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ON THE GENERATION OF RADIATION OF DIFFERENT NATURES

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Inhalt / Content

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INTRODUCTION. ABOUT THE NATURE OF EXCITED FIELDS

As is known, an oscillator whose natural frequency is near the transparency region of the medium is capable of emitting electromagnetic waves. In the region of field opacity, there is no radiation in the middle and far zone. The dominant frequency in the radiation spectrum will be the frequency of the medium closest to the natural frequency of the emitter.

Spontaneous emission. The radiation of many-particle systems often retains the features of spontaneous radiation (which, due to phase mismatch, creates an average radiation intensity proportional to the number of emitters or oscillators), at least at the initial moment of time. In a number of cases, the initially spontaneous field of many emitters or oscillators under noise conditions is not capable of synchronization, that is, of transitioning to the superradiance mode. Because fluctuations in a number of systems of oscillators and emitters often do not allow the latter to synchronize, which preserves precisely this mode of incoherent radiation. A similar situation occurs, for example, in the atmosphere of stars, is represented in seismic vibrations at the boundary of the crust and mantle of planets and in a number of technical devices. For the occurrence of forced (induced) emission modes in such cases, special conditions are required (see, for example, [1–6]). That is, the threshold for the occurrence of forced, induced radiation of a system of emitters and oscillators must be exceeded during the development of processes of their phase or spatial synchronization, for example, due to an external or integral field.

Competition between spontaneous and induced emission. Such competition can arise because the source of radiation is the same system of oscillators or emitters in the system. Of interest is the formation of stimulated emission pulses near the threshold for the occurrence of stimulated emission due to this competition. This threshold for quantum systems was discovered and justified in particular in [7]. Similar phenomena are observed in the atmosphere of stars, where the intensity of pulses of stimulated radiation of a characteristic triangular shape are comparable to the intensity of spontaneous radiation of stars (the main background radiation). And this despite the

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fact that the number of emitters providing the intensity of spontaneous emission is many orders of magnitude greater than the number of emitters forming the field of induced wave packets [8, 9].

Near seismically active zones, where the level of fluctuations at the boundary of the planetary crust and mantle is high, but not sufficient for a powerful earthquake, external natural or artificial disturbances can increase the level of fluctuations, which can exceed the threshold of stimulated emission of seismic waves in this environment. This can cause significantly more powerful earthquakes (see the note at the bottom of page 46 of the book [6]). This, too, is ultimately due to competition between spontaneous and induced processes in fairly large-scale systems. Although in a number of technical electronic devices, in laboratory and other production systems, the levels of spontaneous emission from systems of emitters and oscillators are relatively small and can be neglected. Indeed, in lasers and masers, for example, the intensity of spontaneous radiation is many orders of magnitude lower than the intensity of induced radiation. Therefore, the main attention is paid to the generation of induced, stimulated radiation, which will be discussed below.

Features of the description of systems of quantum and classical emitters. The nature of superradiance. If the active particle-emitters of a quantum system are located at distances where their wave functions do not overlap (the de Broglie lengths of the emitters are less than the distances between them), their interaction is possible only due to the electromagnetic field. In this case, to describe the process of field generation in a system of such emitters, one can use semiclassical theory (see, for example, [10]).

The superradiance field arises as the total field of initially spontaneously emitting emitters and oscillators, even in the absence of a waveguide and resonator. The process of transition of spontaneous emission to the superradiance mode of a system of oscillators or emitters can be considered a phase transition. That is, there is a certain genetic connection between spontaneous radiation and superradiation.

It is obvious that the total (integrated) field of a system of unsynchronized emitters or oscillators at the initial moment is spontaneous. For a system of oscillators, synchronization of their phases leads to greater coherence of the total radiation. The reason for the synchronization of distributed quantum emitters is due to a sufficiently intense electromagnetic field in their volume, capable of suppressing fluctuations existing in the system. This phenomenon was even called collective spontaneous radiation, which can cause misunderstandings, although V.L. Ginzburg definitely classified superradiation as an induced effect [4].

How can you explain the synchronization process? The reason for phase synchronization of individual oscillators is external to them, or the total (integral) field of the system. If the amplitude of the external field relative to the value of the radiation field of a given oscillator particle is large, the phase of its own radiation changes. This leads to synchronization of its phase with the phase of the total field (see, for example, [11,12]) in the region of its localization. For moving emitters, synchronization, leading to the coherence of their radiation, can manifest itself as a result of their spatial grouping (see, in particular, [13]). And here the reason for such grouping is the external field (or the total field of the system). Since there is no phase in the description of quantum emitters, the question arises about the nature of the synchronization of such emitters. Here it should be taken into account that the probability of radiation from a quantum emitter is determined by the amplitude of the electromagnetic field in its volume, and the phase of the radiation corresponds to the phase of this field (see, for example, the books of A. S. Davydov and also work [21]). Let us recall that the superradiance regime discovered for a compact bunch of particles [13–20] manifested itself as the emergence of coherence in the study of most of the emitters. This phenomenon clearly manifested itself both in the quantum case [22] and in the classical case. In distributed systems of gas and solid-state electronics, due to the fairly large distances between particles, the interaction between them, as already noted, occurs only due to their own electromagnetic fields [23–24]. The process of phase synchronization can also arise as a result of large-scale fluctuations, when a small part of emitters or oscillators forms a coherent community that takes on the functions of a forcing field [25], initiating the synchronization process.

That is, with an increase in the level of fluctuations, a regime when the conditions for the development of instability can be met is possible with an exponential increase in the amplitude of the integral field, which is both a consequence and a cause of synchronization of the emitters.

However, this process of self-synchronization is quite slow and, in addition, can be disrupted due to the same fluctuations in the system. Therefore, to accelerate and stimulate the process of synchronization of emitters in the superradiance mode, a fairly large initiating field is used [25], but, of course, much less than the maximum achievable field in the developed mode. The intensity of the average field in such a developed superradiance mode may turn out to be proportional to the square of the number of emitters, that is, this field is forced, induced.

On comparison of fields excited in waveguides with fields in superradiance regimes. A description of the process of generation or amplification of a high-frequency field by moving emitters in waveguides and resonators of electronic devices in a substantially nonlinear regime was proposed in [26] based on the formalism presented there. When using a constant magnetic field in waveguides, electromagnetic waves are excited by electrons[1] rotating in a constant magnetic field - oscillators [27–29]. A significant increase in interest in describing the excitation of oscillations by beams of charged particles and oscillators in waveguide systems has caused numerous publications (see, for example, the bibliography in [29]).

The so-called dissipative generation modes [30–32], where the levels of absorption, as well as the removal of radiation energy from electronic devices, are significant, forced a detailed study of these processes. It is precisely these modes of generation and amplification in open (for energy output) systems that were of interest to the creators of electronic devices. In such devices it was possible to find so-called operating points that ensure maximum energy output from the system or optimal efficiency, as was particularly presented in [34–35]. Typically, such problems were solved under conditions where emitters and oscillators in the active zone of waveguides

[1] In [36], it was shown that the equations describing the excitation of cyclotron oscillations by rotating electrons in the presence of a constant magnetic field in a waveguide can, under certain simplifying conditions, be reduced to a system of equations for oscillators.

and resonators interacted only with a waveguide or resonator field, the type of which was strictly determined by the geometry of the system. The induced field of the resonator or waveguide synchronized the emitters due to phase changes or due to particle grouping, which led to a significant self-consistent increase in the field amplitude. Moreover, the interaction of emitters and oscillators with each other was usually neglected.

The development of high-current electronics has forced us to begin to take into account the changing current and fields of charged flows in the system (see, for example, the work of the author [35]). This was first required to search for the stability of the active zone, that is, beams of charged particles and oscillator systems. Later, attention began to be paid to the high-frequency radiation fields of individual emitters and oscillators. The question arose about the role of the intrinsic radiation of system particles in the process of generation and amplification of oscillations in waveguides and resonators under conditions of sufficient proximity between the frequency of the resonator and the natural frequency of the emitters or oscillators. By placing this ensemble of active elements both inside and outside the resonator or waveguide, it was possible to verify that its total radiation intensity in the superradiance mode was comparable to the intensity of the waveguide and resonator fields [34]. Thus, in existing microwave generators and amplifiers of various types, it is possible to compare the intensities of induced waveguide and resonator radiation and superradiation provoked by the initiating field. Attempts to discover the similarity between the regimes of dissipative instability and superradiance in open systems began to be made in [33], see also [6]. These works also showed that the dependence of the maximum achievable intensity in the superradiance regime of a system of stationary oscillators on the level of nonlinearity (for example, caused by relativistic effects) is almost linear. A similarity between the nature of the process of dissipative instability in the resonator and the superradiance regimes was noticed. The amplitude and characteristic time of development of processes with increasing amplitude of the external field required to stimulate superradiance were estimated. For the same system of emitters (which is either placed in a waveguide or resonator, or located outside the waveguide or resonator), the achievable intensity levels of these two lasing options turn out to be comparable [33, 37]. But for a correct comparison of the resonator excitation and superradiance modes, the areas filled with particles, their number and position must be identical.

Although, as shown in this work, in reality, in waveguides and resonators, the field of oscillators and emitters is first formed at their natural frequencies in the transparency region of these devices. It turns out that it is the intrinsic field of the system particles (essentially, this is the superradiation field) that forms the resonator and waveguide fields due to the effects of reflection from the ends. In this case, the amplitudes of the resonator and waveguide fields can differ significantly from the total field of active particles generating them. Although it is impossible to distinguish resonator and waveguide fields in the structure of the total field of system particles, such fields are nevertheless quickly formed due to reflections from the ends of the system[2].

In different generation modes, as shown in this work, both the waveguide field, determined by the geometry of the system, and the field of the total radiation of system particles, that is, the superradiance field, can dominate. A similar approach to describing the generation of an open system–gyrotron in the superradiance regime was proposed by A.G. Zagorodny and P.I. Fomin [39]. A feature of the operation of devices such as the gyrotron is the very low group velocity of electromagnetic waves, the frequencies of which are located near the cutoff frequency, and the relatively small reflection coefficient from the boundaries. A rational choice of generation modes can provide a high level of output radiation, characteristic of the operation of such devices, which was subsequently confirmed by numerical modeling of such processes [37].

The purpose of this work is to study the competition between such types of radiation as spontaneous, induced waveguide (resonator) and superradiation. If the

[2] Attempts have even been made to take into account simultaneously the field of a resonator or waveguide, as well as the summary radiation field of particles (essentially being a superradiance field [38]). However, such a representation of the field acting on particles formally turned out to be redundant. Although after the formation of a waveguide field due to reflection, one could talk about the presence in the system of two fields simultaneously (see Section 5 of this work) - the waveguide and the summary l radiation field of particles in the core.

competition between spontaneous and induced processes can be noticeable in outer space, then the competition between induced fields: waveguide and superradiation can manifest itself in a number of technical devices of microwave electronics. In addition, the conditions for implementing ultraviolet and x-ray generation modes under superradiance conditions are of considerable interest.

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KAPITEL 1 / *CHAPTER*
COMPETITION BETWEEN SPONTANEOUS AND INDUCED RADIATION

The process of competition between spontaneous emission and stimulated emission can be most simply traced using the example of a two-level quantum system of certain types of particles. As is known, if the difference in the number of excited and unexcited states-energy levels in such a system, which is called population inversion, turns out to be positive, then we can expect the occurrence of stimulated emission [1,2]. However, it turned out that the threshold for the occurrence of stimulated emission is determined by the equality of the square of the population inversion to the total number of states.

The nature of the competition between these two types of radiation is due to the fact that both draw energy from an array of excited particles, the number of states of which for cosmic objects in the equilibrium case is equal to the number of unexcited states. Spontaneous emission is the action of excited particles, while induced is possible when the threshold discovered by A. G. Zagorodniy and V. M. Kuklin is exceeded. It is clear that such competition between spontaneous and stimulated emission can be observed only when the excited and unexcited states of the particles, which form such a two-level quantum system, are approximately equal.

In the photosphere of stars exactly this equality is observed. Moreover, collisional processes form the upper energy levels - excited particles, and relaxation processes - accordingly, the lower energy levels of this system. A slight excess of the number of excited states over the number of states of relaxed-unexcited particles can form characteristic stimulated emission pulses with a steeper leading edge and a flatter trailing edge.

In the vast majority of known cases of implementation of population inversion of energy states of systems, for example, in electronics, the intensities of spontaneous radiation are many orders of magnitude lower than the achieved intensities of induced radiation. Therefore, spontaneous emission of particles is not taken into account, assuming it to be negligible.

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1.1. About the new threshold of induced radiation

In the simplest case [1], the equations describing a quantum system with two energy levels in the presence of radiation at the transition frequency $\varepsilon_2 - \varepsilon_1 = \hbar\omega_{12}$ are as follows:

$$\partial n_2 / \partial t = -(u_{21} + w_{21} \cdot N_k) \cdot n_2 + w_{12} \cdot N_k \cdot n_1, \quad (2.1)$$

$$\partial n_1 / \partial t = -w_{12} \cdot N_k \cdot n_1 + (u_{21} + w_{21} \cdot N_k) \cdot n_2, \quad (2.2)$$

$$\frac{\partial N_k}{\partial t} = (u_{21} + w_{21} \cdot N_k) \cdot n_2 - (w_{12} \cdot N_k) \cdot n_1, \quad (2.3)$$

and the total number of particles of the system is constant $n_1 + n_2 = N = Const$, u_{21} - here is the rate of change in the number of quanta of the excited level due to spontaneous emission processes. The rate of change in the number of quanta (particles) at these levels due to induced processes of radiation $w_{21} \cdot N_k \cdot n_2$ and absorption $w_{12} \cdot N_k \cdot n_1$, N_k is the number of radiation quanta at the transition frequency. First, let's find the threshold of coherent radiation when the inversion $\mu = n_2 - n_1 << n_1, n_2$. This threshold was discovered in [7,8]. This threshold was discovered in [7] and explained in [8].

The system of equations in the absence of energy losses can be rewritten in the form.

$$\partial \mathrm{M} / \partial T = -N_0 - 2\mathrm{M} \cdot \mathrm{N}, \quad (2.4)$$

$$\partial \mathrm{N} / \partial T = (N_0 / 2) + \mathrm{M} \cdot \mathrm{N}, \quad (2.5)$$

where $\mathrm{M} = \mu / \mu_0, \quad T = w_{21} \cdot \mu_0 \cdot t = \mu_0 \cdot \tau, \quad \mathrm{N} = N_k / \mu_0$, the only free parameter is $N_0 = g \cdot N / \mu_0^2$, $\mu_0 = \mu(\tau = 0)$.

Let us show that the nature of the process changes if the initial inversion value is greater or less than a threshold value equal to $\mu_{TH2} = (2N)^{1/2}$. The suppression of the exponential growth of quanta in this case $\mu_0 < \mu_{TH2} = (2N)^{1/2}$ indicates not only a change in the process regime, but also gives grounds to assume that the process of

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stimulated emission is suppressed by the predominant growth of spontaneous emission. In Fig. 1.1. the dynamics of process development is shown for cases of different values of the parameter $N_0 \subset (30 \div 0.01)$. Attention should be paid to the change in the nature of the process when crossing the threshold

$$\mu_{TH2} = \sqrt{2(n_1 + n_2)} . \tag{2.6}$$

The figure 2.1 shows that for large values of the initial inversion, the parameter N_0 is small ($N_0 < 1$), stimulated emission begins to manifest itself, and the regime of exponential growth in the number of quanta becomes more and more clearly visible. So, *it is not enough to require a positive inversion value to realize stimulated emission; the real threshold for stimulated emission (2.6) is somewhat higher.*

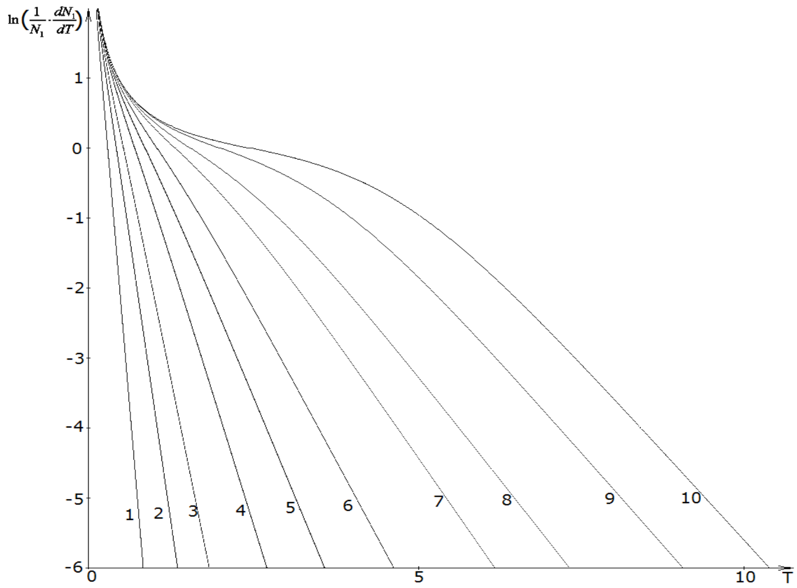

Fig.2.1. Behavior of a quantity over time for the parameter value, $N_0 = (n_1 + n_2)/(n_2 - n_1)^2$**, 1–** $N_0 = 30$**; 2–** $N_0 = 10$**;3–** $N_0 = 5$**;4–** $N_0 = 2$**; 5–** $N_0 = 1$**;6–** $N_0 = 0.5$**; 7** $N_0 = 0.2$**; 8–** $N_0 = 0.1$**; 9–** $N_0 = 0.03$**; 10–** $N_0 = 0.01$**.** *Source [8]*

Let us consider the case of approximately equal number of particles at two levels. If the threshold (2.6) is exceeded during continuous emission of spontaneous radiation from the second energy level, additional generation of an induced radiation pulse is possible. To understand in detail the nature of the process, one can divide the radiation

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quanta by origin. Let the relative number of quanta of spontaneous emission $N_{inc} = N_k^{(incoh)} / \mu_0$ and the relative number of quanta of induced emission $N_c = N_k^{(coh)} / \mu_0$, then it is possible to formulate a qualitative system of equations with the division of quanta according to their origin, along with the traditional description (see Table 2.1)

Table 2.1 Qualitative and traditional systems of equations

Qualitative system of equations with separation of quanta according to their origin	Traditional system equations
$\partial M_1 / \partial T = -N_0 - 2M_1 \cdot N_c$; $\partial N_{inc} / \partial T = (N_0 / 2)$; (2.7) $\partial N_c / \partial T = M_1 \cdot N_c - \theta \cdot N_c$.	$\partial M / \partial T = -N_0 - 2M \cdot N$; (2.8) $\partial N / \partial T = (N_0 / 2) + M \cdot N - \theta \cdot N$.

Source [7,8]

Here, the absorption of field energy is taken into account by the value of $\theta = \delta / \mu_0$, where δ is the decrement of field absorption in the medium. The solution of the traditional model for small absorption is shown in Fig. 2.2 thin lines. We note that the inversion decreases to a negative value, and the growth of the total number of quanta is limited. A qualitative system of equations in a non-absorbing medium ($\theta = 0$) makes it possible, although approximately, to understand in detail the nature of the radiation. According to the qualitative description, after decreasing the amplitude of the coherent radiation pulse N_c (solid curve), the number of spontaneous emission quanta N_{inc} (dashed line) continues to increase. That is, after a decrease in the amplitude of the coherent pulse, the main contribution to the total number of quanta can only come from a spontaneous process.

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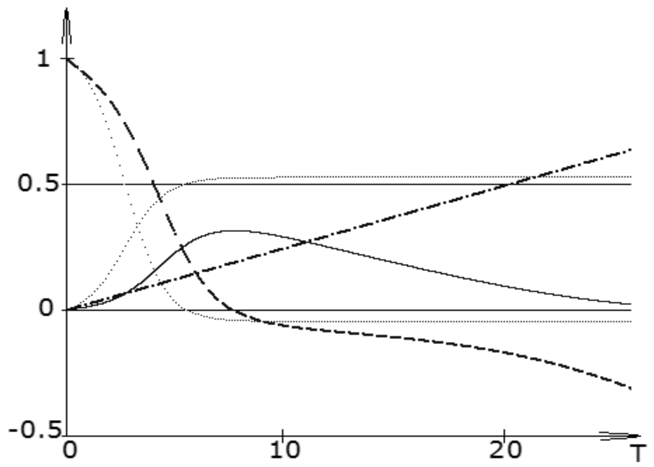

Fig.2.2. The behavior of values of the traditional model, as functions of time, is represented M and N for ($\theta = 0.2$) small dense dots. For a qualitative model, the value M_1 is represented by a dotted line, N_c – solid, N_{inc} – dash-dotted line ($N_0 = N / \mu_0^2 = 0.05$, $\theta = 0$). Source [7,8]

It is important to note that the appearance of the stimulated emission pulse takes on a characteristic triangular shape with a steep leading edge. Spontaneous emission demonstrates its previous intensity. When the threshold (2.6) is slightly exceeded, the intensities of spontaneous and stimulated radiation from the same source turn out to be comparable. Therefore, it is in this range of parameters that competition between these two types of radiation manifests itself.

1.2. On the generation of sawth-shaped pulses in outer space

Astrophysicists know that in the photosphere of stars, the equilibrium conditions for the processes of collisional excitation by free electrons of the main gas of active atoms and their radiative relaxation are satisfied $I_0 = 0$, mainly due to spontaneous radiation. The conditions of this equilibrium for a two-level system can be written in the form , where $[(\nu - u_{21}) / w_{21}]n_1 = \mu_0^2 I_0$, and is the effective frequency of collisions of active trace atoms with fast electrons, which ensures the transition of atoms from the lower energy level of the quantum system to the upper energy level. When $I_0 < 0$ the population inversion is negative and there is no induced radiation, but if the collisional excitation is high or there is a source of excited atoms entering the active zone of the photosphere from the lower heated layers of the star, then $I_0 > 0$. In this case, the population inversion is positive and in the stationary case the emission consists of spontaneous and induced, and the presence of the latter can noticeably increase the brightness of the star.

In most cases, stimulated emission quickly reduces the population inversion and restores the equilibrium state $I_0 = 0$. The process of this restoration leads to the appearance of stimulated radiation pulses, which have a characteristic sawtooth character, and the integral intensity of the radiation sometimes increases several times. Note that if the overlying layers are sufficiently thick, due to the effects of scattering in them of radiation emanating from the active zone of a two-level system, the radiation spectrum of an absolutely black body is formed there, as noted by S. B. Pikelner in his work "Atmospheres of Stars." That is why the presence of a coherent component characteristic of stimulated emission is often not detected.

Let's return to the equations that describe changes in the number of states at two energy levels

$$\partial n_1 / \partial \tau = g \cdot n_2 - \mu \cdot N_k + \nu \cdot n_1 / w_{21} = [(\nu - u_{21}) / w_{21}] \cdot n_1 - \mu \cdot N_k, \qquad (2.9)$$

$$\partial n_2 / \partial \tau = -[(\nu - u_{21}) / w_{21}] \cdot n_1 + \mu \cdot N_k. \qquad (2.10)$$

Some remarks should be made here. First of all, collisions for atoms are a random process, therefore, in the absence of an external coherent field, the radiation of excited atoms is also random in nature, that is, it meets the spontaneous criterion. When a coherent field appears, the latter forces the particles of the upper level to synchronize, which is described by terms of the form $\mathrm{M} \cdot \mathrm{N}_c$, while the induced component of the radiation N_c increases. The reason for the appearance of the induced field is the exceeding of the threshold (2.6). It is obvious that below the threshold (2.6) there is no induced field, but spontaneous emission from the second energy level is present.

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Since for $I_0 = 0$, the qualitative system of equations can be rewritten as

$$\partial \mathrm{M} / \partial T = -2\mathrm{M} \cdot \mathrm{N}_c \,, \tag{2.11}$$

$$\partial \mathrm{N}_{inc} / \partial T = (N_0 / 2) - \theta \cdot \mathrm{N}_{inc} \,, \tag{2.12}$$

$$\partial \mathrm{N}_c / \partial T = \mathrm{M} \cdot \mathrm{N}_c - \theta \cdot \mathrm{N}_c \,. \tag{2.13}$$

In addition to the collisional excitation of atoms in the active medium, the process of transfer of a certain number of excited atoms from the hot lower layers of the star is also possible, which can be written in the form [7–9]

$$-V \cdot \partial \mathrm{M} / \partial X \approx V \cdot \mathrm{M} / L = K \cdot M$$

where V is the rate of such transfer and L is the spatial scale of the change in inversion. When taking such transfer into account, the first of the previous system of equations takes the form

$$\partial \mathrm{M} / \partial T = K \cdot \mathrm{M} - 2\mathrm{M} \cdot \mathrm{N}_c \,. \tag{2.14}$$

A one-parameter system of equations describing the pulse generation process can be presented as follows

$$\partial M_2 / \partial \tau = K_2 \cdot M_2 - 2M_2 \cdot N_2 \,, \tag{2.15}$$

$$\partial N_2 / \partial \tau = M_2 \cdot N_2 - 2N_2 \,, \tag{2.16}$$

where $\tau = \delta t$, $M_2 = \mu w_{21} / \delta$, $N_2 = \frac{< E^2 >}{4\pi\hbar\omega} \frac{\mu_0 w_{21}}{\delta}$, $K_2 = 2K / \theta$.

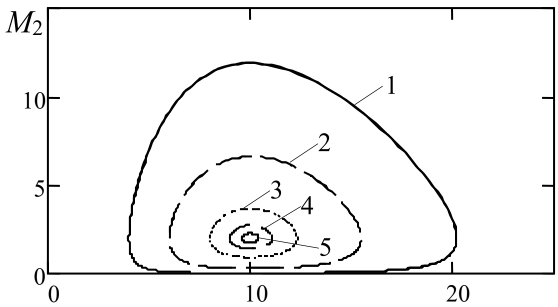

Fig.2.3. Phase diagrams (M_2, N_2) or the system of equations (2.15) – (2.16) when choosing initial conditions in the form 1 – $N_2(0) = 4$, 2 – $N_2(0) = 6$, 3 – $N_2(0) = 8$, 4 – $N_2(0) = 9$, 5 – $N_2(0) = 9.6$. Source [9]

System (2.15) – (2.16) has a singular point $(2, K_2/2)$. The phase trajectories turn out to be closed lines, which indicates the presence of stable periodic solutions.The graph of changes in the number of quanta N_2 has the form of a sinusoid for phase trajectories near a singular point $(2, K_2/2)$. For phase trajectories far from a singular point, the change graph takes the form of a saw: a sharp increase and a slow decrease. Similar behavior is typical for the luminosity of Cepheid stars, where thermonuclear fusion has reached its limiting element - iron: significant sawtooth fluctuations in the luminosity of the star Delta Cepheus and small sinusoidal fluctuations in the luminosity of the Polaris. It is possible to represent solutions (2.15) – (2.16) in variables accepted in astrophysics, for example, the relationship between the apparent magnitude m and luminosity *l* (using the known value of the distance to the star in parsecs, the relationship between the absolute magnitude (visible magnitude from a distance of 10 parsecs) and apparent magnitude m, that is $M_l = m - 5\lg(d/10)$. The solutions to equations (2.15) - (2.16) are presented in these variables in Fig. 2.4 and Fig. 2.5. Here is the change in the magnitude of the star Delta Cepheus and Polaris [40]. Along the ordinate axis the values of the apparent magnitude are plotted, along the abscissa axis the time in fractions of the period of change in the brightness of the star.

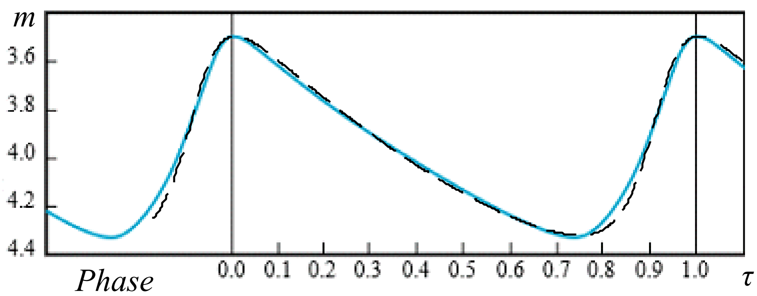

Fig.2.4. Variation in magnitude of the Delta Cephei star over time. (solid curve - obtained in the 1930s by N.F. Florey using a visual photometer), <http://heasarc.gsfc.nasa.gov/W3Browse/all/gcvs.html>. The solution of the equations of the system (2.15) - (2.16) in the same variables, when selecting the level of total spontaneous radiation (already the entire star) and scales (dotted line) Source [40,9]

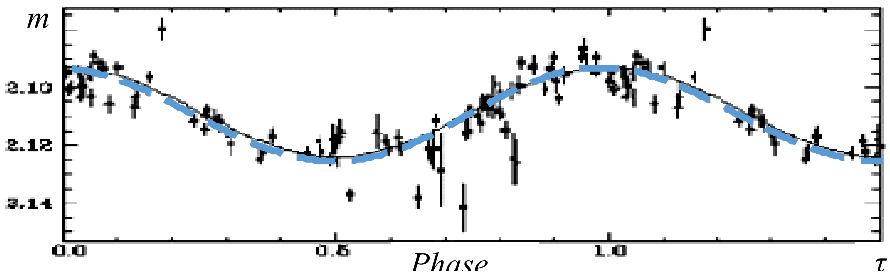

Fig.2.5. Change in the magnitude of the Polar star over time. (solid curve) and solving the equations of the system (2.15) – (2.16) in the same variables, when selecting the level of total spontaneous radiation (already the entire star) and scales (dotted line Source [40, 9]

Since there are a number of other descriptions of these luminosity oscillations of these two almost similar stars, we note the advantages of this model. The pulses here always have a pronounced triangular character with a sharp leading edge. In addition, only a quantum mechanism is capable of providing such a clear periodicity. When the amplitude of the pulses is small, they become sinusoidal. Since the stars are similar, a decrease in the pulse amplitude (for small amplitude Cepheids - DCEPS) leads to a 30% increase in the period, which is important to take into account in astrophysical measurements.

1.3. Processes in electronics in a two-level description

It can be shown that in the equations of the quasilinear theory, which describes the change in the distribution function of an electron beam in a plasma during the generation of oscillations, one can also see the same two-level system, which has properties similar to the quantum system discussed above. The equations of the quasilinear theory look like this (see, for example, [41])

$$\partial \mathrm{N} / \partial t = (2\pi^2 e^2 \omega^2 / \hbar k^3) \cdot \{f_b + \mathrm{N}(\hbar k / m) \partial f_b / \partial v\}, \tag{2.17}$$

where ω, k is the frequency and wave number of oscillations, $\varepsilon(\omega, k)$ is the dielectric constant, m is the electron mass, $f_b = f_b(v)$ is the velocity distribution function of electron beam particles, in particular, for the Maxwellian distribution the expression is valid

$$f_b = [n_{b0} / \sqrt{\pi} v_{Tb}] \cdot \exp\{-(v - v_{0b})^2 / v_{Tb}^2\} . \quad (2.18)$$

Here v_{0b}, $v_{Tb} = \sqrt{2k_B T_b / m_b}$ – are the average and thermal velocities of the beam electrons. The velocity interval in which changes in the distribution function can be traced is chosen to be equal to $(\hbar k / m)$, $\mathrm{N} = \frac{\partial \omega \varepsilon(\omega, k)}{8\pi\hbar\omega \cdot \partial\omega} | E_k |^2 \approx \frac{\omega_{pe}^2}{4\pi\hbar\omega^3(k)} | E_k |^2$ - the number of quanta (plasmons) per unit volume and if we move to another time scale $T = (2\pi^2 e^2 \omega^2 / \hbar k^3) \cdot t$ and use the obvious relation $(\hbar k / m) \cdot df / f_2 dv = (1 - f_1 / f_2)$, equation (2.17) can be represented as

$$d\mathrm{N} / dT = \{f_2 + \mathrm{N} \cdot (f_2 - f_1)\} = \{n_2 + \mathrm{N} \cdot (n_2 - n_1)\} = \{n_2 + \mathrm{N} \cdot \mathrm{M} . \quad (2.19)$$

Here f_2 corresponds to the upper level of the system, and f_1 – to the lower one.

The inverse influence of oscillations on the distribution function is described by an equation that takes into account only the influence of the induced field (for the quasilinear theory this was sufficient)

$$\partial f_b / \partial t = 2\pi^2 \frac{e^2 | \mathrm{E} |^2 \omega_{pe}}{m^2 k} \partial^2 f_b / \partial v^2 . \quad (2.20)$$

From the expression $\partial f / \partial v = (f_2 - f_1) \cdot m / \hbar k$ you can understand how to transform the derivatives in equation (2.19), which takes the form

$$\frac{\partial f_2}{\partial T} = -f_2 - (f_2 - f_1) \cdot \mathrm{N} , \quad (2.21)$$

where on the right side a term is added that takes into account the influence of spontaneous radiation. For the first level the following expression is valid:

$$\frac{\partial f_1}{\partial T} = f_2 - (f_1 - f_2) \cdot \mathrm{N} . \quad (2.22)$$

Using this expression for f_2 and the same for f_1 we obtain for their difference

$$\frac{\partial M}{\partial T} = -N_0 - 2\mathrm{N}\cdot\mathrm{M} \quad (2.23)$$

Thus, as a result of these transformations, it is possible to describe quasilinear relaxation taking into account spontaneous processes in the form of a two-level system.

One remark should be made. In electronics, the case of comparable intensities of the spontaneous field and the field of stimulated radiation is rarely observed under modern conditions. Only superradiance (superluminescence), which inherits the properties of spontaneous emission, is able to compete with waveguide or resonator stimulated emission in generators and amplifiers. First of all, because it is virtually no different from stimulated radiation. But this will be discussed below.

Although at the dawn of the development of plasma electronics, noticeable intensities of spontaneous radiation were discovered for relatively low-current beams of charged particles in the entire spectral region occupied by plasma, even where the conditions for excitation of stimulated radiation were not met [42,43].

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KAPITEL 2 / *CHAPTER 2*
COMPARISON OF INDUCED RADIATION IN RESONATORS AND SUPERRADIATION MODES

At the beginning of the section, we will consider the nature of the process of excitation of electromagnetic waves by an ensemble of emitters, which is a two-level system of dipoles.

2.1. Semi-classical equations, describing the generation of oscillations

Let us pay attention to the fact that in the quantum case we are not talking about synchronizing the phases of the emitters, as in the classical consideration, but about increasing the probability of radiation, which actually leads to the same result. If the active zone is rarefied, the emitters are separated in space, the overlap of their wave functions becomes either weak or completely imperceptible. In this case, their interaction will be determined only by electromagnetic radiation fields.

To describe such a two-level system of dipoles, we will use the semiclassical model of interaction between the field and particles (see, for example, [10]. In this case, the medium is represented quantum mechanically, and the field will be described in the classical representation. As is known, the radiation intensity of a quantum emitter is determined by the amplitude of the electromagnetic field, and the radiation phase corresponds to the phase of this field in its volume [21].

Let us consider a one-dimensional model for electric field disturbances E, polarization P, and population inversion μ slowly varying with time, describing the excitation of electromagnetic oscillations in a two-level active medium, the equations of which can be presented in the form (10]).

$$,\ \frac{\partial^2 E}{\partial t^2} + \delta \frac{\partial E}{\partial t} - c^2 \frac{\partial^2 E}{\partial x^2} = -4\pi \frac{\partial^2 P}{\partial t^2} \qquad (3.1)$$

$$\frac{\partial^2 P}{\partial t^2} + \gamma_{ab} \frac{\partial P}{\partial t} + \omega^2 \cdot P = -\frac{2\omega |d_{ab}|^2}{\hbar} \mu E\ , \qquad (3.2)$$

$$\frac{\partial \mu}{\partial t} + \gamma_1(\mu - \mu^0) = \frac{2}{\hbar\omega} < E\frac{\partial P}{\partial t} >, \qquad (3.3)$$

where the frequency of transition between levels ω corresponds to the field frequency, we neglect the relaxation of inversion due to external causes, δ is the field absorption decrement in the medium, d_{ab} is the matrix element of the dipole moment (more precisely, its projection onto the direction of the electric field), $\mu = n \cdot (\rho_a - \rho_b)$ is the population difference per unit volume, ρ_a and ρ_b is the relative populations levels in the absence of a field, γ_{ab} is the width of the spectral line, $\gamma_1 = \gamma_a = \gamma_b$ is the reciprocal of the lifetime of levels, n is the density of dipoles of the active medium. Here the linewidth is inversely proportional to the lifetime of the states, which is due to relaxation processes. The fields are presented in the form $E = \mathrm{Re}[E(t) \cdot \exp(-i\omega t)]$ and $P = \mathrm{Re}[P(t) \cdot \exp(-i\omega t)]$, where $E(t), P(t)$ are slowly varying amplitudes. Wherein $< E^2 >=| E(t)|^2 /2$.

Let us pay attention to the relationship between the values of the characteristic inverse time of change in slow amplitudes (increment) and the linewidth of the wave packet. When $\gamma_{ab} >>| \partial E(t) / E(t)\partial t |$ relatively low levels of electric field intensity and small values of population inversion are realized. This is the case discussed above. The behavior of a two-level system can be described by so-called balance equations. These equations were analyzed in the previous section. If the line width is less than the inverse time of change in the slow polarization amplitude of the radiation (increment), and the population inversion is significant, the field intensity is high and one should use description (3.1) – (3.3), actively used by Yu. L. Klimontovich and his colleagues.

Generally speaking, a resonator, even an open one, determines the spatial configuration of the excited field. In the first traditional case of describing the process of excitation of oscillations, the interaction of particle-emitters with each other is often neglected, and their interaction only with the resonator field is considered. In the second case of describing generation, in the superradiance mode, on the contrary, the field is formed as the sum of the fields of the emitters. So, either the so-called resonator (waveguide) mode of excitation of oscillations, or the superradiation mode is often considered.

In reality, the fields are first formed by an ensemble of particle-emitters, and only then, due to reflection effects, a resonator (or waveguide) field is formed.

In this total field, the resonator (or waveguide) field can then be isolated.

But to discuss the features of the excitation of these fields, we will consider two modes separately - 1) excitation of the field of intrinsic radiation of active elements-emitters, that is, the superradiance field, and 2) excitation of only the resonator (waveguide) field.

Let us pay attention to the fact that these two modes are considered, firstly, quite artificially, and secondly, they should be separated from each other, because in each there is a different mode of interaction of emitters with fields.

Let us dwell on the case of superradiation. Such a system of dipoles can be located in the resonator or outside it. For the purity of the description, let the system of particle-emitters be located outside the resonator. Then the field that they emit at the initial moment is spontaneous. And only synchronization of the emitters leads to greater coherence of the total radiation. This phenomenon in the absence of a resonator was initially called collective spontaneous radiation, and later superradiation. To accelerate and stimulate the process of synchronization of emitters in the superradiance mode, a fairly large initiating field was often used [25], but, of course, much less than the achievable field in the developed generation mode.

Superradiation field. Let us place quantum emitters along the Oz axis, the field emitted by them is directed along Ox. Their total number $M = b \cdot n$ is divided into S particles, n is the density, b is the length of the system, M / S is the number of emitters in one large modeling particle, and $z_i \; (i = 1, \ldots, S)$ is the coordinates of large particles located evenly along the length of the system. The polarization of the emitter—one molecule [10] is equal to $p = \left(d_{ba}\rho_{ab} + d_{ab}\rho_{ba} \right)$. Equations (3.1–3.3) use polarization density (polarization per unit volume) $P = np = n\left(d_{ba}\rho_{ab} + d_{ab}\rho_{ba} \right)$. The field equation of a polarized molecule (at point z0) has the form.

$$\frac{\partial^2 E_x}{\partial t^2} - c^2 \frac{\partial^2 E_x}{\partial z^2} = 4\pi\omega^2 p(t) \cdot e^{-i\omega t} \cdot \delta(z - z_0) . \quad (3.4)$$

From equation (3.4) for a slowly varying amplitude of the field emitted by one molecule, we obtain

$$E(z,t) = i \cdot 2\pi \cdot p \cdot \omega \cdot c^{-1} \cdot e^{ik|z - z_0|} \quad (3.5)$$

Then, the amplitude of the field emitted by the system (superradiance field) is equal

$$E_{\sup}(z,t) = \frac{i \cdot 2\pi \cdot \omega}{\delta} \cdot \frac{1}{S} \sum_{s=1}^{S} P_s(t) \cdot e^{ik|z - z_s|} . \quad (3.6)$$

Here, $\delta = c / b$ it is actually the ratio of the energy flow from the system to its total energy.

Field in a resonator. If a system of emitters is placed in a resonator, then the latter, in the presence of reflections, will form the spatial structure of the field. Equation (3.4) for one emitter – a polarized molecule at point z_0 can be written as

$$\frac{\partial^2 E}{\partial t^2} + \delta \frac{\partial E}{\partial t} - c^2 \frac{\partial^2 E}{\partial z^2} = 4\pi\omega^2 p e^{-i\omega t} \delta(z - z_0) . \quad (3.7)$$

Assuming that the resonator field has the form

$$E = E_+(t) \cdot e^{-i\omega t + ikz} + E_-(t) \cdot e^{-i\omega t - ikz} , \quad (3.8)$$

and, therefore, for slowly varying field amplitudes we have

$$\frac{\partial E_\pm(t)}{\partial t} + \delta E_\pm(t) = i \cdot 2\pi \cdot \omega \cdot p \cdot \delta(z - z_0) e^{\mp ikz} . \quad (3.9)$$

For a system of particles we obtain equations for the amplitudes of the resonator field

$$\frac{\partial E_\pm(t)}{\partial t} + \delta E_\pm(t) = i \cdot 2\pi \cdot \omega \cdot \frac{1}{S} \sum_s P_s \cdot e^{\mp ikz_s} , \quad (3.10)$$

and, in fact, the field itself

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$$E_{res}(z,t) = E_+(t) \cdot e^{ikz} + E_-(t) \cdot e^{-ikz} . \quad (3.11)$$

Polarization and inversion. From equation (3.2) for a slowly varying polarization amplitude, one can easily obtain the equation for the polarization amplitude

$$\frac{\partial P(z,t)}{\partial t} + \gamma_{ab} P(z,t) = -i \frac{|d_{ab}|^2}{\hbar} \mu(z,t) \cdot E(z,t) . \quad (3.12)$$

Similarly, for a slowly varying inversion, from equation (3.3) we obtain

$$\frac{\partial \mu}{\partial t} + \gamma_1 \left(\mu - \mu^0 \right) = \frac{i}{2\hbar} \left(EP^* - E^* P \right) . \quad (3.13)$$

2.2. Models of generation by a system of quantum emitters

Next, we will compare the fields generated by open systems in the cases of 1) excitation of only the resonator field, 2) excitation of only the field of self-radiation of active elements-emitters, that is, the superradiance field. Let us rewrite equations (3.6–3.10) in dimensionless form using the following notation:

$E_0 = \sqrt{2\pi\omega\hbar\mu_0}$, $P_0 = |d_{ab}| \cdot \mu_0$, $\gamma_0 = \frac{|d_{12}| \cdot E_0}{\hbar}$, $\Theta = \delta / \gamma_0$, $\mathrm{E} = E / E_0$, $\mathrm{P} = P / P_0$, $\hat{\mathrm{P}} = i \mathrm{P}$,

$Z = kz / 2\pi$, $\tau = \gamma_0 t$.

Assuming additionally that $\gamma_{ab} / \gamma_0 \propto 1$ and $\gamma_1 / \gamma_0 \propto 1$,, we obtain the following system of equations.

$$\frac{\partial \hat{\mathrm{P}}}{\partial \tau} = \mathrm{M} \cdot \mathrm{E}, \quad (3.14)$$

$$\frac{\partial \mathrm{M}}{\partial \tau} = -\frac{1}{2} \left(\mathrm{E} \hat{\mathrm{P}}^* + \mathrm{E}^* \hat{\mathrm{P}} \right), \quad (3.15)$$

$$\mathrm{E}_{sup}(Z,\tau) = \frac{1}{\Theta} \cdot \frac{1}{S} \sum_{s=1}^{S} \hat{\mathrm{P}}_s(\tau) \cdot e^{i2\pi|Z - Z_s|} , \quad (3.16)$$

$$\mathrm{E}_{res}(Z,\tau) = \mathrm{E}_+(\tau) \cdot e^{i2\pi Z} + \mathrm{E}_-(\tau) \cdot e^{-i2\pi Z} , \quad (3.17)$$

$$\frac{\partial \mathrm{E}_\pm}{\partial \tau} + \Theta \, \mathrm{E}_\pm = \frac{1}{S} \sum_s \hat{\mathrm{P}}_s \cdot e^{\mp i2\pi Z_s} . \quad (3.18)$$

As already noted, the system is divided into S large particles, therefore equations (3.14–3.15) are solved for each particle. In formulas (3.16–3.17), the fields are also calculated for each large particle. In all cases, equations (3.14–3.15) are solved, but the form of the fields E is different. In the first case (excitation of only the resonator field), equations (3.18) are solved, the field is found according to formula (3.17). In the second case (excitation of only the field of self-radiation of active elements-emitters) $\mathrm{E} = \mathrm{E}_{res}(Z,\tau)$, the field is taken in the form

$$\mathrm{E} = \mathrm{E}_{sup}(Z,\tau) + \mathrm{E}_{ext}(Z), \qquad (3.19)$$

where $\mathrm{E}_{ext}(Z) = \mathrm{E}_{0+} \cdot e^{i2\pi Z} + \mathrm{E}_{0-} \cdot e^{-i2\pi Z}$, $\mathrm{E}_{ext}(Z)$ is the external field initiating the process.

Results of numerical simulation. A comparison was made of the above models for describing generation. For all models, calculations were carried out under the same initial conditions and different values of the loss coefficient Θ, defined as the ratio of the energy flow from the system to the total energy in its volume. Initial conditions: the length of the system is equal to one wavelength ($Z_j \in (0;1)$); number of particles S=800; initial inversion $\mathrm{M}_i(0) = 1$; the initial polarization has an amplitude of 0.1 and random phases φ ($\mathrm{P}_i(0) = 0.1 \cdot \exp(i\varphi)$); $\mathrm{E}_{0\pm} = 0.01$ – initial values for the amplitudes of the resonator field, as well as constants for the external field (3.16). Figures 3.1–3.3 show for two modes the dependence on the level of energy loss Θ of the maximum field intensity in the volume of the emitter $|\mathrm{E}_{\max}|^2$ (Fig.3.1), the rate of energy output $\Theta|\mathrm{E}_{\max}|^2$ (Fig.3.2) and the reverse time (increment) of the process $\gamma = \left(\frac{1}{|E|^2}\frac{d|E|^2}{d\tau}\right)_{\max}$ (Fig.3.3).

On the graphs, the maximum intensity $|E|^2 \approx |E_{\max}|^2$ is selected for each moment in time. The solid line marks the resonator mode, the superradiance mode is represented by dotted lines in the graphs. It is important to note that taking into account the emitters'

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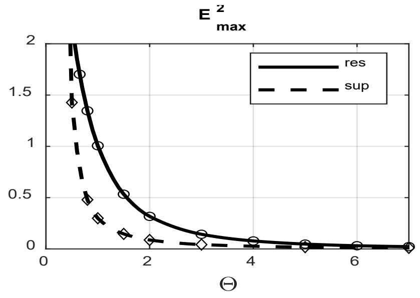

Fig. 3.1. Dependence of the maximum field intensity $\propto |E|^2$ **on Θ for the cases: solid line (res) – generation of a resonator field; dotted line – superradiance mode (sup)** *Source [24.33]*

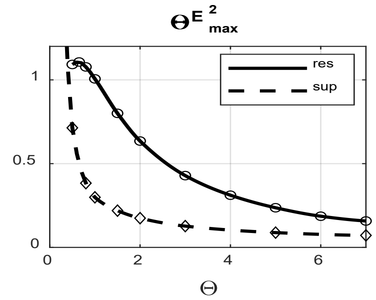

Fig. 3.2. Dependence of the rate of energy output $\Theta |E|^2$ **on Θ for the cases: solid line (res) – generation of a resonator field; dotted line – superradiance mode (sup)** *Source [24.33]*

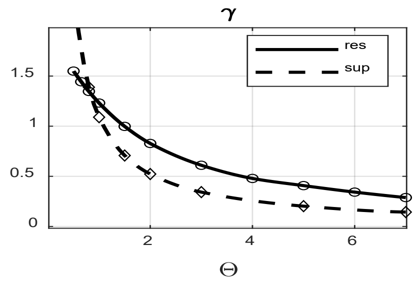

Fig. 3.3. Dependence of the reciprocal time (increment) of the process $\propto |E|^{-2} \cdot \left(d|E|^2/dt\right)$ **on Θ: solid line (res) – generation of the resonator field; dotted line – superradiance mode (sup).** *Source [24.33]*

own fields when excitation of the field leads to a higher field intensity and a higher rate of energy output and reduces the characteristic development time of the process as Θ decreases. For the resonator mode, the maximum rate of energy output occurs at Θ=0.7.

2.3. Excitation of the field by currents of classical oscillators

Let us discuss the nature of field excitation by a system of classical oscillators. Let us consider an oscillator whose charge (electron) moves along the OX axis, that is $\vec{r} = (x(t), 0, z_0)$, where $x(t) = i \cdot a \cdot \exp\{-i\omega t + i\psi\}$, at the same time $\operatorname{Re} x = a \cdot \sin(\omega_0 t - \psi)$. In this case, the speed and current $J_x = -edx/dt = -e \cdot a \cdot \omega_0 \cdot \exp\{-i\omega t + i\psi\}$. can be written as $dx/dt = a \cdot \omega_0 \cdot \exp\{-i\omega t + i\psi\}$ and the equation describing the excitation of the field by the oscillator current

$$\frac{\partial^2 E_x}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2 D_x}{\partial t^2}=\frac{4\pi}{c^2}\frac{\partial J_x}{\partial t}=\frac{4\pi}{c^2}\cdot e\cdot a\cdot {\omega_0}^2\cdot i\cdot \exp\{-i\omega t+i\psi\}\cdot\delta(z-z_0) \qquad (3.20)$$

Let us set the dielectric constant of the medium in the absence of oscillators equal to unity $\varepsilon_0=1$. We will look for a solution for the amplitude of the electric field in the form $\vec{E}=(E\cdot\exp\{-i\omega t+ikz\},0,0)$, that is $E_x=E\cdot\exp\{-i\omega t+ikz\}$, assuming a slow change in the complex amplitude $E_x(t,z)$:

$$\left|\frac{1}{E_x(t,z)}\frac{\partial}{\partial t}E_x(t,z)\right|<<\omega\ ,\quad \left|\frac{1}{E_x(t,z)}\frac{\partial}{\partial z}E_x(t,z)\right|<<k\ . \qquad (3.21)$$

Below we will consider the excitation of the field by an ensemble of resonators, which in our one-dimensional case are uniformly distributed in the active zone over an interval equal to the radiation wavelength. Let us discuss two modes of excitation of fields by currents of an ensemble of oscillators.

The first mode of excitation of the resonator (or waveguide) field corresponds to the traditional description of generation (or amplification) in electronic devices, when all oscillators (emitters) interact only with the field of the resonator in which the active zone is located. In this case, we exclude the interaction of oscillators with each other. That is, we do not take into account the own fields of the emitters in the system. The second mode of field excitation by an ensemble of emitters takes into account only the own fields of the emitters located in the same interval of the active zone, and in this case there is no resonator (waveguide) field. It is not difficult to see that this process corresponds to the superradiance regime. By the way, in this case the resonator or waveguide may be absent altogether.

It is worth recalling that these two modes are considered, firstly, quite artificially, and secondly, they should be separated from each other, because in each there is a different mode of interaction of particles with fields.

The summary field of oscillators (superradiance mode). Generally speaking, the field excited in a system of oscillators consists of the sum of all the fields of individual oscillators. Let us consider the superradiance regime, when there is no resonator field or waveguide field. An important circumstance is the conditions for synchronization by the own radiation fields of the oscillator system in the superradiance mode. It turns out that only when taking into account the nonlinearity of the oscillators, it becomes possible in this case to ensure synchronization of the phases of the field and the oscillator [11,12] (see also [6]). First, let's find the field of one oscillator. For the radiation field amplitude slowly varying in space, the equation holds:

$$\frac{\partial E}{\partial z} = 2ea\omega^2 \frac{\pi}{c^2 k} \cdot \exp\{i\psi + ikz\} \cdot \delta(z - z_0) = \lambda \cdot \delta(z - z_0) \qquad (3.22)$$

the solution of which is $E = C + \lambda \cdot \theta(z - z_0)$, where $\theta(z < 0) = 0, \quad \theta(z \geq 0) = 1$. Since for the wave emitted by the oscillator the equation $D(\omega, k) \equiv (\omega^2 \varepsilon_0 - k^2) = 0$ is valid, the roots of which are $k_{1,2} = \pm(\omega_0 \operatorname{Re} \varepsilon_0 / c)(1 + i \operatorname{Im} \varepsilon_0 / \operatorname{Re} \varepsilon_0) \approx \pm(\omega_0 \varepsilon_0 / c)(1 + i0)$, then for a wave propagating in the direction $z > z_0$, the wave number $k = k_1 > 0$ and the value of the constant *C* should be chosen equal to zero in order to avoid of the field at infinity. For a wave propagating in the direction $z < z_0$, the wave number $k = k_2 < 0$, the value of the constant, for the same reasons, should be chosen equal to –*λ*. The amplitude of the electric field in this case

$$E_x = 2\pi ea\omega_0 M \cdot c^{-1} \exp\{-i\omega t + i\psi\}[\exp\{ik_0(z - z_0) \cdot U(z - z_0) + \\ + \exp\{-ik(z - z_0) \cdot U(z - z_0)\},$$

(3.23)

where $U(z) = 1$ at $z \geq 0$ and $U(z) = 0$ at $z < 0$. For one particle in such a volume of unit cross section and resonator length b, M it is numerically equal to unity.

The equation of motion for an oscillating electron has the form

$$\frac{dx_i}{dt} = v_i, \qquad \frac{d}{dt} \frac{v_i}{\sqrt{1 - \frac{|v_i|^2}{c^2}}} + \omega_0{}^2 x_i = -\frac{e}{m} E_x(z_i, t), \qquad (3.24)$$

where

$$x_i(t) = i \cdot a_i \cdot \exp\{-i\omega t + i\psi\} = iA \cdot \exp\{-i\omega t\}, \quad v_i = \omega \cdot a_i \cdot \exp\{-i\omega t + i\psi\} = \omega A \cdot \exp\{-i\omega t\}$$

Using these notations, we write (3.24) in the form

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$$\frac{dA_j}{dt} - i\frac{|3A_j|^2 \cdot \omega^3}{4c^2} A_j + i\frac{|3A_{j0}|^2 \cdot \omega^3}{4c^2} A_{j0} =$$
$$= -\frac{\pi \cdot e^2 \cdot M}{mc} \cdot \frac{1}{N}\sum_{s=1}^{N} A_s \left(e^{ik(z_j - z_s)} \cdot \theta(z_j - z_s) + e^{-ik(z_j - z_s)} \cdot \theta(z_s - z_j)\right). \quad (3.25)$$

Let's choose dimensionless variables and parameters

$$\mathrm{A} = A / a_0, \quad kz = 2\pi Z, \quad \gamma_0 = \omega_{pe} / 2, \quad \gamma_0^2 = \omega_{pe}^2 / 4 = \frac{\pi n e^2}{m}, \quad \gamma_0 t = \tau, \quad \beta = m / m_1, \quad kb = 2\pi \bar{b},$$
$$\delta = \frac{c}{b}, \quad \Theta = \delta / \gamma_0, \quad E_{01} = \frac{2m \cdot \gamma_0 \cdot \omega \cdot a_0}{e}, \quad \mathrm{E} = E / E_{01}, \quad \alpha = \frac{3\omega}{4\gamma_0}(ka_0)^2.$$

Let us consider the excitation of a field in a resonator whose longitudinal size b is equal to the wavelength (without loss of generality, the results are generalized to the case of several wavelengths), the group radiation velocity is c, and the effective field attenuation decrement is equal to $\delta = \frac{c}{b}$. If we take into account the radiation from the system (which, due to the small size of the system, can be considered distributed), then the process increment is $\gamma \approx \gamma_0^2 / \delta = \gamma_0 / \Theta$, and $\delta > \gamma_0$. Note that in all the cases discussed, due to the chosen placement of the ensemble of oscillators, the increments of the non-dissipative (that is, without energy loss $\Theta = 0$) mode turn out to be the same and equal γ_0 [33]. We obtain the expression for the superradiance field

$$\mathrm{E}_{sr}(Z,\tau) = \frac{1}{\Theta N}\sum_{s=1}^{N} \mathrm{A}_s \cdot e^{i2\pi|Z - Z_s|}, \quad (3.26)$$

and the equation of motion describing the dynamics of the oscillators takes the form

$$\frac{d\,\mathrm{A}_j}{d\tau} = \frac{i\alpha}{2} \cdot \left|\mathrm{A}_j\right|^2 \mathrm{A}_j - \mathrm{E}(Z_j, \tau), \quad (3.27)$$

where for the general field the following expression is valid:

$$\mathrm{E}(Z,\tau) = \mathrm{E}_{sr}(Z,\tau) + \mathrm{E}_{ex}(Z,\tau), \quad (3.28)$$

the second term in (3.28) is the external initiating field, which is often necessary to speed up the process and can be written as

$$\mathrm{E}_{ex}(Z,\tau) = \mathrm{E}_{0+}\, e^{i2\pi Z} + \mathrm{E}_{0-}\, e^{-i2\pi Z}, \quad (3.29)$$

The nature of synchronization of oscillators in this and other modes is quite obvious [12,6]. Let's return to equation (3.27), which can be written differently

$$\frac{d[|\mathrm{A}_j|\exp(i\psi_j)]}{dt}=\frac{i\alpha}{2}\cdot\left|\mathrm{A}_j\right|^2|\mathrm{A}_j|\cdot\exp(i\psi_j)-|\mathrm{E}(Z_j,\tau)|\cdot\exp(i\varphi) \qquad (3.30)$$

.

Then the equation for the oscillator phase, which follows from (3.30), takes the form

$$\frac{d\psi_j}{dt}-\frac{\alpha}{2}\cdot\left|\mathrm{A}_j\right|^2=-\{|\mathrm{E}(Z_j,\tau)|/|\mathrm{A}_j|\}\cdot\sin(\varphi-\psi_j) \qquad (3.31)$$

.

You can pay attention to the fact that the right side of the last equation is quite large $|\mathrm{E}(Z_j,\tau)/\mathrm{A}_j|>>1$. This causes the phase of the individual oscillator to synchronize with the phase of the ensemble's total field $\psi_j\to\varphi$. This explains the need to use the starting initiating field (second term (3.28)) for the development of the superradiation process. The amplitude of such a field should noticeably exceed the amplitude of the spontaneous emission of an individual oscillator.

The value $\frac{i\alpha}{2}\cdot\left|\mathrm{A}_j\right|^2$ provides regularization, that is, a certain spread of phase values.

However, the synchronization process is significantly affected by the spread of oscillator amplitudes. The synchronization process was considered for different distributions of initial amplitudes. The squares of the initial amplitudes were randomly distributed in the range so that the average value of the amplitudes at the initial moment was equal to $\mathrm{A}_{av}^2=\frac{1}{N}\sum_{j=1}^{N}|\mathrm{A}_j^2|=\mathrm{A}_0^2$. Calculations of the system (3.26-3.28) were carried out with such parameters $N=900$, $\alpha=1$, $\mathrm{A}_0=1,\quad \Delta\mathrm{A}^2=1; 0.4; 0.65; 0.75; 0.8; 1$.

The spread of amplitudes of oscillators with a random distribution of their phases at the initial moment can be as follows

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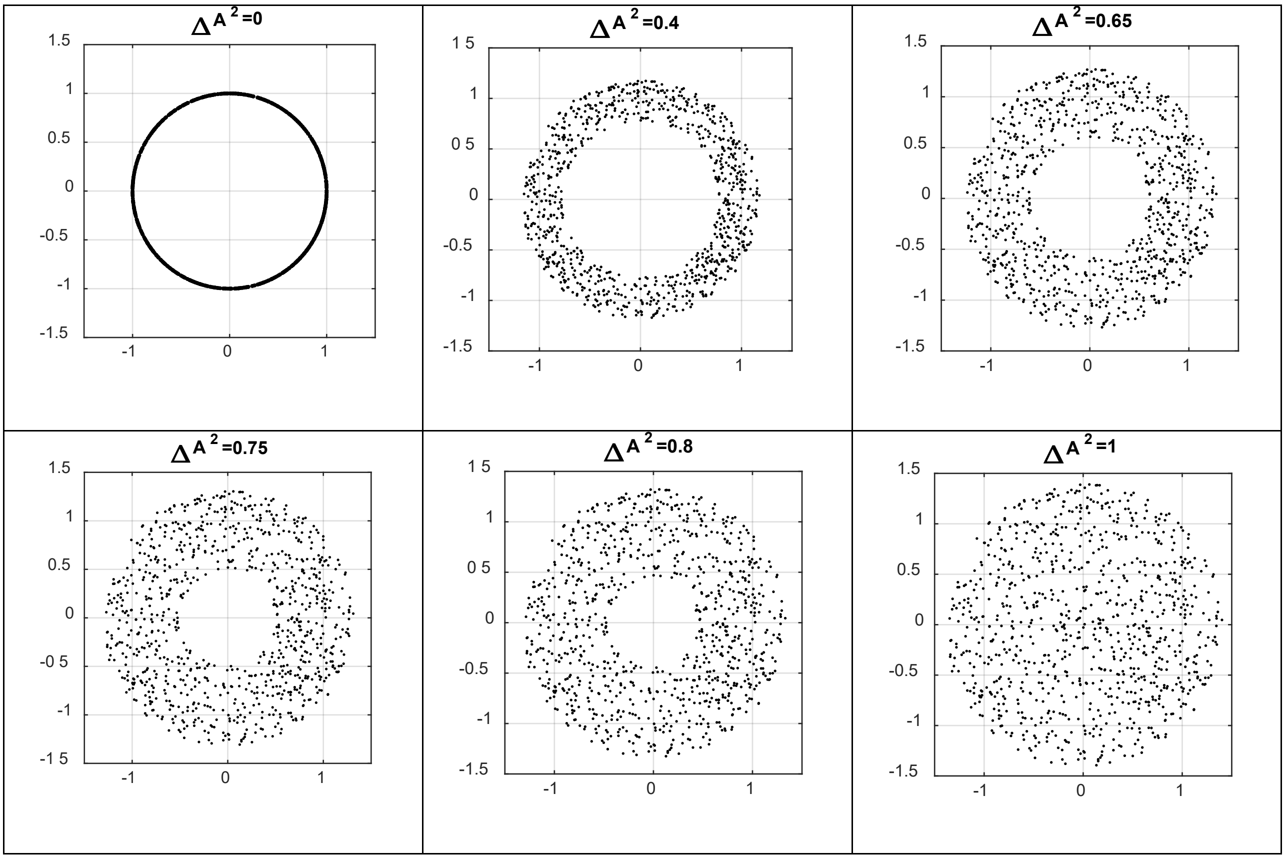

Fig. 3. 1. Phase planes “amplitude-phase” for different cases of initial distribution of oscillators. *Source is an authoring.*

For these cases, we can give the nature of the change in the energy of the oscillator system and the behavior of the field amplitude.

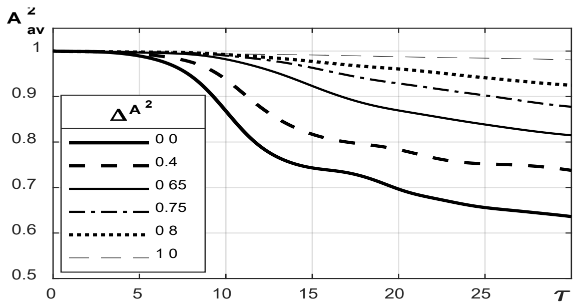

Fig.3.2.a. Changes in the energy of the oscillator system for different levels of amplitude spread. *Source is an authoring.*

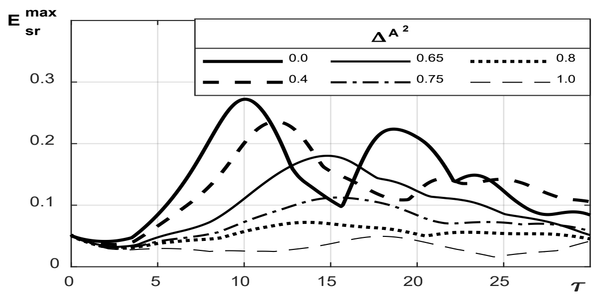

Fig. 3.2.b. Behavior of the field amplitude for different levels of amplitude spread. Note that in all cases, the maximum intensity is selected for each moment in time $|E|^2 \approx |E_{\max}|^2$ **.** *Source is an authoring.*

From Fig. 3.1 and Fig. 3.2 it is clear that for efficient generation, in particular in the superradiance regime, it is necessary to achieve an insignificant spread in the amplitudes (or energy) of the oscillators. Note that the quantity $\frac{i\alpha}{2}\cdot\left|A_j\right|^2$, which characterizes the nonlinearity of the oscillator, thereby ensures regularization, that is, a certain spread of phase values.

Resonator field. In the presence of reflections, a field can be formed in a resonator (or waveguide) in such a way that the type of the field does not depend on the radiation of individual oscillators. Note that such a field, generally speaking, should consist of traveling waves in two directions $(k > 0)$

$$E_x = E_+\cdot\exp\{-i\omega t + ikz\} + E_-\cdot\exp\{-i\omega t - ikz\}, \tag{3.32}$$

where the slowly varying complex wave amplitude has the form $E_\pm = |E_\pm|\cdot\exp\{i\varphi_\pm\}$.

The interaction of oscillators with these fields can be described by the equation [9,25]

$$2i\omega_0\left(\frac{\partial E_\pm}{\partial t} + \delta_D\right) = -e\omega^2{}_0\frac{4\pi n_0}{N\cdot 2i}\int a_s dz\cdot\exp\{i\psi_s \mp ikz\}\cdot\delta(z - z_s) \tag{3.33}$$

,

where added δ_D is the wave absorption decrement in the absence of sources, $A_j = a_j\exp(i\psi_j)$. We use the equations of motion in the form (3.24). We accept the following notation $\frac{e\,|E(t)|\exp\{i\varphi\}}{m\gamma_0\omega_0 a_0} = \mathrm{E}(t)$, $\gamma_0 t = \tau$, $\gamma_0^2 = \pi e^2 n_0/m = \omega_{pe}^2/4$, $\mathrm{A}_j = a_j/a_0$, $k_0 z_j = 2\pi Z_j$, $\theta = \delta/\gamma_0$, $\alpha = \frac{3\omega_0}{4\gamma_0}(k_0 a_0)^2$ – this parameter takes into account the weak dependence of the relativistic mass of a particle on the speed and we write (3.33) and (3.24) in the form

$$\frac{\partial}{\partial\tau}\mathrm{E}_\pm + \theta\cdot\mathrm{E}_\pm = \frac{1}{N}\sum_{j=1}^{N}\mathrm{A}_j\cdot\exp i\{\mp 2\pi Z_j\}, \tag{3.34}$$

$$\frac{d}{d\tau}\mathrm{A}_j - i\alpha(\mathrm{A}_j^3 - \mathrm{A}_{j0}^3) = -\frac{1}{2}[\mathrm{E}_+\cdot\exp i\{2\pi Z_j\} + \mathrm{E}_-\cdot\exp i\{-2\pi Z_j\}]. \tag{3.35}$$

It is useful to set the second term on the left side of (3.35) equal to zero at the initial moment $\Delta_j = \alpha\,|\,\mathrm{A}_j\,|^2$. Then

$$\frac{d}{d\tau}\mathrm{A}_j - i\alpha(\mathrm{A}_j\,|\,\mathrm{A}_j\,|^2) = -\frac{1}{2}[\mathrm{E}_+\cdot\exp i\{2\pi Z_j\} + \mathrm{E}_-\cdot\exp i\{-2\pi Z_j\}] \quad (3.36)$$

.

The law of conservation of energy can be obtained in the form

$$(\frac{\partial}{\partial\tau} + 2\theta)\{|\,\mathrm{E}_+\,|^2 + |\,\mathrm{E}_-\,|^2\} = 2\frac{\partial}{\partial\tau}\sum_{j=1}^{N}|\,\mathrm{A}_j\,|^2 \quad (3.37)$$

.

When choosing dimensionless variables and parameters

$$\mathrm{A} = A/a_0,\quad kz = 2\pi Z,\quad \gamma_0 = \omega_{pe}/2,\quad \gamma_0^2 = \omega_{pe}^2/4 = \frac{\pi n e^2}{m},\quad \gamma_0 t = \tau,\quad \beta = m/m_1,\quad kb = 2\pi\overline{b},$$

$$\delta = \frac{c}{b},\quad \Theta = \delta/\gamma_0,\quad E_{01} = \frac{2m\cdot\gamma_0\cdot\omega\cdot a_0}{e},\quad \mathrm{E} = E/E_{01},\qquad \alpha = \frac{3\omega}{4\gamma_0}(ka_0)^2.$$

For the resonator field in this case we obtain the expression

$$\mathrm{E}_{wg}(Z,\tau) = \mathrm{E}_+(\tau)e^{i2\pi Z} + \mathrm{E}_-(\tau)e^{-i2\pi Z} \quad (3.38)$$

,

where the components corresponding to the propagation of radiation in different directions can be written as

$$\frac{\partial\,\mathrm{E}_+}{\partial\tau} + \Theta\,\mathrm{E}_+ = \frac{1}{N}\sum_s \mathrm{A}_s\cdot e^{-i2\pi Z_s} \quad (3.39)$$

,

$$\frac{\partial\,\mathrm{E}_-}{\partial\tau} + \Theta\,\mathrm{E}_- = \frac{1}{N}\sum_s \mathrm{A}_s\cdot e^{i2\pi Z_s} \quad (3.40)$$

,

and at the initial moment their values should be set

$$\mathrm{E}_+(0) = \mathrm{E}_{0+} \text{ and } \mathrm{E}_-(0) = \mathrm{E}_{0-}. \quad (3.41)$$

The equations of motion for oscillators take the form

$$\frac{d\,\mathrm{A}_j}{dt} = \frac{i\alpha}{2}\cdot\left|\mathrm{A}_j\right|^2\mathrm{A}_j - \mathrm{E}_{wg}(Z_j,\tau) \quad (3.42)$$

.

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On the connection between the description of the generation of a system of oscillators and the excitation of cyclotron oscillations*[3]*. It was shown in [38] that the equations describing the excitation of cyclotron oscillations by rotating electrons in a constant magnetic field in a waveguide can, under certain conditions, be reduced to a system of equations for oscillators. This circumstance indicated the existence of a common phase synchronization mechanism for all these cases. On the other hand, this provided grounds for studying the nature of the excitation of oscillations in a simpler system, which made it possible to more thoroughly identify all the characteristic features of the excitation of a resonator (analogue of a waveguide) field and a superradiance field. It is possible to imagine systems of equations that describe the excitation of two electromagnetic waves of different polarization with frequency ω and wave vector $\vec{k} = (k_{ms}, 0, k_z)$ in a smooth metal cylindrical waveguide of radius r_w by an electron beam in cases of resonance $\omega \approx +n \cdot \omega_B$, where ω_B is the cyclotron frequency of electron rotation. Induction of a constant magnetic field $\vec{B} = (0, 0, B)$. The electron beam occupies a cylindrical layer in the cross section of the waveguide, which we will consider to be quite thin. All centers of Larmor rotation (the radius of which r_B electrons are at the same distance r_C from the axis of the waveguide.

TE wave. The equations describing the excitation of a TE wave by a system of electrons rotating in a magnetic field, the centers of rotation of which are stationary (this wave does not have an electric field component in the direction of its propagation), can be written in the form (see, for example, [45])

$$\frac{d\mathrm{E}_e}{d\tau} + \theta_e \cdot \mathrm{E}_e = N^{-1} \cdot \sum_{j=1}^{N} a_j \cdot J_n^{'}(a_j) \cdot \mathrm{Sin}(2\pi\zeta_j + 2\pi Z_j + \varphi_e), \qquad (3.43)$$

[3] The use of a simplified system of equations for oscillators, which follows from more general systems of equations to describe the generation of cyclotron waves, is rational, because it allows one to understand the nature of the phenomena. However, it is useful to realize that the use of such simplified models is correct only if they are directly derived from proven descriptions and is not permissible when using phenomenology and simulation modeling.

$$\frac{d\varphi_e}{d\tau} - \Delta_e = (\mathrm{E}_e N)^{-1} \cdot \sum_{j=1}^{N} a_j \cdot J_n^{'}(a_j) \cdot \mathrm{Cos}(2\pi\zeta_j + 2\pi Z_j + \varphi_e) \quad . \qquad (3.44)$$

The equations of motion of electrons in the field of this wave in the presence of a constant magnetic field are as follows:

$$2\pi \frac{d\zeta_i}{d\tau} = \eta_i + n\mathrm{E}_e \cdot J_n(a_i) \cdot [1 - \frac{n^2}{a_i^2}] \cdot \mathrm{Cos}(2\pi\zeta_i + 2\pi Z_j + \varphi_e) \quad , \qquad (3.45)$$

$$da_i / d\tau = -n \cdot \mathrm{E}_e \cdot J_n^{'}(a_i) \cdot \mathrm{Sin}(2\pi\zeta_i + 2\pi Z_j + \varphi_e) \quad , \qquad (3.46)$$

where, $\delta_e^2 = 4e^2 \cdot \omega_B \cdot \mathrm{N}_{b0} \cdot [m_e \cdot c \cdot k_{ms}^2 \cdot r_w \cdot J_m^2(x_{ms}) \cdot (1 - m^2 / x_{ms}^2) \cdot D_\omega]^{-1} \cdot J_{m-n}^2(k_{ms} \cdot r_C)$,

$D_\omega = \partial D / \partial \omega = \partial\{[\omega^2 - (k_z^2 + k_{ms}^2)c^2] / [\omega^2 - k_z^2 c^2]\} / \partial\omega |_{D=0}$, $\tau = \delta_e t$, $R_e = k_z^2 \cdot \omega_B / k_{ms}^2 \cdot \delta_e$,

$\mathrm{E}_e = e \cdot b_B \cdot J_{m+n}(k_{ms} \cdot r_C) / m_e \cdot c \cdot \delta_e$, $Z_j = kz_j / 2\pi$ – is the position of the electron rotation center along the waveguide axis, $a = k_{ms} r_B = k_{ms} v_\Phi / \omega_B$, $\omega_B = eB / m_e c$, N_{b0} – is the number of particles of the unperturbed beam per unit length. Here b_B is the amplitude of the wave, and the longitudinal component of the magnetic field of the wave has the form $B_z = b_B \cdot J_m(k_{ms} r) \cdot \exp\{-i\omega t + ik_z z + im\vartheta\}$ in a cylindrical coordinate system (r, ϑ, z), m is an integer, $J_m(x)$ and $J_m^{'}(x) = dJ_m(x) / dx$ is the Bessel function and its derivative. The requirement that the tangential component of the field vanish at the waveguide boundary determines the values of the transverse wave number $k_\perp = k_{ms} = x_{ms} / r_w$, and x_{ms} — s– is the root of the equation $dJ_m(x) / dx = 0$.

TM wave. The equations describing the field of a TM wave (which has an electric field component in the direction of its propagation) can be written in the form (see, for example, [27])

$$\frac{d\mathrm{E}_h}{d\tau} + \theta_h \cdot \mathrm{E}_h = N^{-1} \cdot \sum_{j=1}^{N} J_n(a_j) \cdot \mathrm{Cos}(2\pi\zeta_j + 2\pi Z_j + \varphi_h) \quad , \qquad (3.47)$$

$$\frac{d\varphi_h}{d\tau} - \Delta_h = (\mathrm{E}_h N)^{-1} \cdot \sum_{j=1}^{N} J_n(a_j) \cdot \mathrm{Sin}(2\pi\zeta_j + 2\pi Z_j + \varphi_h) \quad . \qquad (3.48)$$

The equations of motion for electrons in the field of this wave, which do not move along the axis of the waveguide, are as follows:

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$$2\pi \frac{d\zeta_i}{d\tau} = \eta_i + \frac{n}{a_i} \cdot \mathrm{E}_h \cdot J_n^{'}(a_i) \cdot \mathrm{Sin}(2\pi\zeta_i + 2\pi Z_j + \varphi_h), \qquad (3.49)$$

$$da_i / d\tau = -(n / a_j) \cdot \mathrm{E}_h \cdot J_n(a_i) \cdot \mathrm{Cos}(2\pi\zeta_i + 2\pi Z_j + \varphi_h), \qquad (3.50)$$

where $\tau = \delta_h t$, $\delta_h^2 = 4e^2 \cdot \omega_B \cdot \mathrm{N}_{b0} \cdot [m_e \cdot r_w^2 \cdot J_m^{'2}(x_{ms}) \cdot D_\omega]^{-1} \cdot (k_{ms}^2 / \omega_B k_z^2) \cdot J_{m-n}^2(k_{ms} \cdot r_C)$, $R_e = k_z^2 \cdot \omega_B / k_{ms}^2 \cdot \delta_h$, $Z_j = kz_j / 2\pi$ is the position of the center of rotation of the electron along the axis of the waveguide $\mathrm{E}_h = e \cdot h \cdot J_{m+n}(k_{ms} \cdot r) \cdot (k_{ms}^2 / \omega_B k_z) / m_e \cdot \delta_h$, h is the amplitude of the wave, and the longitudinal component of the electric field of the wave has the form $E_z = h \cdot J_m(k_{ms} r) \cdot \exp\{-i\omega t + ik_z z + im\vartheta\}$ in a cylindrical coordinate system (r, ϑ, z), the remaining notations are similar to the previous ones used to describe the TE wave.

Integral. Note that for these two waves the integral is valid

$$|\mathrm{E}|^2 - n^{-1} \cdot N^{-1} \sum_{j=1}^{N} a_j^2 = Const, \qquad (3.51)$$

moreover, this integral (3.38) is valid for $\theta = 0$, if $\theta \neq 0$ on its right side $Const \to Const + \theta \cdot \int_0^t dt' |\mathrm{E}(t')|^2$. It is useful to pay attention to the consequence of the integral in the absence of field energy losses ($\theta = 0$). If the change in transverse motion energy is equal to $\Delta W_\perp = \frac{\omega_B \cdot m_e \cdot \mathrm{N}_{b0}}{2k_{ms}^2} N^{-1} \sum_{j=1}^{N} (a_j^2 - a_{j0}^2)$, then the ratio of the change in field energy to the change in transverse motion energy can be represented as $\omega / n\omega_B$. Note that the system of equations for the TE wave can be transformed into the known system of gyrotron equations under the conditions of neglecting the longitudinal motion of electrons even in the presence of low-density plasma [46-48] [Taking into account relativism (the effect of negative mass, see, for example, [48] leads to changing the nature of particle motion in the equations of motion (3.45) and (3.49), and $\gamma_0 = (1 - v_\Phi^2 / c^2)^{-1/2} |_{\tau=0}$,

Transition to equations for describing superradiance [6,49-52]. Let us limit ourselves to considering the excitation of the TE wave. The case of radiation of an individual particle (out of a total number of particles equal to) should be considered as follows. The equation for the field that an individual particle emits can be written as

$$v_g \frac{\partial B_j}{\partial z} = i\frac{a_j}{N} J_n^{'}(a_j)\exp\{-2\pi i\zeta_j\}\exp\{-ik_z z_j\}\cdot\delta(z-z_j) \tag{3.52}$$

or $\frac{\partial B_j}{\partial z} = \lambda\cdot\delta(z-z_j)$, where $\lambda = i\frac{a_j}{Nv_g} J_n^{'}(a_j)\exp\{-2\pi i\zeta_j\}\exp\{-ik_z z_j\}$,

the solution of which $B_j = C + \lambda\cdot\theta(z-z_j)$, where C is the constant to be determined. Since for a wave emitted by an oscillator the equation $D(\omega,k)=0$ is valid, the roots of which are $k_{z1,2} = \pm\operatorname{Re}D(1+i\operatorname{Im}D/\operatorname{Re}D)\approx\pm(\frac{\omega^2}{c^2}-k_\perp^2)^{1/2}(1+i0)$, then for a wave propagating in the direction $z>z_j$, the wave number $k_z = k_{z1} > 0$ and the value of the constant should be chosen equal to zero in order to avoid unlimited growth of the field at infinity. For a wave propagating in the direction $z<z_j$, the wave number $k_z = k_{z2} < 0$ value of the constant should be chosen equal $-\lambda$ to for the same reasons. The field amplitude in this case

$$B_j(Z) = i\frac{a_j}{NV_g} J_n^{'}(a_j)\exp\{-2\pi i\zeta_j\}\cdot$$
$$\cdot[\exp\{2\pi i(Z-Z_j\}\cdot U(Z-Z_j)+\exp\{-2\pi i(Z-Z_j\}\cdot U(Z_j-Z)], \tag{3.53}$$

where $U(z)=1$ at $z\ge 0$ and $U(z)=0$ at $z<0$. Let us draw attention to the fact that the direction of the strength vector of the longitudinal component of the magnetic field in this case does not depend on the direction of wave propagation. This is due to their suppression of the rotating electron’s own magnetic field when emitting waves in both directions. Obviously, for a system of N oscillators, the equation for the field can be written in the form

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$$B(Z) = i\frac{1}{2N\vartheta}\sum_{j=1}^{N} a_j J_n'(a_j)\exp\{-2\pi i\zeta_j\}\cdot$$
$$\cdot[\exp\{2\pi i(Z - Z_j\}\cdot U(Z - Z_j) + \exp\{-2\pi i(Z - Z_j\}\cdot U(Z_j - Z)], \quad (3.54)$$

where $\vartheta = 2V_g N_{0b} / M = 2v_g / d\cdot\delta$ is the ratio of the maximum increment to the attenuation decrement due to radiation from the ends of the system $2v_g / d$. It is important to note that in such notation the ratio of the wave energy along the length of the system $Z_M = 2\pi k_z d$ to the particle energy is represented by the expression $\int_0^{\xi_M} |B(\xi)|^2 d\xi / \frac{1}{N}\sum_{i=1}^{N} a_{i0}^2$, and since the ratio of the energy emitted from the system to the total field energy during the time $1/\delta$ in the system is equal to ϑ, then the efficiency of the system (if the unit of time is chosen to be $1/\delta$) can be assessed as $\frac{\vartheta}{\xi_M}\int_0^{\xi_M} |B(\xi)|^2 d\xi / \frac{1}{N}\sum_{i=1}^{N} a_{i0}^2$. Equations of motion for beam electrons

$$2\pi\frac{d\zeta_i}{d\tau} = \eta_i - n\cdot J_n(a_i)\cdot\{1 - \frac{n^2}{a_i^2}]\frac{1}{2N\vartheta}\sum_{j=1}^{N} a_j J_n'(a_j)[\mathrm{Sin}\{2\pi(\zeta_i - \zeta_j) +$$
$$+ 2\pi(Z_i - Z_j)\}\cdot U(Z_i - Z_j) + \mathrm{Sin}\{2\pi(\zeta_i - \zeta_j) - 2\pi(Z_i - Z_j)\}\cdot U(Z_j - Z_i)], \quad (3.55)$$

$$da_i / d\tau = -n\cdot J_n'(a_i)\cdot\frac{1}{2N\vartheta}\sum_{j=1}^{N} a_j J_n'(a_j)[\mathrm{Cos}\{2\pi(\zeta_i - \zeta_j) +$$
$$+ 2\pi(Z_i - Z_j)\}\cdot U(Z_i - Z_j) + \mathrm{Cos}\{2\pi(\zeta_i - \zeta_j) - 2\pi(Z_i - Z_j)\}\cdot U(Z_j - Z_i)]. \quad (3.56)$$

Please note that the field values are not required for calculations; they are already taken into account in the right-hand sides of (3.55) - (3.56). Nevertheless, it is possible to calculate the values of the longitudinal magnetic field (3.54), using which you can restore all other wave components at any point in the waveguide and outside of it. It turns out that for linear oscillators that do not move in the direction of wave propagation, subject to the following conditions: , $n = 1$, $\theta = 0$, $\Delta = 0$, $\varphi \to -\varphi$, $\psi \leftrightarrow 2\pi\zeta + \pi/2$, $R \to 0$ for small a_i ($J_1(a_i)_{a_i\to 0} \to a_i/2$,

$J_0(a_i)_{a_i \to 0} \to 1$, $J'_1(a_i)_{a_i \to 0} \to 1/2$) and using the known relations $\mathrm{Sin}\,\alpha = -\mathrm{Cos}(\alpha + \pi/2)$, $\mathrm{Cos}\,\alpha = +\mathrm{Sin}(\alpha + \pi/2)$, the system of equations for TE and TM waves (3.43)–(3.46) and (3.47)–(3.50) coincide with the equations of the system (3.34)–(3.35) or in dimensionless form (3.61)–(3.66) – up to the choice of the field sign and value. Similarly, the system of equations for superradiance of a system of oscillators (3.26)–(3.28) or in dimensionless form (3.57)-(3.59) can be obtained from equations (3.54)–(3.56).

Conditions for the development of generation in the superstudy mode in the presence of noise. The equation describing the change in the complex amplitude of an individual oscillator in the presence of noise takes the form [49]

$$\frac{d\,\mathrm{A}_j}{d\tau} = \frac{i\alpha}{2} \cdot \left|\mathrm{A}_j\right|^2 \mathrm{A}_j - \mathrm{E}(Z_j, \tau) + i \cdot \delta \cdot r_j(\tau) \cdot \mathrm{A}_j \, , \qquad (3.57)$$

where the last term on the right side of (3.57) is additionally introduced, which takes into account the influence of external noise $r_j(\tau)$. Here – takes random values from –1 to +1, changing at intervals $\Delta\tau$ on a selected time scale, δ – is the maximum value of this impact. Expression (3.26) is valid for the field $\mathrm{E}(Z, \tau)$.

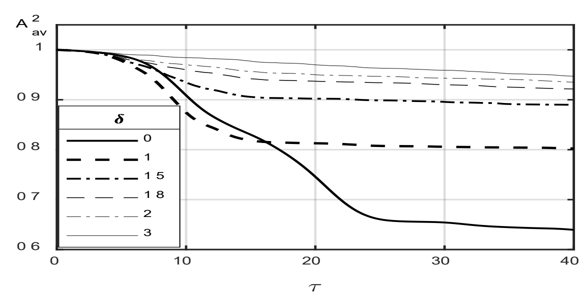

Fig. 3.3 a. Change in the average value of the squared amplitude $A^2_{av} = \frac{1}{N}\sum_{s=1}^{N}\left|\mathrm{A}_s\right|^2$ **in the system over time.** *Source [49]*

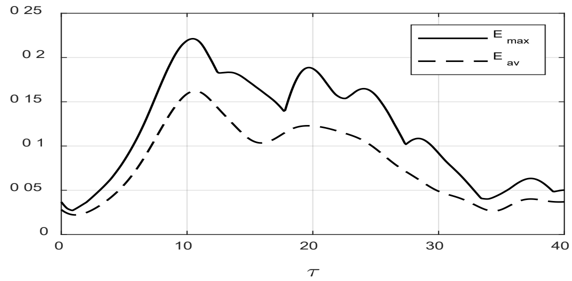

Fig. 3.3 b. Time dependence of the field in the system in the absence of noise $(\delta = 0)$**. Note that in all cases, for each moment in time, the maximum intensity** $|E|^2 \approx |E_{\max}|^2$ **is selected.** *Source [49]*

Random exposure, switched at intervals $\Delta\tau = 0.4$, leads to weakening of synchronization or even complete chaotization of phases. The calculation results below use the following parameters: $N = 900$ number of particles, nonlinearity parameter $\alpha = 1$, noise switching interval $\Delta\tau = 0.4$, system length $b = 1$ (one wavelength).

In Fig. 3.3a one shows the change in the average value of the squared amplitude in the system $A_{av}^2 = \frac{1}{N}\sum_{s=1}^{N}|A_s|^2$ over time for different values of the external noise level δ. In Fig. 3.3 b one shows the behavior of the maximum field amplitude $E_{max} = \max_{Z\in(0,1)} E_{sr}(Z)$ and the volume-average field amplitude $E_{av} = \sqrt{\frac{1}{b}\int_0^b |E_{sr}|^2\, dZ}$ in a system of oscillators in the absence of noise $\delta = 0$. Even from these figures one can see the existence of a threshold: in this case $\delta = 0 \div 1$, the field value reaches 0.22 at the selected scale.

Below the threshold value there is no field growth, the energy extraction from the oscillators is weakened (the average energy remains at the level of 96% of the initial one). In this case, a turbulent state is formed with an average field value close to the spontaneous electric field strength level of 0.03–0.04. The peak level of fluctuations exceeds the average level by two or more times (see Fig. 3.4).

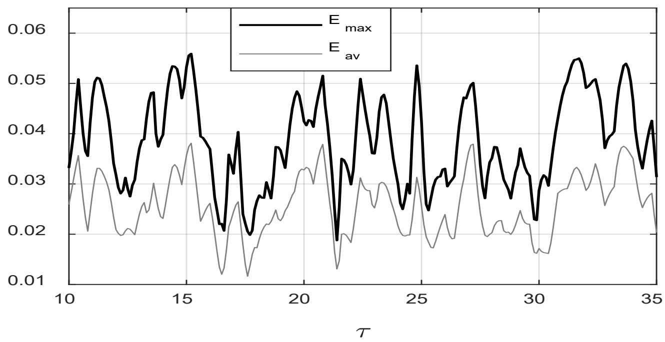

Fig. 3.4. Average field values and peak fluctuation values at noise level $\delta = 3$.

Source [49]

When approaching the threshold, the average values of the amplitudes of the oscillators (at $\delta \propto 1.8$, $A_{av}^2 < 0.97$) change slightly, that is, no noticeable energy extraction from the system of oscillators is observed. however, when approaching the threshold, the peak values of fluctuations increase (see Fig. 3.5)

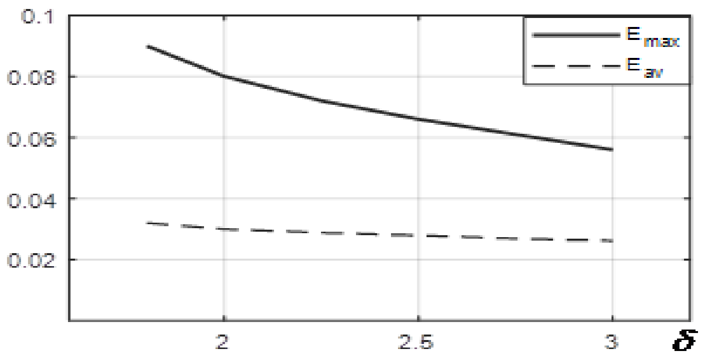

Fig. 3.5. Growth of fluctuations when approaching the threshold of generation development under superradiance conditions. *Source [49]*

By considering the region near and below the threshold, it is possible to find out how the external field influences the occurrence and development of generation under superradiance conditions. The time dependence of the maximum generation field of the oscillator system at different amplitudes of the external field is shown in Fig. 3.6.

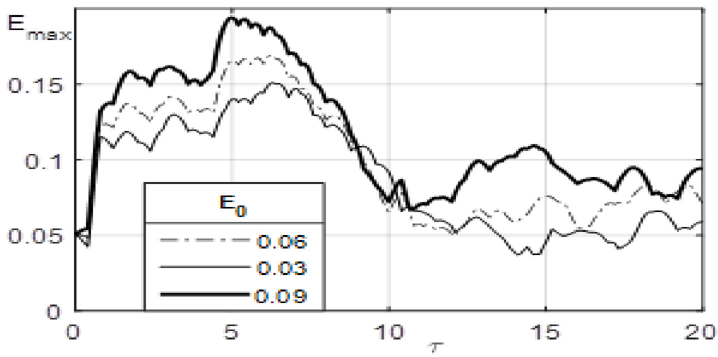

Fig. 3.6 Influence of an external initiating field $E_0 = 0.03; 0.06; 0.09$ on generation at $\delta = 1.8$ under superradiance conditions. Note that in all cases, the maximum intensity $|E|^2 \simeq |E_{\max}|^2$ is selected for each moment in time. *Source [49]*

In Fig. 3.6. It can be seen that an increase in the amplitude of the external initiating field $E_0 = 0.03; 0.06; 0.09$, even in the case of noise ($\delta \approx 1,8$), leads to an increase in the maximum lasing field up to values that are realized in the absence of noise.

Thus, the noise in the system forms the generation threshold. When this threshold is exceeded, even in the absence of an initiating external field, a significant part of the excited oscillators are capable of generating fields whose maximum amplitudes are comparable to the generation amplitudes in the absence of noise. Below the presented threshold, no field growth is observed.

Numerical modeling of superradiance from an ensemble of oscillators. In this section, we will take into account only the own fields of oscillators located in the same interval of the active zone. The resonator field is absent, as is the resonator itself. The dimensionless system of equations in this case takes the following form. For the superradiance field the following expression is valid:

$$\mathrm{E}_{sr}(Z,\tau) = \frac{1}{\Theta N}\sum_{s=1}^{N} \mathrm{A}_s \cdot e^{i2\pi|Z-Z_s|}, \tag{3.58}$$

and the equations of motion describing the dynamics of oscillators can be written as

$$\frac{d\,\mathrm{A}_j}{dt} = \frac{i\alpha}{2}\cdot\left|\mathrm{A}_j\right|^2 \mathrm{A}_j - \mathrm{E}(Z_j,\tau). \tag{3.59}$$

where for the general field the following expression is valid:

$$\mathrm{E}(Z,\tau) = \mathrm{E}_{sr}(Z,\tau) + \mathrm{E}_{ex}(Z,\tau), \tag{3.60}$$

moreover, the second term in (3.60) – the external initiating field, usually used in superradiance modes to accelerate the process, can be represented in the form

$$\mathrm{E}_{ex}(Z,\tau) = \mathrm{E}_{0+}\, e^{i2\pi Z} + \mathrm{E}_{0-}\, e^{-i2\pi Z}. \tag{3.61}$$

Calculations were carried out under the same initial conditions and different values of the loss coefficient Θ, defined as the ratio of the energy flow from the system to the total energy in its volume. Initial conditions: the length of the system is equal to one wavelength $Z_j \in (0;1)$; number of oscillators N=1000; $\mathrm{E}_{0\pm} = 0.01$ – constants for the external field (2.60); taking into account nonlinearity (relativism) is determined by the coefficient α =1.

The case of excitation of oscillations in the superradiance mode, the dependences on the loss coefficient Θ of the field intensity $\propto |E|^2$ (solid line), the reciprocal time (increment) $\gamma = \left(\frac{1}{|E|^2} \frac{d|E|^2}{d\tau} \right)_{\max}$ of the process (dashed-dotted line), and the rate of energy output $\Theta |E|^2$ (dashed line) are presented. In the superradiance regime, the most effective energy extraction would be around the values of Θ =2–2.5, with an output one and a half times greater than in the case of excitation of the resonator field.

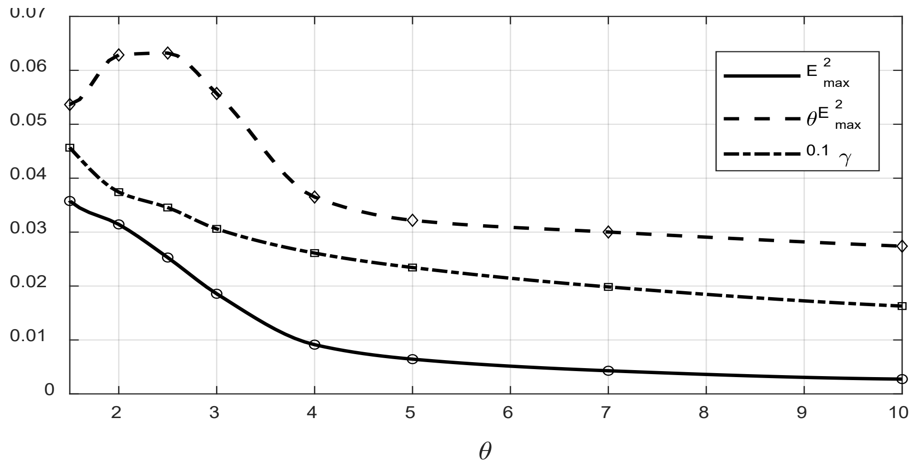

Fig. 3.7. Dependences on the loss coefficient Θ for the case of excitation in the superradiance mode: solid line – field intensity $\propto |E|^2$; dash-dotted line – reverse time $\gamma = \left(|E|^{-2} \, d|E|^2 / d\tau \right)_{\max}$ (increment) of the process; dotted line – rate of energy output $\Theta |E|^2$. Note that in all cases the maximum intensity is selected $|E|^2 \simeq |E_{\max}|^2$. *Source [33]*

Simulation of the process of excitation of the resonator field while excluding the interaction of oscillators with each other. In the traditional description of generation, all oscillators interact only with the field of the resonator in which the active zone is located. In this case, we exclude the interaction of oscillators with each other. We use equations in dimensionless form that describe this process. The resonator fields in this case can be represented as the sum of two wave processes propagating in opposite directions

$$\mathrm{E}_{wg}(Z,\tau) = \mathrm{E}_{wg+}(\tau) e^{i2\pi Z} + \mathrm{E}_{wg-}(\tau) e^{-i2\pi Z}, \qquad (3.62)$$

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Moreover, the components corresponding to the propagation of radiation in different directions can be written as

$$\frac{\partial \mathrm{E}_{wg+}}{\partial \tau} + \Theta \mathrm{E}_{wg+} = \frac{1}{N}\sum_s \mathrm{A}_s \cdot e^{-i2\pi Z_s}, \quad (3.63)$$

$$\frac{\partial \mathrm{E}_{wg-}}{\partial \tau} + \Theta \mathrm{E}_{wg-} = \frac{1}{N}\sum_s \mathrm{A}_s \cdot e^{i2\pi Z_s}. \quad (3.64)$$

At the initial moment you should set their values

$$\mathrm{E}_{wg+}(0) = \mathrm{E}_{0wg+}, \ \mathrm{E}_{wg-}(0) = \mathrm{E}_{0wg-}, \quad (3.65)$$

The equations of motion for oscillators take the form

$$\frac{d\,\mathrm{A}_j}{dt} = \frac{i\alpha}{2}\cdot\left|\mathrm{A}_j\right|^2 \mathrm{A}_j - \mathrm{E}_{wg}(Z_j,\tau). \quad (3.66)$$

Under the same initial conditions ($Z_j \in (0;1)$; N=1000; $\mathrm{E}_{0\pm} = 0.01$; α =1), calculations were carried out for different values of the loss coefficient Θ.

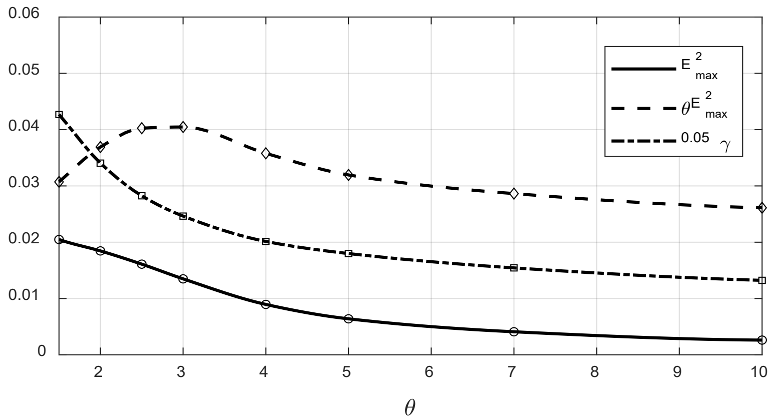

Fig.3.8. Dependences on the loss coefficient Θ for the case of excitation in the resonator field mode: solid line – maximum field intensity $\propto |E|^2$; dash-dotted line – reverse time $\gamma = \left(|E|^{-2}\, d|E|^2/d\tau\right)_{\max}$ (increment) of the process; dotted line – rate of energy output $\Theta|E|^2$. Note that in all cases, the maximum intensity is selected for each moment in time $|E|^2 \simeq |E_{\max}|^2$. Source [49]

For this case of excitation of the resonator field without taking into account the own fields of the oscillators, the field intensity $\propto |E|^2$ (solid line), the increment

$\gamma = \left(\frac{1}{|E|^2} \frac{d|E|^2}{d\tau} \right)_{\max}$ of the process (dashed-dotted line), and the rate of energy output $\Theta |E|^2$ (dotted line) are presented in Fig.3.8. If we focus on the excitation mode of the resonator field without taking into account the oscillator's own fields, then the most effective energy extraction would occur in the region of about Θ=2.5–3 [37].

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KAPITEL 3 / *CHAPTER 3*
DIFFERENT GENERATION MODES

3.1. Generation in a system of oscillators under pumping conditions

System of superradiance equations taking into account pumping. In the previous section, the relaxation of a system of excited oscillators in different regimes (Cauchy problem) was discussed. However, of no less interest are modes with constant energy pumping into the system to ensure the process of continuous generation of the integral field of a system of oscillators in the superradiance mode. Let us consider the development of such generation modes with constant energy pumping, namely, with the restoration of a certain value of the amplitude of the oscillators [50].

The superradiance mode is more sensitive to the nature of the spatiotemporal external influence. Therefore, the influence on the nature of generation of various pumping methods is more pronounced and makes it possible to understand both the mechanisms and the consequences of this influence.

Therefore, below we will limit ourselves to studying the superradiance regime of classical oscillators with the addition of a pumping mechanism (the last term of the second equation):

$$\mathrm{E}(\mathrm{Z},\tau)=\frac{2}{N}\sum_{s=1}^{N}\mathrm{A}_s\cdot e^{i2\pi|\mathrm{Z}-Z_s|}, \qquad (4.1)$$

$$\frac{d\,\mathrm{A}_j}{d\tau}=\frac{i\alpha}{2}\cdot\left|\mathrm{A}_j\right|^2\mathrm{A}_j-\frac{1}{N}\sum_{s=1}^{N}\mathrm{A}_s\cdot e^{i2\pi|\mathrm{Z}-Z_s|}-\mathrm{E}_0\cdot e^{2\pi i Z_j}+\gamma_p\left(\left|\mathrm{A}_0\right|e^{i\cdot\delta_j\tau}-\mathrm{A}_j\right) \qquad (4.2)$$

where $\gamma=\gamma_0^2/\delta_D=\pi e^2M/mc$; $\mathrm{E}=eE/m\omega\gamma a_0$, $\mathrm{A}=A/a_0$; $k_0z=2\pi Z$; $\alpha=3k_0^2a_0^2\omega/4\gamma$; $\tau=\gamma t$; $\gamma_0^2=\pi e^2n_0/m=\omega_{pe}^2/4$; $M=b\cdot n_0$; b is the length of the space under consideration in the longitudinal direction, n_0 is the density of particles per unit volume, E_0 is the amplitude of the external field running in the positive direction of the Z axis. Pumping is represented in (4.2) by the term $\gamma_p\left(\left|\mathrm{A}_0\right|e^{i\cdot\delta_j\tau}-\mathrm{A}_j\right)$

, where γ_p is the pumping rate, here the pumping amplitude is chosen equal to the initial value of the oscillator amplitudes $|A_0|$, and $\delta_{j\tau}$ – varying from zero to 2π pumping phase. The phase in general can vary randomly in time and space.

Random distribution of the pump phase in space. Let the pumping phase $\delta_{j\tau}$ be randomly distributed along the system along the OX axis, which does not change with time. Changes in the pumping phase for fixed points in space are selected from zero to 2π and do not change in time. Calculations were carried out for this case at different pumping rates γ_p. Let's choose the following parameters. Number of particles N=300, nonlinearity parameter $\alpha = 1$, amplitude of the external field initiating the superradiance process, E_0=0.05. Let at the initial moment of time the oscillators be uniformly distributed along the system, the magnitudes of the amplitudes of the oscillators are equal to unity $|A_j(0)| = |A_0| = 1$, their phases $\psi_j(0)$ have random values in the range $0 - 2\pi$ ($A_j = |A_j(\tau)| e^{i\psi_j(\tau)}$).

In Fig. 4.1 shows the time dependence of the average energy of the oscillators $|A|^2_{ev} = \frac{1}{N}\sum |A_j|^2$ (Fig. 4.1a) and the field at the left and right ends of the system, as well as the maximum field in the system (Fig. 4b) in the case of no pumping ($\gamma_p = 0$).

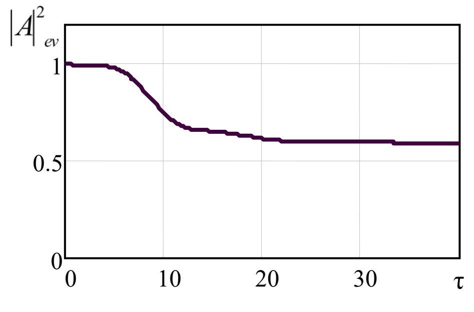

Fig. 4.1a. Time dependence of the average energy of oscillators in the absence of pumping *Source [50-53]*

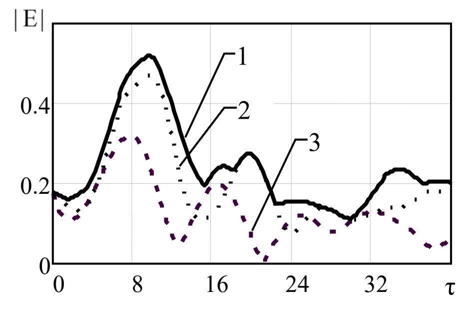

Fig. 4.1b. Dependence of the field modulus on time in the absence of pumping: 1 – maximum field in the system, 2 – right end of the system, 3 – at the left end of the system *Source [50-53]*

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As can be seen from Fig. 4.1, due to the synchronization of the oscillators, a surge in the field is observed and then a slow process of damping of the oscillations. In Fig. 4.2 and Fig. 4.3 show the same dependences at low ($\gamma_p = 0.1$) and high ($\gamma_p = 1$) pumping rates.

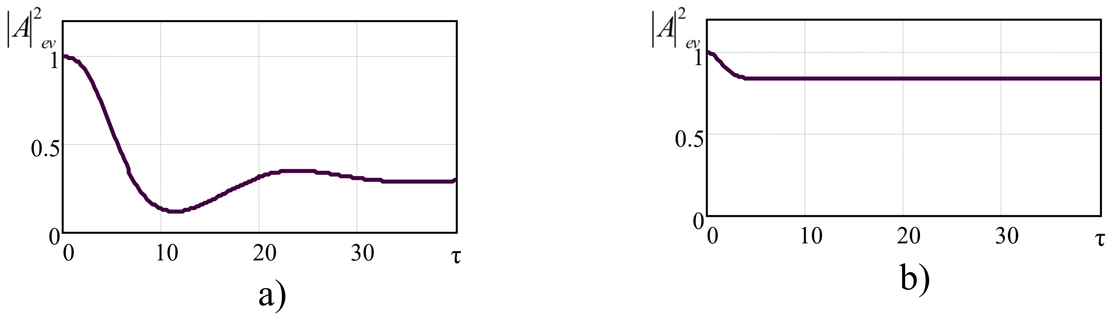

Fig4.2. Time dependence of the average energy of oscillators at different pumping rates: a) $\gamma_p = 0.1$, **б)** *Source [50-53]*

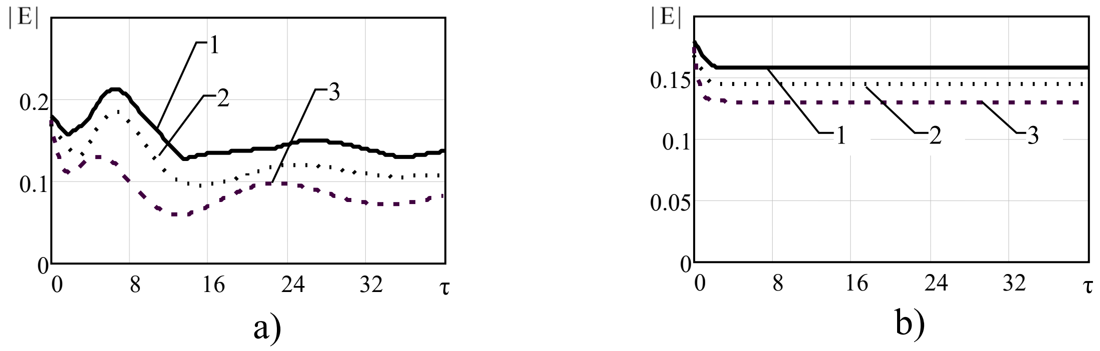

Fig 4.3. Time dependence of the field modulus at different pumping rates: maximum field in the system – 1, field at the right end of the system – 2, field at the left end of the system – 3. Pumping rate: a) $\gamma_p = 0.1$ **б)** $\gamma_p = 1$
Source [50-53]

From Fig. 4.2 and Fig. 4.3 one can see that pumping with random phases that do not change in time along the system prevents the synchronization of the oscillators, as a result of which the field value remains at the "noise" level. At the same time, at a low pumping rate, a more rapid decrease in the average amplitude of the oscillators is observed (Fig. 4.2a) than without pumping (Fig. 4.1a). At a higher pump level, the average amplitude stabilizes at a fairly high level, but, as already noted, due to the suppression of synchronization, this does not lead to an increase in the field.

The pumping phase is constant in space and random in time. It is useful for generality to discuss the case where the pump phase is constant along the system but varies randomly with time. Let the pumping phase $\delta_{j\tau}$ be constant along the system (and that means at any moment in time it is the same for all oscillators) but randomly varies with time in the range from zero to 2π . Calculations were carried out for this case at different pumping rates γ and different periods of random phase changes Δτ.

Fig. 4.4 shows the time dependence of the average energy of the oscillators at a low (γ_p=0.1) pumping rate and different periods of random changes in the pumping phase Δτ. In Fig. 4.5 for the same conditions the fields in the system are presented.

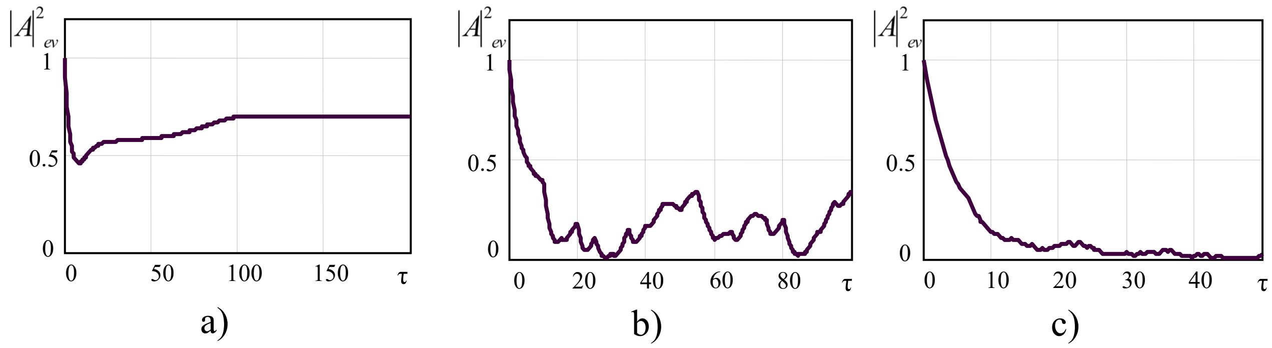

Fig. 4.4. Time dependence of the average energy of oscillators at different periods of pump phase change: a) Δτ = ∞, b) Δτ = 5, c) Δτ = 0.5. *Source [50-53]*

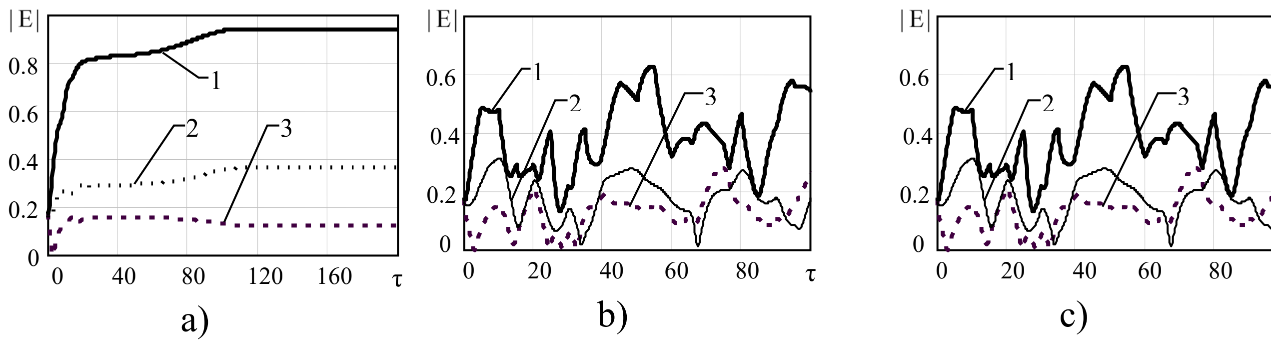

Fig.4.5. Time dependence of the field modulus at and different periods of pump phase change: maximum field in the system – 1, field at the right end of the system – 2, field at the left end of the system – 3. Period of pump phase change: a) Δτ = ∞, b) Δτ = 5, c) Δτ = 0.5. *Source [50-53]*

When the pumping phase is constant both in time and in space (see Fig. 4.4 a and Fig. 4.5 a), the system reaches a stationary mode (τ > 100) with a fairly high level of average amplitude and maximum field. The field level indicates the emerging effective phase synchronization of individual oscillators. An increase in the frequency of phase changes ($\Delta\tau^{-1}$) leads to a noticeable stochastization of processes with a decrease in the

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average amplitude of the oscillators and field in the system (see Fig. 4.4 b, c and Fig. 4.5 b, c). Increasing the pumping rate γ_p does not change the qualitative picture of the process. Thus, for constant pumping ($\Delta\tau = \infty$), an exit to a stationary regime is observed, but in a shorter time and at high levels of average amplitude and field ($\tau > 5$ at $\gamma_p = 1$). Acceleration of the process of stochastization of the system in the modes of changing the pumping phase and increasing the frequency of phase changes ($\Delta\tau^{-1}$) also occurs with increasing pumping rate γ_p. The selected pump control mechanism limits the growth of oscillator amplitudes to a certain fixed value. The most important thing here is the change in the pumping phase. Modeling of these processes is quite complex, so only certain modes of pump phase change were selected. However, by considering these regimes of pump phase shifts, it is quite possible to get an idea of how a change in the pump phase affects the efficiency of field generation.

Inhomogeneity of the pumping phase. In the first mode, the phase at each point along the length of the system is different and, generally speaking, randomly selected, but there are no phase changes over time. In the case of a low injection rate of pump energy (γ_p<0.1), the characteristic time of which is much less than the characteristic time of development of the generation process, the field amplitudes in the system at a low pump injection rate (Fig. 4.3a) turn out to be half as large as in the absence of pumping (Fig. 4.1. b). The high injection rate (γ_p=1) keeps the amplitudes of the oscillators quite large, but the amplitude of the excited field decreases somewhat (see Fig. 4.3 b) compared to the previous version of the low pump injection rate (Fig. 4.3a). With a further increase in the pump injection rate (γ_p>1), the behavior of the system does not change qualitatively.

Slightly different dynamics are observed when the phase of the pump field is constant along the system, but varies randomly with time (Fig. 4.4. b, c and Fig. 4.5. b, c). Let us compare these regimes with the case when changes in time of the pumping phase do not occur at all (Fig. 4.4.a and Fig. 4.5.a). In this case, it can be seen that the

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amplitudes of the oscillators remain significant and the field, at least inside the system, reaches values that are almost twice as large as in the case of no pumping (Fig. 4.1). However, these significant fields are observed only inside the system, and at its ends the field is several times smaller. In particular, if changes in the pump phase occur quite rarely, that is, the time of these changes is many times greater than the characteristic time of the generation process, then significant values of the fields inside the system can be observed (Fig. 4.5.b). At the ends of the system, there is still a tendency for a noticeable decrease in the field amplitude. With frequent changes in the pump phase (Fig. 4.5 c), when the period of change is less than the characteristic generation time, the generation efficiency drops, and the generation mode takes on a characteristic multi-periodic oscillatory appearance. The period of noticeable changes in the field at the ends of the system is an order of magnitude greater than the characteristic generation time. In this case, small quasiperiodic field oscillations are also observed inside the system with a period close to the period of change in the pump phase. Thus, the most effective pumping method remains the mode with a slow input of energy into the system ($\gamma_p < 0.1$), and the nature of the field phase distribution along the system can be either random or with a certain spatial period, but weakly changing with time.

3.2. Generation of oscillations by moving oscillators

System of equations taking into account the movement of oscillators under the influence of HF field pressure [50]. In the previous sections, equations for the slow change in the amplitude of oscillators and their total field are given if the oscillators are stationary. Let's consider the same system of oscillators, but take into account that in addition to the electric field $E_x(z,t)$, which directly affects the oscillator oscillations, there is also a magnetic radiation field $H_y(z,t)$, taking into account which is necessary to determine the Lorentz force. The movement of the oscillators occurs under the influence of the Lorentz force F_z along the longitudinal axis of the system. Let us limit ourselves to considering only the most interesting superradiance regime. In this case, obviously, the system of oscillators is located in a limited space. But oscillators can be pushed by fields beyond the boundaries of the original area.

$$\frac{\partial E_x}{\partial z} = -\frac{1}{c}\frac{\partial H_y}{\partial t} = \frac{i\omega}{c}H_y, \quad H_y = \frac{c}{i\omega}\frac{\partial E_x}{\partial z}, \quad F_z = -e\frac{v_x}{c}H_y. \tag{4.3}$$

The electric field of the oscillator (electron) at a point z_0 is equal to

$$E_x = 2\pi\cdot e\cdot A\cdot\omega\cdot c^{-1}\cdot e^{-i\omega t}\left(e^{ik(z-z_0)}\cdot\theta(z-z_0)+e^{-ik(z-z_0)}\cdot\theta(z_0-z)\right) = 2\pi\cdot e\cdot A\cdot\omega\cdot c^{-1}\cdot e^{-i\omega t}\cdot e^{ik|z-z_0|}$$

Therefore, the magnetic field of this oscillator is equal to

$$H_y = \frac{c}{i\omega}\frac{\partial E_x}{\partial z} = 2\pi\cdot e\cdot A\cdot\omega\cdot c^{-1}\cdot e^{-i\omega t}\left(e^{ik(z-z_0)}\cdot\theta(z-z_0)-e^{-ik(z-z_0)}\cdot\theta(z_0-z)\right) \tag{4.4}$$

and for a system of oscillators we obtain the magnetic field of the system

$$H_y(z,t) = \frac{2\pi\cdot e\cdot\omega\cdot M}{c}\cdot e^{-i\omega t}\cdot\frac{1}{N}\sum_{s=1}^{N}A_s\left(e^{ik(z-z_s)}\cdot\theta(z-z_s)-e^{-ik(z-z_s)}\cdot\theta(z_s-z)\right) \tag{4.5}$$

and the force acting on the oscillator at the point z_j

$$F_{zj} = -\pi\cdot e^2\cdot k^2\cdot M\cdot\mathrm{Re}\left[A_j^*\cdot\frac{1}{N}\sum_{s=1}^{N}A_s\left(e^{ik(z_j-z_s)}\cdot\theta(z_j-z_s)-e^{-ik(z_j-z_s)}\cdot\theta(z_s-z_j)\right)\right] \tag{4.6}$$

Unlike previous cases, the longitudinal coordinates of the oscillators change. The equations of motion of oscillators under the influence of force (4.6) have the form

$$\frac{dv_{zj}}{dt} = -\frac{\pi\cdot e^2\cdot k^2\cdot M}{m_1}\mathrm{Re}\left[A_j^*\cdot\frac{1}{N}\sum_{s=1}^{N}A_s\left(e^{ik(z_j-z_s)}\cdot\theta(z_j-z_s)-e^{-ik(z_j-z_s)}\cdot\theta(z_s-z_j)\right)\right] \tag{4.7}$$

$$\frac{dz_j}{dt} = v_{zj} \tag{4.8}$$

The equation describing longitudinal motion, together with the equation for changing the amplitude of the oscillator, complements the system of equations for describing modes with radiation of a system of moving oscillators.

In dimensionless form, the system of equations takes the form

$$\begin{aligned}\frac{d\,\mathrm{A}_j}{d\tau} &= \frac{i\alpha}{2}\cdot\left|\mathrm{A}_j\right|^2\mathrm{A}_j-\frac{1}{N}\sum_{s=1}^{N}\mathrm{A}_s\cdot e^{i2\pi|Z_j-Z_s|}-\mathrm{E}_0\cdot e^{2\pi iZ_j} = \\ &= \frac{i\alpha}{2}\cdot\left|\mathrm{A}_j\right|^2\mathrm{A}_j-\frac{1}{2}\mathrm{E}(Z_j,\tau)-\mathrm{E}_0\cdot e^{2\pi iZ_j},\end{aligned} \tag{4.9}$$

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$$\frac{dV_{Zj}}{d\tau} = -\beta \cdot \frac{4\alpha}{3} \cdot \mathrm{Re}\left(\mathrm{A}_j^* \cdot \frac{1}{N} \sum_s \mathrm{A}_s \cdot e^{i2\pi|Z_j - Z_s|} \cdot sign(Z_j - Z_s) \right), \tag{4.10}$$

$$\frac{dZ_j}{d\tau} = \frac{1}{2\pi} V_{Zj}. \tag{4.11}$$

Here $\gamma = \gamma_0^2 / \delta_D = \pi e^2 M / mc;\ \mathrm{E} = eE / m\omega\gamma a_0$, $\mathrm{A} = A / a_0;\ k_0 z = 2\pi Z;\ \tau = \gamma t;$ $\gamma_0^2 = \pi e^2 n_0 / m = \omega_{pe}^2 / 4$, $\alpha = 3k_0^2 a_0^2 \omega / 4\gamma$, $\beta = m/m_1$, $M = b \cdot n_0$, b is the length of the space under consideration in the longitudinal direction, n_0 is the density of particles per unit volume, m, m_1 are the masses of the electron and the oscillator, respectively, E_0 is the amplitude of the external field propagating in the positive direction of the Z axis.

The electric field strength of the oscillator radiation in dimensionless units is written by the expression

$$\mathrm{E}_x(Z,\tau) = \frac{2}{N} \sum_{s=1}^{N} \mathrm{A}_s \exp\{i2\pi \,|\, Z - Z_s \,|\}. \tag{4.12}$$

Results of numerical calculations [50]. In the previous sections, calculations were carried out in the case of stationary oscillators. For system (4.9)–(4.12), similar calculations (fixed oscillators) were previously carried out at N=3600, N=10000, N=20000. The results were practically the same, so further all calculations were carried out at N=10000.

The following parameters were selected. Number of particles N =10000, $\alpha = 1$ nonlinearity parameter, amplitude of the additional external field initiating the superradiance process, E_0=0.02. At the initial moment of time, the oscillators are uniformly distributed along the system, their speed is equal to zero $V_{zj}(0) = 0$, the amplitude modulus of the oscillators $|\mathrm{A}_j(0)| = 1$, and their phases ψ_j have random values in the range $(-\pi,\pi)$. additional external field initiating the process of superradiation E_0=0.02. The parameter β (the ratio of the charge mass to the total mass of the oscillator) took values of 0, 0.1, 0.5, 1. Larger values of the parameter correspond to lighter and more mobile oscillators.

Figure 4.6 (a, b, c, d) shows the time dependence of the absolute value of the maximum field value in the system and the field at the edges of the system (Z=0 and Z=1) and for different values of the parameter β.

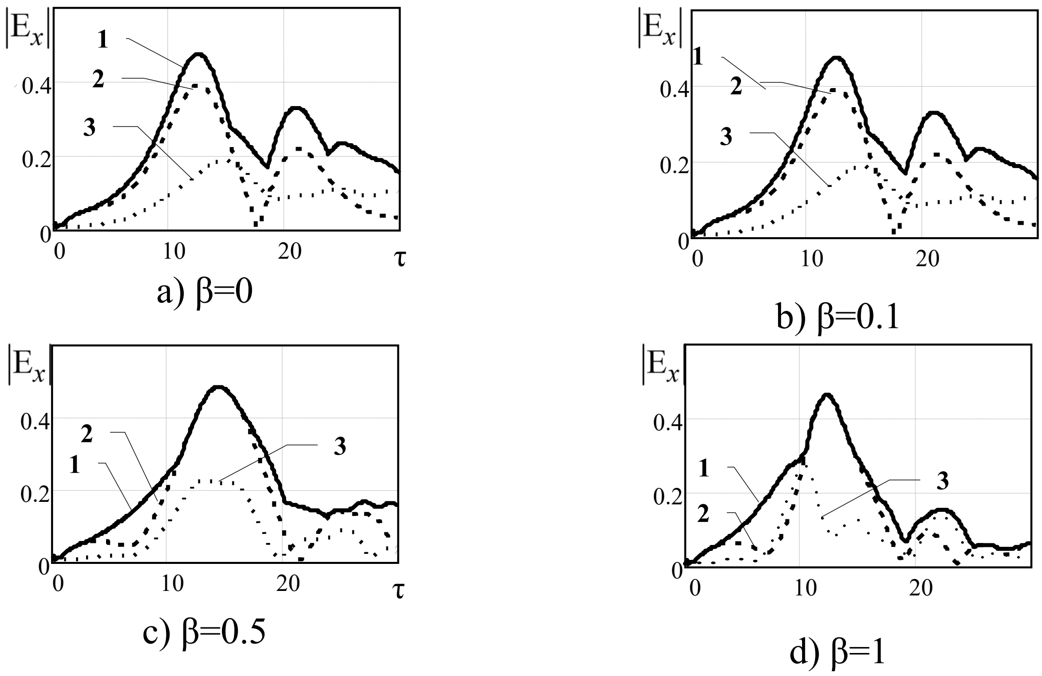

Fig. 4.6. Dependence of the field modulus in different parts of the system on time τ: 1– $\max|E_x|$**, 2 –** $|E_x(Z=1)|$**, 3 –** $|E_x(Z=0)|$ *Source [50-53]*

As can be seen from these figures, taking into account the motion of the oscillators does not affect the maximum field value in the system. At β=0 (the oscillators are stationary) and at β=0.1 (the oscillators are massive and inactive), the field maximum is observed inside the system, which was observed in the previous sections. But at β=0.5 and β=1, the field maximum is observed at the end of the system (Z=1).

Since the energy output from the system is determined by the field value at the ends, in the case when the maximum amplitude is achieved at the end, the radiation efficiency is greatest. The change in efficiency can be assessed by the ratio of the squares of the field amplitude at the end to the corresponding value at the maximum. For β=0 and β=0.1 this ratio is equal to 0.66, for β=0.5 and β=1 this ratio is obviously equal to 1.

In Fig. 4.7 and fig. Fig. 4.8 shows the distribution in space of velocities and amplitude moduli for all particles at β=1 at the moment the field reaches its maximum τ=12.

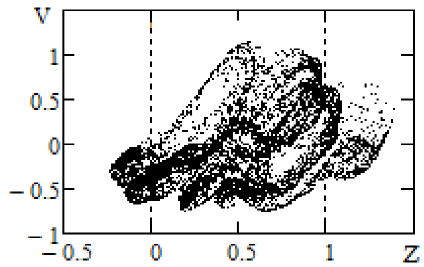

Fig.4.7. Distribution in space of velocities of oscillators V at β=1 at the moment τ=12. *Source [50-53]*

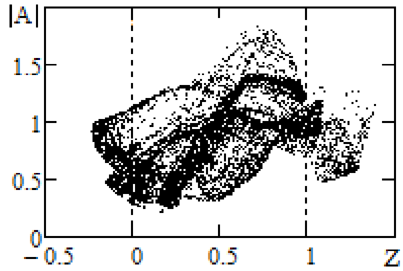

Fig.4.8. Distribution of oscillator amplitudes |E| in space at β=1 at the moment τ=12. *Source [50-53]*

As can be seen from the figures, particles come out from both ends of the limited region we have chosen. Moreover, particles fly out to a greater distance in the direction of movement of the external field (the wave vector of the external field is oriented in the direction of the Z axis) from the end of the system (Z>1), where the field is maximum, and the particles emitted at the beginning of the system (Z<0) are located more compact. By the time the field reaches its maximum τ=12, the coordinate is Z<0 for 15% of particles and Z>1 for 10% of particles. Those. a larger number of particles flew out through the beginning of the system against the external field. There is an expansion of the area occupied by the oscillators and its shift. In Fig. 4.9 and fig. Figure 4.10 shows the dynamics over time of the expansion of oscillators

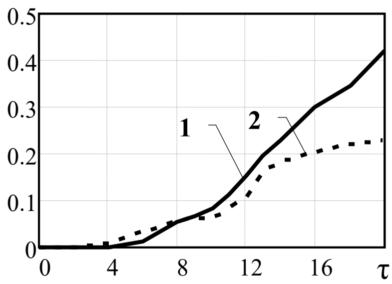

Fig.4.9. Dependence on time of the fraction of particles escaping through the ends of the system, 1 – escaping through the beginning of the system (Z<0), 2 – escaping through the end of the system (Z>1) *Source [50-53]*

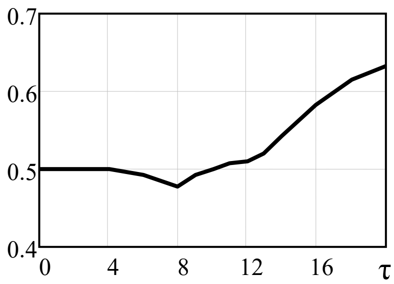

Fig. 4.10. Time dependence of the fraction of particles in the first half of the system (Z<0.5).

Source [50-53]

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From the figures it follows that until the moment τ=8 the proportion of particles in the first half of the system decreases (Fig. 4.9) and a larger number of particles leave through the end of the system, the particles shift as a whole in the direction of movement of the external field. But from the moment τ=8 the dynamics become opposite, the system of particles, continuing to expand, shifts towards the beginning against the direction of the external field. It is important to note that in the units considered, the average amplitude of spontaneous emission of 10,000 oscillators is approximately 0.01. On the other hand, the maximum amplitude of the stimulated emission of these oscillators, the phases of which were completely correlated, would reach unity. Therefore, the amplitude of the initiating field was chosen to be only twice the level of spontaneous emission (E_0=0.02). The achievable field amplitude in the system is approximately 0.5. That is, the same trend is observed - the degree of coherence of radiation is about 0.25. In other words, the energy density of the initiating field is four times higher than the corresponding spontaneous level of radiation of oscillators with a random phase distribution. The achieved radiation level, in turn, is four times less than the maximum energy density of all these oscillators, if their phases were completely correlated.

Mode of constant pumping of energy into the system due to particle injection. Let us consider the process of constant injection of particles from the left side of the resonator and the removal of a corresponding number of them from the right side. In this way, a mode of constant pumping of energy into the system is implemented. Let us recall that the electromagnetic field in the system considered in Section 3 is actually the sum of the fields of individual oscillators and can be interpreted as a superradiance field. A constant speed is set - the coordinates change.

$$\frac{dZ_i}{d\tau} = V_0 \rightarrow Z_i = Z_{0i} + V_0\tau \quad , \tag{4.13}$$

here the phase of the particles changes, but grouping by coordinate does not occur - the distance between the particles remains constant. The effects of field reflection from the ends of the system were not taken into account; the system is completely open.

Calculations were carried out with the following parameters:

$N = 900$, $\alpha = 1$, $\theta = 1$, $A_0 = 1$, $V_0 = 0; 0.1; 0.15; 0.2$.

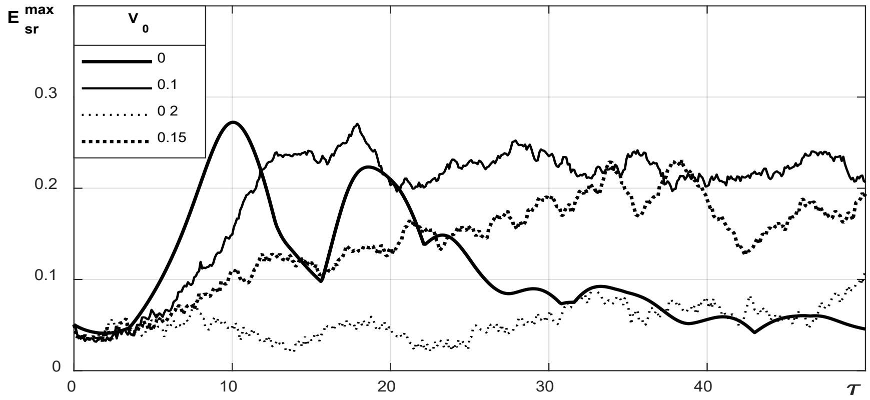

Fig.4.11. Maximum field amplitude for each moment of time in superradiance modes at different oscillator flow rates. *Source [50-53]*

At zero speed, due to the depletion of the energy of the oscillators that do not leave the active zone, the field amplitude naturally decreases with time. At a speed equal to $V_0 = 0.1$ oscillators pass through the active zone in a time $\Delta\tau(0) = 10$ where previously at zero speed at $\tau = 10$ field maximum was formed. However, at $V_0 = 0.1$ the maximum of the field shifts to $\Delta\tau(0.1) \le 20$. At higher speed $V_0 = 0.15$, the maximum of the superradiance field is achieved at longer times $\Delta\tau(0.15) \ge 30$. A further increase in particle velocity does not lead to a noticeable increase in the field in the time interval under consideration. It is important to note that the maximum values of superradiance fields in this interval of changes in particle shear rates $0 \le V \le 1.5$ practically do not change.

3.3. About formation of a resonator field due to reflection effects

On the mechanism of formation of the resonator field. In this section, in addition to particle injection, we will additionally take into account the reflection of the field from the ends of the systems. Previously, two generation modes were considered, and paying tribute to traditions, the first of them excited a resonator field, the shape of which was predetermined (for example, by boundary conditions, although these boundary conditions were not specifically taken into account in the calculation process). It is important that the oscillators of the active zone interacted only with the cavity field. The second mode is when there is no resonator field, and the particles interact only with each other, creating their own total field - the superradiance field. Both of these cases are quite artificial, because in each of them some interactions are excluded. Below we will consider a more realistic case, when most likely the total field of all particles in the active zone first arises, that is, the superradiance field, and then, due to reflections from the ends of the system, a resonator field is formed. It is important that this field is not given, but is the result of a natural process. It is clear that the amplitude of the resonator field may be different, depending on the reflection conditions and the choice of the velocity of the injected particles.

The resonator field in this one-dimensional model can be the result of reflection of radiation from the ends of the system, and it is difficult to isolate this field from the total total radiation field of the system of oscillators. The resonator field can be formed as a result of reflection from the ends, and in this case it will contain two waves propagating in opposite directions. With a sufficiently significant reflection coefficient, the amplitudes of these waves are approximately equal, so the resonator field is a standing wave. We will consider the field reflection from the ends of the system is determined by the linear reflection coefficients. The total radiation field of all particles is equal (cf. (3.26)):

$$\mathrm{E}_{sr}(Z,\tau)=\frac{1}{\Theta\cdot N}\sum_{s=1}^{N}\mathrm{A}_s(\tau)\cdot e^{i2\pi|Z-Z_s|}, \tag{4.14}$$

in this case, the speed of particle-oscillators will be considered constant

$$\frac{dZ_j}{d\tau}=V_0=const, \tag{4.15}$$

moreover, at the initial moment, the particles are distributed uniformly from Z=0 to Z=1, have a speed V_0 are identical in amplitude A_0, but have random phases. The oscillators move at a constant speed V_0 in the direction from Z=0 to Z=1. When the oscillators are removed outside the resonator (Z>1), they are replaced by new ones on the other side of the resonator Z=0, with speed V_0, amplitude modulus A_0, and a random arbitrary phase. Obviously, the waveguide field (here is the resonator field), subject to noticeable reflection, will have the form

$$E_w(Z,\tau)=E_+(\tau)e^{2\pi iZ}+E_-(\tau)e^{-2\pi iZ} \tag{4.16}$$

Thus, the total field in the resonator can be represented as ,

$$E(Z,\tau)=E_+(\tau)e^{2\pi iZ}+E_-(\tau)e^{-2\pi iZ}+E_{sr}(Z,\tau), \tag{4.17}$$

Here $E_{sr}(Z,\tau)$ is the summary radiation field of the oscillators (4.14), the field reflected from the left end is $E_+(\tau)e^{2\pi iZ}$, reflected from the right end is $E_-(\tau)e^{-2\pi iZ}$. It is total field (4.17) that acts on the particles. However, these waves (4.16), the amplitudes of which are formed as a result of reflections from the ends, change under the influence of the fields of moving oscillators when passing through the active zone. Therefore, the effect of oscillators in the active zone on these reflected waves should be taken into account. In this case, the resonator field will already consist of two waves traveling in different directions, where terms are added to the amplitude of the reflected waves at the boundary, qualitatively taking into account the effect of oscillators in the resonator volume on these waves

$$E_+\to E_++0.5\,E_{wg+},\quad E_-\to E_-+0.5\,E_{wg-} \tag{4.18}$$

The resonator field in the active zone is $E_{wg}(Z)=0.5\cdot(E_{wg+}\cdot e^{2\pi iZ}+E_{wg-}\cdot e^{-2\pi iZ})$, where $E_{wg+}=\frac{1}{\Theta N}\sum_s A_s\cdot e^{-i2\pi Z_s},\quad E_{wg-}=\frac{1}{\Theta N}\sum_s A_s\cdot e^{i2\pi Z_s}$. Here we will assume that the condition $\frac{\partial E_\pm}{\partial\tau}<\Theta E_\pm$ is satisfied (see (3.63)-(3.64)). The total field acting on the oscillators in the resonator (4.17) will change taking into account these replacements (4.18).

To find the amplitudes of two differently directed waves (4.16), we will follow the following simplified procedure. The amplitudes of the reflected waves are found from the conditions of reflection at the ends of the resonator based on the condition for the amplitudes on the left (Z=0) and right (Z=1) sides of the resonator of the incident

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and reflected wave: $E_{\rightarrow}/E_{\leftarrow}|_{Z=0,1}=-r_{l,r}$, r_l, r_r, - reflection coefficients at the left and right ends. First, we determine the total radiation field of particle-oscillators at the ends of the system. Obviously, at Z=0 and at Z=1, respectively

$$E_{sr}(Z=0,\tau)=\frac{1}{\Theta\cdot N}\sum_{s=1}^{N}A_s\cdot e^{i2\pi Z_s} \qquad E_{sr}(Z=1,\tau)=\frac{1}{\Theta\cdot N}\sum_{s=1}^{N}A_s\cdot e^{-i2\pi Z_s}$$

(4.19)

Let's consider the fields at the ends of the resonator Z=0 and Z=1

$$E_{+}(\tau)|_{Z=0}=-r_l\left(E_{-}(\tau)|_{Z=0}+E_{sr}(Z=0,\tau)\right)$$

(4.20)

$$E_{-}(\tau)|_{Z=1}=-r_r\left(E_{+}(\tau)|_{Z=1}+E_{sr}(Z=1,\tau)\right)$$

(4.21)

Where $r_l;r_r$ are the reflection coefficients, respectively, at the ends Z=0 and Z=1, from where we obtain the values of the amplitudes of the waves that make up the standing wave

$$E_{+}=\frac{r_l\left(r_r\,E_{sr}(Z=1,\tau)-E_{sr}(Z=0,\tau)\right)}{(1-r_l\cdot r_r)},$$

$$E_{-}=\frac{r_r\left(r_l\,E_{sr}(Z=0,\tau)-E_{sr}(Z=1,\tau)\right)}{(1-r_l\cdot r_r)}$$

(4.22)

If we take into account the influence of oscillators in the resonator volume (4.18) on the reflected waves, then in formulas (4.22) we need to make substitutions $E_{sr}(Z=1,\tau)\rightarrow E_{sr}(Z=1,\tau)+0.5\,E_{wg+}$ and $E_{sr}(Z=0,\tau)\rightarrow E_{sr}(Z=0,\tau)+0.5\,E_{wg-}$

Numerical modeling of the formation of a resonator field. The influence of oscillator speed. Let us first consider how the formation of the resonator field is affected by the drift speed of oscillators, which are injected from the left side and the same amount is removed from the right side of the resonator. The nature of the reflection will be considered unchanged. The reflection of the field from the ends is determined by relations (4.20) and (4.21). Field amplitudes (4.22) are updated after each calculation cycle.

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The calculations, the results of which are shown in Fig. 4.12-4.15, were carried out with the following parameters: $N = 900$, $\alpha = 1,\ \theta = 1$, $A_0 = 1$, reflection coefficients $r_L = 1,\ r_R = 0.8$. From Fig. 4.12. ($V_0 = 0.02$) it is clear that the appearance of the resonator field occurs after the formation of the total field of oscillators

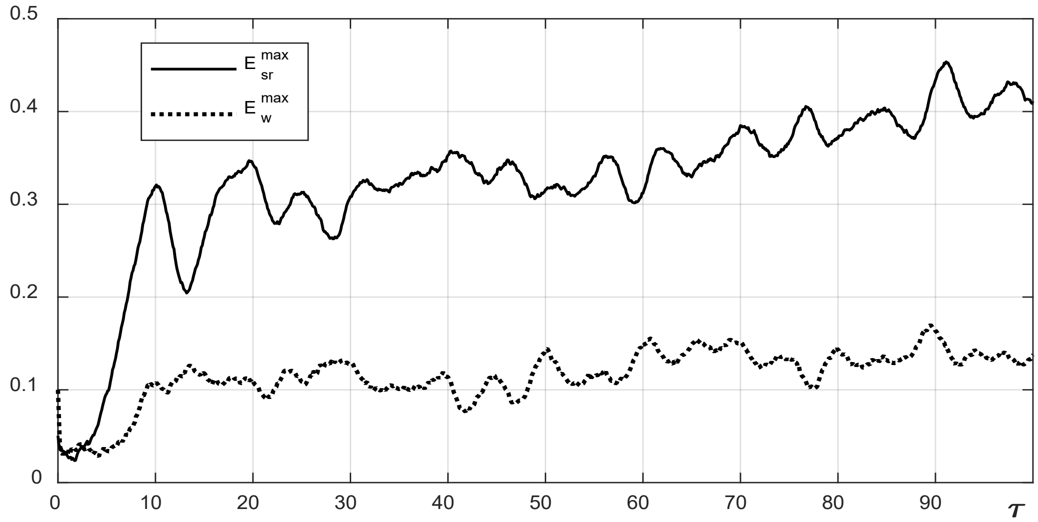

Fig. 4.12 Behavior of the maximum field amplitudes for each moment of time: the radiation field of the oscillators (solid curve) and the resonator field, which owes its existence to the processes of reflection from the ends ($V_0 = 0.02$**,** $r_L = 1,\ r_R = 0.8$**)**

The structure of the fields is shown in Fig. 4.13. It can be seen that the resonator field is a standing wave, and the total field has a maximum at the center of the resonator.

At a low drift speed of the oscillators, the standing wave formed due to reflection effects - the resonator field - has a noticeably lower amplitude than the total field of the oscillators. however, the situation changes if the rate of replacement of spent oscillators in the system is accelerated due to their removal from the system with an increase in the constant flow rate $V_0 = 0.15$. In this case, the amplitude of the resonant mode turns out to be comparable to the amplitude of the radiation field of the oscillators (in fact, the superradiance field).

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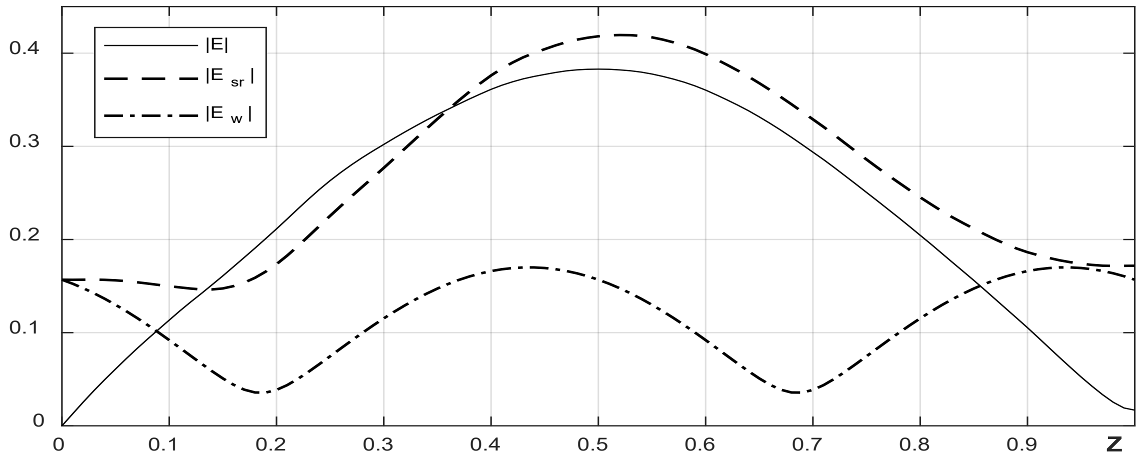

Fig. 4.13. Amplitude at the moment of time of the total radiation field of the oscillators (dashed line) and the resonator field (dashed-dotted line). The solid line is the sum of the oscillator field and the resonator field ($V_0 = 0.02$ **,** $r_L = 1,\ r_R = 0.8$ **)**

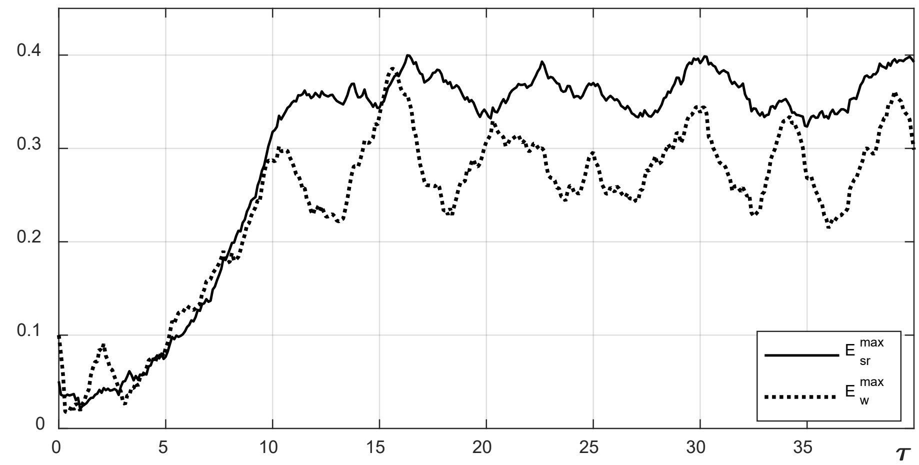

Fig. 4.14. Comparison of the maximum field amplitudes in the system for each moment in time: the radiation field of the oscillators (actually the superradiance fields - solid line) and the resonator field ($V_0 = 0.15\ r_L = 1,\ r_R = 0.8$ **)**

In the mode of constant pumping of energy into the system due to the injection of excited oscillators, the speed of which in the active zone is $V_0 = 0.15$ almost an order of magnitude greater than in the previous case, with sufficiently large reflection coefficients, the resonator field is comparable to the radiation field of the oscillators, which practically corresponds to the superradiance field. A further increase in the flow rate of oscillators, which corresponds to an increase in the constant pumping of energy into the system, leads to the dominance of the waveguide field.

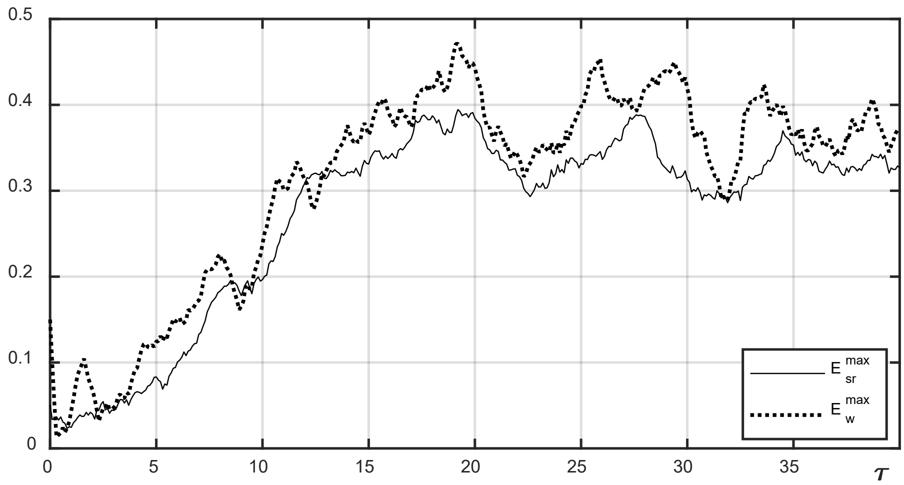

Fig.4.15. Comparison of the maximum field amplitudes in the system for each moment in time: Dominance of the waveguide field formed due to reflection effects in comparison with the particle radiation field (solid line)

$V_0 = 0.2$, $r_L = 1,\ r_R = 0.8$

Thus, with the same character of reflection from the ends, acceleration of the input of new oscillators from the left edge of the resonator and an increase in their drift speed leads to an increase in the resonator field.

Modeling of the formation of a resonator field with a change in the injection rate and reflection conditions. The calculations were performed using constant parameters N=2500, α = θ = 1, rL = 1 (total reflection at the left edge). At the right edge r_R in the region of the left end, particles with a random phase were injected. After the quasi-stationary mode (τ>50) was established, the averaged values over the waveguide volume and time were calculated. $\mathrm{E}_{sr}{}^2 = <(E_{sr}{}^2)_{av}>_t$, $\mathrm{E}_w{}^2 = <(E_w{}^2)_{av}>_t \equiv < \mathrm{E}_+(\tau)e^{2\pi iZ} + \mathrm{E}_-(\tau)e^{-2\pi iZ} >_{av}{}^2$, $K = \mathrm{E}_w{}^2 / \mathrm{E}_{sr}{}^2$

The following operating modes can be observed: 1 – dominance of the oscillators' own field generation - superradiance mode ($\mathrm{E}_w{}^2 < \mathrm{E}_{sr}{}^2,\quad K = \mathrm{E}_w{}^2/\mathrm{E}_{sr}{}^2 < 1$), 2 – generation of the resonator field ($\mathrm{E}_w{}^2 > \mathrm{E}_{sr}{}^2,\quad K = \mathrm{E}_w{}^2/\mathrm{E}_{sr}{}^2 > 1$), 3 – absence (collapse) of generation.

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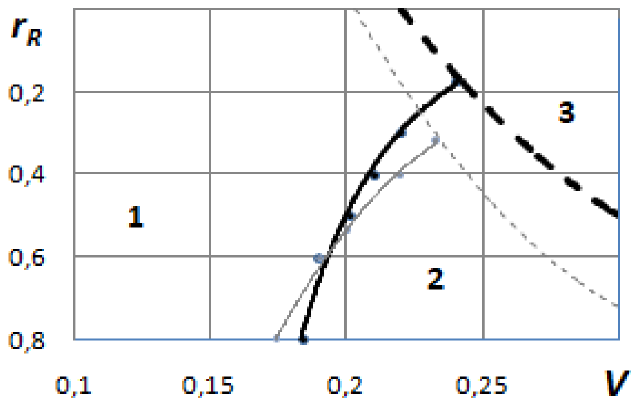

Fig. 4.16.The oundaries between generation modes in the case of correction of reflected waves, according to equations (4.18). Thin lines indicate the boundaries of regions in the absence of such correction. *Source is an authoring.*

An increase in reflection at the right end of the system leads to the appearance of a resonator field, the amplitude of which is already comparable to the total radiation field of the oscillators (see Fig. 4.17).

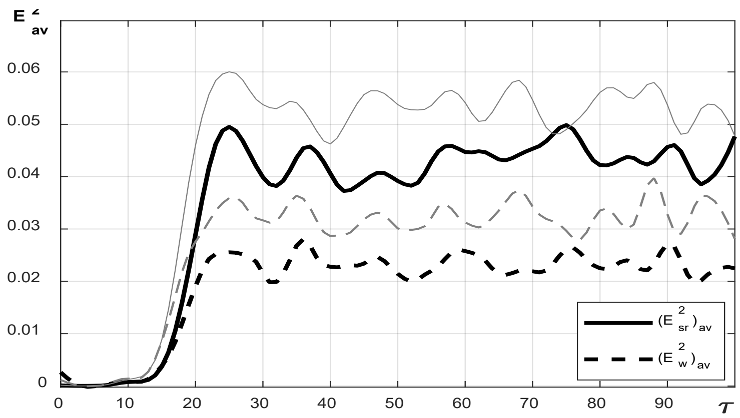

Fig.4.17. Time dependence of the averaged over the waveguide volume squares of the particle field (dotted line) and the waveguide field (solid line) in mode 1 ($V = 0.15, r_R = 0.6$), thick lines in the case of correction of reflected waves, according to equations (4.18). Thin lines are the particle field and the reflected field in the absence of such correction. *Source is an authoring.*

In Fig. 4.18. we see the distributions along the waveguide length of the amplitude moduli of the total field,the oscillator field and the waveguide field in mode 1 at the moment τ=100

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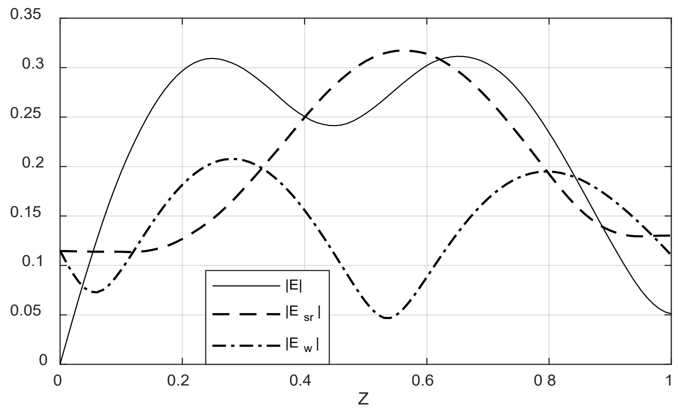

Fig. 4.18. Distributions along the waveguide length of the amplitude moduli of the total field, the oscillator field and the waveguide field in mode 1 at the moment τ=100 for the parameters $V = 0.15, r_R = 0.6$ **in the case of correction of reflected waves, according to equations (4.18).** *Source is an authoring.*

A further increase in the oscillator velocity leads to an excess of the resonator field in relation to the total field of particle radiation, i.e. in relation to the superradiance field. An increase in the reflection coefficient leads to a similar effect.

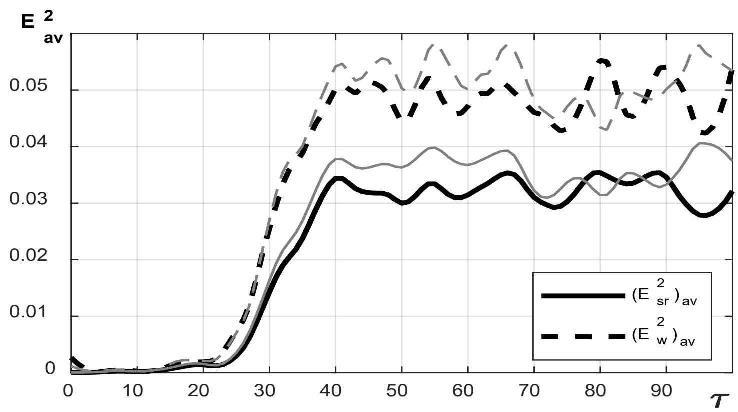

Fig.4.19. Time dependence of the averaged over the waveguide volume squares of the particle field (dotted line) and the waveguide field (solid line) in mode 2 $V = 0.23, r_R = 0.6$ **, thick lines in the case of correction of reflected waves, according to equations (4.18). Thin lines are the particle field and the reflected field in the absence of such correction.** *Source is an authoring.*

In this case, the distribution of the amplitude of the resonator field, the total particle field and the integral field is shown in Fig. 4.20. Thus, even at a constant drift speed of injected particles, one can see an increase in the amplitude of the resonator

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field with increasing reflection coefficients from the left end of the system

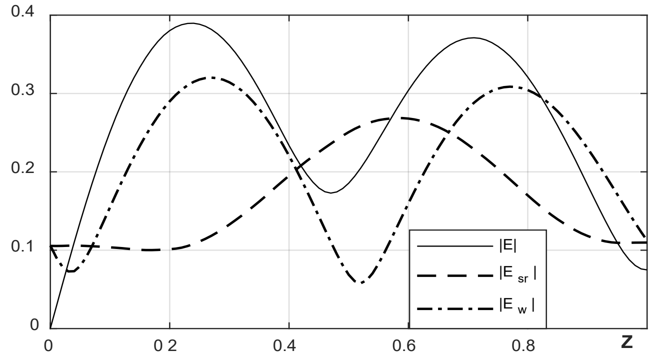

Fig. 4.20. Distributions along the waveguide length of the amplitude moduli of the total field, the oscillator field and the waveguide field in mode 2 at the moment τ=100 for the parameters $V = 0.23, r_R = 0.6$ **in the case of correction of reflected waves, according to equations (4.18)** *Source is an authoring.*

KAPITEL 4 / *CHAPTER 4*
GENERATION OF CYCLOTRON OSCILLATIONS BY AN ELECTRON BEAM IN THE PRESENCE OF A CONSTANT MAGNETIC FIELD

Let us consider the generation of electromagnetic waves of two polarizations, discussed in Section 4 of this work, but we will take into account the possibility of longitudinal motion of rotating electron oscillators in the active zone [6, 45].

4.1. Models descriptions

First, let's look at two generation modes. The first is the generation of a waveguide field, in conditions where the oscillators do not interact with each other, but only with the waveguide field. The second case corresponds to the generation of oscillator fields and the formation of their integral field, that is, the superradiance field.

Such consideration is in a certain sense artificial. Generally speaking, often the integral radiation field (essentially a superradiance field) first arises completely or partially, and only then, due to the reflection of waves due to the boundary conditions, the waveguide field is finally formed. In the future, most likely, only the waveguide field remains in the stationary mode. But a comparison of the fields of a waveguide and superradiance makes it possible to understand the nature of the development of field generation in such regimes and to understand the features of excitation of open waveguides.

TE wave. Let us use the notation of Section 3. Let us present the equations that describe the generation of the field of a TE wave, the electric vector of which is perpendicular to the axis of the waveguide. The structure of the waveguide field is formed by the boundary conditions in the transparency zone and, generally speaking, its spatial form weakly depends on the active zone filled with rotating electrons-oscillators. The traditional description of the process of excitation of a waveguide field assumes that the interaction of particles with each other in the active zone is neglected. Particles interact only with the waveguide field.

The equations for the longitudinal component of the magnetic field and electromagnetic waves propagating in both directions have the form [6, 28, 52,53]

$$\frac{d\mathrm{B}_{\pm}}{d\tau}+\theta\cdot\mathrm{B}_{\pm}=\frac{i}{2}N^{-1}\cdot\sum_{j=1}^{N}a_{j}\cdot J_{1}^{'}(a_{j})\cdot\exp(-2\pi \mathrm{i}\zeta_{j}\mp 2\pi \mathrm{i}Z_{j}), \tag{5.1}$$

where the summary value of the magnetic field of the waves is equal to

$$\mathrm{B}_{W}(Z_{i})=\frac{1}{2}[\mathrm{B}_{+}\exp(2\pi iZ_{i})+\mathrm{B}_{-}\exp(-2\pi iZ_{i})], \tag{5.2}$$

and $B_{\pm}=|B_{\pm}|\exp\{i\varphi_{\pm}\}$.

Equations of motion for electrons rotating in a constant magnetic field

$$2\pi\frac{d\zeta_{i}}{d\tau}=\eta_{i}(1-\alpha)+\mathrm{Re}\{J_{1}(a_{i})\cdot[1-\frac{1}{a_{i}^{2}}]\cdot\exp(2\pi i\zeta_{i})\cdot[\mathrm{B}_{W}(Z_{i})]\}, \tag{5.3}$$

$$da_{i}/d\tau=\mathrm{Re}\{i\cdot J_{1}^{'}(a_{i})\cdot\exp(2\pi i\zeta_{i})\cdot[\mathrm{B}_{W}(Z_{i})]\}, \tag{5.4}$$

should be supplemented with two more equations that take into account the longitudinal motion of the oscillators.

$$d\eta_{i}/d\tau=2\pi d^{2}Z_{i}/d\tau^{2}=\mathrm{Re}\{iR\cdot a_{i}\cdot J_{1}^{'}(a_{i})\exp(2\pi i\zeta_{i})\cdot[\mathrm{B}_{W}(Z_{i})]\}, \tag{5.5}$$

$$2\pi dZ_{i}/d\tau=\eta_{i} \tag{5.6}$$

Another approach to describing the process of field generation by the same system of oscillators in this waveguide assumes that each such oscillator emits the same wave in both directions. Because the oscillator is capable of emitting only waves located in the zone of transparency of the medium (or system - in this case, a waveguide). In this case, the field of this radiation will act on all particles of the ensemble of oscillators. In other words, all oscillators interact with each other. This mode can be considered a superradiation mode, if, of course, the phase synchronization of the oscillators occurs in the future.

Let us assume that there is no waveguide field, but only a superradiance field, that is, a field that is determined only by the interaction of rotating electrons. To describe the magnetic field of a wave, you can use the expression

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$$\mathrm{B}_{sr}(Z) = i\frac{1}{2N\theta}\sum_{j=1}^{N} a_j J_1^{'}(a_j)\exp\{-2\pi i\zeta_j\}[\exp\{2\pi i(Z - Z_j\}\cdot U(Z - Z_j) + \\ + \exp\{-2\pi i(Z - Z_j\}\cdot U(Z_j - Z)]. \quad (5.7)$$

The equations of electron motion will retain the same form, but with the replacement of the expression for the field

$$2\pi\frac{d\zeta_i}{d\tau} = \eta_i(1-\alpha) + \mathrm{Re}\{J_1(a_i)\cdot[1-\frac{1}{a_i^{\,2}}]\cdot\exp(2\pi i\zeta_i)\cdot[\mathrm{B}_{sr}(Z_i)]\}, \quad (5.8)$$

$$da_i / d\tau = \mathrm{Re}\{i\cdot \mathrm{B}\cdot J_1^{'}(a_i)\cdot\exp(2\pi i\zeta_i)\cdot[\mathrm{B}_{sr}(Z_i)]\} \quad (5.9)$$

$$d\eta_i / d\tau = 2\pi d^2 Z_i / d\tau^2 = \mathrm{Re}\{iR\cdot a_i\cdot J_1^{'}(a_i)\exp(2\pi i\zeta_i)\cdot[\mathrm{B}_{sr}(Z_i)]\} \quad (5.10)$$

TM wave. The excitation of this wave can also be described as a mode of generation of a waveguide field, with which all electrons rotating in the same constant magnetic field in the same waveguide interact. And in this case, in the traditional description, the interaction of oscillators with each other is neglected; the particles interact only with the field of the waveguide.

Let us present the equations for two TM waves propagating in opposite directions, the electric vector of which has a component along the direction of the waveguide axis and for which these equations are written [6, 28, 52,53]

$$[\frac{d}{d\tau} + \Theta]\mathrm{E}_{\pm} = \frac{1}{N}\sum_{j=1}^{N} J_1(a_j)\cdot\exp(-2\pi i\zeta_j \mp 2\pi i Z_j), \quad (5.11)$$

and the longitudinal electric field of the waves has the form

$$\mathrm{E}_W = \frac{1}{2}[\mathrm{E}_+\exp(2\pi i Z_i) + \mathrm{E}_-\exp(-2\pi i Z_i)]. \quad (5.12)$$

The equations of motion can also be written for the case of excitation of the waveguide field

$$2\pi\frac{d\zeta_i}{d\tau} = \eta_i(1-\alpha) + \mathrm{Re}\{-i\cdot J_1^{'}(a_i)\cdot\exp(2\pi i\zeta_i)\cdot[\mathrm{E}_W(Z_i)]\}, \quad (5.13)$$

$$d\eta_i / d\tau = 2\pi d^2 Z_i / d\tau^2 = -\mathrm{Re}\{R\cdot J_1(a_i)\cdot\exp(2\pi\zeta_i)[\mathrm{E}_W(Z_i)]\} \quad (5.14)$$

$$\eta_i = 2\pi dZ_i / d\tau \quad (5.15)$$

$$da_i / d\tau = -\operatorname{Re}\{(1/a_i)\cdot J_1(a_i)\cdot \exp(2\pi i\zeta_i)\cdot[\mathrm{E}_W(Z_i)]\} \tag{5.16}$$

For a superradiance field, when the oscillators interact only with each other, if we assume that there is no waveguide field, we can give the total longitudinal electric field of the system

$$\mathrm{E}_{sr}(Z) = \frac{1}{2N\theta}\sum_{j=1}^{N} J_1(a_j)\exp\{-2\pi i\zeta_j\}[\exp\{2\pi i(Z-Z_j\}\cdot U(Z-Z_j)+ \\ +\exp\{-2\pi i(Z-Z_j\}\cdot U(Z_j-Z)] \tag{5.17}$$

and, accordingly, the equations of motion, taking into account a different value of the wave field

$$2\pi\frac{d\zeta_i}{d\tau} = \eta_i(1-\alpha)+\operatorname{Re}\{-i\cdot J_1^{'}(a_i)\cdot \exp(2\pi i\zeta_i)\cdot[\mathrm{E}_{sr}(Z_i)]\}, \tag{5.18}$$

$$d\eta_i / d\tau = 2\pi d^2 Z_i / d\tau^2 = -\operatorname{Re}\{R\cdot J_1(a_i)\cdot \exp(2\pi\zeta_i)[\mathrm{E}_{sr}(Z_i)]\} \tag{5.19}$$

$$\eta_i = 2\pi dZ_i / d\tau \tag{5.20}$$

$$da_i / d\tau = -\operatorname{Re}\{(1/a_i)\cdot J_1(a_i)\cdot \exp(2\pi i\zeta_i)\cdot[\mathrm{E}_{sr}(Z_i)]\} \tag{5.21}$$

4.2. Results of simulation of generation of waveguide field and super-radiation field of the tm wave [51]

Let's consider two generation modes. The first is the generation of a waveguide field, under conditions where the oscillators do not interact with each other, but interact only with the waveguide field. To do this, we use the system of equations (5.1)–(5.6). The second case corresponds to the generation of a superradiance field, that is, the system of equations (5.6) – (5.11). Let us compare waveguide field generation models (5.1)–(5.6). and superradiance fields (5.6)–(5.11). Note that the structure of the waveguide field is already determined in the initial conditions. Below we will discuss in detail the nature of the formation of the waveguide field due to the effects of reflection from the ends of the waveguide. The following parameters were selected for calculation. Number of particles simulating electrons of the beam $N = 1600,\ \theta = 1,$

$R = 0.05,\ \alpha = 0.5.$ At the initial moment, the dimensionless radius of rotation of the electrons $a_j(0) = 1,$ the integral phase of rotation $\zeta_j(0)$ is a random value in the range (0,1), the initial longitudinal velocity $\eta_j(0) = 0.025$ is chosen such that the oscillators, for the most part, remain in the volume of the waveguide during the characteristic time of development of the process. At the initial moment, the particles are uniformly distributed along the length of the waveguide. Initial conditions for the waveguide field equation (5.2) $B_{0+} = 0.01, \quad B_{0-} = 0.$

$$\mathrm{B}_{ex}(Z) = \mathrm{B}_{0+} \cdot \exp i\{2\pi Z\} + \mathrm{B}_{0-} \cdot \exp i\{-2\pi Z\}. \qquad (5.22)$$

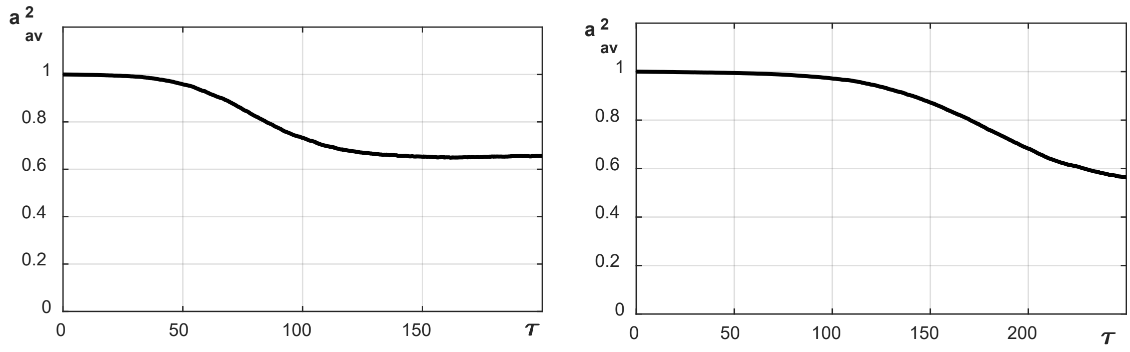
In addition, the same values $B_{0\pm}$ were used to specify an additional initiating field when calculating the superradiance field: During the calculation, particles extending beyond the waveguide (Z>1) were replaced by new ones with coordinate Z=0, phase and velocity as in the initial conditions. The total number of particles in the waveguide remained unchanged. Note that if in the superradiance field formula (5.7) all particles have an amplitude $a_j = 1$ and are synchronized (for each point Z), then the theoretically possible maximum of the superradiance field

$$|\mathrm{B}_{sr}|_{\max} = \frac{1}{2N\theta} \sum_{j=1}^{N} 1 \cdot J_1^{'}(1) = \frac{J_1^{'}(1)}{2\theta} \approx \frac{0.325}{2\theta} \approx 0.16 \cdot \theta^{-1}. \qquad (5.23)$$

When $\theta = 1 \ - \ |\mathrm{B}_{sr}|_{\max} \approx 0.16$. At the initial moment, the magnitude of the amplitude of the spontaneous field is approximately equal to $|\mathrm{B}_{sr}|_{rand} \approx 0.16 / \sqrt{N} = 0.16 / \sqrt{1600} = 0.004$. Thus, the initiating field (and the initial waveguide) is 2.5 times greater than the spontaneous one and 16 times less than the “theoretically possible” one (for a larger difference between the initiating and maximum fields, it is necessary to take a larger number of particles). To compare models when calculating the generation of a waveguide field for a configuration of particles formed under the influence of a waveguide field, the intrinsic field of these particles was also estimated using formula (5.7). It is clear that this field was not taken into account when calculating the waveguide generation model in the traditional description, but for evaluations such a calculated field was of interest. Similarly, when calculating a model of superradiance generation for a configuration of particles formed under the influence of the particles’ own field, the waveguide field of these particles was also estimated, which in reality was not actually taken into account in this model. The comparison results are shown in the figures below.Fig. 5.1 shows the time dependence of the mean square of particle amplitudes

$$a_{av}^2 = \frac{1}{N}\sum_j a_j^2 \quad (5.24)$$

.



a) b)

Fig. 5.1. Mean square of particle amplitudes, a – superradiance model, b – waveguide model *Source [54]*

Figure 5.2 for the two models of the generation shows the time dependence of the modulus of the waveguide field and the intrinsic field of particles (actually the superradiance field) at the edges of the system (left – Z=0 and right – Z=1) and the maximum value of the field modulus in the system (max). As can be seen from the figure, the maximum of the superradiance field with the selected calculation parameters is 25% of the “theoretical” maximum and 4 times greater than the initiating field. From a comparison of the two models (Fig. 5.2 a) and Fig. 5.2 b)) it follows that the maximum of the particles’ own field is approximately one and a half times greater than the maximum of the waveguide field. In addition, the maximum of the superradiance field is located on the right edge of the system (Z=1), which is a consequence of the fact that the initiating field is specified in the form of a wave traveling to the right: $B_{ex}(Z) = 0.01 \cdot \exp i\{2\pi Z\}$.

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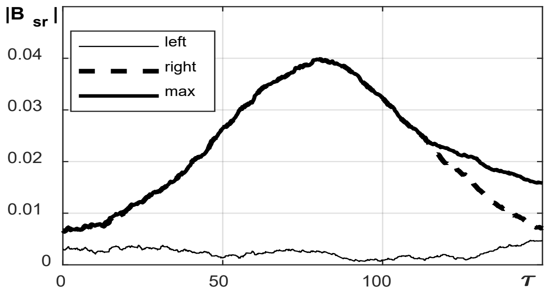
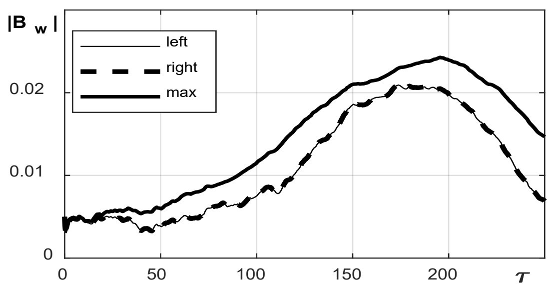


a)

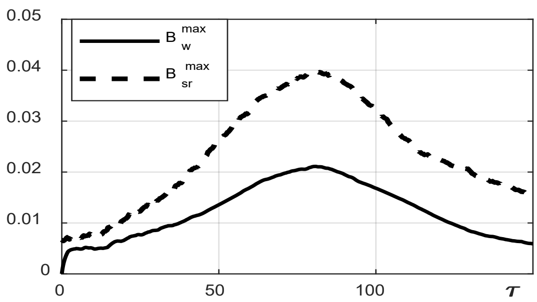


б)

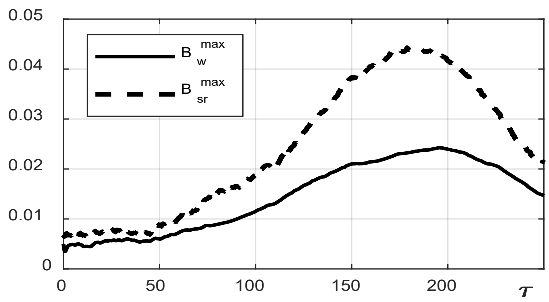
Fig. 5.2. Dependences on time of the modulus of the modulus field at the edges of the system (left – Z=0 and right – Z=1) and the maximum value in the system (max), a – superradiance model, b – waveguide model. Source [54]

In Fig. 5.3 for both models (superradiance and waveguide generation models), a comparison is shown of the time dependence of the maximum field used in the model with the field estimate for another model calculated for a folding configuration of oscillators. In other words, the configuration of oscillators with their amplitudes and phases obtained in the calculations of one model allows one to calculate the field for another model.

Although only one of the models is actually implemented.



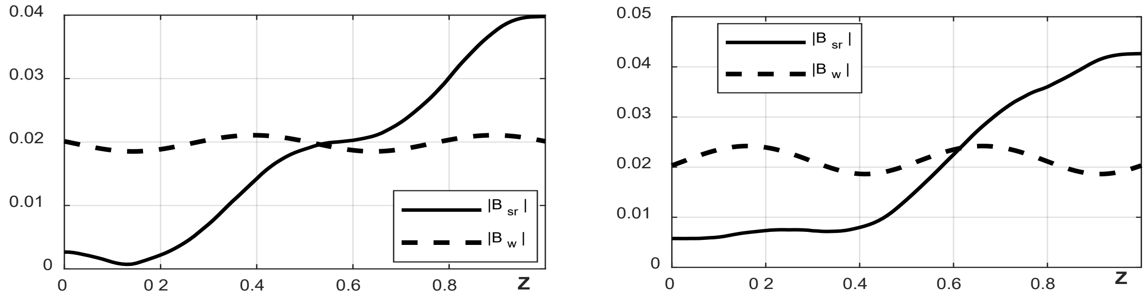
a)



b)

Fig. 5.3. Comparison of the time dependence of the field maximum: a) model of superradiance, B_{sr} – integral field of particles (used in the model), B_W – estimate of the waveguide field that could exist for the same configuration of oscillators; b) waveguide model, B_W – amplitude of the waveguide field (used in the model), B_{sr} – estimate of the superradiance field, which could be for the same configuration of oscillators. *Source [54]*

As can be seen from Fig. 5.3, in both cases the maximum of the integral particle field (in fact, the superradiance field) turns out to be greater than the maximum of the waveguide field. In Fig. 5.4. for both models (superradiance and waveguide model), a comparison is shown of the distribution along the length of the field system used in the model with the field estimate for another model, calculated for the folding configuration of particles. The distributions were taken at the moments when the field maximum was reached, τ=80 for the superradiance model, τ=197 for the waveguide model.



a) b)

Fig. 5.4. Comparison of the field distribution along the length of the system at the moments of maximum field: a) superradiance model, τ=80, B_{sr} – integral field of oscillators (used in the model), B_{W} – estimate of the waveguide field that could exist for the same configuration of oscillators; b) waveguide model, τ=197, B_{W} – amplitude of the waveguide field (used in the model), B_{sr} – estimate of the integral field of oscillators, which could be for the same configuration of oscillators. *Source [54]*

Note that the amplitude of the superradiance field and the proportion of electrons synchronized with the field increase in the longitudinal direction. That is, the advantage of the superradiance mode is the growth of the field amplitude toward the end of the system, which simplifies energy extraction and increases the generation efficiency.

As follows from formula (5.2), the waveguide field consists of two waves traveling towards them with constant (at a specific moment in time) amplitudes (actually a standing wave):

$$\mathrm{B}_W(Z) = \mathrm{B}_{W+}\exp\{2\pi iZ\} + \mathrm{B}_{W-}\exp\{-2\pi iZ\}. \quad (5.25)$$

where $\mathrm{B}_{W+} = 0.5\cdot\mathrm{B}_+,\ \ \mathrm{B}_{W-} = 0.5\cdot\mathrm{B}_-$, $\mathrm{B}_+,\ \ \mathrm{B}_-$ are found from expression (5.1).

The intrinsic field of particles (5.7) can also present in a formal way of two counter traveling waves, but their amplitudes depend on the coordinate

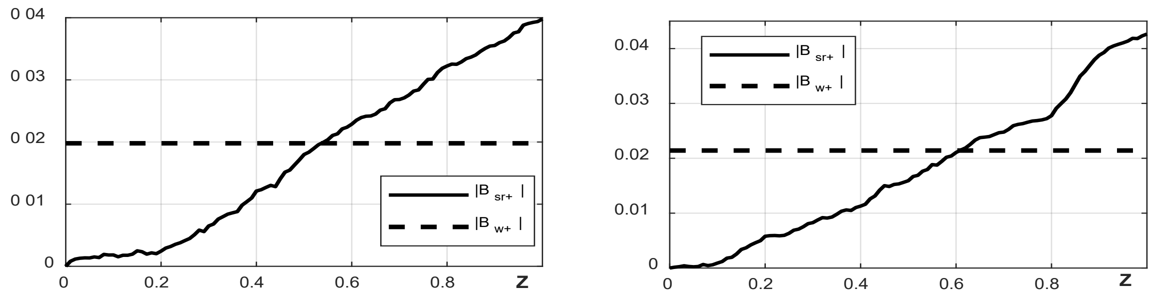
$$\mathrm{B}_{sr}(Z) = \mathrm{B}_{sr+}(Z)\exp\{2\pi iZ\} + \mathrm{B}_{sr-}(Z)\exp\{-2\pi iZ\}, \qquad (5.26)$$

where

$$\mathrm{B}_{sr+}(Z) = i\frac{1}{2N\theta}\sum_{j=1}^{N} a_j J_1'(a_j)\exp\{-2\pi i\zeta_j - 2\pi iZ_j\}\cdot U(Z - Z_j) \qquad (5.27)$$

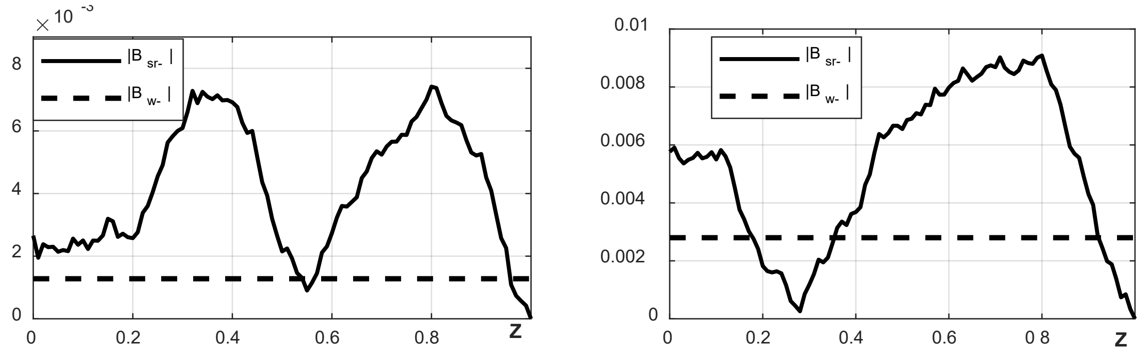
$$\mathrm{B}_{sr-}(Z) = i\frac{1}{2N\theta}\sum_{j=1}^{N} a_j J_1'(a_j)\exp\{-2\pi i\zeta_j + 2\pi iZ_j\}\cdot U(Z_j - Z) \qquad (5.28)$$

In Fig. 5.5 and fig. 5.6. For both models (superradiation and waveguide generation model), a comparison is shown of the distribution along the length of the system of traveling field components used in the model with the assessment of these components for another model, calculated for the folding configuration of particles. The distributions were taken at the moments when the field maximum was reached, τ = 80 for the superradiance model, τ = 197 for the waveguide model.



a) b)

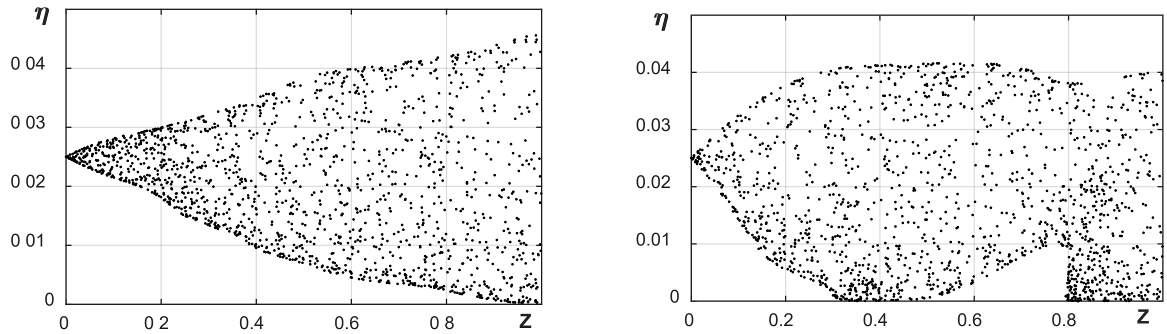
Fig. 5.5. Comparison of field components running in a positive direction along the length of the system at moments of field maximum: a) superradiance model, τ=80, B_{sr+} – for the oscillator field (used in the model), B_{W+} – estimate for the waveguide field, which could be for the same configuration of oscillators; b) waveguide model, τ=197, B_{W+} – for the waveguide field (used in the model), B_{sr+} – assessment of the oscillator field, which could be for the same configuration of oscillators. *Source [54]*



a) b)

Fig. 5.6. Comparison of field components running in a negative direction along the length of the system at moments of field maximum: a) superradiance model, τ=80, B_{sr-} **for the particle field (used in the model),** B_{W-} **– estimate for the waveguide field. b) waveguide model, τ=197,** B_{W-}**– for the waveguide field (used in the model),** B_{sr-}**– assessment of the oscillator field, which could be for the same configuration of oscillators.** *Source [54]*

Figure 5.7 shows the distribution along the length of the system of particle velocities at the moments when the field reaches its maximum.



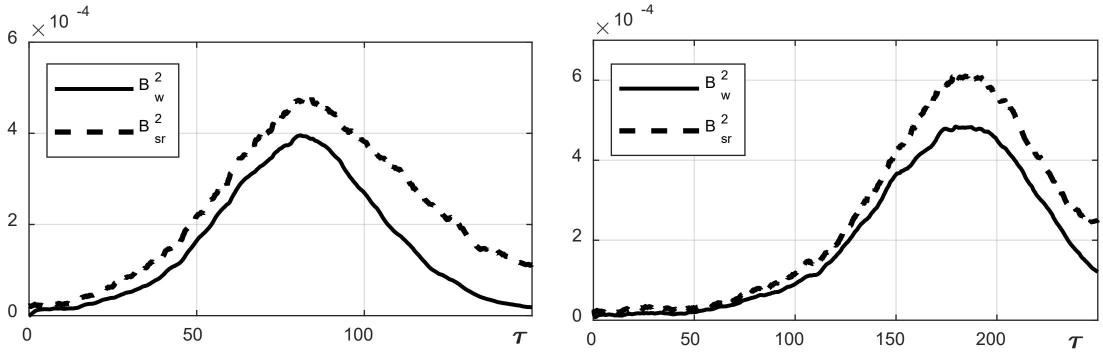
a) b)

Fig. 5.7. Distribution of the longitudinal velocity of particles along the length of the waveguide: a – superradiance model, τ=80; b – waveguide model, τ=197. *Source [54]*

Thus, the component of the integral field of oscillators (actually the superradiance field) running in the positive direction increases towards the end of the system. The component of the integral field of the oscillators running in the negative direction is much smaller than the component of the field running in the positive direction and remained practically at the level of spontaneous emission. This predominance of one direction is apparently a consequence of the openness of the system, the absence of reflection of part of the field at the edge. Fig. 5.8 compares the time dependences of the total field energy in the resonator volume for two models, calculated using the formulas (5.29).

$$|B_w^2| = \int_0^1 |B_W(Z)|^2\,dZ, \quad |B_{sr}^2| = \int_0^1 |B_{sr}(Z)|^2\,dZ \qquad (5.29)$$

,



a) b)

Fig. 5.8. Comparison of the time dependence of the total field energy in the resonator volume:a) superradiance model, $|B_{sr}^2|$**– energy density of the oscillator field (used in the model),** $|B_W^2|$**– estimate for the waveguide field, which could be for the same configuration of oscillators; b) waveguide model,** $|B_W^2|$**– the energy density of the waveguide field (used in the model),–** $|B_{sr}^2|$ **an estimate of the energy density of the oscillator field, which could be for the same configuration of oscillators** *Source [54]*

The maximum summary energy density of the particle's own field (dotted line in Fig. 5.8 a)) is approximately equal to the maximum energy of the waveguide field (solid line in Fig. 5.8.b)), but, as noted above, at the right end of the waveguide Z = 1 the particle's own field more than the waveguide (obviously when generating a field traveling in the positive direction).

Note that the relationships between the amplitudes and energies in the two modes were determined by the choice of the shape of the resonator field and could be different for a different shape of the resonator field.

On taking* into *account the effect of negative mass. Similar calculations were performed at α=1.5. In Fig. 5.9 and fig. Fig. 5.10 shows the distributions of the field and particle velocities at the moment of maximum field in the volume of the waveguide for the superradiance model. The field maximum occurs a little later: τ=80 at α=0.5 and τ=107 at α=1.5.

A comparison of the field amplitude distribution along the length of the waveguide at the moments when the maximum is reached (Fig. 5.9 and Fig. 5.4.a) shows a tendency towards greater field uniformity with increasing α. Comparison of Fig. 5.10 with Fig. 5.7a shows that the effect of negative mass changes the nature of the grouping of particles.

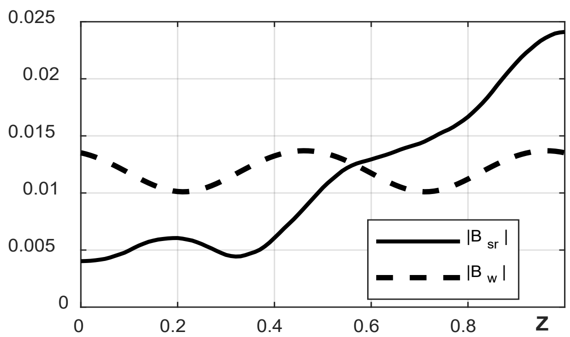


Fig.5.9. Field amplitude distribution along the waveguide length at τ =107 and α =1.5 (superradiance model)

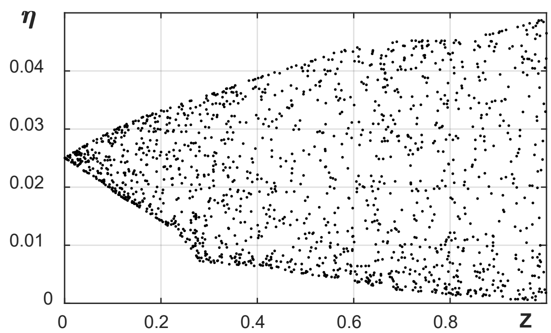


Fig.5.10. Distribution of velocities of particle-oscillators along the length of the waveguide at τ=107 and α=1.5 (superradiance model). *Source [54]*

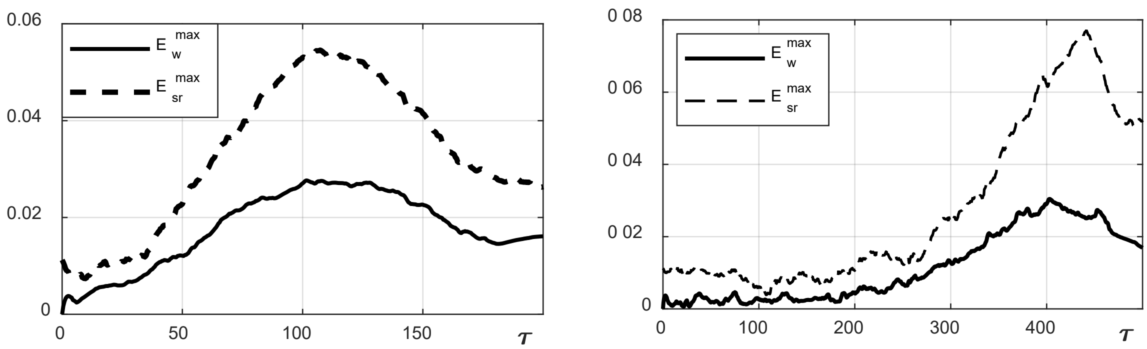
Simulation of TE wave generation in a cylindrical waveguide shows that for a short open system the field inhomogeneity along the length of the waveguide is significantly manifested. Therefore, for modeling pulsed processes in such systems, the superradiance model, which uses the integral intrinsic field of model particles, is perhaps more adequate.

4.3. About the generation of tm waves in a waveguide [52].

Let us consider and compare the generation models of the waveguide field (5.11)–(5.16) and the superradiance field (5.17)–(5.29). The following parameters were selected for calculation. The number of particles simulating beam electrons $N = 2500,\ \theta = 1,\ R = 0.05.$ The parameter α was set equal to 0.5 and 1.5. This parameter determines the electron dynamics. If this value is greater than unity, this leads to a negative mass effect and a change in the particle grouping mechanism [34].

At the initial moment, the dimensionless radius of rotation of the electrons $a_j(0) = 1,$ the integral phase of rotation is a random value $\zeta_j(0)$ in the range (0,1), the initial longitudinal velocity is chosen such $\eta_j(0) = 0.025$ that the oscillators, for the most part, remain in the volume of the waveguide during the characteristic time of the process development. At the initial moment the particles are uniformly distributed along the length of the waveguide. Initial conditions for the waveguide field equation $E_{0+} = E_{0-} = 0.001$. In addition, the same values $E_{0\pm}$ were used to specify an additional initiating field when calculating the superradiance field:

$$E_{ex}(Z) = E_{0+} \cdot \exp i\{2\pi Z\} + E_{0-} \cdot \exp i\{-2\pi Z\}, \quad (5.30)$$

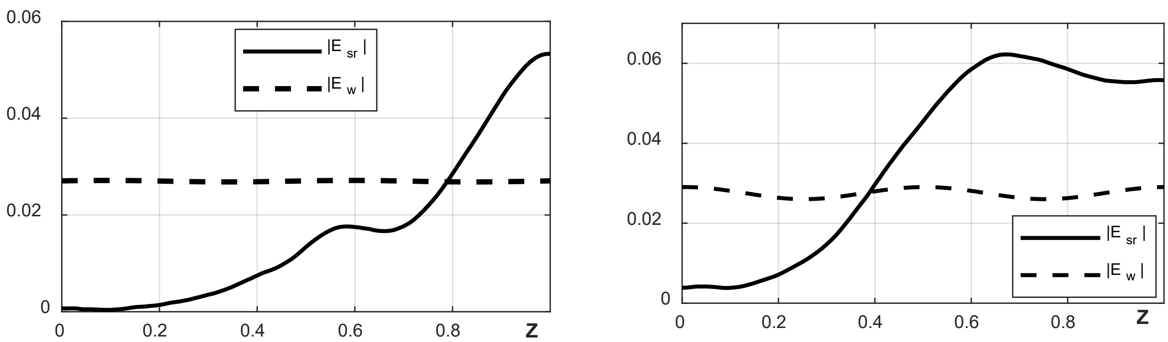


a) b)

Fig.5.11. Time dependence of the maximum field in the volume of the waveguide at α=0.5: a – model of superradiance; b – waveguide model. *Source [54]*

Figure 5.11.a shows the dependence of the maximum field in the volume of the waveguide for the superradiance model. The particles moved under the influence of

their own field, which was calculated using formula (5.17). The model corresponds to the integral field of particles $E_{sr\max}$ (solid line), but at the same time the waveguide field $E_w^{\max}$ is estimated for the current values of amplitudes and phases of particles (dashed-dotted line). As can be seen from the figure, the maximum of the particles' own integral field always exceeds the maximum of the waveguide field. Figure 5.11.b similarly shows the dependence of the maximum field in the volume of the waveguide for the waveguide model $E_w^{\max}$. The particles moved under the influence of a waveguide field $E_w^{\max}$. The model corresponds to a waveguide field (solid line) $E_w^{\max}$, but at the same time the integral field of particles is estimated for the current values of amplitudes and phases of particles $E_{sr\max}$ (dashed-dotted line). Again, the maximum of the particles' own integral field exceeds the maximum of the waveguide field. Figure 5.12 shows the distributions of the amplitude modulus of the model field and estimates from an alternative model at the moments when the field reaches its maximum value (maximum moments in Fig. 5.11).



a) b)

Fig.5.12.Distribution of field amplitude along the length of the waveguide at α=0.5: a – superradiance model, τ =110; b – waveguide model, τ =400.

Source [54]

As can be seen in the figures, the intrinsic field of the particles is very inhomogeneous along the length of the waveguide, while the waveguide field is an almost constant value and is represented to some extent as the average value of the field in the volume of the waveguide. Figure 5.13 shows the distribution of particle

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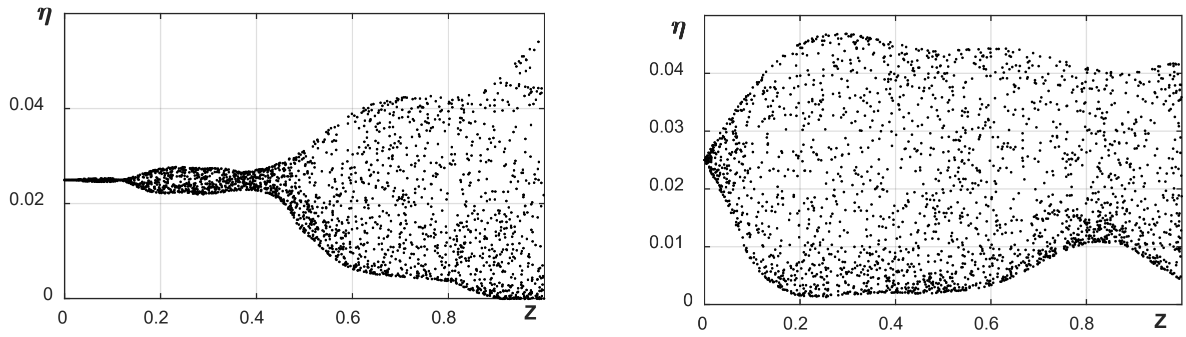
velocities along the waveguide at these moments.



a) б)

Fig.5.13. Distribution of particle velocities along the length of the waveguide at α=0.5: a – superradiance model, τ =110; b – waveguide model, τ =400.

Source [54]

From Figure 5.13 it is clear that in the superradiance model, in contrast to the waveguide model, a significant effect on the parameters of the oscillators occurs only at a distance of approximately half the wavelength from the right end of the waveguide. This is due to the obviously uneven distribution of the field along the length of the waveguide (Fig. 5.12.a).

Similar calculations were performed at α=1.5. In Fig. 5.14 and fig. Figure 5.15 shows the distributions of the field and particle velocities at the moment of maximum field in the volume of the waveguide for the superradiance model. Comparison of Fig. 5.14 from fig. 5.12.a and especially Fig. 5.15 with Fig. 5.13a shows that the effect of negative mass significantly changes the nature of the grouping of particles.

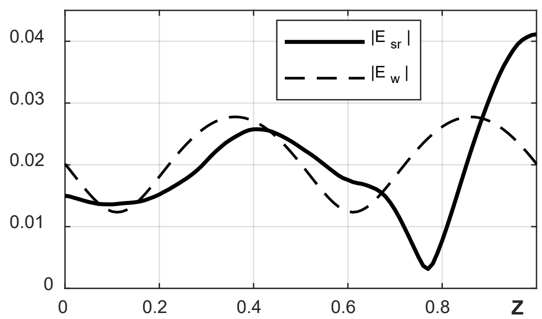


Fig.5.14. Field amplitude distribution along the length of the waveguide at τ=150 and α =1.5 (superradiance model)

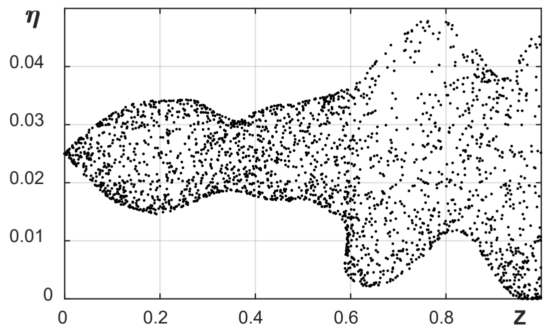


Fig.5.15. Distribution of velocities of particle-oscillators along the length of the waveguide at τ=150 and α=1.5 (superradiance model). *Source [54]*

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Thus, modeling of TM wave generation in a cylindrical waveguide showed that for a completely open system, the field inhomogeneity along the length of the waveguide is significantly manifested. Therefore, for modeling processes in such systems, the superradiance model, which uses the own field of model particles, is more adequate. Let us recall that a similar approach to describing the generation of TE waves in an open waveguide - a gyrotron in the superradiance mode was proposed by A.G. Zagorodny and P.I. Fomin [39]. It was in this device that the influence of boundary conditions was weakened. In addition, generation was realized near the cutoff frequency of a circular waveguide, where the group velocity of the wave was very low and synchronized with the drift speed of particles oscillating in the magnetic field of electrons[4].

4.4. Formation of a wave guide field due to reflection effects [54]

In the absence of field reflection from the boundaries of the waveguide when the excitation threshold is exceeded, we can assume that only the superradiance regime, studied in detail above, is realized. The presence of reflection at the ends of the waveguide leads to the appearance of reflected waves and the formation of a standing wave, which is actually a waveguide mode. Earlier, in the previous sections of this chapter, the generation of the waveguide field was quite artificial. The shape of this field was specified by the initial conditions and, in addition, the interaction of particles (electrons oscillating in the magnetic field) in the active zone occurred only with this waveguide mode. In this case, the particles did not interact with each other. Such discrimination of the interaction of system particles with each other prevented the formation of a total field of particles in the system—superradiance in the generation description models used.

In reality, each particle, an electron (oscillator) rotating in a magnetic field, emits waves that are transparent to the waveguide and which propagate in two directions.

[4] We also note the use of a corrugated central rod for mode selection, which, however, has virtually no effect on the nature of the excitation of oscillations.

Thus, first the total field of radiating particles is formed, essentially a superradiance field. his is easy to understand if we first assume that there is no reflection at all from the ends of the waveguide. Moreover, it is important to note that in the superradiation field it is difficult to identify integral waves that propagate in the volume of the waveguide in two directions and form a standing wave—the waveguide field. Only later, due to the effects of multiple reflection of the field, can such two integral waves traveling in two directions be formed.

The traditional description of the process of excitation of the resonator waveguide field (under conditions of its formation due to reflection from the boundaries-ends of the waveguide) assumes that the interaction of particles with each other in the active zone is neglected. The particles interact only with the waveguide field. The equations for the longitudinal component of the magnetic field, electromagnetic waves propagating in both directions (the interference of which is the resonator waveguide field) [11-13] have the form (5.1) and (5.2), With a sufficiently significant reflection coefficient, the amplitudes of these waves are approximately equal, so the resonator field is a standing wave.

The summary radiation field of all particles $\mathrm{B}_{sr}(Z)$ is found according to formula (5.7). Particles move along the waveguide with a speed $\eta_i(\tau)$ that is found from (5.10). When particles are removed outside the resonator (0<Z<1), they are replaced by new ones with an initial coordinate Z=0, an initial velocity η_0, an initial amplitude a_0, and a random random phase. It is important to note that if the noise level is low, the initial amplitudes of the oscillators - the radii of the rotating electrons - are the same, due to synchronization we have a fairly significant superradiance field (similar to that considered in the fourth section), which at the initial stage of the process creates reflected waves, which form the resonant field of the waveguide.

Obviously, the waveguide field, subject to noticeable reflection, will have the form

$$\mathrm{B}_w(Z,\tau) = \mathrm{B}_+(\tau)e^{2\pi iZ} + \mathrm{B}_-(\tau)e^{-2\pi iZ} \tag{5.30}$$

Thus, the total field in the waveguide can be represented as ,

$$B(Z,\tau)=B_{+}(\tau)e^{2\pi iZ}+B_{-}(\tau)e^{-2\pi iZ}+B_{sr}(Z,\tau). \tag{5.31}$$

Here is the summary radiation field of the oscillators $B_{sr}(Z,\tau)$ (5.7), the field reflected from the left end: $B_{+}(\tau)e^{2\pi iZ}$, reflected from the right end: $B_{-}(\tau)e^{-2\pi iZ}$. It is this total field (5.31) that acts on the particles. To find the amplitudes of two differently directed waves, we will follow the following simplified procedure. The amplitudes of the reflcted waves are found from the conditions of reflection at the ends of the waveguide based on the condition for the amplitudes on the left (Z=0) and right (Z=1) sides of the incident and reflected wave: $B_{\rightarrow}/B_{\leftarrow}|_{Z=0,1}=-r_{l,r}$, r_l, r_r – reflection coefficients at the left and right ends.

The total radiation field of particles at the ends of the system is obtained from (5.7)

$$B_{sr}(Z=0)=i\frac{1}{2N\theta}\sum_{j=1}^{N}a_jJ_1'(a_j)\exp\{-2\pi i\zeta_j\}\exp\{2\pi iZ_j\} \tag{5.32}$$

$$B_{sr}(Z=1)=i\frac{1}{2N\theta}\sum_{j=1}^{N}a_jJ_1'(a_j)\exp\{-2\pi i\zeta_j\}\exp\{-2\pi iZ_j\} \tag{5.33}$$

Let's consider the fields at the ends of the resonator Z=0 and Z=1. The field incident on the Z=0 end is equal to $B_{-}(\tau)e^{-2\pi iZ}|_{Z=0}+B_{sr}(Z=0,\tau)$, the reflected field is equal to $B_{+}(\tau)e^{2\pi iZ}|_{Z=0}$. Similarly for Z=1: $B_{+}(\tau)e^{2\pi iZ}|_{Z=1}+B_{sr}(Z=1,\tau)$ incident field, reflected field -$B_{-}(\tau)e^{2\pi iZ}|_{Z=1}$. From here we obtain equations for the amplitudes of reflected waves

$$B_{+}(\tau)=-r_l\left(B_{-}(\tau)+B_{sr}(Z=0,\tau)\right) \tag{5.34}$$

$$B_{-}(\tau)=-r_r\left(B_{+}(\tau)+B_{sr}(Z=1,\tau)\right) \tag{5.35}$$

where $r_l;r_r$ are the reflection coefficients, respectively, at the ends Z=0 and Z=1.

Solving system (5.34-5.35), we obtain the values of the amplitudes of the waves that make up the standing wave

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$$B_+ = \frac{r_l \left(r_r \, B_{sr}(Z=1,\tau) - B_{sr}(Z=0,\tau) \right)}{(1 - r_l \cdot r_r)}, \quad B_- = \frac{r_r \left(r_l \, B_{sr}(Z=0,\tau) - B_{sr}(Z=1,\tau) \right)}{(1 - r_l \cdot r_r)}$$

(5.36)

Equations (5.36) are obtained under the condition that the influence of particles in the volume on the reflected waves, the sum of which is the waveguide field, is not taken into account. If the influence of particles-electrons in the waveguide volume on the reflected waves is taken into account, then in expressions (5.36) the following substitutions should be made:

$B_{sr}(Z=1,\tau) \rightarrow B_{sr}(Z=1,\tau) + 0.5 B_{wg+}$ and $B_{sr}(Z=0,\tau) \rightarrow B_{sr}(Z=0,\tau) + 0.5 B_{wg-}$ (5.37)

where at $|\partial B_{wg\pm} / \partial \tau| << \theta \, |B_{wg\pm}|$, $B_{wg\pm} = \frac{i}{2N\theta \cdot} \cdot \sum_{j=1}^{N} a_j \cdot J_1^{'}(a_j) \cdot \exp(-2\pi i \zeta_j \mp 2\pi i Z_j)$

, see (5.1).

Numerical modeling of the formation of a resonator field. Influence of oscillator speed and reflection coefficients. The waveguide contains a given number of particles simulating the ensemble of electrons - N. At the initial moment, all particles have a given amplitude, speed and random phase. Instead of particles leaving the waveguide, particles are injected at the beginning of the waveguide in such a way that their total number in the waveguide remains constant.

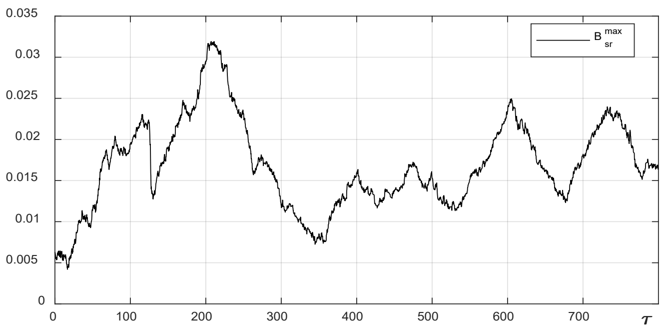

Fig.5.16. Time dependence of the particle field maximum for each moment of time at zero reflection coefficients $r_L = 0, \; r_R = 0$ *Source [54].*

The amplitude, speed and phase of the new (replacement) particles are the same as those of the particles at the initial moment: $N = 500, \quad \theta = 1,$ $\alpha = 0.5, \; a_0 = 1, \quad R = 0.1, \quad r_L = 1.$ Constant parameters were used in the calculations (total reflection at the left edge).It is worth noting that in the absence of reflection there is no wave field; only the presence of reflection effects allows the formation of a waveguide field.-Phase synchronization and formation of noticeable coherence of radiation of an ensemble of particles in the active zone of an open waveguide with a large dispersion (scattering) of the initial amplitudes of the oscillators (here these are the Larmor rotation radii of the electrons) generally requires an initiating external field and noise attenuation [49] to accelerate the generation process. It turned out that in the absence of a spread of the initial amplitudes of the oscillators, an external field is not required, the development of the generation process (see Fig. 5.16) occurs noticeably faster (a similar phenomenon was discussed in Section 3, where Fig. 3.2a and 3.2c show the influence of the dispersion of the initial velocities on the generation process). Under the considered conditions, the proper field of the particles-rotating electrons turns out to be large enough and can effectively form reflected waves of noticeable amplitude due to reflection from the ends. Reflection from the ends of the resonator gives rise to reflected waves traveling in the opposite direction. Reflected waves can be formed due to field reflections from the ends according to (5.36). But generally speaking, these waves can change when passing through the active zone under the influence of the fields of moving oscillators in the volume of the active zone. Therefore, it is useful to take into account the effect of oscillators in the active zone on these reflected waves. And then the resonator field will consist of two waves traveling in different directions, where the amplitude of the reflected waves at the boundary is supplemented by a term that qualitatively takes into account the effect of oscillators in the resonator volume on these waves (see the corrections (5.36)).

Below, two mechanisms for the formation of reflected waves that make up the waveguide field are considered. The first is only due to reflections from the ends of the waveguide according to (5.36) and the second, where the effect of particles in the volume of the active zone on the reflected waves is additionally taken into account, that is, taking into account the corrections after formula (5.36).

After a certain period of establishing generation in the waveguide, a quasi-stationary regime is formed. Therefore, after this mode is established, the values of the

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squares of the resonator waveguide field and the total particle field, averaged over the waveguide volume and time, as well as their ratio K, are calculated.

$$E_{sr}^2 = <(E_{sr}^2)_{av}>_t, \quad E_w^2 = <(E_w^2)_{av}>_t, \quad K = E_w^2 / E_{sr}^2$$

In Fig. 5.17, the boundaries between the generation modes are shown depending on the parameters changed in the calculations: the injection rate and the reflection coefficient at the right edge of the waveguide.

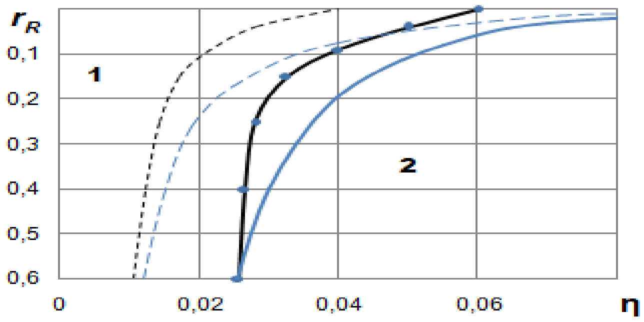

Fig. 5.17. The boundaries of the generation modes in the case of correction of reflected waves, according to the conditions of the influence of particles on reflected waves (3.57) (black line), the blue line indicates the boundary of the regions in the absence of such accounting, according to calculations (3.56).
Source is an authoring.

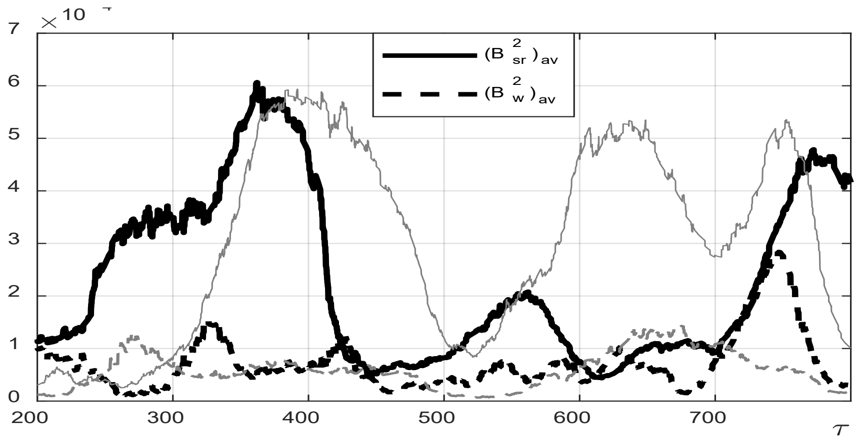

Fig.5.18. Time dependence of the averaged over the waveguide volume squares of the particle field (dotted line) and the resonator waveguide field (solid line) in mode 1 ($\eta_0 = 0.02, r_R = 0.2$) thick lines in the case of taking into account the influence of particles in the waveguide volume on the reflected waves, according to conditions (5.37). Thin lines are the particle field and the reflected field in the absence of such an influence of particles in the volume on the reflected waves according to conditions (5.36). Source is an authoring.

The dotted lines correspond to the boundary of the regions in the case when the average particle velocity in the waveguide volume is used, rather than the initial velocity of the injected particles. In region 1, the generation of the intrinsic particle field dominates (in fact, the superradiance mode), in region 2, the generation of the resonator waveguide field of the traditional type, caused by reflection processes from the ends. In the developed mode 1 - dominance of superradiance - the intensity of the total particle field exceeds the intensity of the resonator waveguide field.

A decrease in the drift velocity of electrons along the system in developed generation modes is characteristic. For mode 1 - superradiation:

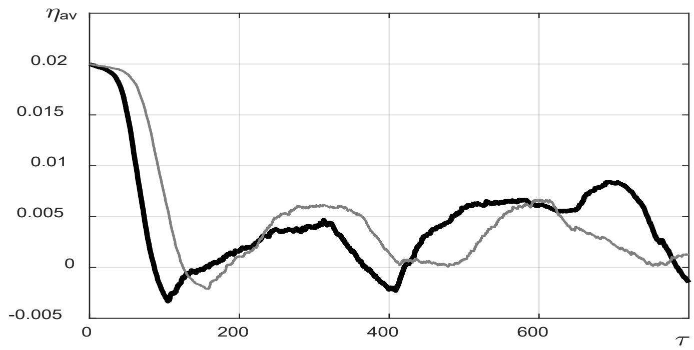

Fig. 5.19. Time dependence of the average particle velocity over the waveguide volume in mode 1 ($\eta_0 = 0.02, r_R = 0.2$) the bold line in the case of taking into account the influence of particles in the waveguide volume on the reflected waves, according to conditions (5.37). The thin line - in the absence of such a provision, according to conditions (5.36). *Source is an authoring.*

We can give the form of the fields for this mode. It is seen that the resonant waveguide field retains the sinusoidal shape of the standing wave, the resonant waveguide field increases towards the left end of the system. The resonator waveguide field in this mode is less than the total particle field, the attenuation coefficient is $K = 0.32$.

In the mode of traditional waveguide generation 2, the radiation field of particles changes weakly in relation to the field of the same type in the superradiance mode1. However, due to the increase in reflection and acceleration of injection, the amplitudes of the resonator waveguide field increase noticeably.

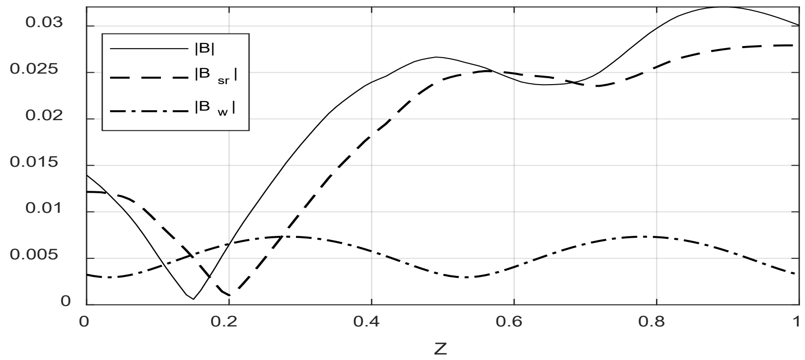

Fig. 5.20. Distributions along the waveguide length of the amplitude moduli of the total field, particle field and waveguide field in mode 1 at the moment τ=800 for the parameters $\eta_0 = 0.02, r_R = 0.2$ in the case of the influence of particles in the waveguide volume on the reflected waves, according to conditions (5.37).
Source is an authoring.

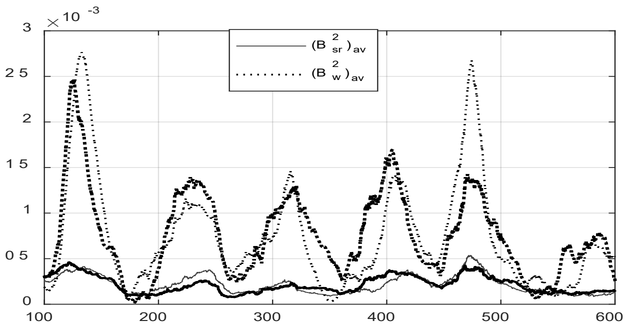

Fig. 5.21. Time dependence of the averaged over the waveguide volume squares of the particle field (dotted line) and the waveguide field (solid line) in the waveguide generation mode 2 ($\eta_0 = 0.06, r_R = 0.2$), thick lines in the case of the influence of particles in the waveguide volume on the reflected waves, according to conditions (5.37). Thin lines are the particle field and the reflected field in the absence of such influence according to conditions (5.36). *Source is an authoring.*

In the waveguide generation mode, i.e. in mode 2, the initial longitudinal velocity of the electrons also decreases noticeably.

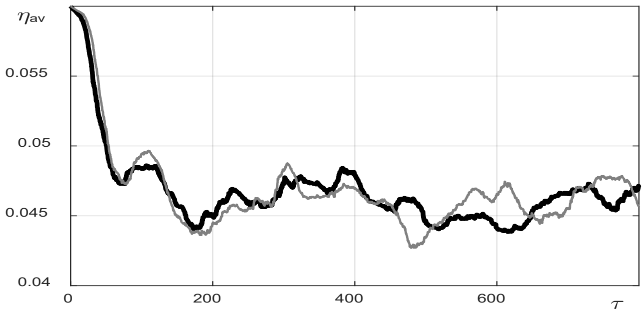

Fig. 5.22. Time dependence of the average particle velocity over the waveguide volume in mode 2 ($\eta_0 = 0.06, r_R = 0.6$) the bold line in the case of taking into account the influence of particles on reflected waves, according to conditions (5.37). The thin line - in the absence of such influence, according to conditions (5.36). *Source is an authoring.*

The waveguide-resonator field in this case is greater than the total particle field, the excess factor is $K = 2.77$.

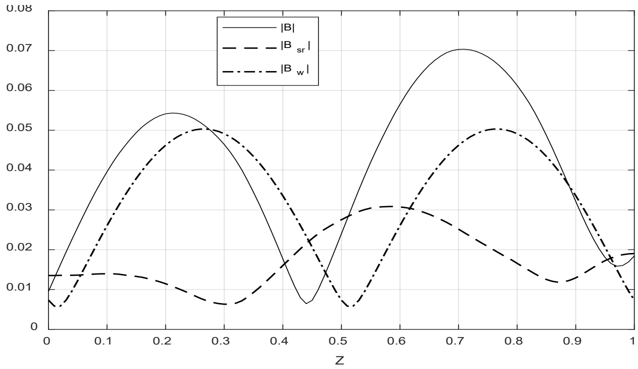

Fig. 5.23. Distributions along the length of the waveguide of the amplitude moduli of the total field, particle field and waveguide field in mode 2 at the moment τ=800 for the parameters $\eta_0 = 0.06, r_R = 0.6$ in the case of the influence of particles in the waveguide volume on the reflected waves, according to conditions (5.37). *Source is an authoring.*

CONCLUSION

The work discusses various manifestations of spontaneous emission, superradiation and stimulated emission and their competition. The introduction presents problems associated with competition between different types of radiation that have the same energy source. The second section of the work examines the competition between spontaneous and induced radiation, which manifests itself especially clearly in the surrounding outer space. At a low radiation density and at a close to zero population inversion of the energy levels of excited atoms and molecules of a substance, the generation of characteristic triangular pulses of stimulated radiation is possible, which manifest themselves noticeably against the background of spontaneous radiation from sources. For example, in interstellar space the intensity of such pulses may be comparable or even exceed the intensity of the mostly spontaneous radiation of stars. Based on these ideas, a model is proposed to describe the periodic change in the luminosity of Cepheid stars. The features of this model are the formation of pulses of a characteristic type and the clear periodicity of their appearance characteristic of quantum systems. In laboratory conditions, levels of spontaneous emission are usually much lower than levels of stimulated emission, although spontaneous effects are quite capable of manifesting themselves.

Much attention is paid in the third section to the comparison in open systems of microwave electronics of the generation modes of resonator (waveguide) radiation and generation modes under superradiance conditions. In the first case of the traditional description of generation, emitters or oscillators in the active zone interact only with the field of the resonator or waveguide. Their interaction with each other directly is ignored in traditional electronics models. As shown in this work, in vain. A connection is presented between an ensemble of electrons rotating in a constant magnetic field, which excite TE and TM cyclotron electromagnetic oscillations of the waveguide, and a system of nonlinear oscillators. In the superradiance regime, which is genetically close to the nature of spontaneous emission, it is the interaction of emitters and oscillators in completely open systems that is dominant. As a result of this interaction,

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the effect of synchronization of emitters and oscillators and the appearance of intense coherent radiation in both classical and quantum systems arises. The similarity of the dynamics of dissipative generation modes caused by the removal of energy from the system and the development of superradiance modes in the same system of oscillators is shown. In this case, for the same ensembles of oscillators, the characteristic time of field rise in these two cases is almost the same, as are the achieved maximum field amplitudes.

The fourth section discusses the influence of various types of pumping on the nature of lasing in the superradiance regime in completely open resonators. For a system of nonlinear oscillators, constant pumping modes are considered and the RF field pressure, which causes the movement of the oscillators, is taken into account. The nature of the formation of the resonator field due to particle injection and reflection effects from the ends of the system is also discussed.

In the fifth section, the conditions for the generation of TE and TM cyclotron waves in a magnetoactive waveguide by an electron flow, the rotation speed in the cross-sectional plane of which significantly exceeds the longitudinal speed of movement, are studied in detail. The modes of traditional excitation of oscillations are discussed, when oscillators - rotating electrons interact only with the field of the waveguide. Superradiance regimes when their interaction is decisive are also considered. Of greatest interest are the studied conditions for the generation of superradiance and the formation of a waveguide field when taking into account the injection of particles and reflections from the ends of the system.

In different generation modes, as shown in this work, both the waveguide field, determined by the geometry of the system, and the field of the total radiation of system particles, that is, the superradiance field, can dominate. A similar flexible approach to describing the generation of an open system–gyrotron, in particular in the superradiance regime, was proposed by A.G. Zagorodny and P.I. Fomin [39]. This review shows that in the absence of field reflection from the ends of the system, the formation of a waveguide field does not occur. In the volume of the active zone, only the total field of oscillating electron-oscillators appears. When taking into account the effects of reflection and additional particle injection, a waveguide field appears. A further increase in the reflection of fields from the ends increases the amplitude of the reflected waves, which form a waveguide field in the form of a standing wave. At the same time, the amplitude of the own fields (essentially, the superradiance field) remains practically unchanged.

The resonator field can be supported by particles in the waveguide volume, or it can be formed only due to reflection effects. It is important to note that the zones of dominance of superradiation and traditional resonator generation are always formed under different conditions of energy exchange between reflected waves and oscillators in the waveguide volume. Note that even with a decrease in the direct effect of oscillators in the volume on reflected waves, which meets conditions (5.36), the zones of different types of generation shift slightly. This is due to the presence of a total field of oscillators, which is capable of acting as an intermediary between the oscillators in the active zone and the waves reflected from the ends of the resonator, forming the waveguide resonator field. It is noted that the amplitudes of the eigenfields of electron oscillators in different modes differ slightly.

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