UDC: 519.85 https://doi.org/10.15407/jai2025.04.___

A. Chuhai [1, 2], G. Yaskov [1, 3], O. Starkova [2], A. Zhuravka [4], M. Shcherbyna [1]

[1]Anatolii Pidhornyi Institute of Power Machines and Systems of the National Academy of Sciences of Ukraine
Komunalnykiv St., 2/10, Kharkiv, 61046

[2] Simon Kuznets Kharkiv National University of Economics, Ukraine
Nauky Ave, 9A, Kharkiv, 61166

[3] Kharkiv National University of Radio Electronics, Ukraine
Nauky Ave., 14, Kharkiv, 61166

[4] Lviv Polytechnic National University, Ukraine
Stepana Bandery St., 12, Lviv, 79000

INTELLIGENT ADAPTIVE PACKING SYSTEM BASED ON PHI-FUNCTIONS AND AGENT-CONTROLLED INTERACTIONS

Abstract. This paper introduces an innovative intelligent system called Adaptive Packing Intelligence (API) for spatial optimization. It builds upon traditional packing models by enabling agent-controlled interactions among geometric objects. Unlike standard methods that enforce strict non-overlap constraints, API offers flexible control over object placement through dynamic agent parameters. These parameters are integrated into phi-functions within the mathematical framework, regulating spatial relationships and supporting three modes: enforced separation, exact contact, and controlled non-overlapping. The system's architecture features a unified mathematical core based on phi-functions for geometric representation, an adaptive Agent for managing real-time interactions, and an optimization engine utilizing gradient-based and heuristic algorithms to identify optimal configurations. API can address various objective functions, including volume minimization, packing density maximization, and energy-based interaction modeling. Its applications extend across medicine, biology, and engineering. By unifying diverse disciplines under a single framework, API enhances adaptability and computational efficiency in solving complex packing and simulation problems.

Keywords: phi-function, agent-controlled interactions, mathematical modelling, computational geometry, nonlinear programming.

Introduction

The growing use of artificial intelligence in medical, biological, and engineering systems has led to the development of intelligent frameworks that precisely optimize the placement of objects. These systems sit at the intersection of computational geometry, data-driven modeling, and adaptive control theory, opening up new possibilities for solving complex optimization challenges that were previously too difficult for traditional methods. In medicine and biology, they enable the automated design of cellular structures, implants, and biomaterials, ensuring they fit well and reducing conflicts within limited spaces. In engineering, intelligent spatial optimization enables the design of efficient layouts for mechanical parts, additive manufacturing, and material structures, thereby saving resources and enhancing reliability. By combining AI decision-making with mathematical models of spatial interactions, these systems can adapt to uncertainties, disturbances, and changing constraints, balancing accuracy, flexibility, and computational speed. This synergy of AI, geometry, and physical modeling paves the way for advanced optimization tools that support independent reasoning and spatial understanding across various scientific and technological fields [1-3].

One of the most difficult tasks in these fields is the automated and adaptable placement of geometric objects, such as therapeutic agents, mechanical parts, or molecular structures, within confined spaces. Traditional packing models, which depend on strict non-overlap rules, often fail to represent the dynamic and interactive nature of real-world situations [4]. Intelligent packing systems have been developed, providing flexible control over object placement through advanced optimization methods and agent-based interactions [5].

Mathematical modeling plays a key role in these systems, enabling the simulation and control of spatial relationships. Techniques based on phi-functions offer a continuous representation of object boundaries and distances, allowing control over separation, contact, and overlap [6]. When combined with agent-controlled parameters, these models enable adaptive behavior, allowing for the simulation of complex interactions and the real-time optimization of configurations.

As shown by Fonseca et al. [7], agent-based systems can be effectively transformed into surrogate models that preserve key dynamic features and simplify optimization. This approach enables the application of classical control theory to complex, stochastic environments, particularly for spatial systems involving interacting geometric entities. Using surrogate modeling in this manner can lead to quicker convergence during optimization and reduce computational demands, particularly for large-scale simulations.

In this investigation, we propose an intelligent system called Adaptive Packing Intelligence (API). API integrates a phi-function core, an adaptive agent module, and a hybrid optimization engine. This architecture accommodates various objective functions, including volume minimization, packing density maximization, and energy-based interaction modeling.

API demonstrates significant potential in areas such as engineering, robotics, materials science, and medicine, where arranging components optimally in space is crucial for system efficiency. It can be applied to microchip layout design, coordinating robotic swarms, and planning modular assemblies. Additionally, in logistics and manufacturing, APIs help optimize packing and placement of goods, decreasing space requirements and transportation expenses.

In physics and chemistry, the system models how particles interact, how molecules pack, and how energy affects their spatial arrangements, offering insights into stability and dynamics. The API provides adaptive control over object placement and interactions, creating a unified approach to complex packing and simulation challenges across scientific and industrial fields. While the full architecture encompasses modular components for simulation and control, this paper primarily focuses on the core modeling framework and its practical applications. Selected examples demonstrate how agent-driven adaptability enables precise and efficient spatial arrangements in various applications.

Special universal mathematical model

At the core of the API system is a universal mathematical model designed to optimize the spatial arrangement of geometric objects. This model frames the problem as a nonlinear optimization task, where a set of objects (e.g., capsules, mechanical parts, molecules) with predefined geometric properties are placed within a bounded region.

We consider a set geometric object in T_i in the mD Euclidean space ($m = 2, 3, ..., 24$). Let metric characteristics a_i, $i \in I_N = \{1, 2, ..., N\}$ give sizes of T_i. We denote $\mathbf{u}_i = (\mathbf{v}_i, \mathbf{\Theta}_i) \in \mathbf{R}^l$, $l \le 2m+1$, as placement parameters of T_i, $i \in I_N$, where $\mathbf{v}_i$ are coordinates of T_i and $\mathbf{\Theta}_i$ are rotation angles (if any). The location of all objects is defined as $\mathbf{u} = (\mathbf{u}_1, \mathbf{u}_2, ..., \mathbf{u}_N)$.

The core mathematical model of API is expanded by adding an adaptive agent parameter that controls the spatial interaction mode between objects and the boundary of the region.

Objects from the set T_i, $i \in I_N$, are arranged with respect to a designated region P in one of three Agent-Controlled Configurations, depending on the value of the adaptive agent parameters:

– minimum admissible separation $d_{ij} \ge 0$, $1 \le i < j \in I_N$ between themselves and the minimum admissible distances $d_i \ge 0$, $i \in I_N$ to the boundary of P;

– precise contact, where objects may be positioned to touch each other or the boundary of the region without overlapping (in this case, minimum admissible separation $d_{ij} = 0$, $1 \le i < j \in I_N$ between themselves and the minimum admissible distances $d_i = 0$, $i \in I_N$ to the boundary of P);

– controlled overlap and overhang, i.e. $d_{ij} < 0$, $1 \le i < j \in I_N$, and allowing objects to extend beyond the boundary P, regulated by $d_i < 0$, $i \in I_N$.

Our goal is to identify a subset from the set O_i, $i \in I_N$, that can be arranged to maximize a total cost (area, volume, etc.) of the objects. The mathematical model of the problem is as follows:

$$C^* = \max \sum_{i \in I_N} t_i C(O_i) \text{ s.t. } \mathbf{u} \in G \quad (1)$$

where

$$t_i = \begin{cases} 1 \text{ if } \Phi_i(\mathbf{u}_i) \geq d_i, \\ 0 \text{ otherwise,} \end{cases} \quad (2)$$

$$G = \left\{ \begin{array}{l} \mathbf{u} \in \mathbf{R}^{lN} : t_i t_j (\Phi_{ij}(\mathbf{u}_i, \mathbf{u}_j) \text{-} d_{ij}) \geq 0, \\ i < j \in I_N \end{array} \right\}. \quad (3)$$

Here, t_i, $i \in I_N$, indicates whether O_i belongs to the selected subset. The inequality $\Phi_i(\mathbf{u}_i) \geq d_i$ specifies the placement condition relative to the boundary of P, while the inequality $\Phi_{ij}(\mathbf{u}_i, \mathbf{u}_j) \geq d_{ij}$ defines the nonoverlapping or distance condition.

The main challenge is to construct normalized phi-functions. Some basic phi-functions have already been built [17-18].

Initially, we perform exhaustive testing on all possible subsets of the object set. Then, we place each subset in the region. Each subset is positioned within the target region, and its placement is evaluated. Finally, we select the placement that yields the best value of the objective function.

Referring to the typology of Cutting and Packing Problems [15], Problems (1) – (3) can be classified as either a Knapsack Problem or an Identical Item Packing Problem, depending on the objects' metric properties. To solve these problems, a sequential addition strategy [16] (also known as block optimization [17]) is typically employed. A method for solving the Knapsack Problem is discussed in [18]. An additional challenge arises when objects have orientation angles, which must be taken into account during placement.

Agent-Controlled Separation Configuration

When the adaptive parameters have positive values, the system enforces minimum allowable distances between objects and between objects and the boundary of the region. This setup ensures all elements are positioned with enough clearance, preventing contact or overlap. For instance, in brachytherapy planning, such a mode allows for the safe placement of cylindrical radioactive capsules within the treatment area. By maintaining controlled separation, the system reduces the risk of excessive radiation exposure to healthy tissues while ensuring even coverage of the target zone. In this case, the adaptive parameters serve as safety margins, guiding the optimization process toward clinically acceptable configurations.

We consider a set of objects $T_i = \mathbf{R}^3$ as cylinders with given radius r, and height h, $i \in I_N = \{1, 2, ..., N\}$, $\mathbf{u}_i = \mathbf{v}_i = (x_i, y_i)$. Rotation is defined by two angles $\mathbf{\Theta}_i = (\varphi_i, \omega_i)$. Adaptive parameters $d_{ij} = d_1 > 0$, $1 \leq i < j \in I_N$, mean distance between cylinders and $d_i = d_2 > 0$, $i \in I_N$, define the minimum admissible distances to the boundary of P. The targeted region P is a convex polyhedron specified by the inequalities $A_l x + B_l y + C_l z + D_l \geq 0,\ l \in L$ with normal equations left parts.

The problem (1) – (3) transforms to:

$$V^* = \pi r^2 h \max \sum_{i \in I_N} t_i \text{ s.t. } u \in G \quad (4)$$

where

$$t_i = \begin{cases} 1 \text{ if } \Phi_i(\mathbf{u}_i) \geq d_2, \\ 0 \text{ otherwise,} \end{cases} \quad (5)$$

$$G = \left\{ \begin{array}{l} \mathbf{u} \in \mathbf{R}^{5N} : t_i t_j (\Phi_{ij}(\mathbf{u}_i, \mathbf{u}_j) \text{-} d_1) \geq 0, \\ i < j \in I_N \end{array} \right\}. \quad (6)$$

The problem (4) – (6) is the identical item problem. Cylinders are presented as convex right prisms for which the normalized phi-functions are used [19, 20].

An example placement of 20 cylinders within a cuboid is illustrated in Fig. 1.

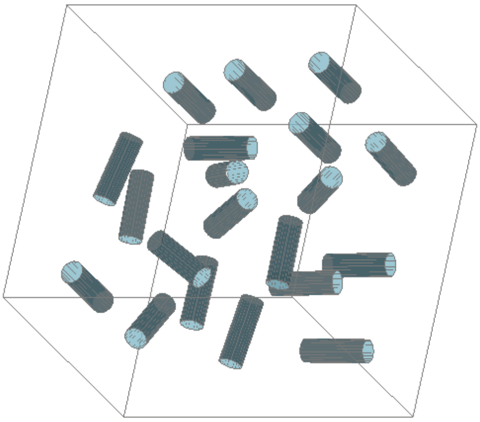

Fig. 1. Placement of 20 cylinders

Agent-Controlled Contact Configuration

When the adaptive parameters are positive, the system enforces minimum allowable distances between objects and between objects and the boundary of the region. This setup ensures that all elements are spaced with enough clearance, preventing contact or overlap.

A clear example of the contact configuration appears in the modeling of composite materials for additive manufacturing. In applications like aerospace and defense, titanium alloy powders are packed within a specified container to form structural components. To ensure mechanical stability and uniformity, particles must be positioned in close contact without overlapping.

We consider a set of objects $T_i = \mathbf{R}^3$ as spheres with given radius r_i $i \in I_N = \{1, 2, ..., N\}$, $\mathbf{u}_i = \mathbf{v}_i = (x_i, y_i)$. Rotation is not defined Adaptive parameters are set $d_{ij} = d_i = 0$, $1 \le i < j \in I_N$ The targeted region P is a cuboid with given sizes.

The problem (1) – (3) transforms to:

$$V^* = \pi \max \sum_{i \in I_N} t_i r_i^3 \text{ s.t. } u \in G \quad (7)$$

where

$$t_i = \begin{cases} 1 \text{ if } \Phi_i(\mathbf{u}_i) \ge 0, \\ 0 \text{ otherwise,} \end{cases} \quad (8)$$

$$G = \left\{ \mathbf{u} \in \mathbf{R}^{3N} : t_i t_j \Phi_{ij}(\mathbf{u}_i, \mathbf{u}_j) \ge 0,\ i < j \in I_N \right\}. \quad (9)$$

The problem (7) – (9) is the knapsack problem.

An example placement of 11048 spheres within a cuboid is illustrated in Fig. 2.

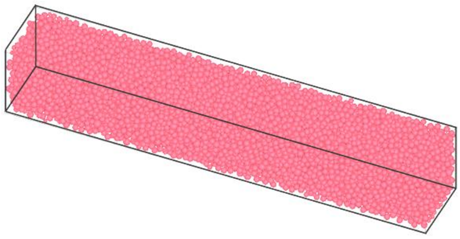

Fig. 2. Placement of 11048 spheres

Agent-Controlled Overlap Configuration

In certain biological systems, spatial organization involves partial overlap between entities. A representative case is the modeling of chromosome territories within the cell nucleus. Although chromosomes are inherently three-dimensional structures, their spatial distribution during interphase can be approximated in two dimensions for computational efficiency. In this simplified model, chromosome territories are represented as overlapping ellipses placed within a polygon that approximates the nuclear boundary. The adaptive packing system enables controlled overlap between these ellipses, allowing the simulation to reflect biologically plausible configurations.

This configuration corresponds to the agent-controlled overlap mode, where the adaptive parameters permit regulated intrusion beyond object boundaries and into the surrounding space, enabling biologically accurate representations of chromosomal positioning during interphase.

According to the problem (1)–(3) we suppose $T_i \in \mathbf{R}^2$ are ellipses with half-axis a_i, and height b_i, $i \in I_N = \{1, 2, ..., N\}$, $\mathbf{u}_i = (x_i, y_i, \varphi_i)$, parameters are $d_{ij} = d < 0$, $1 \le i < j \in I_N$ and $d_i = 0$, $i \in I_N$. The placement region P is considered as a polygon. In order to improve the packing efficiency we consider a_i and b_i as variables.

The problem (1) – (3) takes the following form:

$$V^{*}=\pi \max \sum_{i\in I_N} t_i a_i b_i \text{ s.t. } u\in G \qquad (10)$$

where

$$t_i=\begin{cases} 1 \text{ if } \Phi_i(\mathbf{u}_i)\geq 0, \\ 0 \text{ otherwise,} \end{cases} \qquad (11)$$

$$G=\left\{\mathbf{u}\in \mathbf{R}^{3N} : t_i t_j \Phi_{ij}(\mathbf{u}_i,\mathbf{u}_j)\geq d,\ i<j\in I_N\right\}. \qquad (12)$$

We set $d_1=-3$. Normalized phi-functions are constructed in [19,20].

An example of the placement of 10 ellipses is shown in Figure 3.

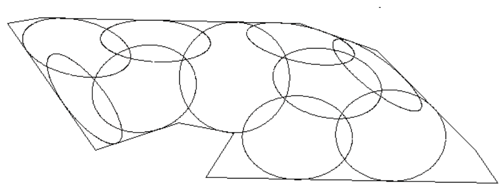

Fig. 3. Illustration of the packing of 10 ellipses

Conclusion

This paper introduces an intelligent adaptive system for solving complex spatial optimization problems involving the placement of geometric objects. The proposed Adaptive Packing Intelligence (API) system integrates a universal mathematical model based on normalized phi-functions with agent-controlled interaction parameters. This combination enables flexible control over object positioning, supporting various placement modes: from strict separation to precise contact and controlled overlap. The model effectively considers both continuous and combinatorial aspects of packing problems, taking into account the geometry, orientation, and spatial constraints of objects. Its adaptability allows it to be applied across diverse fields, including medicine, biology, materials science, and engineering.

By incorporating adaptive parameters, the model can flexibly adjust to different interaction regimes, making it suitable for a wide range of real-world applications. The use of mixed-integer nonlinear programming, constraint programming, and heuristic methods allows the system to efficiently find optimal or near-optimal configurations.

Furthermore, integrating artificial intelligence opens the door to predictive modeling and real-time adaptation. Future developments will aim to improve the system's responsiveness to environmental changes, expand its ability to manage more complex constraints, and enhance the accuracy and scalability of the solutions.

References

1. T. A. Cohen, V. L. Patel, E. H. Shortliffe, Intelligent Systems in Medicine and Health: The Role of AI, Springer, (2022). doi:10.1007/978-3-031-09108-7.
2. R. H. Chowdhury, Intelligent systems for healthcare diagnostics and treatment, World Journal of Advanced Research and Reviews 23(01) (2024) 007–015. doi: 10.30574/wjarr.2024.23.1.2015
3. Y. Kumar, A. Koul, R. Singla, M. F. Ijaz, Artificial intelligence in disease diagnosis: a systematic literature review, synthesizing framework and future research agenda, Journal of Ambient Intelligence and Humanized Computing 14 (2023) 8459–8486. doi: 10.1007/s12652-021-03612-z.
4. U. Wolter, Logics of Statements in Context-Category Independent Basics, Mathematics 10(7) (2022) 1085. doi: 10.3390/math10071085.
5. O. Tunç, New Results on the Ulam-Hyers-Mittag-Leffler Stability of Caputo Fractional-Order Delay Differential Equations, Mathematics 12(9) (2024) 1342. doi: 10.3390/math12091342.
6. Y. Liu, R. Wu, A. Yang, Research on Medical Problems Based on Mathematical Models, Mathematics, 11(13) (2023) 2842. doi: 10.3390/math11132842.
7. S. Jones, K. Thompson, B. Porter, et al., Automation and artificial intelligence in radiation therapy treatment planning, J Med Radiat Sci. 71(2) (2024) 290–298. doi: 10.1002/jmrs.729.
8. J. I. Lim, A. V. Rachitskaya, J. A. Hallak, S. Gholami, M. N. Alam, Artificial intelligence for retinal diseases, Asia Pac J Ophthalmol (Phila) 13(4) (2024) 100096. doi: 10.1016/j.apjo.2024.100096.
9. P. McGeachy, J. Madamesila, A. Beauchamp, R. Khan, An open-source genetic algorithm for determining optimal seed distributions for low-dose-rate prostate brachytherapy, Brachytherapy 14(5) (2015) 692–702. doi:10.1016/j.brachy.2015.04.007.
10. N. Sehgal, A. J. Fritz, J. Vecerova, et al., Large-scale probabilistic 3D organization of human chromosome territories, Hum Mol Genet. 25(3) (2016) 419-36. doi:10.1093/hmg/ddv479.
11. J. D. Pintér, I. Castillo, F. J. Kampas, Nonlinear Optimization and Adaptive Heuristics for Solving Irregular Object Packing Problems, Algorithms 17 (2024) 480. doi:10.3390/a17110480.
12. X. Zhao, Y. Rao, P. Qi, Q. Lyu, P. Yang, S. Yu, An efficient constructive heuristic for the rectangular packing problem with rotations. PLoS ONE 18(12) (2023) e0295206. doi:10.1371/journal.pone.0295206.
13. J. Lee, S. Leyffer, Mixed Integer Nonlinear Programming, Springer, 2012.

doi:10.1007/978-1-4614-1927-3.

14. T. Romanova, A. Pankratov, I. Litvinchev, et al., Sparsest packing of two-dimensional objects. International Journal of Production Research 59(13) (2020) 3900–3915. doi:10.1080/00207543.2020.1755471.

15. G. Wäscher, H. Haußner, H. Schumann, An improved typology of cutting and packing problems. European Journal of Operational Research 183(3) (2007) 1109–1130. doi: 10.1016/j.ejor.2005.12.047.

16. F. Wang, K. Hauser, Stable Bin Packing of Non-convex 3D Objects with a Robot Manipulator, in: 2019 International Conference on Robotics and Automation (ICRA), Montreal, QC, Canada, 2019, pp. 8698–8704, doi: 10.1109/ICRA.2019.8794049.

17. G. Yaskov, A. Chugay, Packing Equal Spheres by Means of the Block Coordinate Descent Method, CEUR Workshop Proceedings 2608 (2020) 150–160. doi: 10.32782/cmis/2608-13.

18. G. Yaskov, T. Romanova, I. Litvinchev, S. Shekhovtsov, Optimal Packing Problems: From Knapsack Problem to Open Dimension Problem, in: Vasant, P., Zelinka, I., Weber, GW. (eds.) Intelligent Computing and Optimization, ICO 2019, Advances in Intelligent Systems and Computing, vol 1072, Springer, Cham, 2020, pp. 671–678. doi: 10.1007/978-3-030-33585-4_65.

19. T. Romanova, J. Bennell, Y. Stoyan, A. Pankratov, Packing of concave polyhedra with continuous rotations using nonlinear optimisation, European Journal of Operational Research 268(1) (2018) 37–53. doi: 10.1016/j.ejor.2018.01.025.

20. A. M. Chugay, A. V. Zhuravka, Packing Optimization Problems and Their Application in 3D Printing, in: Z. Hu, S. Petoukhov, I. Dychka, M. He (eds.), Advances in Computer Science for Engineering and Education III. ICCSEEA 2020, Advances in Intelligent Systems and Computing, vol. 1247, Springer, Cham, 2021. doi:10.1007/978-3-030-55506-1_7.

The article has been sent to the editors 13.11.25.
After processing 10.12.25.
Submitted for printing 30.12.25.