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# COMPLEX DYNAMICS OF INNOVATION DIFFUSION

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**Complex Dynamics of Innovation Diffusion**

*To search for innovative ways of sustainable growth within the trend of global economic expansion, it is necessary to have formalized approaches within the framework of the synergetic paradigm – the theory of self-organization in open non-equilibrium systems. One of the most important directions in this regard is the conception of innovation diffusion. In this study, the classical logistic model of the spread of an innovative product was considered. The development of the mathematical model was implemented based on the dynamic balance of «supply – demand» in the innovation market, both in discrete and continuous time. At the same time, the linear dependence of demand on the total volume of innovative products was taken into account, while on the supply side, the possibility of technological production constraints was considered, which is reflected in the form of a quadratic dependence of the supply function on the quantity of innovative products. In developing the discrete dynamic model, the basic balance equation was transformed into the form of the classical logistic equation with known properties, with a further detailed analysis provided in the study. The theoretical results were confirmed through corresponding numerical computations and simulation modeling, which illustrated important dynamic regimes such as limit cycles with period doubling, irregular chaotic behavior, and others. In continuous time, a mathematical model of innovation diffusion was constructed taking into account delays (distributed time lag), considered as a second-order dynamic process. The model was reduced to a system of two differential equations, in which limit cycles with varying stability characteristics can exist. Both mathematical models – discrete and continuous – have the same equilibrium states (fixed points), and the dynamics near these points significantly depend on the initial conditions.*

***Keywords:*** *dynamic equilibrium of demand and supply for innovations, logistic curve, dynamic memory, stability of equilibrium positions (fixed points), limit cycle, attractor and repulsor, bifurcation, chaos.*

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*Для пошуку інноваційних шляхів сталого зростання у тренді глобального економічного зростання необхідно мати формалізовані підходи у межах синергетичної парадигми – теорії самоорганізації у відкритих нерівноважних системах. У якості одного з найважливіших таких напрямків можна виділити концепцію дифузії інновацій. У даному дослідженні було розглянуто класичну логістичну модель розповсюдження інноваційного продукту. Розбудова математичної моделі було реалізовано на основі динамічного балансу «попит – пропозиція» на ринку інновацій як у дискретному, так і у неперервному часі. При цьому було враховано наявність лінійної залежності попиту від загального обсягу інноваційної продукції, а з боку пропозиції – розглянуто можливість існування технологічних обмежень виробництва. що відображається у формі квадратичної залежності функції пропозиції від кількості інноваційної продукції. При побудові дискретної динамічної моделі було виконано перетворення базового балансового співвідношення до вигляду класичного логістичного рівняння з відомими властивостями, ретельний аналіз наведено в роботі. Теоретичні результати було підтверджено завдяки проведенню відповідних числових розрахунків та імітаційного моделювання, завдяки чому проілюстровано такі важливі динамічні режими як граничні цикли з подвоєнням періодів, нерегулярна хаотична поведінка та інші. У неперервному часі побудовано математичну модель дифузії інновацій з урахуванням запізнення (розподіленого часового лагу), що розглядається як динамічний процес другого порядку. Модель приведено до вигляду системи двох диференціальних рівнянь, в якій може існувати граничні цикли з різним характером стійкості. Обидві математичні моделі – дискретна і неперервна – мають ті ж самі рівноважні стани (нерухомі точки), а динаміка поведінки в околі цих точок суттєво залежить від початкових умов.*

***Ключові слова:*** *динамічна рівновага попиту і пропозиції за інноваціями, логістична крива, динамічна пам'ять, стійкість положень рівноваги (нерухомих точок), граничний цикл, атрактор і репульсор, біфуркація, хаос.*

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**Introduction.** Under current conditions of global multidirectional challenges, the transition to a knowledge economy and innovative activity specifically become the key to successfully implementing the paradigm of sustainable economic development at the level of individual enterprises, industries, and the country as a whole. One of the most critical issues in ensuring efficient innovative development is the synthesis of economic and social evolution, which is accompanied by a synergistic effect. Effective resolution of this issue is possible only when the management of innovation implementation processes is based on mathematical models that allow forecasting the characteristics of the system's complex dynamics and identifying ways to overcome crisis phenomena. Since this concerns the development of scenarios for the evolution of an open non-equilibrium system, such as the process of innovation diffusion, and forecasting needs to be done over the long term, it is necessary to use dynamic mathematical models to construct a model capable of adequately describing the state of such a system. Particularly, these are models in which time is present not only as an independent variable, but the model parameters, explicitly or implicitly, are also functions of time.

Both linear and nonlinear models can be used as dynamic mathematical models. Linear models are the simplest both in construction and interpretation, which is why they are the most widespread. The basis of linear dynamic models is the assumption that the predictors, that is, the parameters used to build the model, maintain the constant values within the time period for which the model is developed [1-5, etc.]. However, linear models do not account for random factors and uncertainties, which significantly reduces their accuracy even for short-term forecasting. Therefore, when forecasting over a sufficiently long period, it is advisable to use nonlinear dynamic models to adequately represent the processes occurring in the system and ensure forecast accuracy.

Unlike linear models, which assume that during the system's operational period its future state is determined by a simple superposition of the influences of various factors, whose numerical characteristics remain constant over time, nonlinear models take into account the complexity of internal processes within the system and the non-additivity of their interactions. Such models recognize the presence of synergistic effects in the system, that is, effects in which the result of the simultaneous influence of factors due to their interaction exceeds the sum of the results of each factor's individual influence [6-12 etc.]. Accordingly, nonlinear models are capable of predicting and describing phenomena such as loss of equilibrium, the emergence of limit cycles, bifurcations, and the transition to a chaotic state [13-21 etc.]. This approach becomes especially important in studying innovation processes, since the innovative development of a system is nonlinear in nature [22-28 etc.].

Attention should be paid to certain features of the mathematical instruments used in building mathematical models of nonlinear dynamics. Economic processes are usually considered in discrete time (month, quarter, year), so difference equations are used for their modeling [29-32 etc.]. However, quite often, a system approach using the mathematical apparatus of differential equation theory is applied in constructing dynamic mathematical models [33-36 etc.], although within these models, time must be considered continuous, which is also acceptable. In addition to real variables, mathematical models of dynamics can include lag variables. By introducing lag variables that describe delays, such models allow the influence of external variables on processes to be considered, taking into account that the system's response to this influence may be postponed [37-40, etc.].

It should also be emphasized that general systems theory allows for the consideration of the interaction of interconnected system elements, taking into account feedbacks that can be either positive or negative and are nonlinear in nature [41-45, etc.]. While positive feedback amplifies the system's response to external influence, the presence of negative feedbacks to a certain extent ensures the implementation of system self-regulation, thereby maintaining its steady development in the intended direction. It should be noted that the nonlinear nature of feedback in a system can give rise to an entire hierarchy of unstable states, which during the system's development may lead to bifurcations, the emergence of limit cycles, homoclinic structures, and even chaos [46].

The aim of the present study is to apply mathematical dynamic models to explore the phenomenon of innovation diffusion as a fundamentally nonlinear process within a system, which may have several equilibrium positions, around which markedly different innovation behavior dynamics are observed.

**Results and discussion**. To ensure qualitative forecasting of the innovation diffusion process, the mathematical model of the object must be constructed taking into account the regularities of the system's functional balance in which this process occurs. Let us assume that the market for a certain innovative product is characterized by two main variables: $p = p(t)$ – the unit price of the product, which depends on the time *t*, and $y = y(t)$ – the volume of innovative product output, which also depends on the time *t*. Within the framework of the approach of Léon Walras and Paul Samuelson [47, 48], we will assume that there is a static equilibrium – a balance of supply $D(p,y)$ and demand $S(p,y)$, and therefore the satisfying equation is:

$$D(p,y) = S(p,y). \quad (1)$$

Without violating generality, to simplify further transformations using expressions for demand and supply, it is reasonable to assume that the price is constant, i. e. $p = const$, and we will assume that it is equal to one (conditionally). Furthermore, let the volume of demand equal the volume of output, i. e., we have $D(p,y) = y$. To determine supply, we apply a function $S = \alpha(L - y)y$, where *L* – the growth limit, realization of which is determined by available resources and technological constraints, $\alpha$ – a dimensionality parameter responsible for consistency. Such a specification of the function implies that there are limits to the growth of innovative product output. Considering these assumptions and based on the balance equation (1), we obtain the following relationship:

$$y = \alpha(L - y)y \quad (2)$$

The equation (2) has two particular solutions (singular points):

$$y_1^* = 0 \text{ and } y_2^* = L - \frac{1}{\alpha}. \quad (3)$$

The relation (3) define two equilibrium positions (the so-called fixed points) of the innovation process, with $L > \frac{1}{\alpha}$.

A dynamic process arises in the presence of a delay factor (time lag). This factor can manifest on either the demand side or the supply side. In the simplest case, it is reasonable to assume a single-step delay from the supply side at discrete time points: $n = 0,1,2,3,...$. Taking into account the balance equation (1) and all previous assumptions, this delay can be presented in the following form:

$$D(y_{n=1}) = S(y_n). \quad (4)$$

Without loss of generality, with respect to the parameter *L* it is convenient to assume that *L* = 1. Then the structure of the dynamic process can be presented as follows:

$$y_{n+1} = \alpha y_n (1 - y_n). \quad (5)$$

This process is characterized by two equilibrium positions in the form of two fixed points:

$$y_1^* = 0 \text{ and } y_2^* = \frac{\alpha - 1}{\alpha}, \text{ where } \alpha > 1. \quad (6)$$

It should be noted that at $\alpha = 1$ both equilibrium positions merge into one. The nonlinear recurrence relation (5) is, in fact, a logistic map with complex dynamic behavior [49, 50]. The diversity of trajectories inherent in the dynamic equation is quite heterogeneous. They can correspond to virtually any evolutionary processes in various scientific fields: physics, chemistry, biology, sociology, and economics. One may observe numerical sequences whose terms continually increase over time, or, conversely, continually decrease. Periodic processes with different periods may also occur. Furthermore, there may exist solutions where no signs of any regular behavior can be detected.

The right-hand side of the difference equation (5) is a quadratic mapping, which has the form

$$F_\alpha(y) = \alpha y(1-y), \; y \in [0;1], \quad (7)$$

transforming the segment $y \in [0;1]$ into the segment $y \in [0; 0{,}25\alpha]$. Hence, it follows that $1 < \alpha \le 4$.

Before carrying out a detailed analysis of the properties of the quadratic mapping (7), let us consider some examples of some individual values of the parameter $\alpha$, for which exact recursive solutions of the equation (5) can be determined.

Suppose that $\alpha = 2$. Then, the equation (5) takes the form:

$$y_{n+1} = 2y_n(1-y_n). \quad (8)$$

If an initial condition is set, that is, a certain value of $y_0$, then the equation (8) has the following solution:

$$y_n = \frac{1}{2}(1-(1-2y_0)^{2^n}). \quad (9)$$

The solution (9) exhibits stable behavior since it has a finite limit as the number of steps approaches infinity:

$$\lim_{n\to\infty} y_n = \frac{1}{2}. \quad (10)$$

From the recursive equation (8), it is evident that it has two equilibrium positions (fixed points) $y_1^* = 0$ and $y_2^* = \frac{1}{2}$. At this, the point $y_2^* = \frac{1}{2}$ is an attractor, meaning it defines a stable state towards which the dynamic system approaches over time, while the point $y_1^* = 0$ is unstable, that is, a repulsor.

Let us consider another interesting case of the behavior of the equation (5), namely, at $\alpha = 4$. Let us rewrite the equation (5) in the following form:

$$y_{n+1} = 4y_n(1-y_n). \quad (11)$$

The equation (11) has the general solution:

$$y_n = \sin^2(2^n \arcsin\sqrt{y_0}). \quad (12)$$

It is important to emphasize that solution (12) largely depends on the initial condition, that is, on the value of $y_0$. At this, both fixed points $y_1^* = 0$ and $y_2^* = \frac{3}{4}$ are unstable, and analyzing the properties of periodic solutions (11) and (12) appears very complex.

From the properties of mapping (7), it follows that, at $\alpha = 4$, it transforms the segment $y \in [0;1]$ into itself. Obviously, in the general case, relation (7) has two fixed points:

$$y_1^* = 0 \text{ and } y_2^* = \frac{\alpha - 1}{\alpha}.$$

Let us find the first derivative of the quadratic mapping. It can be put down in the following form:

$$\frac{dF}{dy} = \alpha - 2\alpha y. \quad (13)$$

From this, it follows that for all values of $1 < \alpha \le 4$ the point $y_1^* = 0$ is repelling, i. e., indicates a repulsor. For the second fixed point $y_2^*$, the expression (13) takes the following form:

$$\left.\frac{dF}{dy}\right|_{y=y_2^*} = 2 - \alpha. \quad (14)$$

Then, from the inequality $|2-\alpha| < 1$ it follows that the point $y_2^* = \frac{\alpha-1}{\alpha}$ is either an attracting point (attractor) at $1 < \alpha \le 3$, or a repelling point (repulsor) at $3 < \alpha < 4$.

The given examples indicate that when analyzing the stability of the fixed points of the recurrence equation (5) and, accordingly, of the mapping (7), the key factor is the value of the dimension parameter $\alpha$.

Let us examine in more detail the properties of the solutions of the equation (5) for all values of the parameter $\alpha$ in the range $1 < \alpha \le 4$, based on the data presented in the study [49].

At $1 < \alpha \le 3$, the equilibrium position $y_1^* = 0$ is a repulsor, and $y_2^* = \frac{\alpha-1}{\alpha}$ is an attractor. In this case, the trajectory monotonically converges to the point $y_2^*$ at $1 < \alpha \le 2$, but oscillates at $2 < \alpha \le 3$.

If the condition $3 < \alpha < 1 + \sqrt{6} \approx 3{,}449$ is satisfied, then there are two pairs of fixed points, namely the pair $y_1^* = 0$ and $y_2^* = \frac{2}{3}$, also the pair $y_1^* = 0$ and $y_2^* = \frac{6-\sqrt{6}}{5}$, which are repulsors. In addition, an attracting cycle with a period equal to 2 arises, formed by the two points $\hat{y}_1 = \frac{\alpha - \sqrt{\alpha^2 - 2\alpha - 3}}{2\alpha}$ and $\hat{y}_2 = \frac{\alpha + \sqrt{\alpha^2 - 2\alpha - 3}}{2\alpha}$, note that these points are not stationary ones.

If $1+\sqrt{6} \le \alpha < 3{,}596,$ then all the points mentioned above, namely $y_1^*$, $y_2^*$, $\hat{y}_1$ and $\hat{y}_2$, become unstable, but a new attractor appears, which is a cycle with a period equal to 4, provided that $\alpha < 3{,}54$. At $\alpha > 3{,}54$, this new cycle changes its stability character and an attracting cycle appears with a period equal to 8.

Thus, a period-doubling bifurcation occurs. And such a state of the system can be observed until the parameter $\alpha$ reaches the value of $\alpha \approx 3{,}596$.

At $3{,}596 \le \alpha < 3{,}83$, all equilibrium positions and limit cycles become repelling.

If $3{,}83 \le \alpha < 4,$ then cycles with an arbitrary period arise.

At $\alpha = 4,$ besides monotonic and periodic trajectories, sequences that have no regularity may appear. This indicates the regime of dynamic chaos.

Let us illustrate the features of the system's dynamic behavior using simulation modeling depending on the parameter value $\alpha$. Calculations were performed according to the recurrent equation (5) in the MS Excel environment. The calculations showed that, at $\alpha = 1,$ the volume of innovative product output is described by a function that monotonically decreases over time (Fig. 1).

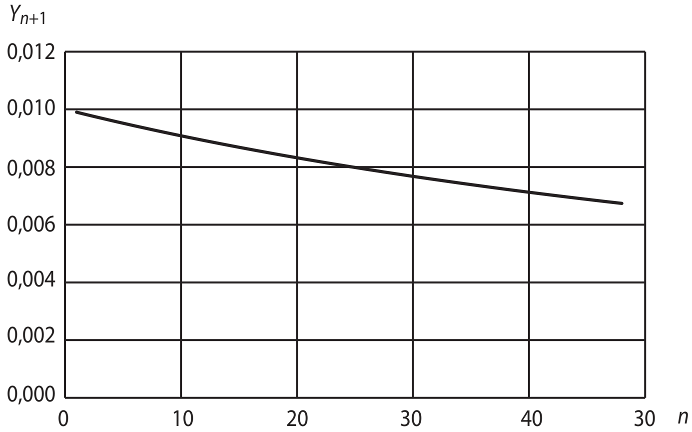


**Fig. 1. Dynamics of the volume of innovative product output at $\alpha = 1$**

Within the range of $1 < \alpha < 2{,}5$, a monotonic increase in the function of innovative product output is observed, and this growth is described by an S-shaped curve (Fig. 2). As the parameter $\alpha$ increases within this range, the growth rate of the function rises, the number of steps it takes for the function to reach its maximum decreases, while the maximum value of the function itself increases.

At $\alpha = 2{,}5,$ a monotonic increase in the function is also observed until it reaches its maximum, but at this step, cyclic oscillations occur, which quickly dampen (Fig. 3).

It should be noted that, at $\alpha = 2{,}5$, the initial maximum value occurs already at $n = 11$, and this step corresponds to the largest deviation from the maximum value of the function. The amplitude of the oscillations decreases rapidly, but cyclicity is still observed up to the 20th step. Compared to the range of $1 < \alpha < 2{,}5,$ the maximum value of the innovative product output function increases, and the speed of reaching the maximum also increases, but this increase is not as significant as in the range of $1 < \alpha < 2{,}5.$

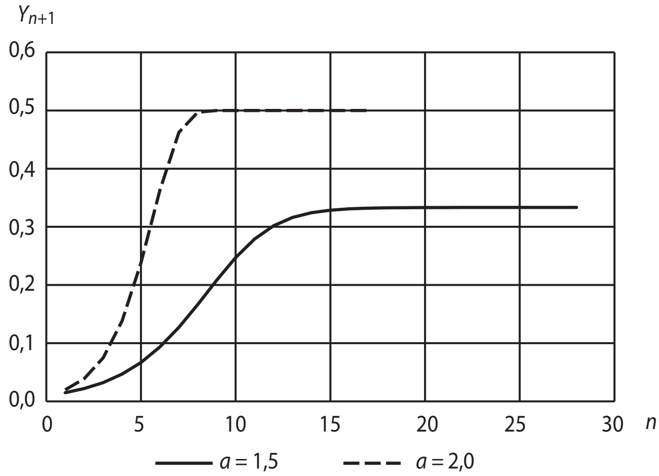


**Fig. 2. Dynamics of innovative product output at $1 < \alpha < 2{,}5$**

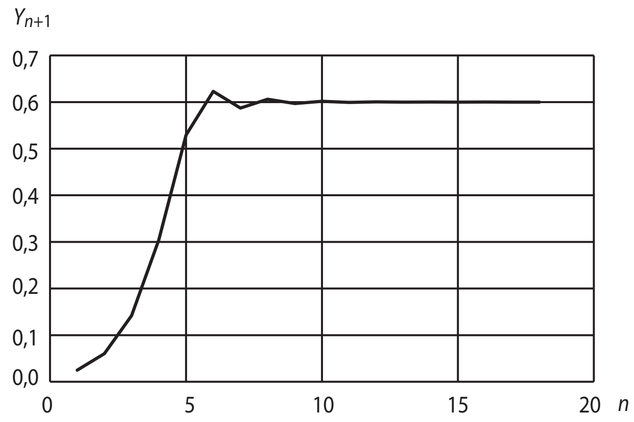


**Fig. 3. Dynamics of innovative product output at $\alpha = 2{,}5$**

At $\alpha = 3$, for the first 5 steps, there is a monotonic increase in the output of innovative products and reaching the maximum value, which is again slightly higher than at the previous values of the parameter $\alpha$ (Fig. 4).

However, after reaching the maximum value, cyclical oscillations appear in the system. Over 30 steps, their amplitude slightly decreases and reaches a steady value. These cyclical oscillations then continue indefinitely.

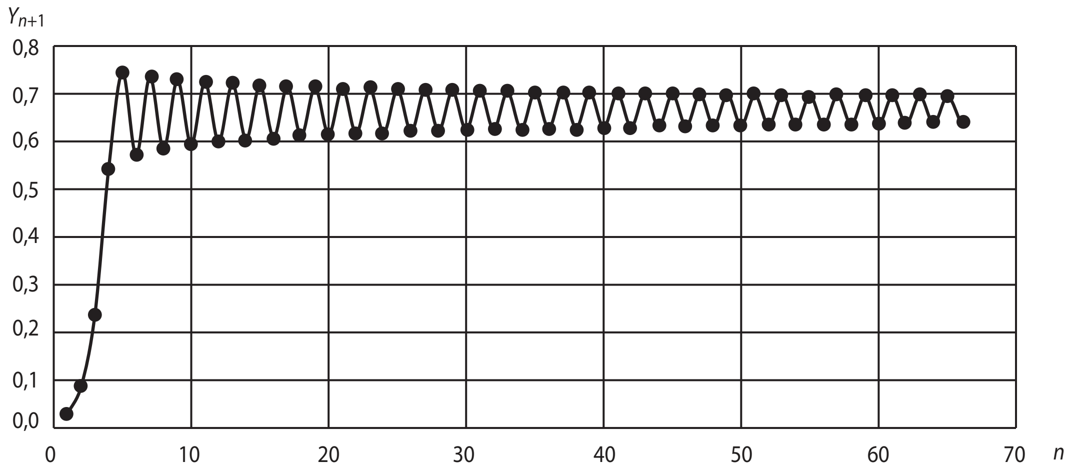


**Fig. 4. Dynamics of innovative product output at $\alpha = 3$**

When the parameter is increased to the value of $\alpha = 3{,}5,$ a slight increase is again observed in the rate at which the innovative product output function reaches its maximum level, as well as in the maximum level itself compared to the previous parameter value of $\alpha$ (Fig. 5). However, there is an oscillation around the maximum level, which refers to its average value.

It should be noted that the amplitude of the oscillations remains constant from the moment the function reaches its maximum level, although the period consists of two types of oscillations (larger and smaller amplitudes). These cyclical oscillations continue indefinitely.

At $\alpha = 4,$ the system is in a state of chaos (Fig. 6).

Thus, Figures 1-6 convincingly illustrate the features of the dynamic behavior of the nonlinear system, which were formulated in the theoretical analysis presented above.

Now, we consider a somewhat different version of the method for introducing a delay on the supply side that exceeds demand. Suppose that the dynamic balance of supply and demand contains information about all past values of supply.

This can be defined as follows:

$$D(y_n) = \sum_{i=0}^{n-1} K_{n-i-1} \cdot S(y_i), \tag{15}$$

where $K_{n-i-1}$ is a way of mapping the «dynamic memory» of previous values of the supply function. Let, for example, $K_{n-i-1} = (1-b)b^{n-i-1}$ be in a decreasing geometric progression with denominator $0 < b < 1.$

The equation (15) can be rewritten with a shift from $n$ to $n+1$:

$$D(y_{n+1}) = \sum_{i=0}^{n} K_{n-i} \cdot S(y_i). \tag{16}$$

Let us transform (16) to isolate the supply at the moment corresponding to step number $n$, i. e.

$$D(y_{n+1}) = b\sum_{i=0}^{n} (1-b)b^{n-i-1} \cdot S(y_i) + (1-b) \cdot S(y_n), \tag{17}$$

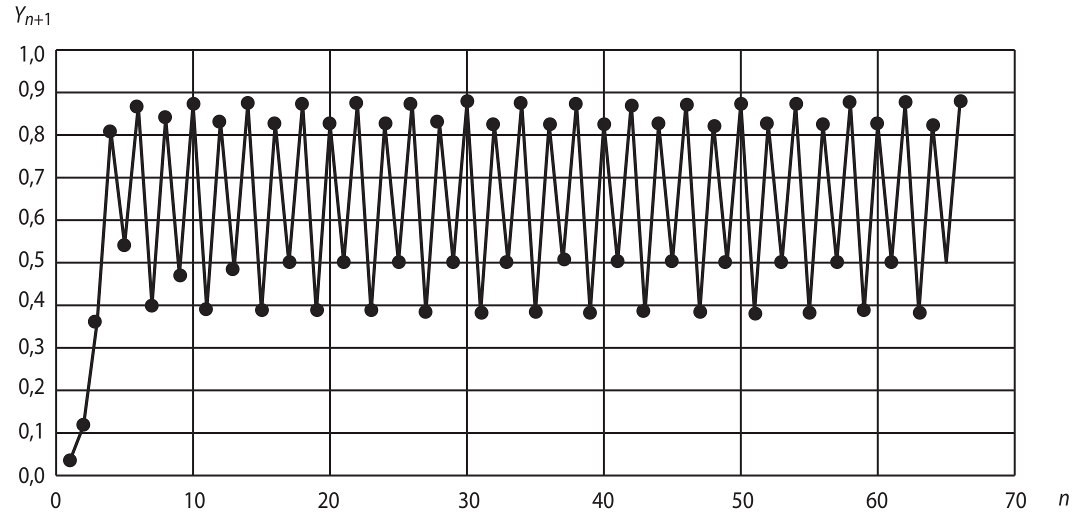


**Fig. 5. Dynamics of the innovative product output at $\alpha = 3{,}5$**

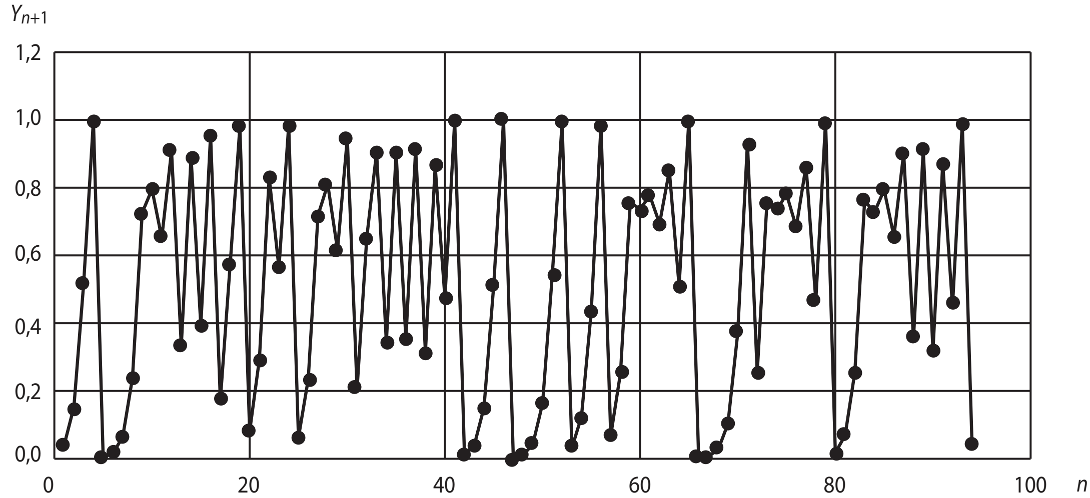


**Fig. 6. Dynamics of the innovative product output at $\alpha = 4$**

or

$$D(y_{n+1}) = b \cdot D(y_n) + (1-b) \cdot S(y_n). \qquad (18)$$

The relationship (18) can be expressed in the following form:

$$D(y_{n+1}) - D(y_n) = (1-b)\big(S(y_n) - D(y_n)\big). \qquad (19)$$

It is clear that the equation (19) has the same equilibrium positions as the equation (5), namely $y_1^* = 0$ and $y_2^* = \dfrac{\alpha - 1}{\alpha}$.

After substituting into equation (19) the expressions for the demand and supply functions at $L = 1$, we obtain:

$$y_{n+1} = y_n (b + (1-b)\alpha - (1-b)\alpha y_n). \qquad (20)$$

It is evident that the equation (20) is structurally similar to the recurrence equation (5). Let us demonstrate this by making a change of variable: $y = \dfrac{b + (1-b)\alpha}{(1-b)\alpha} x$. Then the equation (20) takes the form:

$$x_{n+1} = \beta x_n (1 - x_n), \text{ where } \beta = b + (1-b)\alpha. \qquad (21)$$

Thus, up to parameter values, the equation (21) is also a logistic mapping, just like the equation (5). This allows for a complete analysis of the behavioral properties of solutions to the equation (21) based on the results of the analysis of the equation (5).

Let us demonstrate another approach. We will consider the dynamic process of innovation diffusion as continuous in time, which allows the use of the mathematical apparatus of integral and differential calculus. Simultaneously, we will also assume that the demand function for the innovation at a given time moment $t$ depends on all previous values of the supply. This can be expressed in the following form:

$$D\big(y(t)\big) = \int_0^t K(t-\tau) \cdot S\big(y(\tau)\big) d\tau. \qquad (22)$$

We will choose a «memory» function $K(t-\tau)$ in the form of a second-order dynamic chain, which can be put down as:

$$K(s)=\frac{c_1 s+a_0}{s^2+a_1 s+a_0}. \qquad (23)$$

where $K$ is the generating function of a second-order differential operator; $s$ is the operator variable; $a_0, a_1, c_1$ are the parameters of the differential operator.

After formal transformations of the integral relation (22), we obtain a second-order nonlinear differential equation:

$$\frac{d^2D}{dt^2}+a_1\frac{dD}{dt}+a_0 D=c_1\frac{dS}{dt}+a_0 S, \qquad (24)$$

taking into account the explicit form of the demand and supply functions, this equation takes the form:

$$\frac{d^2y}{dt^2}+(a_1-c_1\alpha)\frac{dy}{dt}+a_0(1-\alpha)y+2c_1\alpha y\frac{dy}{dt}+a_0\alpha y^2=0. \qquad (25)$$

The equation (25) has two particular solutions $y_1^*=0$ and $y_2^*=\frac{\alpha-1}{\alpha}$, each solution corresponds to an equilibrium state of a dynamic system in which the process of spreading an innovative product occurs.

Let us analyze the behavioral properties of the differential equation (25) near the equilibrium position $y_2^*=\frac{\alpha-1}{\alpha}$. Let us introduce new variables $u_1=y-y_2^*$ and $u_2=\frac{dy}{dt}$, , which characterize deviations from the equilibrium position. Then equation (25) can be viewed as a system of two differential equations:

$$\begin{cases}\frac{du_1}{dt}=u_2, \\ \frac{du_2}{dt}=a_0(1-\alpha)u_1+\big(c_1(2-\alpha)-a_1\big)u_2-2c_1\alpha u_1 u_2-a_0\alpha u_1^2.\end{cases} \qquad (26)$$

The linear part of the system (26) corresponds to the following characteristic equation:

$$\lambda^2+\big(a_1-c_1(2-\alpha)\big)\lambda+a_0(\alpha-1)=0. \qquad (27)$$

It follows from the condition $\alpha>1$ that the roots of quadratic equation (27) have the same sign; therefore, the system (26) has no saddle-node singular points, and consequently, a saddle-node bifurcation cannot occur.

Suppose the condition: $a_1=c_1(2-\alpha)-\mu$, is satisfied, where $\mu$ is a small sign-changing parameter. Let us introduce the notation $a_0(\alpha-1)=\omega^2$ and rewrite the characteristic equation (27) in the following form:

$$\lambda^2-\mu\lambda+\omega^2=0. \qquad (28)$$

At $\mu\to 0$, the equation (28) has complex conjugate roots $\lambda_{1,2}=\pm i\omega$ $(i^2=-1)$. Differentiating equation (28) with respect to the variable $\mu$, at $\mu=0$, we obtain $\frac{d\lambda}{d\mu}=\frac{1}{2}$. This result indicates the presence of a Hopf bifurcation in the system.

To further analyze the properties of the bifurcation, as well as the birth or death of a limit cycle, we reduce system (26) to the Poincaré normal form [51]. For this purpose, we make the change of variables: $u_1=x_1$ and $u_2=-\omega x_2$. After performing certain transformations, we obtain the system of differential equations:

$$\begin{cases}\frac{dx_1}{dt}=-\omega x_2; \\ \frac{dx_2}{dt}=\omega x_1-2c_1\alpha x_1 x_2+\frac{a_0\alpha}{\omega}x_1^2.\end{cases} \qquad (29)$$

Let us now rewrite the system (29) in complex form using the variable $z=x_1+ix_2$.

$$\frac{dz}{dt}=i\omega z+g_{20}\frac{z^2}{2}+g_{11}z\cdot\bar{z}+g_{02}\frac{\bar{z}^2}{2}, \qquad (30)$$

where $\bar{z}=x_1-ix_2$; $g_{11}=i\frac{a_0\alpha}{2\omega}$; $g_{20}=-c_1\alpha+i\frac{a_0\alpha}{2\omega}$; $g_{02}=c_1\alpha+i\frac{a_0\alpha}{2\omega}$.

To obtain information about the stability of a limit cycle, it is necessary to determine the sign of the first Lyapunov coefficient, which is as follows:

$$l_1=-\frac{1}{2}\operatorname{Im}(g_{20}\cdot g_{11}). \qquad (31)$$

For the system under study, we have

$$l_1=\frac{c_1 a_0\alpha^2}{4\omega^2}. \qquad (32)$$

It can be seen that, at $c_1>0$, the limit cycle is unstable and the self-oscillation regime is hard. Conversely, at $c_1<0$, a stable limit cycle emerges, and a soft self-oscillation excitation regime is observed.

**Conclusions.** The results presented in this study, both in discrete and continuous time, reflect the uneven evolution of the system in which the diffusion of innovations occurs, namely, the periodicity of phase changes in cycles and the hysteresis inherent in complex systems itself.

This fully aligns with the patterns of cyclic dynamics. Having such information allows for accurate forecasting (based on phase trajectories) of the cyclical nature of the processes of the spread of an innovative product, as well as predicting crisis phenomena such as the emergence of chaos in the innovation market, and timely identifying and recognizing undesirable trends in the development of innovation processes. The contemporary, powerful mathematical framework of nonlinear dynamics, synergetics, and oscillation theory makes it possible, with considerable reliability, to predict and identify the crisis state of a system in which innovation diffusion occurs, and to determine efficient ways to overcome crisis phenomena.

This can serve as the foundation for an anti-crisis program when developing the government's innovation policy.

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