Analysis of vibrations of the working body of the vibrating separator in the resonant mode

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Abstract — **This paper analyzes the nonlinear vibrations of the working body of a vibrating separator in resonant modes, focusing on the redistribution of energy between oscillation coordinates. Using the asymptotic averaging method, approximate solutions and auxiliary equations are derived to study the spatial stability of oscillations under combination resonances. It is shown that resonance relations involving the sum of natural frequencies and the excitation frequency do not disturb the planned technological process, ensuring stable operation of vibratory grain cleaning machines. The obtained results define parameter conditions that ensure stable operation of vibratory grain cleaning machines with helical oscillations of the working body.**

Keywords — vibration separators; nonlinear oscillations; natural frequencies; working body; resonant mode; external force; spatial stability (key words)

I. Introduction

The working body of the vibrating separator must perform oscillations that are provided for by the technological process. Therefore, external disturbing forces act in the direction of not all, but only some coordinates. In nonlinear systems, which also include vibration separators, it is possible to redistribute the energy supplied from the outside in such a way that intense vibrations occur in the direction of the undisturbed coordinates [1].

In this paper, we consider the redistribution of energy between coordinates if resonant ratios arise between the natural frequencies of the oscillating system and the frequencies of external action on the oscillating system. Resonant modes associated with coordinates in the direction of which external disturbing forces do not act can change the law of motion of the working body of a vibrating separator.

In this regard, it is necessary to know the set of parameters of the oscillatory system under which unfavorable conditions of energy distribution between the coordinates arise.

II. Problem formulation

We study the nonlinear oscillations of a vibratory grain cleaning machine with helical oscillations of the working body in the resonant case.

A. Mathematical model

The differential equations of motion of the oscillating system can be written in the following form:

$$\begin{gathered}
\ddot{\xi} + P_\xi{}^2 \xi = \mu \Phi_\xi\left(\dot{\xi}, \phi, \dot{\phi}, \ddot{\phi}, t\right); \\
\ddot{\eta} + P_\eta{}^2 \eta = \mu \Phi_\eta\left(\dot{\eta}, \theta, \dot{\theta}, \ddot{\theta}, t\right); \\
\ddot{\zeta} + P_\zeta^2 \zeta = \mu \Phi_\zeta\left(\zeta, \theta, \phi, \dot{\theta}, \dot{\phi}, \ddot{\theta}, \ddot{\phi}, t\right) + F_\zeta \cos \omega t; \quad (1) \\
\ddot{\theta} + P_\theta^2 \theta = \mu \Phi_\theta\left(\eta, \zeta, \theta, \phi, \varphi, \dot{\theta}, \dot{\phi}, \dot{\varphi}, \ddot{\eta}, \ddot{\zeta}, \ddot{\theta}, \ddot{\phi}, t\right); \\
\ddot{\phi} + P_\phi^2 \phi = \mu \Phi_\phi\left(\xi, \zeta, \theta, \phi, \varphi, \dot{\theta}, \dot{\phi}, \dot{\varphi}, \ddot{\xi}, \ddot{\zeta}, \ddot{\theta}, \ddot{\phi}, \ddot{\varphi}, t\right); \\
\ddot{\varphi} + P_\varphi^2 \varphi = \mu \Phi_\varphi\left(\theta, \phi, \dot{\theta}, \dot{\phi}, \dot{\varphi}, \ddot{\phi}, \ddot{\varphi}, t\right) + F_\varphi \cos(\omega t + \pi).
\end{gathered}$$

Let us consider the cases of combination resonances, when the resonance relations include two unperturbed coordinates. To this end, asymptotic solutions are found for system (1) by the method of averaging [2].

Assuming $\mu = 0$, the solutions of system (1) are found in the form

$$\begin{gathered}
\xi = C_\xi e^{iP_\xi t} + D_\xi e^{-iP_\xi t}; \\
\dot{\xi} = iP_\xi\left(C_\xi e^{iP_\xi t} - D_\xi e^{-iP_\xi t}\right); \\
\eta = C_\eta e^{iP_\eta t} + D_\eta e^{-iP\eta t}; \\
\dot{\eta} = iP_\eta\left(C_\eta e^{iP_\eta t} - D_\eta e^{-iP_\eta t}\right);
\end{gathered}$$

$$\zeta = C_\zeta e^{iP_\zeta t} + D_\zeta e^{-iP_\zeta t} + A_\zeta e^{i\omega t} + B_\zeta e^{-i\omega t};$$

$$\dot{\zeta} = iP_\zeta\left(C_\zeta e^{iP_\zeta t} - D_\zeta e^{-iP_\zeta t}\right) + i\omega\left(A_\zeta e^{i\omega t} - B_\zeta e^{-i\omega t}\right);$$

$$\theta = C_\theta e^{iP_\theta t} + D_\theta e^{-iP_\theta t}; \quad \dot{\theta} = iP_\theta\left(C_\theta e^{iP_\theta t} - D_\theta e^{-iP_\theta t}\right);$$

$$\phi = C_\phi e^{iP_\phi t} + D_\phi e^{-iP_\phi t}; \tag{2}$$

$$\dot{\phi} = iP_\phi\left(C_\phi e^{iP_\phi t} - D_\xi e^{-iP_\phi t}\right);$$

$$\varphi = C_\varphi e^{iP_\zeta t} + D_\varphi e^{-iP_\varphi t} - A_\varphi e^{i\omega t} - B_\varphi e^{-i\omega t};$$

$$\dot{\varphi} = iP_\varphi\left(C_\varphi e^{iP_\varphi t} - D_\varphi e^{-iP_\varphi t}\right) -$$

$$- i\omega\left(A_\varphi e^{i\omega t} - B_\varphi e^{-i\omega t}\right);$$

$$\dot{\zeta} = iP_\zeta\left(C_\zeta e^{iP_\zeta t} - D_\zeta e^{-iP_\zeta t}\right) + i\omega\left(A_\zeta e^{i\omega t} - B_\zeta e^{-i\omega t}\right).$$

In the solutions of (2), the values C_k, D_k $(k = \xi, \eta, \zeta, \theta, \psi, \varphi)$ are arbitrary constants; ω is the frequency of the external disturbing force, $\sqrt{-1} = \boldsymbol{i}$ is an imaginary unit; A_s, B_s $(s = \zeta, \varphi)$ are the amplitudes of the forced oscillations of the system at $\Phi_\zeta = 0$, $\Phi_\varphi = 0$.

If we put in (2) $C_k = 0, D_k = 0$ $(k = \xi, \eta, \zeta, \theta, \psi, \varphi)$, the initial system (1) will have the following solution:

$$\xi = \eta = \theta = \psi = 0, \qquad \dot{\xi} = \dot{\eta} = \dot{\theta} = \dot{\psi} = 0 \tag{3}$$

$$\varsigma = \frac{F_\varsigma}{\left(P_\varsigma^2 - \omega^2\right)}\cos\omega t; \quad \varphi = \frac{F_\varphi}{\left(P_\varphi^2 - \omega^2\right)}\cos(\omega t + \pi).$$

B. Stability Analysis

Oscillations (3) are provided for by the design of vibratory separators and the technological process. In the following, we will investigate the spatial stability of such oscillations.

We take arbitrary constants C_k, D_k $(k = \xi, \eta, \zeta, \theta, \psi, \varphi)$ as new unknowns instead of $\xi, \dot{\xi}, \eta, \dot{\eta}, \zeta, \dot{\zeta}, \theta, \dot{\theta}, \phi, \dot{\phi}, \varphi, \dot{\varphi}$, and introduce the replacement of variables (2). Assuming C_k, D_k functions of time t, we differentiate $\dot{\xi}, \dot{\eta}, \dot{\zeta}, \dot{\theta}, \dot{\psi}, \dot{\varphi}$, taken from (2), by t. We obtain the expressions for the second derivatives

$$\ddot{\xi} = -P_\xi^2\left(C_\xi e^{iP_\xi t} + D_\xi e^{-iP_\xi t}\right) +$$

$$+ iP_\xi\left(\dot{C}_\xi e^{iP_\xi t} - \dot{D}_\xi e^{-iP_\xi t}\right);$$

$$\ddot{\eta} = -P_\eta^2\left(C_\eta e^{iP_\eta t} + D_\eta e^{-iP_\eta t}\right) +$$

$$+ iP_\eta\left(\dot{C}_\eta e^{iP_\eta t} - \dot{D}_\eta e^{-iP_\eta t}\right);$$

$$\ddot{\zeta} = -P_\zeta^2\left(C_\zeta e^{iP_\zeta t} + D_\zeta e^{-iP_\zeta t}\right) +$$

$$+ iP_\zeta\left(\dot{C}_\zeta e^{iP_\zeta t} - \dot{D}_\zeta e^{-iP_\zeta t}\right) -$$

$$- \omega^2\left(A_\zeta e^{i\omega t} + B_\zeta e^{-i\omega t}\right). \tag{4}$$

Solving (4) with respect to $\dot{C}_k, \dot{D}_k$, we obtain a system of differential equations in the standard form with respect to the envelope C_k, D_k

$$\dot{C}_k = \frac{1}{2iP_k} e^{-iP_k t}\left[\mu\Phi_k^*\right];$$

$$\dot{D}_k = \frac{1}{2iP_k} e^{-iP_k t}\left[\mu\Phi_k^*\right];$$

$$k = \xi, \eta, \xi, \theta, \psi, \varphi. \tag{5}$$

The right-hand sides of equations (5) are functions of order of smallness μ, and therefore the functions C_k, D_k, $k = \xi, \eta, \xi, \theta, \psi, \varphi$ will be slowly changing functions.

Approximate values of C_k, D_k, as functions of time are determined from the auxiliary equations obtained by averaging equations (5) over time *t*, which is explicitly included in these equations. These auxiliary equations have the following form in the first approximation

$$\dot{C}_k = \frac{\mu}{2iP_k} \times$$

$$\times \lim_{T\to\infty} \frac{1}{T}\int_0^T \Phi_k^*\left(C_\xi, \ldots, C_\varphi, D_\xi, \ldots, D_\varphi\right) e^{-iP_k t} dt; \tag{6}$$

$$\dot{D}_k = \frac{\mu}{2iP_k} \times \lim_{T\to\infty} \frac{1}{T}\int_0^T \Phi_k^*\left(C_\xi, \ldots, C_\varphi, D_\xi, \ldots, D_\varphi\right) e^{iP_k t} dt.$$

Equations (6) are used to study the spatial stability of vibrations of the working body of a vibration separator in the presence of resonant ratios between the natural frequencies of the oscillating system P_k $(k = \xi, \eta, \theta, \psi)$ and the frequencies ω of an external force.

IV. Conclusions

Resonance relations, which contain the sum of two frequencies of the oscillating system and the frequency of the external force, are considered. It is proved that at the considered resonance relations the planned technological process will not be disturbed.

References

[1] N.V. Smetankina, O.V. Postnyi, S.Yu. Misura, A.I. Merkulova and D.O. Merkulov Optimal design of layered cylindrical shells with minimum weight under impulse loading. In: 2021 IEEE 2nd KhPI Week on Advanced Technology (KhPIWeek). 2021. P. 506–509.

[2] N. N. Bogoliubov and Y. A. Mitropolski, Asymptotic Methods in the Theory of Non-Linear Oscillations. - New York : Gordon & Breach Science Publishers, 1961, 537 p.