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«ІНСТИТУТ МОДЕРНІЗАЦІЇ ЗМІСТУ ОСВІТИ»

УНІВЕРСИТЕТ ЕКОНОМІКИ В БИДГОЩІ
(РЕСПУБЛІКА ПОЛЬЩА)

КУЯВСЬКИЙ УНІВЕРСИТЕТ У ВЛОЦЛАВЕКУ
(РЕСПУБЛІКА ПОЛЬЩА)

ЕКОНОМІЧНО-ГУМАНІТАРНИЙ УНІВЕРСИТЕТ У ВАРШАВІ
(РЕСПУБЛІКА ПОЛЬЩА)

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COBB–DOUGLAS FUNCTION AS A TOOL FOR ASSESSING THE EFFICIENCY OF UKRAINE'S MANUFACTURING INDUSTRY IN THE CONDITIONS OF ECONOMIC CHALLENGES

Stability and sustainability of industrial development are key conditions for the country's economic security. In conditions of structural shifts, military risks, increasing uncertainty and limited resources, the use of scientifically based approaches to assessing production efficiency and determining the factors of its growth becomes especially relevant. The manufacturing industry plays a significant role in shaping Ukraine's GDP and added value, so the analysis of its productivity and the changes taking place in this sector of the economy is of important practical importance. The purpose of the study is to develop a methodology for constructing a mathematical model that reflects the influence of various factors on the state of the processing industry in Ukraine, with the subsequent use of this model to ensure the reliability of forecasting changes in economic indicators in the medium term.

In world economics, one of the most common models that determines the influence of factors on the efficiency of production processes is the Cobb–Douglas function [1–3 et al.]. It was first proposed by Charles Cobb and Paul Douglas in 1928, based on data from the USA manufacturing industry. In its classical form, this function describes the dependence of the volume of output (Y) on the cost of fixed assets (K) and labor costs (L):

$$Y = A \cdot L^{\alpha} \cdot K^{\beta} \tag{1}$$

where A is the technological coefficient corresponding to the output volume at unit labor and capital costs; α, β are elasticities of the output function with respect to changes in relevant external factors.

Taking into account the change in technology during the studied period, the Cobb-Douglas function takes the form:

$$Y = A_0 \cdot L^{\alpha} \cdot K^{\beta} \cdot e^{\gamma t} \tag{2}$$

where A_0 is the initial level of productivity; γ is the indicator of the rate of technological progress; t is the time.

At the current stage of software development, linearization of functions (1) and (2) using logarithmic transformation allows processing large arrays of empirical data and applying the least squares method to estimate model parameters, which provides the smallest variance of statistical estimates. That is why the study of the possibility of using a mathematical model based on the Cobb–Douglas function for Ukrainian realities is important from a scientific and practical point of view.

The fundamental tool for building mathematical models in economics is statistical methods, in particular methods of correlation and regression analysis [4 et al.]. Calculations were performed in MS Excel using the built-in functions MMULT, MINVERSE, and TRANSPOSE, which are used to work with data arrays provided in matrix form, as well as using the Data Analysis add-in. The data of the State Statistics Service of Ukraine [5] for the period 2010–2024 were selected for the study.

As a result of estimating the parameters of model (1), the equation was obtained:

$$\hat{Y} = 5.006 \cdot L^{0.048} \cdot K^{0.862} \quad (3)$$

According to this model, the elasticity of capital is $\beta = 0.862$, , which indicates its leading role in shaping the production potential of the industry, while the impact of labor $\alpha = 0.048$ is significantly lower. These conclusions coincide with actual trends: over the last decade, Ukrainian industry has been undergoing modernization, equipment and technology are being updated, which increases productivity much faster than the change in the number of employees. However, relative to the sum of elasticities we have $\alpha + \beta = 0.91 < 1$, that is, this sum is less than unity. This means that the return in this industry is gradually declining, which is typical for an industry operating under conditions of limited investment opportunities. For this model, the coefficient of determination is equal to $R^2 = 0.615$, i.e., 61.5% of the variability in the volume of output of the manufacturing industry is due to the influence of the cost of fixed assets and labor costs.

As a result of estimating the parameters of the extended model (2), the following equation was obtained:

$$\hat{Y} = 219309.042 \cdot L^{0.009} \cdot K^{0.072} \cdot e^{0.100} \quad (4)$$

For this model, the coefficient of determination is equal to $R^2 = 0.948$. Therefore, this model provides a higher level of explanatory power. However, it should be noted that according to this model, the variability of output is largely related to the time variable, that is, it is determined by technological change, while the effects of labor and capital are statistically weak. Such an effect may be inherent in an economy undergoing rapid structural change.

A detailed analysis of the structure of production factors showed that capital in the processing industry of Ukraine has not only a quantitative, but also a qualitative dimension. According to the State Statistics Service of Ukraine, in 2023, the share of fixed assets in operation for more than 20 years exceeded 55%, and their renewal rate remained below 10%. This means that even a small increase in investment in equipment renewal and modernization can provide a significant increase in the productivity of the processing industry. Therefore, according to model (3), the capital elasticity of the production function is sufficiently high (β = 0.862), i.e., if the volume of capital changes by 1%, provided that other factors (labor and technology) remain unchanged, the volume of production will increase by 0.862%. This reflects not only the direct impact of capital, but also the compensatory effect of overcoming technological lag.

In addition to two mathematical models based on the Cobb–Douglas function, the paper also built a linear multivariate regression model, which, like the extended model (2), takes into account the impact of the cost of fixed assets, labor costs, and changes in production technology, which is reflected as a time dependence. There are known examples of the use of models of this type for analyzing and forecasting the dynamics of production volume in the manufacturing industry [6 et al.]. When estimating the parameters of this model, the following regression equation was obtained:

$$\hat{Y} = 483618.10 - 0.908L - 0.021K + 150551.29t \qquad (5)$$

This linear regression model has a fairly high coefficient of determination, which is equal to R^2 = 0.8831, but the coefficients on the variables K and L turned out to be negative and statistically insignificant, which makes it less suitable for economic interpretation.

To build forecasts for 2026 and 2027, based on the results of the sample, we calculated the average growth rates of fixed assets, which are 1.01, and labor costs, which are 1.10. To assess the accuracy of the forecast, the results of theoretical calculations using models (4) and (5) were compared with real statistical data for the period 2020–2024. A comparison of the accuracy of the forecasts showed that the average error for the extended Cobb–Douglas model is approximately 4%, while for the linear regression model it is about 14%. This demonstrates the advantage of the mathematical model based on the Cobb–Douglas function in its use for forecasting the dynamics of the manufacturing industry even in the short term. Therefore, according to the Cobb–Douglas model, it can be expected that the production volume in 2026 will be in the range from 3.54 to 3.85 trillion UAH, and in 2027 it will be from 4.04 to 4.47 trillion UAH, that is, an increase in production volume in the processing industry should be expected.

The paper also estimated the parameters of the extended Cobb–Douglas function based only on the pre-war period (2010–2021). A comparison of the resulting model with a model built using the full volume of statistical data showed the absence of significant differences in the values of elasticities and the scale effect. This indicates that even during the war period, the dynamics of the processing industry retained the general patterns that were inherent in previous years. Moreover, the calculations of forecasts based on pre-war data demonstrate a growth trajectory that is fully consistent with the trends of the full sample. This suggests that the structure of the processing industry in Ukraine remains stable even during wartime.

Thus, in the context of modern economic challenges, the use of the Cobb–Douglas function to build a mathematical model of the dynamics of the production function makes it possible to assess the real contribution of various types of resources to the development of the processing industry of Ukraine and to obtain a tool for making informed management and investment decisions. The results of the study can be used in planning the modernization of fixed assets, optimizing the cost structure, and forming a policy of economic security for the industrial sector.

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