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BIFURCATIONS IN MODELS OF ECONOMIC DYNAMICS

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Understanding the specifics of the functioning of an open economic system, the possible paths of its development and, accordingly, the formation of targeted management of the state of this system - the solution to all these issues is impossible without constructing a mathematical model of the system, which would establish the relationship between the changes that occur in the system and the causes of these changes. Management of the economic system both at the level of an individual enterprise and at the level of an industry, region and the state as a whole, based on the use of adequate mathematical models of a dynamic system, allows for the optimization of economic development in accordance with the goals of sustainable development.

Mathematical economics is characterized by two fundamentally different methods of modeling: static and dynamic. Static theory considers economic phenomena in their instantaneous manifestation, neglecting inertial changes in time space. The static approach to modeling an economic system is based on the concept of equilibrium of its interconnected elements. Dynamic analysis of economic processes was formed in parallel with the development of economic theory itself. The opposition of static and dynamic approaches in an explicit form runs through the entire history of economic thought. Probably, the main reason lies in the fundamental differences between the understanding of equilibrium balance and causal dynamics.

The most effective, albeit rather complex, mathematical models in this regard are nonlinear dynamics. A distinctive feature of these models is that changes in some factors lead to disproportionate changes in related factors, i.e., the relationships between economic indicators are described by nonlinear functions. The theory of economic dynamics has a very wide range of directions of various research, among which the most significant are the problems of economic growth and business cycles. The given distinctions of dynamic processes can be interpreted as evolutionary (unrepeatable, irreversible) and oscillatory (repeated, reversible).

Nonlinear dynamics is one of the most important and promising areas of development in economic science. The powerful modern apparatus of the theory of differential equations and related sections of mathematical topology provides many opportunities for obtaining meaningful results, primarily of a qualitative nature, in solving problems of economic forecasting. The formation of dynamic analysis in economics occurred in parallel with the development of economic theory itself [1-3 etc.]. The opposition between dynamic and static approaches is easily traced throughout the history of economic thought. It can be assumed that the main reason for this lies in the fundamental differences between the understanding of the balance of power and causal dynamics.

A great contribution to the development of economic systems modeling was made by Joseph Schumpeter. According to his interpretation [4], all types of economic dynamics processes can be divided into two groups: evolutionary (irreversible) and wave-like (reversible). Evolutionary processes are understood as changes in the development of an economic system that, in the absence of external influences (disturbances), proceed in only one direction. Wave-like, or reversible, are processes of change that from time to time change the direction of development and sooner or later may return to the initial state. Also, in many economic models, it is necessary to take into account the delay in compiling balance sheet ratios that take into account interactions of various nature.

According to the theory of nonlinear dynamics, when moving towards a goal in the presence of significantly nonlinear relationships, a hierarchy of instability appears, which produces the appearance of limit cycles, homoclinic structures and the spontaneous formation of chaotic regimes. As a result of such transformations (bifurcations), several different final states of economic equilibrium may appear - the so-called bistability effect. Using the methods of economic dynamics, it is possible to predict the moment of the appearance of a chaotic regime in the system under study, determine the number of possible equilibrium states and identify the nature of their stability. All this, in turn, generates the general problem of constructing alternative scenarios for the development of complex systems.

Let us consider this issue using the example of a mathematical model that describes the formation of prices in a competitive market. In modern economic literature, there is a fairly detailed qualitative description of the mechanism of formation of domestic prices for products, which is based on the analysis of the dynamics of financial flows during export-import operations. Before considering the mathematical model of price dynamics based on the classical macroeconomic Fisher equation [5, 6], a number of assumptions must be made:

1) a free trade scheme is considered without the influence of governments and monopoly structures;

2) the level of national income is considered to be given and the price indicator is allocated on the basis of the quantity theory of monetary circulation;

3) during the period under consideration, changes in the money supply are due only to the surplus (or deficit) of the trade balance;

4) exchange rates are assumed to be fixed, which is equivalent to the complete unification of international settlements;

5) transport costs, insurance and other costs are not taken into account for either commodity or financial flows.

In the following, the following notations will be used: Q is money supply; V is velocity of money circulation (constant value); Y is level of national income (constant value); P is domestic price index; PM is external price index (constant value); M is quantity of imports; X is quantity of exports.

In this definition, the basic equation of the Fisher model has the form:

$$QV = PV \,. \tag{1}$$

The volume of exports $X = X(P)$ is a decreasing function of the domestic price $\partial X / \partial P < 0$, and the volume of imports $M = M(P)$, on the contrary, serves as an increasing function of the domestic price $\partial M / \partial P > 0$.

The relationship $P^* X(P^*) - P_M M(P^*) = 0$ is characterized by the condition of equilibrium of the trade balance and it is assumed that there are positive solutions of this algebraic equation that determine the equilibrium values for the domestic price $P.^*$ Disruption of equilibrium is accompanied by a change in the money supply and is expressed by the equation:

$$\frac{dQ}{dt} = PX(P) - P_M M(P). \tag{2}$$

From the relation (1) it follows that a change in the money supply leads to a change in the internal price $P = P(t)$. We will assume that such a change does not occur instantaneously, that is, there is a time delay, which is determined by a constant positive value τ. In this case, equation (1) is written in the following form:

$$\frac{dP(t)}{dt} = \frac{V}{Y} \cdot \frac{dQ(t-\tau)}{dt}. \tag{3}$$

From relations (2) and (3) follows a differential equation that describes the dynamics of the internal price:

$$\frac{dP(t)}{dt} = \frac{V}{Y} \{P(t-\tau) \cdot X[P(t-\tau)] - P_M M[P(t-\tau)]\}. \tag{4}$$

Let us assume that the export $X(P)$ and import $M(P)$ functions are characterized by a nonlinear dependence on the domestic price P and that their expansion in a Taylor series up to the third degree inclusive in the vicinity of the equilibrium position P^* has the form:

$$\begin{aligned} X(P) &= X_0 + X_1(P - P^*) + X_2(P - P^*)^2 + X_3(P - P^*)^3 + O(P^4), \\ M(P) &= M_0 + M_1(P - P^*) + M_2(P - P^*)^2 + M_3(P - P^*)^3 + O(P^4), \end{aligned} \tag{5}$$

where $X_i = \frac{\partial^i (P^*)}{i! \partial P^i}$, $M_i = \frac{\partial^i M(P^*)}{i! \partial P^i}$ $(i = \overline{1,3})$ are the corresponding derivatives of the functions $X(P)$ and $M(P)$ at the equilibrium point P^*. Let us introduce a new variable, which has the meaning of the deviation of the internal price from the equilibrium value. In this case, equation (4) taking into account (5) is transformed into the form:

$$\frac{d\overline{P}}{d\bar{t}}=\frac{\tau V}{Y}\left[G_1\overline{P}(\bar{t}-1)+G_2\overline{P}^2(\bar{t}-1)+G_3\overline{P}^3(\bar{t}-1)+O\left(\overline{P}^4\right)\right], \tag{6}$$

where

$$\bar{t}=\tau\cdot t,\quad G_i=X_{i-1}+P\cdot X_i-P_M M_i,\quad i=\overline{1,3}.$$

It is advisable to begin the analysis of the dynamics of the process described by (6) by studying the conditions of local stability, limiting ourselves only to the linear part:

$$\frac{d\overline{P}}{d\bar{t}}=\frac{\tau V}{Y}G_1\cdot\overline{P}(\bar{t}-1). \tag{7}$$

The characteristic equation for the slip ratio (7) has the form:

$$\lambda-\frac{\tau V G_1}{Y}e^{-\lambda}=0. \tag{8}$$

Using the results from the theory of stability of differential-difference equations in relation to equation (7), we obtain the necessary and sufficient conditions for the linear stability of the economic system:

$$0<-\frac{\tau V G_1}{Y}<\frac{\pi}{2} \tag{9}$$

From the form of the left-hand side of the double inequality (9) it follows that the quantity G_1 is negative, and the right-hand side formalizes the value of the upper bound for the module G_1.

Condition (9) has a fairly transparent meaningful interpretation, to demonstrate which we will perform the transformation of the initial parameters of the studied model (6).

Let $G_1=X_0\left[1-\eta_X-\eta_M\right]$, where $\eta_X=\frac{P^*X_1}{X_0}$, $\eta_M=\frac{P^*M_1}{M_0}$ under the condition $P^*X_0=P_M M_0$. The quantities η_X and η_M are the elasticity of export and import functions at price P. Since $X_0>0$, the condition $G_1<0$ is equivalent to $\eta_x+\eta_M>1$, which corresponds to the so-called Marshall-Lerner conditions [7]. In this case, condition (9) is transformed to the form:

$$1<\eta_X+\eta_M<1+\frac{Y\pi}{2X_0V\tau}. \tag{10}$$

Therefore, the sum of elasticities must not only be greater than one, but also less than another critical value. In other words, instability in the economic model under study can arise not only when the sum of elasticities is sufficiently small, but also in the case of a significantly large value of this sum.

We investigate the behavioral properties of the initial dynamic system (6) in a small neighborhood of the boundary values of inequality (10). We consider the loss of stability at the lower boundary. We introduce a small parameter $\nu_1 = 1 - \eta_X - \eta_M$. In this case, with a change in sign, the eigenvalue of the linearized problem passes through zero, and the stationary value of the price P^* may not exist or split into several stationary states. That is, there is a bifurcation of stationary solutions. Differential-difference equations (6) can be represented as follows:

$$\frac{d\overline{P}}{d\bar{t}} = A_1 \cdot \overline{P}(\bar{t}-1) + A_2\overline{P}^2(\bar{t}-1) + A_3\overline{P}^3(\bar{t}-1), \tag{11}$$

where $A_i = \frac{\tau V}{Y} G_i$, $i = \overline{1,3}$. It should be remembered that the quantity A_1 is small, i.e., $G_1 = X_0\nu_1$. Additionally, let us assume that and A_2 will also be small if we introduce a small quantity $\nu_2 = G_2 / X_0$. Using the technique of the central manifold method, we can prove that the differential-difference equation (11) under the condition of A_1, A_2 smallness and finite delay time is topologically equivalent in the neighborhood of the point $\overline{P} = 0$ to the differential equation:

$$\frac{d\overline{P}}{d\bar{t}} = A_1 \cdot \overline{P}(\bar{t}) + A_2\overline{P}^2(\bar{t}) + A_3\overline{P}^3(\bar{t}). \tag{12}$$

Using linear substitution of variables in equation (12), we obtain:

$$\frac{d\widetilde{P}}{d\bar{t}} = \alpha_1 + \alpha_2\widetilde{P} + A_3\widetilde{P}^3 \tag{13}$$

where

$$\alpha_1 = \frac{2A_2^3}{27A_3^2} - \frac{A_1A_2}{3A_3}, \quad \alpha_2 = A_1 - \frac{A_2^2}{3A_3}.$$

The transformation $\widetilde{P}(\tilde{t}) = |\beta| W(\bar{t})$ gives an imaginary view of the Poincaré normal form for the differential equation (13):

$$\frac{dW}{d\bar{t}} = \beta_1 + \beta_2 W + SW^3, \tag{14}$$

where

$$\beta = \frac{1}{\sqrt{A_3}}, \quad \beta_1 = \frac{\alpha_1}{|\beta|}, \quad \beta_2 = \alpha_2, \quad S = sign\beta = \pm 1.$$

Let for definiteness $S = -1$. Then equation (14) can have three equilibrium positions. The bifurcation "fold" (or saddle-knot bifurcation) is determined by the curve R on the plane $\beta_1 0 \beta_2$ of the given projection of the line

$$\Gamma:\begin{cases}\beta_1+\beta_2 V-V^3=0,\\ \beta_2-3V^2=0.\end{cases} \tag{15}$$

Eliminating V from (15), we have $R=\{(\beta_1,\beta_2): 4\beta_2^3+27\beta_1^2=0\}$. The curve R is a semi-cubic parabola and has two branches R_1 and R_2, which meet at the "beak tip" point at $\beta_1=\beta_2=0$. Figure 1 shows the corresponding bifurcation diagram.

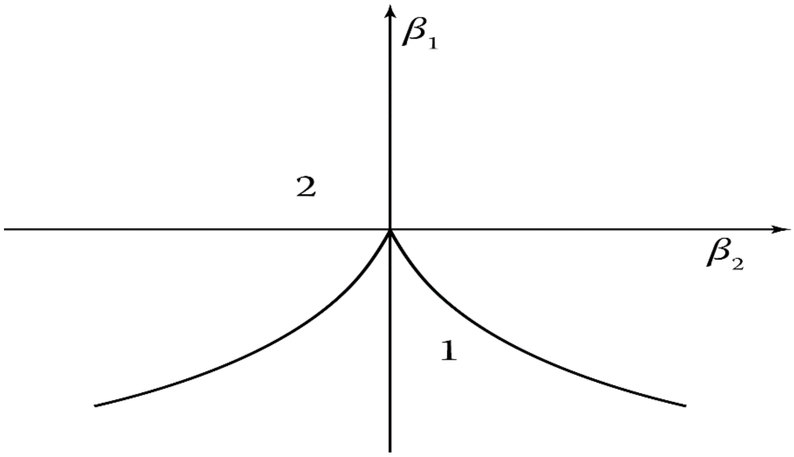

Figure 1. Diagram of a "beak tip" bifurcation
Source of the figure: author's development

In region 1 of this diagram, there are three equilibrium positions before the boundary line: two stable and one unstable. In region 2, there is a single equilibrium position behind the dividing line, which is stable. The nondegenerate bifurcation "fold" occurs at the intersection R_1 or R_2 at any point of the parameter plane $\beta_1 0 \beta_2$ other than the origin. If the system passes from region 1 to region 2, the right stable equilibrium position merges with the unstable one and both disappear.

Similarly, the left stable equilibrium position merges with the unstable one on the line R_2.

When approaching the "beak tip" point in front of region 1, all three equilibrium states merge as the triple root of the right-hand side of the original equation (14). It is also important to note that when transitioning from a stable regime to an unstable one in (14), a hysteresis phenomenon is observed and a catastrophe occurs. The case $S=1$ can be considered analogously.

Next, we will clarify the situation when the sum of elasticities $\eta_X+\eta_M$ is close to the value on the right-hand side of inequality (10). Let us introduce a small parameter $\mu=-(A_1+0.5\pi)$ into consideration. Then equation (6) admits the transformation:

$$\frac{d\overline{P}}{d\overline{t}}=-(\mu+0.5\pi)\overline{P}(\overline{t}-1)+A_2\cdot\overline{P}^2(\overline{t}-1)+A_3\cdot\overline{P}^3(\overline{t}-1). \tag{16}$$

The linear part (16) has the characteristic equation: $\lambda+(\mu+0.5\pi)e^{-\lambda}=0$.

Let us determine when this equation has a pair of purely imaginary roots. If $\lambda = \pm i\omega$, then the conditions are true:

$$(\mu + 0.5\pi)\cos\omega = 0, \quad \omega - (\mu + 0.5\pi)\sin\omega = 0. \tag{17}$$

It follows that when $\mu = 0$ equation (17) has a pair of purely imaginary roots at $\omega = 0.5\pi$. Since λ is analytic with respect to μ, differentiation (17) gives the derivative $\frac{\partial\lambda}{\partial\mu} = (0.5\pi + i) / (0.25\pi^2 + i)$ with respect to $\mu = 0$. Thus, all conditions of the Hopf theorem on the existence of periodic solutions are satisfied, since the real part of the derivative of the eigenvalue with respect to the parameter is not equal to zero. Based on this, we show that equation (16) corresponds to the family of periodic solutions $\overline{P}_\varepsilon(\bar{t})(\varepsilon > 0)$, where ε is the measure of the amplitude $\max\left|\overline{P}_\varepsilon(\bar{t})\right|$ and ε is small. The problem is reduced to studying the bifurcation of the birth (death) of a cycle in the differential-difference equation (16).

To reduce the functional equation (16) to a complex differential equation, we will use the central manifold method. Equation (16) contains a large number of parameters, so to simplify further research, we will perform the change of variable: $\overline{P}_\varepsilon(\bar{t}) = -(A_1 / A_2) \cdot u(\bar{t})$. Then if $\mu = 0$, equation (16) has the form:

$$\frac{du(\bar{t})}{d\bar{t}} = -\frac{\pi}{2}(u(\bar{t}-1) + u^2(\bar{t}-1) + \gamma u^3(\bar{t}-1)), \quad \gamma = X_0 G_3 / G_2^2. \tag{18}$$

By the central manifold theorem, equation (18) reduces to a differential equation with respect to a complex variable:

$$\frac{dZ}{dt} = \frac{i\pi}{2}Z + g_{20}\frac{Z^2}{2} + g_{11}Z\overline{Z} + g_{02}\frac{\overline{Z}^2}{2} + g_{21}\frac{Z^2 Z}{2} + \ldots, \tag{19}$$

where

$$g_{20} = -g_{11} = g_{02} = \pi\overline{D}, \quad g_{21} = 2\pi\left[\left(\frac{2-11i}{5} - i\frac{3\gamma\pi}{4}\right)\overline{D} + \frac{7}{3}D\overline{D} + i\overline{D}^2\right], \tag{20}$$

and

$$D = \frac{1 + i \cdot 0.5\pi}{1 + 0.25\pi^2}, \quad \overline{D} = \frac{1 - i \cdot 0.5\pi}{1 + 0.25\pi^2}.$$

The presence of specific values of the coefficients of the nonlinear part of equation (19) makes it possible to use formulas to determine the stability, direction of birth, period and asymptotic form of periodic solutions of small amplitude of the limit cycle, which leads to the appearance of the Andronov-Hopf bifurcation from the stationary state.

Using (20), we obtain an explicit expression for the first Lyapunov quantity? which determines the type of Hopf bifurcation when passing through the critical value of the parameter:

$$C_1(0)=\frac{\pi}{1+0.25\pi^2}\left\{\frac{2}{5}-\frac{\pi}{2}\left(\frac{11}{5}+\frac{3\gamma\pi}{4}\right)-i\left(\frac{\pi}{5}+\frac{11}{5}+\frac{3\gamma\pi}{4}\right)\right\}. \quad (21)$$

The real part of expression (21) is non-negative if the condition is met:

$$\gamma>\gamma_0=\frac{16-44\pi}{15\pi^2}\approx-0{,}826... \,. \quad (22)$$

This means that the limit cycle will be stable if $r>r_0$ and unstable if condition (22) is not met. For a stable limit cycle, we have the following expressions for its main characteristics:

1) $\varepsilon=\left(\frac{20\mu}{15\gamma\pi^2+44\pi-16}\right)^{0,5}$ – amplitude;

2) $T_\varepsilon=4\left(1+\frac{2}{5\pi}\varepsilon^2\right)$ – period;

3) $u_\varepsilon(\bar{t})=2\varepsilon\cos\left(0.5\pi\bar{t}\right)+2\varepsilon^2\left(0.4\sin\left(\pi\bar{t}\right)-0.2\cos\left(\pi\bar{t}\right)-1\right)$ – asymptotic form of a periodic solution.

In this case, the cycle is born in the direction $\mu>0$, and the periodic solutions that appear are asymptotically stable. This mode of self-oscillations is called soft. If the condition $\gamma<\gamma_0$, is met, then an unstable limit cycle occurs. The loss of stability with the emergence of self-oscillations occurs hard, a sharp breakdown (jump) into a new stationary mode is possible. In a real system, this is accompanied by a catastrophe.

The most complex behaviour of system (18) is when the parameter γ is close to its critical value γ_0, i.e., the value $\xi=\gamma-\gamma_0$ is small. Then the so-called Bautin bifurcation [8], which is a degenerate Hopf bifurcation, can be observed, in which stable and unstable limit cycles can coexist.

To analyse the qualitative properties of the above bifurcation, it is necessary to have an expansion of the right-hand side of equation (20) into a Taylor series up to and including the fifth order, and then, using the central manifold method, reduce the functional equation to a complex differential equation containing nonlinear terms up to the fifth degree, necessary for calculating the second Lyapunov quantity $C_2(0)\neq 0$. In this case, the complex differential equation is represented in the form:

$$\frac{dZ}{dt}=Z(i\omega+\varepsilon_1+\varepsilon_2 Z\overline{Z}+C_2 Z^2\overline{Z}^2), \quad (23)$$

where

$$\varepsilon_1=\varepsilon_1(\mu),\ \varepsilon_2=\varepsilon_2(\xi).$$

The second Lyapunov value determines the stability of the degenerate complex focus and the nature of the Hopf bifurcation (provided that $C_1 = 0$)

Depending on the signs $\varepsilon_1, \varepsilon_2, C_2$, the following scenarios for the evolution of the economic system are possible:

1) $C_2 < 0,\ \varepsilon_2 < 0.$ When ε_1 moving from negative to positive values, the system smoothly enters a stable self-oscillating mode;

2) $C_2 < 0,\ \varepsilon_2 > 0.$ When ε_1 moving from negative to positive values, the system rigidly enters a stable periodic self-oscillation mode, which was born even before the loss of equilibrium state, together with an unstable oscillatory mode, which comes to replace the equilibrium position upon loss of stability;

3) $C_2 > 0,\ \varepsilon_2 < 0.$ The loss of stability is mild, but the emerging limit cycle quickly dies, merging with the unstable one that came from afar, after which a new regime is violently excited in the system;

4) $C_2 > 0,\ \varepsilon_2 > 0.$ Classic hard arousal takes place.

Thus, whatever the sign of $C_2(0)$, for the corresponding sign ε_2, the above analysis describes a qualitatively different phenomenon compared to the one-parameter analysis: for $C_2(0) < 0$ there is a stable regime under hard excitation, and for $C_2(0) > 0$ the transient nature of the softly excited regime is revealed. To establish which of the two cases ($C_2 < 0$ or $C_2 > 0$) actually occurs, it is necessary to calculate the second Lyapunov quantity.

Thus, as a result of the study of the dynamic properties of the differential equation (4) in a small neighborhood of the boundaries of the local stability region, we can conclude that there are bifurcations of codimensionality two (which in itself is a fact that is far from trivial):

1) on the left border $(\eta_X + \eta_M = 1)$ there is a "beak tip" bifurcation;

2) on the right boundary $\left(\eta_X + \eta_M = 1 + \dfrac{Y\pi}{2X_0 V\tau}\right)$ there is a bifurcation of the limit Bautin cycle.

It should be noted that the qualitative theory of differential equations focuses on finding the characteristic features of the phenomenon as a whole, on qualitative prediction of its behavior. In this case, this phenomenon is the evolution of the economic system either at the macro or micro level. The applied problem is to compare these structures of the phase space with the studied economic processes together with the bifurcation analysis. In this case, it is necessary to take into account the properties of the real object, which impose restrictions on both the phase variables and the parameters of equations (4) – (6), which describe the evolution of the economic system.

Although the paper presents a sufficiently substantive study of the qualitative peculiarities of market dynamics for two participants in the competitive struggle, it is clear that these are only individual cases of a complex organizational economic system.

The improvement of market relations, apparently, should be aimed at increasing the number of market participants. It is well known that the appearance of a third participant in the competitive struggle on the market can initiate a chaotic regime in the system with the appearance of a new type of attractor. This "strange" attractor radically modifies the dynamics of competitive relations in the economic system, significantly narrowing the horizon of economic forecasting. Therefore, the most important idea arising from synergetics is that to achieve the goals of sustainable development, the process of market evolution requires a certain amount of chaos, spontaneity of development and self-government, as well as a certain amount of external management by state institutions, which must be coordinated among themselves. It should be emphasized that the attention to the bifurcation properties of economic dynamics is not accidental. It is here that the properties of instability of economic systems with respect to small deviations of parameters are most clearly manifested. In nonlinear systems near the bifurcation boundaries, a different (jump) change in the state of the system is observed.

The complexity of the processes of self-regulation of markets is due to the presence of feedback loops in the structural schemes, both positive and negative, which, in turn, is related to the issue of structural stability of economic objects and systems. In differential and difference equations describing local competitive interaction, feedbacks are determined by nonlinear functions, the nature of which initiates the emergence of complex dynamic regimes with the emergence of attractors of the appropriate type. The fact of the existence of essentially nonlinear relationships and instabilities implies the use of the synergistic para-paradigm as a means of describing competitive interactions in the economic environment. It must be stated that the classical nonlinear principle of superposition loses its force in the complex and nonlinear world that is the market.

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