribution, their parameters. Continuous random variable, its differential distribution function. The main distribution laws of a continuous random variable (uniform, normal, exponential) and their parameters.

Topic 6. Multidimensional random variables

Two-dimensional random variable, its basic numerical characteristics. Correlation moment. Correlation coefficient and its properties. Conditional distribution law of the system of two random variables

Content module 2: Mathematical Statistics

Topic 7. Limit theorems of probability theory. Primary processing of statistical data

The concept of the law of large numbers and the central limit theorem. Chebyshov's theorem and a stability of means. Bernoulli's theorem and a stability of relative frequencies. Central limit theorem. A population and a sample. Ways of a sample formation. Statistical distribution of a sample, ways of its defining. Polygon and histogram. Empirical distribution function. Basic numerical characteristics of statistical distribution. Empirical initial and central moments.

Topic 8. Statistical estimations of basic numerical characteristics of the population and their properties. Point and interval estimations.

Definition of a point estimation. Method of moments for estimation of distribution parameters. Initial and central moments. Point estimation of a mathematical expectation, its properties. Point estimation of a variance, its properties. Asymmetry coefficient. The coefficient of excess. Coefficient of variation. Interval statistical estimations. Estimation accuracy and confidence probability (reliability) of an estimation. Confidence interval. Construction of confidence intervals for estimations of normal distribution parameters.

Topic 9. Verification of statistical hypotheses

Concept of a statistical hypothesis. Null and competing simple and complex hypotheses. Errors of the first and second types. Statistical criterion for testing the null hypothesis. Critical region and hypothesis acceptance region. Pearson's criterion.

Topic 10. Elements of correlation theory

Correlation dependence as a particular case of a statistical dependence. Calculation of the sample correlation coefficient. Sample correlation ratio and its properties.

Topic 11. Elements of variance analysis

Factorial or between-group variance. Residual, or within-group variance. Fisher-Snedeker fitting test.

Topic 12. Elements of regression theory

Conditional means. Regression equation. The least squares method. Finding the parameters of a sample regression equation based on ungrouped data. A correlation table. Finding the parameters of the sample regression equation based on grouped data. The list of practical (seminar) and laboratory studies in the course is given in Table 2.

Table 2

The list of practical (seminar) and laboratory studies

Name of the topic and/or task	Content
Topic 1. Practical study 1	Empirical and logical foundations of probability theory
Topic 2. Laboratory study 1	Basic concepts of probability theory. Basic theorems of probability theory, their economic interpretation
Topic 3. Practical study 2	Scheme of independent trials (Bernoulli's scheme)
Topic 4. Laboratory study 2	Random variables and their economic interpretation
Topic 5. Practical study 3	Distribution laws of one-dimensional discrete random variable and its numerical characteristics
Topic 6. Laboratory study 3	Distribution laws and numerical characteristics of two-dimensional discrete random variable
Topic 7. Practical study 4	Limit theorems of probability theory. Primary processing of statistical data
Topic 8. Laboratory study 4	Statistical estimations of the parameters of the distribution of discrete and continuous random variables
Topic 8. Practical study 5	Statistical estimation of the basic numerical characteristics of a population
Topic 9. Laboratory study 5	Point and interval statistical estimations. Testing statistical hypotheses
Topic 10. Practical study 6	Elements of correlation theory
Topics 11, 12. Laboratory study 6	Elements of regression theory and variance analysis on the example of a two-dimensional random variable

The list of self-studies in the course is given in table 3.

Table 3

List of self-studies

Name of the topic and/or task	Content
Topics 1-12	Learning the lecture material
Topics 1-12	Preparation for practical and laboratory studies
Topics 1-12	Preparation for written tests and colloquiums
Topics 1-12	Performing tasks of self-study
Topics 1, 2 or 12	Performing a creative task
Topics 1-12	Preparation for the final exam

The number of hours of lectures, practical (seminar) and laboratory studies and hours of self-study is given in the technological card of the course.

TEACHING METHODS

In the process of teaching the course, in order to acquire certain learning outcomes, to activate the educational process, it is envisaged to use such teaching methods as:

Verbal (lectures (Topics 1, 3 – 5, 7 – 12), problem lectures (Topics 2, 6).

Visual (demonstration (Topics 1–12)).

Practical (practical work (Topics 1–12), laboratory work (Topics 1–12)).

Presentation (Topics 1, 2 or 12).

Didactic game (Topic 12).

FORMS AND METHODS OF ASSESSMENT

The University uses a 100-point cumulative system for assessing the learning outcomes of students.

Current control is carried out during lectures, practical, laboratory and seminar classes and is aimed at checking the level of readiness of the student to perform a specific job and is evaluated by the amount of points scored:

− for courses with a form of semester control as an exam: maximum amount is 60 points; minimum amount required is 35 points.

The final control includes current control and an exam.

Semester control is carried out in the form of a semester exam.

The final grade in the course is determined:

− for disciplines with a form of exam, the final grade is the amount of all points received during the current control and the exam grade.

During the teaching of the course, the following control measures are used:

Current control: colloquiums (estimated at 7 points (two colloquiums during the semester – the total maximum number of points – 14)); written tests (maximum score – 8 points (two written tests during the semester, total maximum number of points – 16)); homework (maximum score – 2 points (six homework during the semester, total maximum number of points – 12 points)); laboratory work (maximum score – 2 points (six laboratory work during the semester, total maximum number of points – 12 points)); an independent creative task (maximum score – 6 points).

Semester control: Grading including Exam (40 points).

More detailed information on the assessment system is provided in the technological card of the course.

An example of an exam card and assessment criteria.

An example of examination card
SEMEN KUZNETS KHARKIV NATIONAL UNIVERSITY OF ECONOMICS
Study cycle: first (bachelor)
Term 2
The course: "Probability theory and mathematical statistics"

Examination paper (EXAMPLE)

Task 1 (7 points). The probability of successful solving a Math test by the first student is 0.9, by the second student is 0.6, by the third student is 0.8. What is the probability of solving this test by: a) one student only; b) only the third student; c) all of the students; d) at least one of students?

Task 2 (7 points). The pizza delivery department receives 75% brand pizza orders. Find the probability that among 100 received orders there will be (a) 72 brand pizza orders; (b) from 70 to 78 brand pizza orders. (c) Find the most probable number of brand pizza orders.

Task 3 (8 points). The integral function of X is given:

$$F(x)=\begin{cases}0, & x\le 2\\ \frac{1}{16}\left(x^2-4x+4\right), & 2<x\le 6\\ 1, & x>6\end{cases}$$

Calculate: a) the density function $f(x)$; b) the mathematical expectation $M(X)$ and the variance; c) the probability P(4<X<7).

Task 4 (8 points). The continuous statistical series is given in the table:

$[x_i, x_{i+1})$	13 - 15	15 - 17	17 - 19	19 - 21	21 - 23
m_i	8	10	23	6	3

a) Find the mean, the variance and the root-mean-square deviation for this sample. b) Plot the histogram and make the assumption about the distribution law. c) Find the confidence interval for the population mean with the probability 0,95. d) Make an analysis of the obtained values.

Task 5 (10 points). Data: $\sum x=1432,\ \sum y=1714,\ \sum y^2=29553,\ n=100.$

$\sum xy=24816,\ \sum x^2=21756,$

a) Construct a pair linear equation of a regression: $\hat{y}_x=b_0+b_1x$ and make an analysis of coefficients.

b) Calculate a correlation coefficient r, a determination coefficient R^2 and explain obtained results. c) Verify a hypothesis about a statistical significance of a regression equation at the significance level α =0,01.

It was approved at the meeting of the department of higher mathematics and economic mathematical methods
Protocol № _ from ___, 20__

The head of the department L. Malyarets
The lecturer Ie. Misiura

An assessment criteria

Each examination paper contains 5 practical tasks, including two first-level (diagnostic) tasks, two second level (situational) tasks and one third level (diagnostic and heuristic) task.

The structure of the examination paper is given in Table 5.

Table 5

The structure of the examination paper

Task level	The content of tasks after the themes
First	Task 1. A classical definition of a probability and its calculation. Basic formulas of combinatorics. Venn-Euler diagram. Addition theorems of probabilities. Dependent and independent events. A conditional probability. Joint (compatible) and disjoint (incompatible) events. Multiplication theorems of probabilities. A complete group of events. Complementary events. The probability of at least one event. The probability that an event will occur at least one. Formula of a total probability. Bayes formula (the theorem of hypothesis). Task 2. A scheme of Repeated independent trials. Bernoulli's formulas. Local theorem of Moivre–Laplace. Integral theorem of Moivre–Laplace. Poisson's theorem.
Second	Task 3. Discrete and continuous random variables. Distribution laws of probabilities for a random variable. A distribution function of probabilities. Basic numerical characteristics of a random variable: a mathematical expectation, a variance and a root-mean-square deviation. Distribution laws of a discrete random variable, which are used often in social and economic phenomena: a binomial distribution, a geometrical distribution, a hypergeometric distribution. Task 4. Distribution laws of a continuous random variable, which are used often in social and economic phenomena: a uniform distribution, a normal distribution and an exponential distribution. Discrete and interval statistical series. A polygon and a histogram. Basic sampling numerical characteristics. Point and interval estimations. A confidence interval for a mathematical expectation of a normal population. A checking of a statistical hypothesis about defining of a distribution law for a population using results of an investigation of a sample.
Third	Task 5. A sampling coefficient of a correlation. A coefficient of determination. A correlation ratio. A correlation dependence. A correlation table. Empirical lines of a regression. An estimation of parameters of a pair regression equation using the least-squares method. Point estimations. A checking of the significance of parameters of a pair regression equation. A confidence interval for a line of a pair regression.

The task of the first level is assessed as follows (7 points):

7 points, in the case of the exact use of the scientific terminology and symbols in the necessary logical sequence; a creative approach to solving original problems which require a high level of knowledge;

6 points, in the case of the exact use of the mathematical terminology and symbols; irreproachable mastery of mathematical instruments; correct use of mathematical methods, facts, formulas and relations for solving the task of the third level;

5 points, if a logically right sequence of steps of solution has been made. All the key moments of the solution have been grounded. 1–2 slight mistakes or slips are possible in the calculations which don't influence the correctness of the further solution;

4 points, if mastery of a small part of obligatory skills and attainments which are intended for the syllabus of the academic course has been demonstrated; in solving the tasks, the conclusions, reproduction of the syllabus material of the course has not been always shown;

3 points, if the task has been solved only partially with initial right considerations, but there are mistakes, which considerably influenced the process of the right solution of the task;

2 points, if the task fulfillment has been begun, there are separate correct considerations, but a logical mistake has been made, which resulted in an incorrect solution.

1 point, if the condition has been written;

0 point, if no task has been fulfilled.

The task of the second level is assessed as follows (8 points):

8 points, if solving the assigned tasks is characterized by a creative use
of the theoretical instrument, logical correctness, precision, explanation of conclusions, rationality or using original approaches to solving the tasks;

7 points, if perfect mastery of the skill in the use of mathematical instruments application of information from the other educational courses and discipline has been demonstrated; one slight mistake has been made; a high level of standards of carrying out the tasks;

6 points, if a logically right sequence of steps of solution has been chosen. All the key points of solution have been grounded. 1–2 slight mistakes or slips are possible in the calculations, which don't influence the correctness of the further solution;

5 points, in the case of correct using the terminology of the course and the basic methods for solving standard problems; showing the ability to use theoretical knowledge for solving standard (multistep) tasks, availability of some mistakes or deficiencies on the calculating stage of presentation of the solution, the ability to conclude;

4 points, in the case of more than one mistake and one or two deficienies in the calculations, the graphs, the choice of the method of solution, which have caused a wrong final result in the individual cases;

3 points, if task fulfillment has been begun, there are separate correct considerations, but a logical mistake has been made, which resulted in an incorrect solution;

2 points, if numerical gross mistakes have been made in the process of using the concepts of the discipline in the formulas which prove the absence of a minimally

necessary part of the compulsory skills and the practical attainments provided for the discipline syllabus;
1 point, if no task fulfillment has been begun, but the condition has been written;
0 point, if no task fulfillment has been begun.

The task of the third level is assessed as follows (10 points):

10 points, if the ability for scientific investigative developments on the problems of the discipline has been shown; perfect skills in the use of mathematical instruments using modern scientific theoretical approaches; a high level of standards of carrying out tasks has been demonstrated;
9 points, in the case of using scientific terminology and symbols in the necessary logical sequence; solving the assigned tasks is characterized by precision, explanation, a creative approach, rationality of the choice of method of solution, correct necessary calculations and transformations;
8 points, if systematic, deep and full knowledge of all the parts of the academic discipline and the basic questions which go beyond the discipline has been shown; a high level of standards of carrying out the tasks has been demonstrated;
7 points, in the case of solitary slight deficiencies which don't influence the final result; correct use of mathematical methods, facts, formulas and relations for solving the task of different level of complexity;
6 points, if the ability to conclude and compare the theoretical and practical material has been demonstrated; correct (but not always rational) use of mathematical methods of solution, facts, formulas and relations has been shown;
5 points, if half of the tasks have been done, the interpretation of the obtained results is absent; level of standards of carrying out tasks is acceptable;
4 points, if the tasks have been carried out without any logical relationship of the mathematical concepts and practical solutions have not been given sufficient theoretical explanation;
3 points, if acceptable volume of knowledge has been shown within the educational standard; the use of mathematical symbols and terminology has been insufficient and in exact, the knowledge of the basic formulas and concepts on the discipline has not been demonstrated;
2 points, in the case of solving tasks with the theoretical material used only on the level of concepts; the inability to understand the connection of the theoretical material with the practical tasks;
1 point, if the condition has been written;
0 point, if no task fulfillment has been begun.

RECOMMENDED LITERATURE

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